3.16 KARATSUBA MULTIPLICATION ALGORITHM FOR LARGE INTEGERS

Question:

Given two integers X=1234 and Y=5678: Use the Karatsuba algorithm to compute the product Z=X x Y

AIM

To compute the product of two large integers using the Karatsuba algorithm, which reduces the number of multiplications from 4 to 3 by recursively splitting the numbers

ALGORITHM

1. Split the numbers

We split both numbers at the middle:

- X = 1234 \Rightarrow A = 12,\ B = 34
- Y = 5678 \Rightarrow C = 56,\ D = 78

Here, m = 2, since we split after 2 digits.

2. Compute the three products

- AC = 12 × 56 = 672
- BD = 34 × 78 = 2652
- (A + B)(C + D) = (12 + 34) × (56 + 78) = 46 × 134 = 6164

3. Compute the cross term

AD + BC = (A + B)(C + D) - AC - BD = 6164 - 672 - 2652 = **2840**

4. Combine the results

Z = AC \cdot 10^{4} + (AD + BC) \cdot 10^{2} + BD

Z = 672 \cdot 10^4 + 2840 \cdot 10^2 + 2652

Z = 6720000 + 284000 + 2652 = **6996652**

PROGRAM

```
def karatsuba(x, y):
    if x < 10 or y < 10:
        return x * y

    n = max(len(str(x)), len(str(y)))
    m = n // 2

    high1, low1 = divmod(x, 10**m)
    high2, low2 = divmod(y, 10**m)

    z0 = karatsuba(low1, low2)
    z1 = karatsuba(low1 + high1, low2 + high2)
    z2 = karatsuba(high1, high2)

    return z2 * 10**(2*m) + (z1 - z2 - z0) * 10**m + z0

def run_karatsuba():
    x = int(input("Enter X: "))
    y = int(input("Enter Y: "))
    print("Product using Karatsuba:", karatsuba(x, y))

run_karatsuba()
```

Input:

1234 x 5678

Output:

```
Enter X: 1234
Enter Y: 5678
Product using Karatsuba: 7006652
>>>
```

RESULT:

Thus the program is successfully executed and the output is verified.

PERFORMANCE ANALYSIS:

- Time Complexity: O(n^2)
- Space Complexity: O(n)