POSTAL Study Course 2018

Computer Science & IT

Objective Practice Sets

Theory of Computation

Contents

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CHAPTER

Computer Science & IT

Grammars, Languages & Automata

Q.1 Consider the following mealy machine represented with transition table.

$\Sigma = \{a, b\}$. $Q = \{q_0, q_1, q_2\}$, $\Delta = \{A, B\}$.

	a	b
$\rightarrow q_0$	q_0, B	q_1, B
q_1	q_2, A	q_1, B
q_2	q_0, B	q_1, B

It produces output A, if ________ otherwise it produces output B.

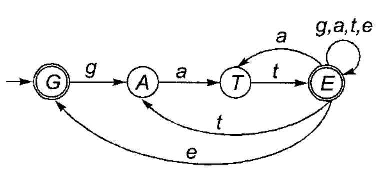
(a) the string ends with *a*

(b) the string contains *baba*

(c) the string contains *ab*

(d) it sees every occurrence of *ba* in the string

Q.2 Consider the following NFA for $\Sigma = \{g, a, t, e\}$

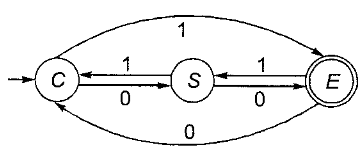


Find the number of states in equivalent DFA.

(a) 4 (b) 5

(c) 6 (d) 8

Q.3 Consider the following finite automata $F1$ that accepts a language 'L'

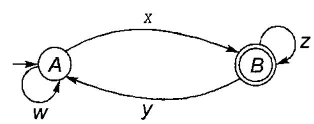


Let $F2$ be a finite automata which is obtained by the reversal of $F1$. Then which of the following is correct?

(a) $L(F1) \neq L(F2)$ (b) $L(F1) = L(F2)$

(c) $L(F1) \subseteq L(F2)$ (d) $L(F2) \subseteq L(F1)$

Q.4 Consider the following state diagram

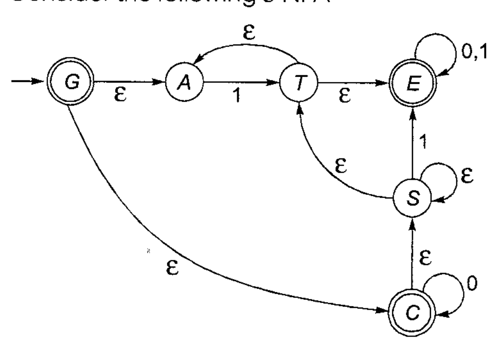


Assume $w, x, y, z \in \{0, 1\}$. If above state diagram is possible to make a DFA with deterministic inputs w,x,y,z then find number of possible DFA's. [Assume all given states are fixed]

(a) 1 (b) 2

(c) 3 (d) 4

Q.5 Consider the following ε-NFA

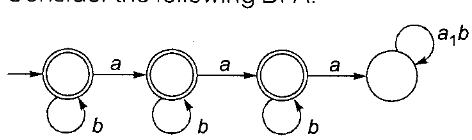


Find the cardinality of epsilon closure of state 'S' [ε-closure (q) is the set which contain all the states those are reachable from state q without reading any input symbol]

(a) 4 (b) 5

(c) 6 (d) 7

Q.6 Consider the following DFA:



Identify the language accepted by the DFA?

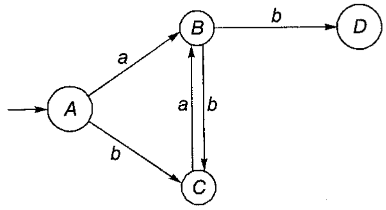
(a) All strings not containing *aaa*.

(b) All strings not starting with *aaa*.

(c) All strings with no more than two *a*'s.

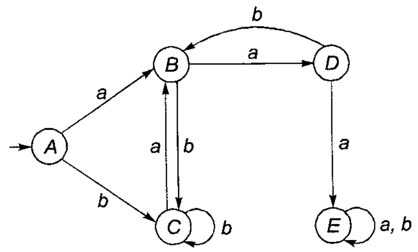
(d) None of these

Q.7 Consider the following NFA over $\Sigma = \{a, b\}$.

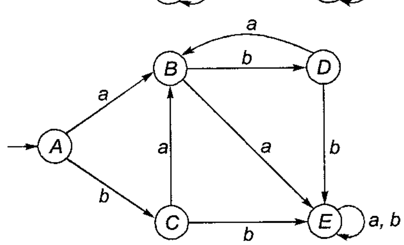


Identify the equivalent DFA for the above NFA.

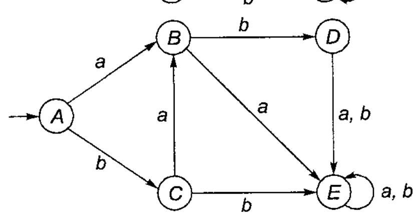
(a)



(b)



(c)



(d) None of the above

Q.8 Which of the following is false?

(a) The languages accepted by FAs are regular languages

(b) Every DFA is an NFA

(c) There are some NFAs for which no DFA can be constructed

(d) If L is accepted by an NFA with $\in$ transition then L is accepted by an NFA without $\in$ transition

Q.9 The Moore machine has six tuples $(Q, \Sigma, \Delta, \delta, \lambda, d_0)$. Which of the following is true?

(a) δ is the output function

(b) δ is the transition function Σ into Q

(c) λ is the transition function $\Sigma \times Q$ into Q

(d) λ is the output function mapping Q into Δ

Q.10 Consider the following table of an FA

δ	a	b
start	q_1	q_0
q_0	q_1	q_0
q_1	q_2	q_1
q_2	q_3	q_2
q_3	q_4	q_3
q_4	q_4	q_4

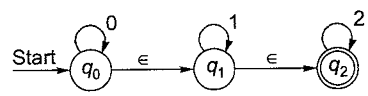
If the final state is q_4, the which of the following strings will be accepted?

1. *aaaaa*
2. *aabbaabbbbb*
3. *bbababab bb*

(a) 1 and 2 (b) 2 and 3

(c) 3 and 1 (d) all of these

Q.11 Consider the following NFA with $\in$ moves.

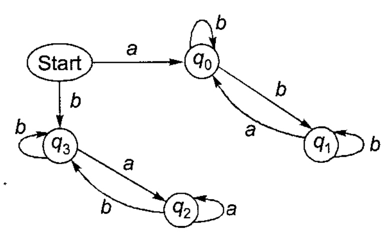


If the given NFA is converted to NFA without $\in$-moves, which of the following denotes the set of final states?

(a) $\{q_2\}$ (b) $\{q_1, q_2\}$

(c) $\{q_0, q_1, q_2\}$ (d) can't be determined

Q.12 Consider the transition diagram of an DFA as given below:



Which of the following represents the set of accepting states if the language to be accepted contains strings having the same starting and ending symbols?

(a) $\{q_0\}$ (b) $\{q_0, q_3\}$

(c) $\{q_3\}$ (d) $\{q_3, q_1\}$

Q.13 Given an arbitrary DFA with 2^N states, what will be the number of states of the corresponding NFA?

(a) $N \times N$ (b) 2^N

(c) $2N$ (d) $N!$

Q.14 Consider the transition table as given below:

δ	0	1	2
(A)	A	B	C
(B)	–	B	C
(C)	–	–	C

How many strings ending with 0 will be accepted by the given DFA if the maximum possible length of the string is n.

(a) 0 (b) 1
(c) $n-1$ (d) n

Q.15 A mealy machine has m states for n-length input alphabet. If k-length input string run on the mealy machine, what is the output length produced by the machine?

(a) m (b) n
(c) k (d) None of these

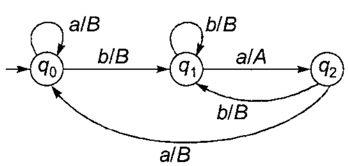
■■■■

Answers Grammars, Languages & Automata

1.	(d)	2.	(b)	3.	(b)	4.	(d)	5.	(a)
6.	(c)	7.	(b)	8.	(c)	9.	(a)	10.	(a)
11.	(c)	12.	(c)	13.	(b)	14.	(d)	15.	(c)

Explanations Grammars, Languages & Automata

1. (d)

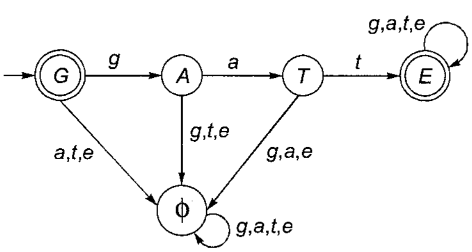


For every occurrence of '*ba*' in the string, it produces *A*.

2. (b)

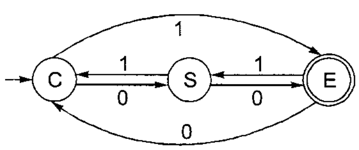
The given NFA accepts a language where each string starts with 'gat' [including Null string]

∴ Number of states required in DFA = 4+1 = 5 states



3. (b)

*F*2 = Reversal of *F*1 :



This is same as *F*1 except the state names. Reversal of *F*1 is equivalent to *F*1, Both accept same language.

So, $L(F2) = L(F1)$.

4. (d)

state	A: w	A: x
Input : 0		1
Input : 1		0

state	B: y	B: z
Input : 0		1
Input : 1		0

From state *A*, there are two choices

From state *B*, there are two choices

∴ 2 × 2 = 4 possible selections

Hence 4 DFA's are possible.

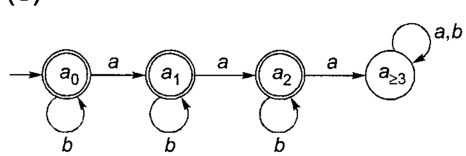
5. (a)

ε-closure (S) = $\{S, T, A, E\}$

Cardinality of ε-closure (S) = 4

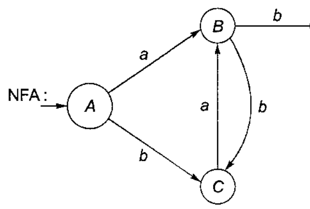
[Cardinality is number of elements in the set]

6. (c)



All strings with atmost two a's are accepted by DFA.

