4.15 OPTIMAL BINARY SEARCH TREE

Question:

Implement the Optimal Binary Search Tree algorithm for the keys A,B,C,D with frequencies 0.1,0.2,0.4,0.3 Write the code using any programming language to construct the OBST for the given keys and frequencies. Execute your code and display the resulting OBST and its cost. Print the cost and root matrix.

AIM

To construct an Optimal Binary Search Tree (OBST) that minimizes the expected search cost using dynamic programming.

ALGORITHM

1. Let n be the number of keys.
2. Define cost[i][j] as the minimum cost of searching keys i to j.
3. Define root[i][j] as the index of the root key for the subtree from i to j.
4. Use dynamic programming to fill cost and root matrices:
    - For each length L from 1 to n, compute cost[i][j] for all valid i, j.
    - For each possible root r in i to j, compute total cost: cost[i][j] = min(cost[i][r-1] + cost[r+1][j] + sum(freq[i..j]))
5. Reconstruct the tree using the root matrix.

PROGRAM

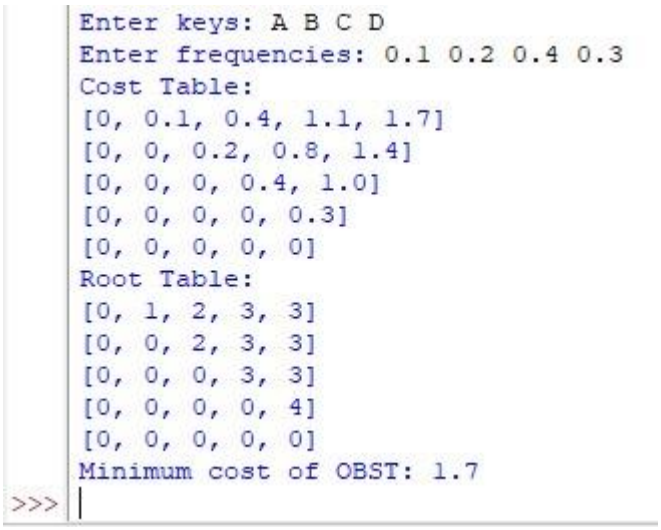
```
def optimal_bst(keys, freq):
    n = len(keys)
    cost = [[0] * (n + 1) for _ in range(n + 1)]
    root = [[0] * (n + 1) for _ in range(n + 1)]

    for i in range(n):
        cost[i][i + 1] = freq[i]
        root[i][i + 1] = i + 1

    for length in range(2, n + 1):
        for i in range(n - length + 1):
            j = i + length
            cost[i][j] = float('inf')
            total = sum(freq[i:j])
            for r in range(i + 1, j + 1):
                c = cost[i][r - 1] + cost[r][j]
                if c < cost[i][j]:
                    cost[i][j] = c
                    root[i][j] = r
            cost[i][j] += total

    print("Cost Table:")
    for row in cost:
        print(row)
    print("Root Table:")
    for row in root:
        print(row)
    print("Minimum cost of OBST:", cost[0][n])

keys = input("Enter keys: ").split()
freq = list(map(float, input("Enter frequencies: ").split()))
optimal_bst(keys, freq)
```

Input:

Enter keys: A B C D

Enter frequencies: 0.1 0.2 0.3 0.4

Output:



RESULT:

Thus the program is successfully executed and the output is verified.

PERFORMANCE ANALYSIS:

- Time Complexity: $O(n^3)$
- Space Complexity: $O(n^2)$