7. (b)



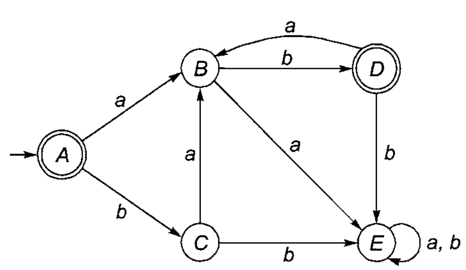
δ	a	b
→A	{B}	{C}
B	{ }	{C,D}
C	{B}	{ }
D	{ }	{ }

Convert NFA into DFA as following

DFA: δ′	a	b
→{A}	{B}	{C}
{B}	{ }	{C,D}
{C}	{B}	{ }
{C,D}	{B}	{ }
{ }	{ }	{ }

After renaming the above states equivalent DFA is:

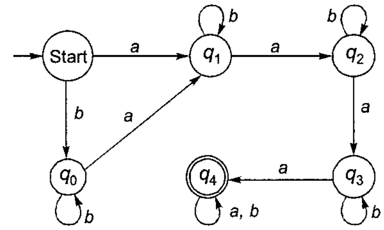
DFA: δ′	a	b
→A	B	C
B	E	D
C	B	E
D	B	E
F	F	E



8. (c)
For every NFA there exist an equivalent DFA and vice versa. Power of both NFA and DFA for recognization of language is same.
So (c) is false.

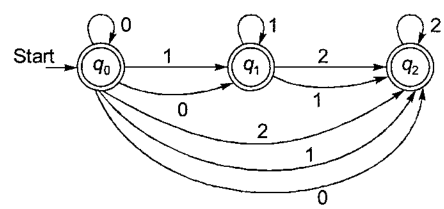
9. (a)
According to the definition of Moore machine (d) is the only correct option.

10. (a)



Drawing the FA we have
we can clearly see that only (i) *aaaaa* and (ii) *aabbaabbbbb* are accepted.

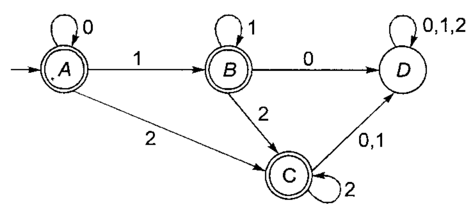
11. (c)
The given NFA when converted to NFA without $\in$-moves



Note: The previous state from which there is an $\in$-Move, if it goes to the final state then it also becomes the final state as well.

13. (b)
Since every DFA is also an NFA hence number of states of corresponding (maximum states) NFA is 2^N.

14. (d)



from the given DFA if maximum possible length of the string is '*n*' then number of strings ending with 0 are '*n*'. Because state '*A*' will only accept this. The strings are {0, 00, 000, 0000,..., 0^{n-1}, 0^n}

15. (c)
k-length input string produces k-length output in mealy machine.

■■■■

2 CHAPTER

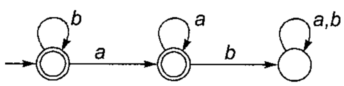
Computer Science & IT

Regular Languages & Finite Automata

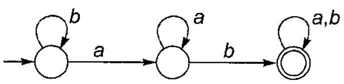
Q.1 Let $h : \{a, b\} \rightarrow \{0, 1\}$ be defined by $h(a) = 01$, and $h(b) = 10$. If $L = L((00 + 1)^*)$ then $h^{-1}(L) =$ ______.

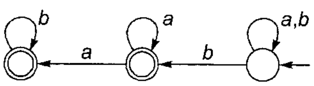
(a) $L((ab)^*)$ (b) $L((aa)^*)$
(c) $L((a + b)^*)$ (d) $L((ba)^*)$

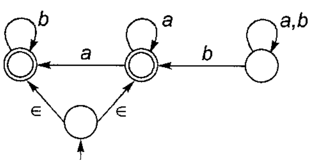
Q.2 Let L be the language accepted by the dfa given below:

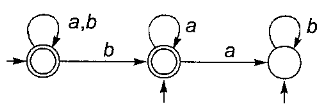


Which of the following is the state diagram of the NFA which accepts L^R?

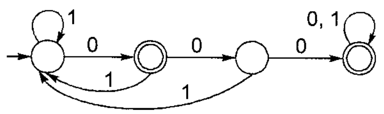
(a) 

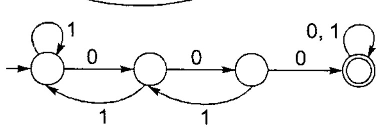
(b) 

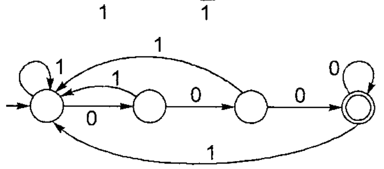
(c) 

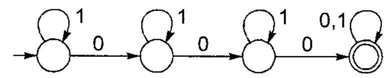
(d) 

Q.3 Which of the following is the minimum *dfa* for accepting
$L = ((0^* \ 1^*)^* \ 000 \ (0 + 1)^*)$

(a) 

(b) 

(c) 

(d) 

Q.4 Let L be a regular language on alphabet Σ. The union of the myhill-nerode equivalence classes is always ______, and the pairwise intersection of the myhill-nerode equivalence classes is always ______. Fill up the blanks

(a) ϕ, ϕ (b) ϕ, Σ^*
(c) Σ^*, ϕ (d) L, ϕ

Q.5 Match the following

Language
P. Odd number of a's
Q. Number of a's multiple of 3
R. A string of two or more a's
S. A string of any number of a's

Regular Expression
1. $(aaa)^*$
2. a^*aa
3. $(aa)^*a$
4. $(a + aa)^*$

Codes:
(a) P-1, Q-2, R-3, S-4
(b) P-3, Q-2, R-1, S-4
(c) P-2, Q-1, R-3, S-4
(d) P-3, Q-1, R-2, S-4

Q.6 $L = \{w \mid w$ is a binary number whose decimal value is a multiple of $n\}$. How many states are there in a minimal *dfa* that accepts L?

(a) n^2 (b) n
(c) $n + 1$ (d) $n + 2$

Q.7 Which of the following are equivalent to $(r + s)^*$?
I. $(rs + sr)^*$
II. $(rs^* + sr^*)^*$
III. $(r + s)^* (rs + sr)^*$
IV. $(r^* s^*) r^* s^*$

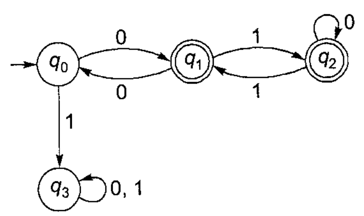
(a) II only (b) II & III only
(c) II, III & IV only (d) All of these

Q.8 Consider the dfa given below:

	u	d
$\rightarrow q_0$	q_1	q_2
q_1	q_2	q_1
$\boxed{q_2}$	q_3	q_2
q_3	q_3	q_3

Which of the following is true about the dfa given?
(a) The dfa accepts no strings on alphabet $\{u, d\}$
(b) The dfa accepts infinite number of strings on the alphabet $\{u, d\}$
(c) The dfa accepts a finite number of strings on the alphabet $\{u, d\}$
(d) None of the above

Q.9 Which of the following pairs of string belonging to Σ^* are distinguishable by the following *dfa*?



(a) "010", "01110" (b) "011", "00000"
(c) "001", "1011101" (d) "000", "010110"

Q.10 Find the number of states in DFA that recognizes the language where all strings that do not contain the substring bab, for $\Sigma = \{a, b\}$
(a) 3 (b) 4
(c) 5 (d) 6

Q.11 Let L = {madeeasy} over $\Sigma = \{m, a, d, e, s, y\}$, $L1$ = prefix(L) and $L2 = L1/\Sigma^*$. Find the number of strings in $L2$. Assume $L1$ and $L2$ do not include empty string.
(a) 5 (b) 6
(c) 7 (d) 8

Q.12 Let $L1$ be the language accepted by DFA $D1$ and $L2$ be the language accepted by DFA $D2$. Similarly $L3$ and $L4$ languages are accepted by NFA $N1$ and $x2$ respectively.
1. $D2$ is obtained by swapping the accepting and non-accepting states of $D1$
2. $N2$ is obtained by swapping the accepting and non-accepting states of $N1$

Which of the following statement is incorrect?
(a) $L2$ is complement of $L1$.
(b) Class of languages recognized by NFA's is not closed under complement.
(c) Class of languages recognized by DFA's is closed under complement.
(d) None of these.

Q.13 Consider a regular expression $R = (a + \varepsilon)(bb^*a)^*$. What is the language generated by R over $\Sigma = \{a, b\}$
(a) Set of all strings that do not contain *aa*.
(b) Set of all strings that do not contain two or more consecutive a's.
(c) Set of all strings that do not end with b and do not contain two or more consecutive a's.
(d) None of the above

Q.14 Find the number of states in minimized DFA for the regular expression $a^+b^+ + b^+a^+$
(a) 4 (b) 5
(c) 6 (d) None of these

Q.15 Identify the non regular language from the following.
(a) $\{x \mid x \in (0+1)^*,\ |x| \text{ is odd, First symbol of } x \text{ is } 1\}$
(b) $\{x \mid x \in (0+1)^*,\ |x| \text{ is odd, Middle symbol of } x \text{ is } 1\}$
(c) $\{x \mid x \in (0+1)^*,\ |x| \text{ is odd, Last symbol of } x \text{ is } 1\}$
(d) None of the above

Q.16 Find the number of states in minimized DFA that accepts a language over $\Sigma = \{a, b\}$ where each string has exactly three a's and atleast two b's.
(a) 12 (b) 13
(c) 14 (d) 15

Q.17 Let $L_1 = a^*b^*$ and $L_2 = \{ab\}$. L_3 = Prefix $(L_1^* \cap L_2)$, where prefix $(L) = \{u \mid uv \in L$ for any $v\}$. Find the number of strings in L_3?

(a) 3 (b) 4
(c) 5 (d) 6

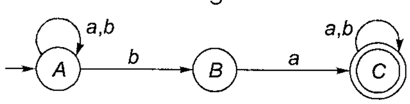
Q.18 Consider the following regular expression R.

$$R = a^*b^* + b^*a^*$$

How many final states exist in the minimized DFA that accepts a language equivalent to R.

(a) 4 (b) 5
(c) 6 (d) 7

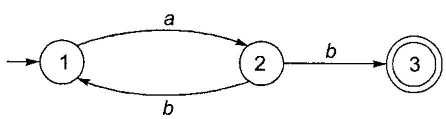
Q.19 Consider the following NFA.

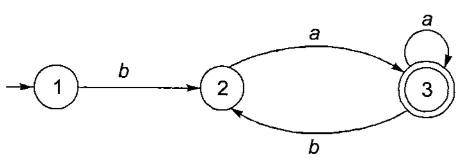


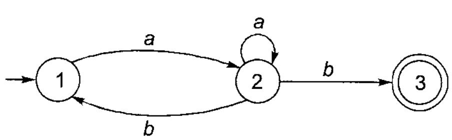
How many final states required in the equivalent DFA?

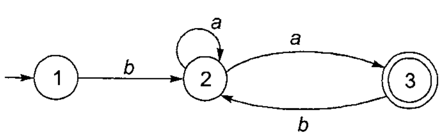
Q.20 Match the following groups.

List-I (Finite Automata)

A. 

B. 

C. 

D. 

List-II (Regular Expression)

1. $b(a + ab)^*a$
2. $a(a + ba)^*b$
3. $ba(a + ba)^*$
4. $a(ba)^*b$

Codes:

	A	B	C	D
(a)	1	2	3	4
(b)	1	3	2	4
(c)	4	3	2	1
(d)	4	2	3	1

Q.21 Find the number of equivalence classes for the following language L.

$$L = \{awb \mid w \in (a + b)^*\}$$

Q.22 Which of the following are regular languages?

(i) The language $\{W \mid W \in \{a, b\}^*$, W has an odd number of b's$\}$
(ii) The language $\{W \mid W \in \{a, b\}^*$, W has an even number of b's$\}$
(iii) The language $\{W \mid W \in \{a, b\}^*$, W has an even number of b's and odd number of a's$\}$

(a) (i) and (ii) only (b) (i) only
(c) (ii) only (d) all of these

Q.23 Which of the following regular expression corresponds to the language of all strings over the alphabet $\{a, b\}$ that contains exactly two a'S

(i) aa (ii) ab^*a
(iii) $b^*\ ab^*a$

(a) (i) and (ii) only (b) (ii) and (iii) only
(c) (i) and (iii) only (d) None of these

Q.24 Which of the following regular expression corresponds to the language of all strings over the alphabet $\{a, b\}$ that do not end with ab.

(a) $(a + b)^*\ (aa + ba + bb)$
(b) $(a + b)^*\ (aa + ba + bb) + a + b + \lambda$
(c) $b^*\ ab^*\ a$
(d) $b^*\ aa\ b^*$

Q.25 How many minimum number of states will be there in the DFA accepting all strings (over the alphabet $\{a, b\}$) that do not contain two consecutive a's

(a) 2 (b) 3
(c) 4 (d) 5

Q.26 How many minimum number of states are required in the DFA (over the alphabet $\{a, b\}$) accepting all the strings with the number of a'S divisible by 4 and number of b'S divisible by 5?

(a) 20 (b) 9
(c) 7 (d) 15

Q.27 Let $X = \{0, 1\}$, $L = X^*$ and $R = \{0^n1^n/n>0\}$ then the language $L \cup R$ and R respectively

(a) Regular, Regular
(b) None regular, Regular
(c) Regular, Not regular
(d) Not regular, Not regular

Q.28 Let $L = \{W : W \text{ has } 3k + 1\ b\text{'s}\ \forall k \geq 0\}$, construct a minimized finite automata D accepting L. How many states are there in D?
(a) 4
(b) 3
(c) 2
(d) The language is not regular

Q.29 Suppose $L_1 = \{10, 1\}$ and $L_2 = \{011, 11\}$. How many distinct elements are there in $L = L_1L_2$.
(a) 4 (b) 3
(c) 2 (d) None of these

Q.30 Which is the equivalent regular expression (RE) for the following? Strings in which every group of three symbols should contain atleast one a.
(a) $[(a + b)(a+b)a]^*$
(b) $[(a+b)(a+b)a]^* [(a+b)(a+b)\ a]^*$
(c) $[(\in + b + bb)\ a]^* [\in + b + bb]$
(d) $(abb)^* (bab)\ b^* (bba)^*$

Q.31 Let L be the set of all strings over $\{0, 1\}$ of length 6 or less. Write a simple RE corresponding to L.
(a) $(a + 1)^*$ (b) $(0 + 1)^6$
(c) $(0 + 1 + \in)^*$ (d) $(0 + 1 + \in)^6$

Q.32 Let L be the language, $L = \{x \in \{0, 1\} \mid x$ ends with 1 and does not contain the substring 0 0$\}$. Give the proper R.E. for the above language.
(a) $(1 + 01)^* (1 + 01)$
(b) $(1 + 01)^*$
(c) Both (a) and (b)
(d) None of these

Q.33 Which of the following statements are true?
(i) $L(((ab)^* (ba)^*) \cap ((ba)^* (ab)^*)) = \{\in\}$
(ii) $L((ab^*\ ba^*) \cap (ba^*\ ab)) = \{\in\}$
(iii) $L((a^*b^*b)^* \cap (b^*a^*a)^*) = \{\in\}$
(a) (i) and (ii) only (b) (ii) and (iii) only
(c) (i) and (iii) only (d) (iii) only

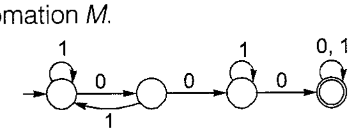
Q.34 $G = \{(a,b), \{S\}, S, \{S \to b/Sa/aS/SS\})$
Which of the following are true?
(i) $aabbaa \in G$
(ii) G is ambiguous
(iii) regular expression corresponding to G is ba^*
(a) (i) and (ii) only (b) (ii) and (iii) only
(c) (i) and (iii) only (d) all of these

Q.35 Consider the following regular grammar,
$R_1 = (a|b)^*$
$R_2 = (a^* \mid b^*)^*$
$R_3 = (\in \mid a \mid b^*)^*$
Minimized deterministic finite automata of which R_1, R_2 and R_3 are exactly same except state names?
(a) DFA for R_1 and R_2 are similar
(b) DFA for R_2 and R_3 are similar
(c) DFAs of R_1, R_2 and R_3 are different
(d) DFAs of R_1, R_2 and R_3 are similar

Q.36 $r_1 = (b^*\ ab^*\ ab^*\ ab^*)^*$, $r_2 = (b^*\ ab^*\ ab^*)^*$. What is $L(r_1) \cap L(r_2)$?
(a) $L[(b^*\ ab^*\ ab^*\ ab^*)^*]$
(b) $L[(b^*\ ab^*\ ab^*)^*]$
(c) $L[(b^*\ ab^*\ ab^*)^6]$
(d) $L[(b^*\ ab^*\ ab^*\ ab^*\ ab^*\ ab^*\ ab^*)^*]$

Q.37 Consider the grammar: $S \to aaaS \mid a \mid aa$
$L(G) = $ ______?
(a) $L(G) = \{w : |w| \bmod 3 = 0\}$
(b) $L(G) = \{w : |w| \bmod 3 = 1 \text{ or } 2\}$
(c) $L(G) = L(a^*)$
(d) $L(G) = L(a^*) - \{\lambda\}$

Q.38 Consider the following deterministic finite state automation M.



Let S denote the set of seven bit binary strings in which the first, the forth and the last bits are 1. The humber of strings is S that are accepted by M is
(a) 1 (b) 5
(c) 7 (d) 8

Q.39 Which is the regular expression of the given language

$$L = \{a^n b^n : n \geq 0\}$$

(a) $a^* b^*$ (b) $(a b)^*$
(c) Not possible (d) $a^+ b^+$

Q.40 Which of the following can be recognized by a DFA?

(a) The number 1, 2, 4, ... 2^n ... written in binary
(b) The number 1, 2, 4, ... 2^n ... written in unary
(c) The set of binary strings in which the number of 0's is same as the number of 1's
(d) The set {0, 101, 11011, 1110111, ...}

Q.41 Consider the transition table of a DFA as given below:

δ	a	b
start	q_0	q_4
q_0	q_0	q_1
q_1	q_0	q_2
q_2	q_0	q_2
q_3	q_3	q_4
q_4	q_3	q_4

Which of the following is the most precise interpretation of state q_3?

(a) Accepts strings starting with a and ending with b
(b) Accepts strings starting with a and ending with bb
(c) Accepts strings starting with a and ending with ab
(d) Accepts strings starting with b and ending with a

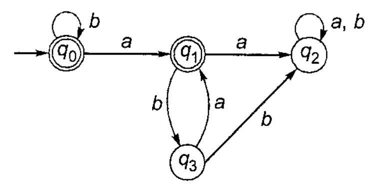
Q.42 Give a RE for the set of strings which are either all b's or else there is an 'a' followed by some b's also containing $\in$.

(a) $b^* + ab^*$ (b) $(\in + a)\, b^+$
(c) $b^* + ab^* + \in$ (d) $(\in + a)\, b^+ + \in$

Q.43 Consider the language $L = \{w \mid w$ contain even number of 0's and each string ending with 1 over alphabet {0, 1}}. What will the minimum number of state in DFA of complement language L?

(a) 2 (b) 3
(c) 4 (d) 5

Q.44 Which of the following regular expression represents the language accepted by given DFA.



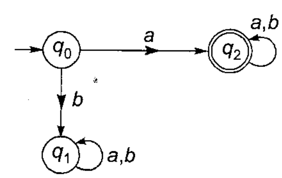
(a) $b^* + b^*a\,(ba)^*$ (b) $b + b^*a\,(ba)^*$
(c) $b^* + ba\,(ba)^*$ (d) $b^* + b^*\,(ab)a^*$

Q.45 Find the number of states in DFA that accepts binary language where each binary string has number of 0's divisible by 2 and number of 1's divisible by 4?

Q.46 Find a regular expression for the language $L = \{w \mid w \in (0+1)^*$, w has no pair of consecutive zeros}

(a) $(1^*011^*)^*(0 + \lambda) + 1^*(0 + \lambda)$
(b) $(1 + 01)^*\,(0 + \lambda)$
(c) $(1^*011^*)^*$
(d) both (a) and (b)

Q.47 Consider the following state transition diagram



Number of Myhill Nerode equivalence classes for the above FA is ________.

Q.48 Which of the following is regular language?

(a) Set of all strings of form $\{0^k 1^\ell \mid k + \ell = 100\}$ over alphabet $\Sigma = \{0, 1\}$
(b) Set of all strings of form $\{0^k 1^\ell \mid k - \ell = 100\}$ over alphabet $\Sigma = \{0, 1\}$
(c) Set of all strings of form $\{0^k 1^\ell \mid k \div \ell = 2\}$ over alphabet $\Sigma = \{0, 1\}$
(d) All of the above

■■■■

Answers Regular Languages & Finite Automata

1.	(d)	2.	(c)	3.	(a)	4.	(c)	5.	(d)	6.	(b)	7.	(c)	8.	(b)	9.	(d)
10.	(b)	11.	(d)	12.	(b)	13.	(c)	14.	(c)	15.	(b)	16.	(b)	17.	(a)	18.	(b)
20.	(c)	22.	(d)	23.	(d)	24.	(b)	25.	(b)	26.	(a)	27.	(c)	28.	(b)	29.	(b)
30.	(c)	31.	(d)	32.	(a)	33.	(d)	34.	(a)	35.	(d)	36.	(d)	37.	(b)	38.	(c)
39.	(c)	40.	(a)	41.	(d)	42.	(d)	43.	(b)	44.	(a)	46.	(d)	48.	(a)		

Explanations Regular Languages & Finite Automata

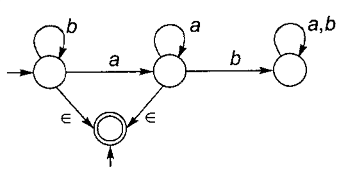
1. (d)

$h((ab)^*)$ can generate $h(ab) = 0110 \notin (00+1)^*$
$h((aa)^*)$ can generate $h(aa) = 0101 \notin (00+1)^*$
$h((a+b)^*)$ can generate $h(a) = 01 \notin (00+1)^*$
only $h((ba)^*)$ can generate $(1001)^* \in (00+1)^*$
So correct answer is choice (d).

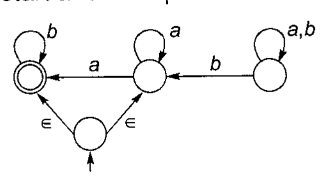
2. (c)

To construct the machine for L^R, from the machine for L.

Step 1: If more than one accepting state is there, make a single accepting state and make epsilon transitions from old accept states to this new accept state.



Step 2: Then, reverse all transitions in M and swap start and accept states.



This diagram corresponds to choice (c).

3. (a)

$(0^* 1^*)^* 000 (0+1)^* = (0^* + 1^*)^* 000(0+1)^*$
$= (0+1)^* 000(0+1)^*$
which is the set of strings containing the substring "000".

4. (c)

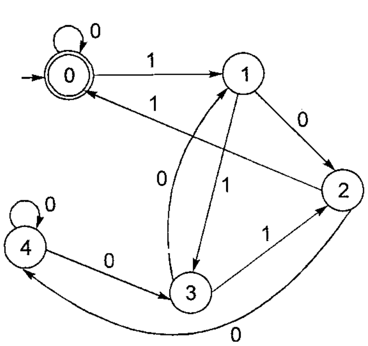
Myhill-nerode equivalence classes are pairwise disjoint and whose union is always Σ^*.

5. (d)

Odd no. of a = (aa)*a {a, aaa, aaaaa,....}
No. of a's multiple of 3 = (aaa)*
A string of 2 or a's = (aaa*) or (a*aa)
Two compulsory a's.
A string of any number of a's
= (a + aa)* accpets 0, 1, 2, ... any no. of a's.
ε = 0 a's, a = a's, aa = a's
(a + aa)* any number of a's.

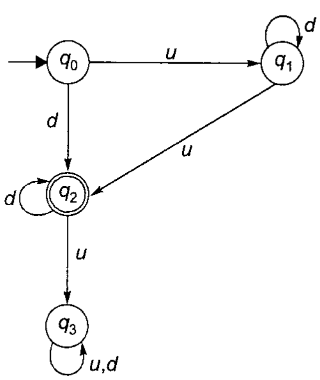
6. (b)

The dfa will have n states, one each to handle the n distinct residues when divided by n. The accepting state will be the starting state corresponding to residue 0.
For example for n = 5, the dfa with 5 states is shown below:



7. (c)
$r \notin (rs + sr)^*$ bur $r \in (r + s)^*$
so $(rs + sr)^* \neq (r + s)^*$
In regular expression II, r & s can be produced inside the brackets so it is $= (r + s)^*$
In regular expression III, putting $(rs + sr)^* = \lambda$, we get $(r + s)^*$ as a subset. So the reg. expression includes all strings & hence $= (r + s)^*$
In IV, $(r^*s^*)^*r^*s^* = (r + s^*)r^*s^*$
by putting r^* and s^* as λ, we can make $(r + s)^*$ as a subset of this. So it includes all strings on r and s and hence $= (r + s)^*$

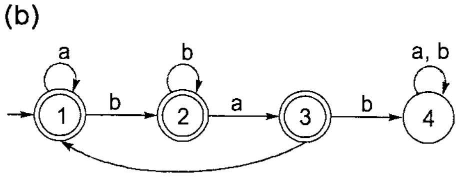
8. (b)
The state diagram for given dfa is



Since there is a loop in the path from start state q_0 to the final state q_2, the language accepted has infinite number of strings.

9. (d)
Two strings $w_1, w_2 \in \Sigma^*$ are distinguishable iff
$\delta^*(q_0, w_1) \neq \delta^*(q_0, w_2)$
by this definition, "000" and "010110" are distinguishable since
$\delta^*(q_0, 000) = q_1$
$\delta^*(q_0, 010110) = q_2$
Clearly $\delta^*(q_0, 000) \neq \delta^*(q_0, 010110)$

10. (b)

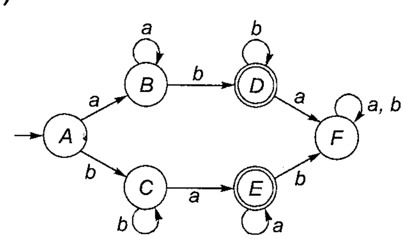


11. (d)
L = {madeeasy} $\Rightarrow L1$ = {m, ma, mad, made, madee, madeea, madeeas, madeeasy}
$L2 = L1/\Sigma^*$ = {m, ma, mad, made, madee, madeea, madeeas, madeeasy} = $L1$
$\therefore$ Total 8 strings [$\because$ Prefix $(L)/\Sigma^*$ = Prefix (L)]

12. (b)
Class of languages recognized by NFA's
$\cong$
Class of languages recognized by DFA's
$\cong$
Class of regular languages
$\Downarrow$
Closed under complement.
So option (b) is not correct statement.
Note: Given data is applicable to option (a) only.

13. (c)
$R = (a + \varepsilon)(bb^*a)^*$. R generates the language that do not contain two or more consecutive a's and do not end with 'b'.

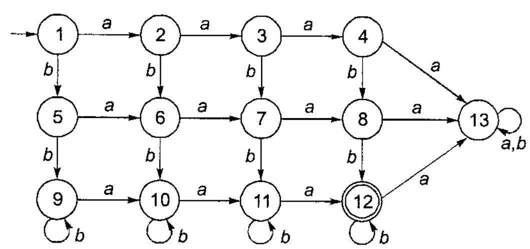
14. (c)



$\therefore$ 6 states are required to accept $a^+b^+ + b^+a^+$.

15. (b)
(a) Regular language: $1[(0 + 1)(0 + 1)]^*$
(b) Non regular language (Finding middle symbol is not possible)
(c) Regular language: $[(0 + 1)(0 + 1)]^*1$

16. (b)



Number of states = 13 states.

17. (a)

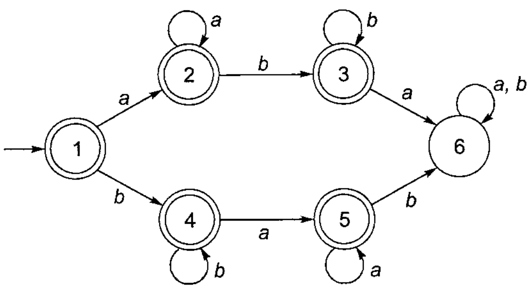
$L_1 = a^*b^* \Rightarrow L_1^* = (a^*b^*)^* = (a+b)^*$

$L_2 = \{ab\}$

$L_1^* \cap L_2 = (a+b)^* \cap \{ab\} = \{ab\}$

$L_3 = \text{Prefix}\,(L_1^* \cap L_2) = \{\epsilon, a, ab\}$

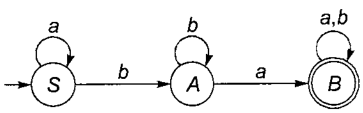
18. (b)



19. (1)

Given NFA accepts a language of all strings which contain 'ba' as a substring.

Equivalent DFA is:

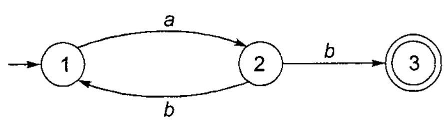


Number of final states = 1.

20. (c)

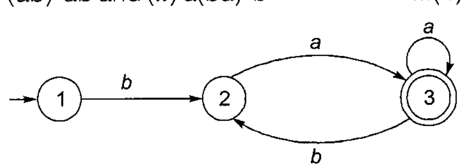
A–4, B–3, C–2, D–1

(a)



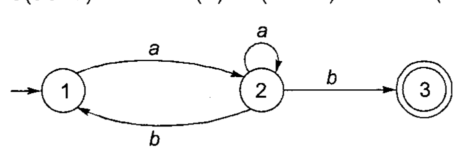
Equivalent regular expressions are: (*i*) $(ab)^*ab$ and (*ii*) $a(ba)^*b$ ...(4)

(b)



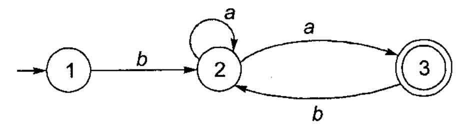
Equivalent regular expressions are: (*i*) $b(aa^*b)^*aa^*$ and (*ii*) $ba(a+ba)^*$ ...(3)

(c)



Equivalent regular expressions are: (*i*) $a(a+ba)^*b$ and (*ii*) $(aa^*b)^*aa^*b$ ...(2)

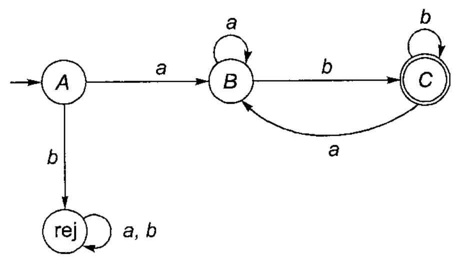
(d)



Equivalent regular expressions are: (*i*) $b(a+ab)^*a$ and (*ii*) $ba^*a(ba^*a)^*$ ...(1)

21. (4)

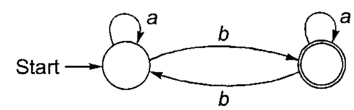
$L = \{awb \mid w \in (a+b)^*\} = a(a+b)^*b$



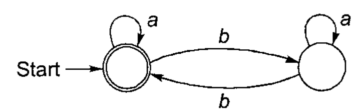
4 states in minimized DFA. Hence four equivalence classes for L.

22. (d)

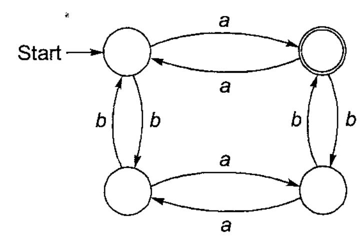
(i)



(ii)



(iii)



As we can construct the DFA for the given languages, hence all of the given languages are regular.

23. (d)

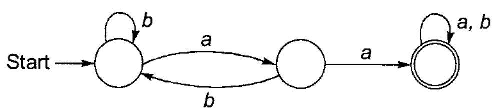
All languages are accepting strings over the alphabet {a, b} that contains exactly two a's but they are not accepting all strings like the first choice just accepts 'aa'. The second choice can't accept 'baa'. The third choice can't accept 'baab'. The correct regular expression is $b^*ab^*ab^*$.

24. (b)

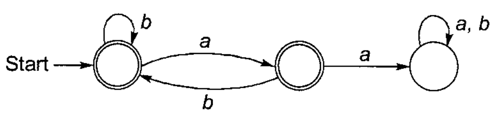
Choice '*a*' is incorrect since it does not include the string "*a*", "*b*" and "λ" (all of which do not end with ab).

None of choices '*c*' or '*d*' accept the string '*a*', So they can't represent specified language.

25. (b)



First draw FSM for accepting all strings containing consecutive a's, as shown above. Now change the final states to non final states and non final states to final states to get the required DFA shown below.



26. (a)

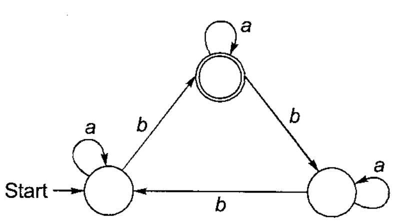
For DFA accepting all the strings with number of a's divisible by 4, four states are required similarly for DFA accepting all the strings with number of b's divisible by 5, five states are required and for their combination, states will be multiplied So $5 \times 4 = 20$ states will be required.

27. (c)

Since $L = X^* = \{0, 1\}^* = (0 + 1)^*$ and $R = \{0^n 1^n \mid n > 0\}$, LUR produces $(0 + 1)^*$ which is regular language and R is not regular as there is no regular expression for that.

28. (b)

The automata D can be constructed as



Hence number of states in the minimised automata is 3.

29. (b)

$L_1 = \{10, 1\}, L_2 = \{011, 111\}$

By concatenation of L_1 and L_2 we get

$L_1.L_2 = \{10011, 1011, 1011, 111\}$

Here only 3 distinct elements are there.

30. (c)

Option (a) cannot generate the valid string "aaaba."

Option (b) cannot generate the valid string "bbababba."

Option (d) generate the invalid string "abbbabbba".

32. (a)

As we know that

$(1 + 01)^* (1 + 01)$ is R.E. with all strings of '0' and '1' starting with 'a' and not contain the substring '00'.

Choice (b) $(1 + 01)^*$ is incorrect, since $(1 + 01)^*$ will generate "λ" which is not ending with 1.

33. (d)

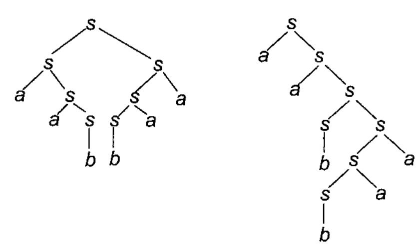
For (i) intersection of $(ab)^*(ba)^*$ and $(ba)^*(ab)^*$ is not $\{\epsilon\}$. For eg. "*ab*" is one more string which satisfies '∩'. Therefore (i) is false.

For (ii) let $L_1 = (ab^*ba^*)$ and $L_2 = (ba^*ab)$. The string in L_1 should always start with '*a*', where as for L_2 it always starts with '*b*'. ϵ is not part of either language. Hence intersection is {}. Hence (ii) is also false.

For (iii) both $(a^*b^*b)^*$ and $(b^*a^*a)^*$ contain "ϵ". No other string is common. Hence only (iii) is true.

34. (a)

Two derivation trees are possible for aabba as given below



Therefore (i) and (ii) are both true.
Choice (iii): 'aba' is also accepted by the given grammar. Therefore (iii) is false.

35. (d)

$$R_1 = (a \mid b)^* = (a + b)^* \quad ...(1)$$
$$R_2 = (a^* \mid b^*)^* = (a^* + b^*) = (a + b)^* \quad ...(2)$$
$$R_3 = (\epsilon \mid a \mid b^*)^* = (\epsilon + a + b)^* = (a + b)^* \quad ...(3)$$

Clearly from (1), (2) and (3)
Hence option (d) is correct.

36. (d)

$r_1 = (b^* ab^* ab^* ab^*)^*$
$r_2 = (b^* ab^* ab^*)^*$
r_1 denotes multiple of 3 a's, with any number of b's.
r_2 denotes multiple of 2 a's, with any number of b's. $L(r_1) \cap L(r_2)$ denotes multiple of 6 a's, with any number of b's.
Hence, $L(r_1) \cap L(r_2) = L[(b^*ab^*ab^*ab^*ab^*ab^*ab^*)^*]$

38. (c)

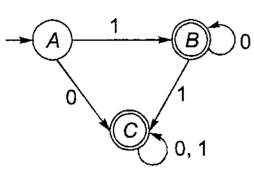
The strings with the first, 4th and 7th bits as '1' will look in the following format
1_ _1_ _1.
So, there can be 16 possible combinations for the above format. But in the given DFA, only 7 strings of these will be accepted. They are,

1. 1001001
2. 1001011
3. 1001101
4. 1001111
5. 1011001
6. 1101001
7. 111001

Hence option (c) is correct.

40. (a)

For (a) we can construct a DFA



which will recognize the series
1, 2, 4,.....2^n ..... in binary as
1, 10, 100, 1000,.....

Choice (b) is $\{1^{2^n}; n \geq 0\}$
Choice (c) is $\{w \mid n_0(w) = n_1(w)\}$
Choice (d) is $\{1^n 0 1^n, n \geq 0\}$
DFA's cannot be constructed for b,c and d since these are not regular languages.
Hence option (a) is correct.

41. (d)

From the DFA constructed above we can clearly see that state q_3 accepts strings starting with b and ending with a.

42. (d)

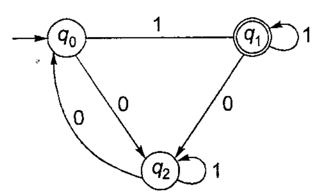
The required regular expression is

$$r = b^+ + ab^+ + \epsilon$$
$$= (\epsilon + a)\, b^+ + \epsilon$$
$$r = b^* + ab^+$$

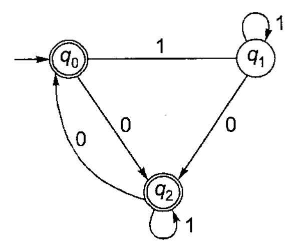
Hence (d) is the correct option.

43. (b)

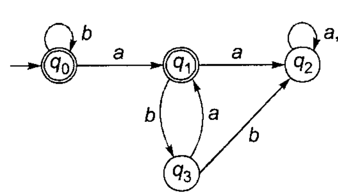
DFA accepting strings of even number of 0's with ending with 1 is



Complement of DFA



44. (a)



$$q_0 = b^* \quad \ldots (0)$$
$$q_1 = q_0 a + q_3 a \quad \ldots (1)$$
$$q_2 = q_1 b \quad \ldots (2)$$

put equation (0) and (2) in (1)

$$q_1 = b^* a + q_1\, ba$$
$$q_1 = b^* a (ba)^*$$

Since two final states

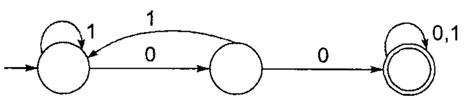
So, final exzpression is = $b^* + b^* a\,(ba)^*$

45. (8)

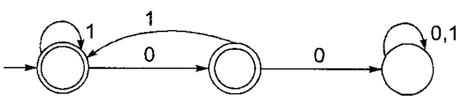
$2 \times 4 = 8$ states.

46. (d)

State diagram for language L = {w| w has atleast one pair of consecutive zeros}



Its complement L = {w ∈ {0, 1}}* : w has no pair of consecutive zeros.



Its regular expression:

$(1^*011^*)^* (0 + \lambda) + 1^*(0 + \lambda) = (1 + 01^*)(0 + \lambda)$

47. (3)

Number of equivalence classes = Number of states in minimal DFA = 3.

48. (a)

- Set of all string of form $\{0^k 1^\ell \mid k + \ell = 100\}$ over $\Sigma = \{0, 1\}$.
  Since $k + \ell = 100$, there are finite value of k and ℓ satisfied the condition so regular.
- Set of all string of form $\{0^k 1^\ell \mid k - \ell = 100\}$ over $\Sigma = \{0, 1\}$.
  Since $k - \ell = 100$, there are infinite values of k and ℓ satisfied the condition, so non-regular. Since there is no finite automata.
- Set of all strings of form $\{0^k 1^\ell \mid k \div \ell = 2\}$ over alphabet $\Sigma = \{0, 1\}$.
  Since $k \div \ell = 2$, there are infinite values of k and ℓ which satisfied the equation so finite automata not possible. So, non-regular.

■■■■

3 CHAPTER

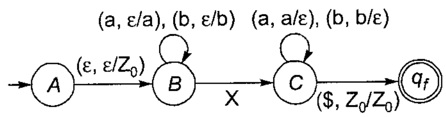
Context Free Languages & Push Down Automata

Q.1 Consider the PDA M as defined below:
$M = \{\{q_0, q_1, q_2, q_3\}, \{a, b\}, \{a, b, z\}, \delta, q_0, \{q_3\}\}$
where δ is defined by
$\delta(q_0, a, z) = \{(q_1, az)\}$
$\delta(q_1, a, a) = \{(q_1, aa)\}$
$\delta(q_1, b, a) = \{(q_1, a)\}$
$\delta(q_1, c, a) = \{(q_2, \lambda)\}$
$\delta(q_2, c, a) = \{(q_2, \lambda)\}$
$\delta(q_2, \lambda, z) = \{(q_3, \lambda)\}$
The above pda accepts which language?
(a) $L(M) = \{a^n b^n c^m \mid n \geq 1, m \geq 0\}$
(b) $L(M) = \{a^n b^m c^n \mid n \geq 1, m \geq 0\}$
(c) $L(M) = \{a^n b^m c^m \mid n \geq 1, m \geq 0\}$
(d) $L(M) = \{a^n b^m c^n \mid n \geq 1, m \geq 1\}$

Q.2 Let L be the language which is accepted by DPDA. Then identify L from the following
(a) $L = \{a^P \mid P \text{ is prime}\}$
(b) $L = \{a^m b^n c^k \mid m = n \text{ or } n = k\}$
(c) $L = \{a^m b^n c^k \mid m < n \text{ or } m > n\}$
(d) None of these

Q.3 Which of the following is true?
(a) In an unambiguous grammar every string has exactly one derivation.
(b) There may exist an unambiguous grammar G, even though, the language $L(G)$ is ambiguous.
(c) If M is a PDA accepting L, by exchanging initial and final states we get $\bar{M}$ which accepts $\bar{L}$.
(d) If L_1 is a CFL and L_2 is regular, then $L_1 - L_2$ is a CFL.

Q.4 The following machine is designed with PDA acceptance by final state mechanism to accept the language L where all strings of L are odd length palindromes.



What are the transitions at X to accept L?
(a) $(a, a/\varepsilon), (b, b/b)$ (b) $(a, a/\varepsilon), (b, b/\varepsilon)$
(c) $(a, \varepsilon/\varepsilon), (b, \varepsilon/\varepsilon)$ (d) None of these

Q.5 Assume PDA stack is limited to 2^{10} symbols. Now stack can contain maximum of 2^{10} symbols. The language accepted by such PDA is________.
(a) Regular language but not finite
(b) DCFL but not regular
(c) CFL but not DCFL
(d) None of these

Q.6 Which of the following is not true?
(a) The set of languages accepted by deterministic and non deterministic PDAs are not equal
(b) $L = \{wcw^R \mid w \text{ in } (0+1)^* \text{ and } c \notin \{0,1\}\}$ can be accepted by a deterministic PDA
(c) $L = \{ww^R \mid w \text{ in } (0+1)^*\}$ can be accepted by a deterministic PDA
(d) $L\{0^n 1^n \mid n \geq 0\}$ can be accepted by a deterministic PDA

Q.7 Which of the following languages over $\{a, b, c\}$ is accepted by deterministic push down automata?
(a) $\{w \mid w \text{ is palindrome over } \{a, b, c\}\}$
(b) $\{ww^R \mid w \in \{a, b, c\}^*\}$
(c) $\{a^n b^n c^n \mid n \geq 0\}$
(d) $\{wcw^R \mid w \in \{a, b\}^*\}$

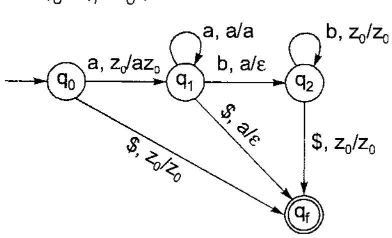
Q.8 Which language does M accept if the following transition is added?
$$\delta(q_0, \in, z_0) = (q_0, \in)$$

(a) $L = \{0^n 1^n \mid n \geq 0\}$
(b) $L = \{0^n 1^n \mid n \geq 1\}$
(c) $L = \{0^n 1^{n+1} \mid n \geq 0\}$
(d) $L = \{0^n 1^{n+1} \mid n \geq 1\}$

Q.9 Which of the following statement is not correct?
(a) If L is accepted by NFA, then there exists a DFA that accepts L
(b) If L is accepted by an NFA with ϵ transition, then L can be accepted by NFA without ϵ transition
(c) If L is accepted by a non-deterministic PDA then it is not always true that L is also accepted by deterministic PDA
(d) If L is accepted by turing Machine, which halts on every w in L then L' is recursively enumerable but not recursive

Q.10 Which of the following language is not having an equivalent PDA.
(a) $\{a^n b^{m+n} c^m \mid m, n \geq 10\}$
(b) $\{a^m b^n a^n b^m\}$
(c) $\{a^{m+n} b^{m+n} c^k\}$
(d) $\{a^m b^n a^m b^n\}$

Q.11 Consider the following PDA $(\{q_0, q_1, q_2, q_f\}, \{a, b\}, d, q_0, q_f, z_0)$



Identify the regular expression which is equivalent to the language accepted by above PDA
(a) $a^*b^*+\epsilon$ (b) $a^*b^+ +\epsilon$
(c) $a^+b^*+\epsilon$ (d) None of these

Q.12 Consider the following statements.
S_1: PDA acceptance by Final state
S_2: PDA acceptance by Empty stack
S_3: PDA acceptance by both Final state and Empty stack.

Find the equivalent statements from the above
(a) S_1, S_2 and S_3 (b) S_1 and S_2
(c) S_2 and S_3 (d) None of these

Q.13 Consider the following 3 languages
$L_1 = \{w \mid w \in \{a, b\}^* \text{ and } w = w^R\}$
$L_2 = \{ww^R \mid w \in \{a, b\}^*\}$
$L_3 = \{w(a + b) w^R \mid w \in \{a, b\}^*\}$
What is the relation between L_1, L_2, L_3 and L_4?
(a) $L_2 \subset L_1$ and $L_3 \subset L_1$ and $L_1 = L_2 \cup L_3$
(b) $(L_2 = L_3) \subset L_1$
(c) $L_2 \cap L_1 = L_3$
(d) $L_2 \subset L_1$ and $L_3 \subset L_1$ but $L_1 \neq L_2 \cup L_3$

Q.14 $S \to SS \mid (S) \mid \lambda$ generates which of the following language?
(a) Language on $\{(,)\}^*$ such that the number of left and right parenthesis are equal
(b) Language of properly balanced parenthesis on $\{(,)\}^*$
(c) Language in which, number of left parenthesis is at least equal to number of right parenthesis
(d) Language in which, number of right parenthesis is at least equal to number of left parenthesis

Q.15 Let $L_1 = \{a^i b^j c^k \mid i < j\}$, $L_2 = \{a^i b^j c^k \mid i < k\}$. L_1, L_2 and $L_1 \cap L_2$ are
(a) CFL, CFL and CFL respectively
(b) regular, regular and not regular respectively
(c) CFL, CFL and not CFL respectively
(d) DCFL, DCFL, CFL respectively

Q.16 The following CFG,
$S \to aB \mid bA$
$A \to a \mid aS \mid bAA$
$B \to b \mid bS \mid aBB$ generates strings with
(a) odd number of $a'S$ and odd number of $b's$
(b) even number of $a's$ and even number of $b's$
(c) equal number of $a's$ and $b's$
(d) odd number of a's and even number of $b's$

Q.17 Identify the correct statement from the following.
(a) Every CFG may not have equivalent PDA
(b) DPDA and NPDA have same power
(c) Membership problem is decidable for CFL
(d) None of these

Q.18 Consider $L = \{ww^Rw \mid w \in \{(00)^*\}\}$ (w^R is reverse string of w). Then L is
(a) Regular (b) DCFL
(c) CFL (d) None of these

Q.19 Identify CFL generated by following CFG

$A \to aB \mid C \mid \varepsilon$
$B \to Ad \mid Cd$
$C \to bD \mid \varepsilon$
$D \to Cc$

(a) $\{a^m b^n c^k d^l \mid m = n, k = l\}$
(b) $\{a^m b^n c^k d^l \mid m = k, n = l\}$
(c) $\{a^m b^n c^k d^l \mid m = l, n = k\}$
(d) None of these

Q.20 Let $L_1 = \{a^*\}$, $L_2 = \{ba\}$, $L_3 = \{b^n a^n\}$, $L_4 = \{b^*\}$. Then $((L_2 / L_1) - L_4) - \overline{L_3} =$
(a) ϕ (b) L_1
(c) L_2 (d) L_3

Note: '/' is quotient, '–' is difference and '¯' is complement.

Q.21 Which of the following is false?
(a) Every regular grammar is linear
(b) Every linear grammar generates a language accepts by some DPDA
(c) A language is DCFL iff it is generated by some LR(K) grammar
(d) All DCFL's are unambiguous

Q.22 Let $L = \left\{a^m b^n b^k d^\ell \mid \text{if}(n+k = \text{even}) \text{ then } m = \ell\right\}$.
Which of the following is true about L?
(a) L is CFL but not DCFL
(b) L is regular but not CFL
(c) L is DCFL but not regular
(d) None of these

Q.23 Consider the following CFG 'G':

$S \to aA \mid bSS \mid SS$
$A \to aAb \mid bAa \mid AA \mid \varepsilon$

The language generated by G is ______.
(a) Set of all strings with atleast one 'a'
(b) Set of all strings with atleast two a's
(c) Set of all strings with atleast one more 'a' than number of b's
(d) None of these

Q.24 Consider the following CFG with a start symbol S.

$S \to AAaSb \mid \varepsilon$
$A \to a \mid \varepsilon$

Which of the following language is generated by G?
(a) $\{a^m b^n \mid m \geq n \geq 0\}$
(b) $\{a^m b^n \mid 0 \leq n \leq m \leq 2n\}$
(c) $\{a^m b^n \mid 0 \leq n \leq m \leq 3n\}$
(d) None of these

Q.25 Let $L = \{w\, x\, w^R \mid w \in (a+b)^*, x \in (a+b)\}$. The complement of language L is ______.
(a) Regular
(b) DCFL but not regular
(c) CFL but not DCFL
(d) None of these

Q.26 Let $L = \{ab^n a^{2n} \mid n > 0\} \cup \{aab^n a^{3n} \mid n > 0\}$. Then L is
(a) Regular (b) DCFL
(c) CFL (d) Not CFL

Q.27 Identify the Regular Language from the following
(a) $L = \{ww^R x \mid w, x \in (a+b)^+\}$
(b) $L = \{xww^R \mid w, x \in (a+b)^*\}$
(c) $L = \{wxw \mid w, x \in (a+b)^+\}$
(d) $L = \{w_1 w_2 x \mid w_1 = w_2, w_1, w_2, \in (a+b)^+, x \in (a+b)^*\}$

Q.28 Which of the following languages are context free

$L_1 = \{a^m\, b^m\, c^n \mid m \geq 1 \text{ and } n \geq 1\}$
$L_2 = \{a^m\, b^m\, c^n \mid n \geq m\}$
$L_3 = \{a^m\, b^m\, c^m \mid m \geq 1\}$

(a) only L_1 (b) L_2 and L_3
(c) only L_2 (d) L_3

Q.29 Which of the following languages is/are context free?
1. $\{a^n\, b^n\, c^m\, d^m \mid n \geq 1, m \geq 1\}$
2. $\{a^n\, b^m\, c^m\, d^n \mid n \geq 1, m \geq 1\}$
3. $\{a^n\, b^m\, c^n\, d^m \mid n \geq 1, m \geq 1\}$
4. $\{a^m\, b^n\, c^m\, d^n \mid n \geq 1, m \geq 1\}$

(a) 1 and 2 (b) 3 and 4
(c) 2 and 4 (d) 1, 2, 3 and 4

Q.30 The following GFG:
$S \to aS|bS|a|b$ and $S \to aS|bS|a|b|\lambda$ is equivalent to regular expressions
(a) $(a + b)$ and $\lambda + a + b$ respectively
(b) $(a + b)(a + b)^*$ and $(a + b)^*$ respectively
(c) $(a + b)(a + b)$ and $(\lambda + a + b)(\lambda + a + b)$ respectively
(d) None of these

Q.31 The set $\{a^n b^n \mid n = 1, 2, 3, ...\}$ can be generated by the CFG
(a) $S \to ab|aSb|\in$ (b) $S \to aaSbb|ab$
(c) $S \to ab|aSb$ (d) None of these

Q.32 The grammar $S \to aaSbb|ab$ can generate the set
(a) $\{a^n b^n | n = 1, 2, 3, ...\}$
(b) $\{a^{2n+1} b^{2n+1} | n = 0, 1, 2, ...\}$
(c) $\{a^{2n+1} b^{2n+1} | n = 1, 2, 3, ...\}$
(d) None of these

Q.33 Write the grammar for the regular expression a^*b^*.
(a) $S \to AB$, $A \to aA|bB|\in$, $B \to bB|aA|\in$
(b) $S \to AB$, $A \to aA|\in$, $B \to bB|\in$
(c) $S \to ab|\in$, $A \to aA|\in$, $B \to bB|\in$
(d) None of these

Q.34 For the grammar given below, what is the equivalent CFG without useless symbols
$S \to AB/a$, $A \to a$
(a) $S \to A$, $A \to a$ (b) $S \to a$
(c) $A \to a$ (d) $S \to A/a$, $A \to a$

Q.35 What is the equivalent CFL for the following CFG?
$$S \to aS/aSbS/\in$$
(a) $\{x|x$ is a palindrome$\}$
(b) $\{x|x = a^n b^n$ for $n \geq 0\}$
(c) $\{x|$ each prefix of x has atleast as many $a'S$ as $b'S\}$
(d) $\{x/x$ has equal number of $a'S$ and $b'S\}$

Q.36 L_1 has the following grammar
$S \to aB/BA$
$A \to bAA/aS/a$
$B \to b/bS/aBB$
L_2 has the following grammar : $S \to Sba/a$.
Which of the following statement is true about?
$L_3 = L_1 \cap L_2$ and $L_4 = L_1.L_1^*$?
(a) Both L_3 and L_4 are not context free
(b) L_3 is context free but L_4 is not
(c) Both L_3 and L_4 are context free
(d) L_4 is context free, but not L_3

Q.37 Consider a grammar G as follows
$S \to aA$, $A \to bbA$, $A \to c$
$L(G) = ?$
(a) $L(G) = \{abbc\}$
(b) $L(G) = \{ab^nc\,;\, n \geq 0\}$
(c) $L(G) = \{ab^{2n}c\,;\, n > 0\}$
(d) $L(G) = \{ab^{2n}c\,;\, n \geq 0\}$

Q.38 Which of the following is not true?
(a) Any LL grammar is unambiguous.
(b) DCFL's are never inherently ambiguous.
(c) If G is an $LR(k)$ grammar, then $L(G)$ is a DCFL.
(d) All CFL's which are not DCFL's are ambiguous.

Q.39 Consider the grammar consisting of 7 productions
$S \to aA \mid aBB$
$A \to aaA \mid \lambda$
$B \to bB \mid bbC$
$C \to B$
After elimination of Unit, useless and λ-productions, how may production remain in the resulting grammar?
(a) 2 (b) 3
(c) 4 (d) 5

Q.40 Consider grammar G
$S \to AB$, $A \to aAA/\in$, $B \to bBB/\in$
If G_1 is constructed from G, after eliminating $\in$-productions, then G_1 is given by
(a) $S \to AB$, $A \to aAA|\alpha A$, $B \to bBB|\beta B$
(b) $S \to AB|A|B$, $A \to aAA|\alpha A$, $B \to bBB|\beta B$
(c) $S \to AB$, $A \to aAA|\alpha A|\alpha$, $B \to bBB|\beta B|\beta$
(d) $S \to AB|A|B$, $A \to aAA|\alpha A|\alpha$, $B \to bBB|\beta B|\beta$

Q.41 If $G = (\{S\}, \{a\}, \{S \to SS\}, S)$, find language generated by G.
(a) $L(G) = \phi$
(b) $L(G) = a^n$
(c) $L(G) = a^*$
(d) $L(G) = a^n b a^n$

Q.42 Consider the grammar $S \to PQ \mid SQ \mid PS$, $P \to x$, $Q \to y$. To get string of n terminals, the number of productions to be used is
(a) n^2
(b) $2n$
(c) 2^{n+1}
(d) $2n - 1$

Answers Context Free Languages & Push Down Automata

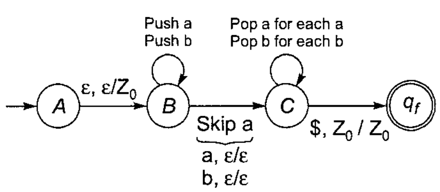
1.	(b)	2.	(c)	3.	(d)	4.	(c)	5.	(c)	6.	(c)	7.	(d)	8.	(a)	9.	(d)
10.	(d)	11.	(c)	12.	(a)	13.	(a)	14.	(b)	15.	(c)	16.	(c)	17.	(c)	18.	(a)
19.	(c)	20.	(c)	21.	(b)	22.	(c)	23.	(c)	24.	(c)	25.	(c)	26.	(b)	27.	(b)
28.	(b)	29.	(a)	30.	(a)	31.	(b)	32.	(c)	33.	(b)	34.	(b)	35.	(b)	36.	(c)
37.	(c)	38.	(d)	39.	(d)	40.	(c)	41.	(a)	42.	(d)						

Explanations Context Free Languages & Push Down Automata

1. (b)
The PDA program can be interpreted as follows: a's are being pushed into stack, then 0 or more b's are being ignored, when c's arrive, the a's are being popped. At end of input, stack must be empty for acceptance, meaning that no of a's must be equal to number of c's.
So, $L(M) = \{a^n b^m c^n ; n \geq 1 ; m \geq 0\}$

2. (c)
(a) $L = \{a^P \mid P \text{ is prime}\}$ is not CFL
(b) $L = \{a^m b^n c^k \mid m = n \text{ or } n = k\}$
$= \{a^i b^i c^j\} \cup \{a^p b^j c^j\}$ is CFL but not DCFL
(c) $L = \{a^m b^n c^k \mid m < n \text{ or } m > n\}$
$= \{a^m b^n c^k \mid m \,!= n\}$ is DCFL

4. (c)



$$L = \left\{ \underbrace{w}_{\text{push}} \ \underbrace{x}_{\text{skip}} \ \underbrace{w^R}_{\text{pop}} \middle| w \in (a+b)^* \ x \in (a+b) \right\}$$

$\therefore$ Option (c) is correct.

6. (c)
$L = \{WW^R \mid W \text{ in } (0+1)^*\}$ cannot be accepted by the deterministic PDA since guessing of the word is required. The word cannot be determined deterministically as $w \in (0+1)^*$ which includes all combinations.

7. (d)
Here
(a) is not correct because we can't find the middle of palindrome, without guessing.
(b) is also not correct because of same reason.
(c) is not correct because 2 stacks are required. Hence it is not even CFL.
(d) is the correct option because $c \notin \{a, b\}^*$. hence we can use "c" recognize the ending of w and starting of w^R.

8. (a)
$\delta(q_0, \in, z_0) = (q_0, \in)$
represents that if no input is there and stack is empty and state is q_0 then it remains at q_0 with accepting $\in$. Hence, $L = \{0^n 1^n \mid n \geq 1\}$ is the language accepted.

9. (d)
Since L is accepted by TM, which halts on every W in L, it is recursive. The complement must also be then recursive. So statement (1) is false.

10. (d)

(a) $\{a^n b^{m+n} c^m \mid m, n \geq 10\}$

$= \{\underline{a^n b^n}\, \underline{b^m c^m} \mid m, n \geq 10\}$ is CFL

(b) $\{a^m \underline{b^n a^n} b^m\}$ is CFL

(c) $\{a^{m+n} b^{m+n} c^k\} = \{a^i b c^k\}$ is CFL

(d) $\{a^m b^n a^m b^n\}$ is not CFL

$\therefore$ Option (d) is correct, which does not contain an equivalent PDA.

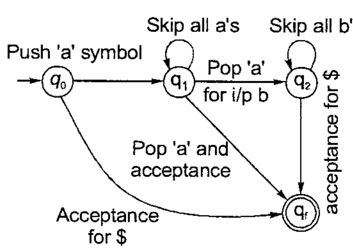
11. (c)

q_0: It can reach q_f without any string. So which can accept ε.

q_1: It can be reached from q_0 by reading atleast one 'a'. First 'a' is pushed onto stack, but after every 'a' is skipped/ignored with by the self loop of q_1.
From q_1, there are two outgoing transitions. By reading \$ it can reach final by poping one symbol 'a' or by reading 'b' it can goto q_2 by poping one symbol 'a'. [only one symbol is pushed from q_0 to q_1]

q_2: All b's are skipped/ignored and once the string ends then it goes to final.

The following PDA shows brief information about given PDA



$$\left.\begin{array}{ll} q_0 \to q_f & : \varepsilon \\ q_0 \to q_1 \to q_f & : a^+ \\ q_0 \to q_1 \to q_2 \to q_f & : a^+b^* \end{array}\right\} \in +a^+ + a^+b^* = \in +a^+b^*$$

12. (a)

$$S_1 \cong S_3 \cong S_3.$$

13. (a)

L_2 is even palindromes on $\{a, b\}^*$
L_3 is odd palindromes on $\{a, b\}^*$
L_1 is any palindrome on $\{a, b\}^*$
Clearly $L_2 \subset L_1$, $L_3 \subset L_1$ and $L_1 = L_2 \cup L_3$.

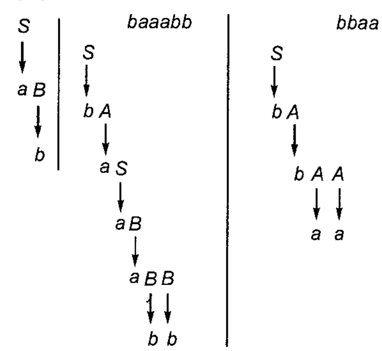
14. (b)

$S \to SS \mid aSb \mid \lambda$ generates the language $L(G) = \{w \mid w \in \{a, b\}^*$ and $n_a(v) \geq n_b(v)$, where v is any prefix of $w\}$.
Now, by replacing "a" with "(" and b with ")", in above grammar, we get $S \to SS \mid (S) \mid \lambda$.
This will generate L(G) = {w | w ∈ {(,)}* and no of left parenthesis ≥ number of right parenthesis in every prefix of w}, which is nothing but the language of properly balanced (matched) parenthesis.

15. (c)

Clearly L_1, L_2 are DCFL's and hence CFL's.
$L_1 \cap L_2 = \{a^i b^j c^k \mid i < j$ and $i < k\}$
is not a CFL, since 2 comparisons must be made before acceptance and this is not possible using a single stack.

16. (c)



17. (c)

Membership problem can be solved using CKY or CYK algorithm.

18. (a)

$L = \{ww^Rw \mid w \in (00)^*\}$ [*Ex.* w = 00 $\Rightarrow$ w^R = 0000]
$L = \{(000000)^*\}$
$\therefore$ L is regular.

19. (c)

$$\left.\begin{array}{l} A \to aB \mid C \mid \varepsilon \\ B \to Ad \mid Cd \end{array}\right\} \Rightarrow L(A) = \{a^m b^n c^n d^m \mid m, n \geq 0\}$$

$$\left.\begin{array}{l} C \to bD \mid \varepsilon \\ D \to Cc \end{array}\right\} \Rightarrow L(C) = \{b^n c^n \mid m, n \geq 0\}$$

$$L(A) = \{a^m b^n c^n d^m\}$$
$$= \{a^m b^n c^k d^l \mid m = l, n = k\}$$

20. (c)

$$L_1 = a^*,\ L_2 = ba,\ L_3 = b^n a^n,\ L_4 = b^*$$

$$((L_2 / L_1) - L_4) - \overline{L}_3 = ?$$

$$L_2 / L_1 = \{ba\} / \{a^*\} = \{ba/\varepsilon,\ ba/a\}$$
$$= \{ba, b\}$$
$$(L_2/L_1) - L_4 = \{ba, b\} - \{b^*\} = \{ba\}$$
$$((L_2/L_1) - L_4) - \overline{L}_3$$
$$= \{ba\} - \left\{\overline{b^n a^n}\right\} = \{ba\}$$
$$= L_2$$

21. (b)

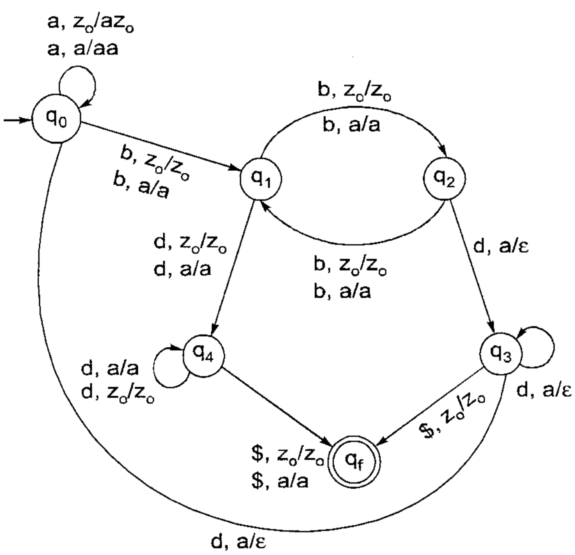
Some linear languages are not DCFL's

Example: The palindrome language.

It is NCFL.

22. (c)

$$L = \{a^m b^n b^k d^\ell \mid \text{if } n = k \text{ then } m = \ell\}$$
$$= \{a^m b^{2n} d^m\} \cup \{a^m b^{2n+1} d^k\}$$
$$= \text{DCFL} \cup \text{regular} = \text{DCFL}$$



23. (c)

$$S \to aA \mid bSS \mid SS$$
$$A \to aAb \mid bAa \mid AA \mid \varepsilon$$

'A' generates equal number of a's and b's

'S' generates atleast one more a than A generates

$\therefore$ Grammar G generates all strings with atleast one more 'a' than number of b's.

24. (c)

$$\left.\begin{array}{l} S \to AAaSb \mid \varepsilon \\ A \to a \mid \varepsilon \end{array}\right\} \cong S \to aSb \mid aaSb \mid aaaSb \mid \varepsilon$$

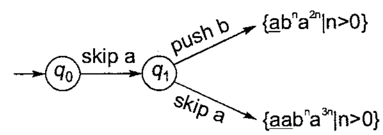
$$L(G) = \left\{a^m b^n \mid n \le m \le 3n\right\}$$

25. (c)

$\overline{L}$ has every even length string and it contain all odd length strings which are not in the form of wxw^R. [It can be implemented by selecting non-deterministic mismatch symbols of w and w^R] $\overline{L}$ is NCFL but not DCFL.

26. (b)

$L = \{ab^n a^{2n} \mid n > 0\} \cup \{aab^n a^{3n} \mid n > 0\}$



The above PDA is possible to construct deterministically.

$\therefore$ L is DCFL.

27. (b)

(a) $L = \{ww^R x \mid w, x \in (a + b)^+\}$ is not regular

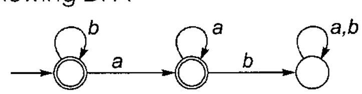
(b) $L = \{xww^R \mid w, x \in (a + b)^*\}$ is regular

(c) $L = \{wxw \mid w, x \in (a + b)^+\}$ is not regular

(d) $L = \{w_1 w_2 x \mid w_1 = w_2,\ w_1,\ w_2, \in (a + b)^+,\ x \in (a + b)^*\}$ is not regular

28. (d)

Each string which do not contain ab is accepted by following DFA



Regular expression equivalent to above DFA is

$$RE = \varepsilon + b^* + b^* aa^* = b^* a^*$$

Given grammar itself is wrong

$S \to AaB \mid \varepsilon \mid a$

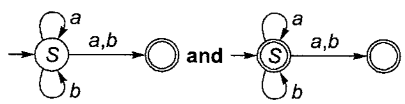
S cannot generate b.

29. (a)

In L_1 only one stack is required while for L_2 and L_3 two stacks are required, so only L_1 is context free language as it is accepted by PDA.

30. (a)
Here for the languages (i) and (ii) we can design a PDA Hence they are CFL.
In (iii) and (iv) we cannot conclude that no(a) = no(c) and no(b) = no(d) using only one stack memory so they can't CFL.

31. (b)
The DFA's construction for the grammars
$S \to aS \mid bS \mid a \mid b$ and $S \to aS \mid bS \mid a \mid b \mid \lambda$ are respectively



The required languages are
$L_1 = (a + b)(a + b)^*$
$L_2 = (a + b)^*$

32. (c)
The language set $\{a^n b^n \mid n = 1, 2, 3, ...\}$ can be generated by (c) $\to$ ab | aSb
(a) is false since it is generating $\in$ also.
(b) is false since aabb is not generated.

33. (b)
$S \to aa\ S\ bb \mid ab\ S \to aaSbb$ generates the sentential from $a^{2n}Sb^{2n}$. Now by substituting $S \to ab$, we get $\{a^{2n+1}\ b^{2n+1}, n \geq 1\}$. $S \to ab$ can directely generate "ab".
So $\quad L = \{a^{2n+1}\ b^{2n+1}, n \geq 1\}$

34. (b)
The language $L = \{W \mid W \in$ any number of a's followed by any number of b's$\}$ has regular expression $a^*\ b^*$. The corresponding grammar is $S \to AB, A \to aA \mid \in, B \to bB \mid \in$
Here $A \to aA \mid \in \quad B \to bB \mid \in$ denotes any number of a's and any number of b's respectively. $S \to AB$ gives the no. of a's followed by number of b's.

35. (b)
Since in grammar $S \to AB/a, A \to a$, B is useless and so is A. Hence $S \to AB$ is useless. Therefore the equivalent CFG is $S \to a$.

36. (c)
The given CFG
$S \to aS/aSbS/\in$
denotes each prefix of x has atleast as many a's as b's.

37. (c)
L_1 and L_2 are context free languages. We can see that L_2 is in fact a regular language having regular expression a (ba)*.
L_1 is a language containing strings having equal number of a's and b's.
$\Rightarrow L_3 = L_1 \cap L_2$ is a context free language.
$\because$ class of context free languages is closed under concatenation and kleene closure.
Therefore $L_4 = L_1 . L_1^*$ is a context free.

38. (d)
The given grammar G
$S \to aA,\ A \to bbA,\ A \to c,\ A \to bbA \mid c$
generates $\{b^{2n}c, n \geq 0\}$
Then $S \to aA$ generates $\{ab^{2n}c, n \geq 0\}$ then the language corresponding to it is.
So, $L(G) = \{ab^{2n}c : n \geq 0\}$

40. (c)
B and C are useless, since they are not generating any terminal.
Therefore the remaining productions left after removing useless, unit and λ-productions
$$S \to aA \mid a,\ A \to aa \mid aaA$$

41. (a)
$\therefore\ G = (\{S\}, \{a\}, \{S \to SS\}, s)$
Since the production $S \to SS$ is never generating any string hence the language $L = \phi$.

42. (d)
Let start with S and develop some string
1. $S \to PQ$ using $S \to PQ$
2. $S \to xQ$ using $P \to x$
3. $S \to xy$ using $Q \to y$

Hence $n = 2$ and number of productions required is 3. Now check n^2 i.e. $2^2 \neq 3$ eliminates (a)
$2n = 2.2 = 4 \neq 3\ \therefore$ eliminates (b)
$2n + 1 = 2.2 + 1 = 8 \neq 3\ \therefore$ eliminates (c)
$2n - 1 = 2.2 - 1 = 3 = 3$

■■■■

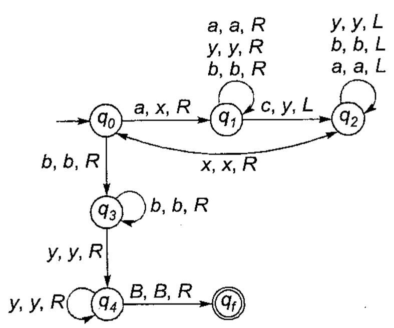
4
CHAPTER

REC, RE Languages & Turing Machines, Decidability

Q.1 Identify which of the following problem can not be constructed by TM?

(a) Generating Set of prime numbers
(b) Computing n^n
(c) Multiplication of any two arbitrary numbers
(d) None of these

Q.2 Consider the following TM.



Note: (*a*, *b*, *c*) represents: by reading input '*a*', it replaces '*a*' by '*b*' and moves to '*c*' direction.

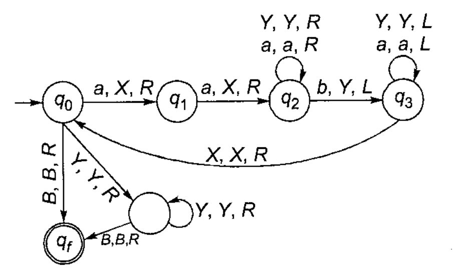
Identify the language accepted by above TM.

(a) $\{a^m b^n c^k | m, n, k \geq 0, m = k\}$
(b) $\{a^m b^n c^k | m, n, k \geq 0, m = n\}$
(c) $\{a^m b^n c^k | m, n, k > 0, m = k\}$
(d) $\{a^m b^n c^k | m, n, k > 0, m = n\}$

Q.3 Which of the following is decidable. Assume given a turing machine *M*, a state *q*, a symbol '*x*' and a string '*w*'.

(a) Whether *M* ever reaches state q when started with input *w* from its initial state.
(b) Whether *M* ever writes the symbol '*x*' when started with an empty tape.
(c) Whether *M* ever moves its head to the left when started with input *w*.
(d) Whether the language accepted by *M* is finite.

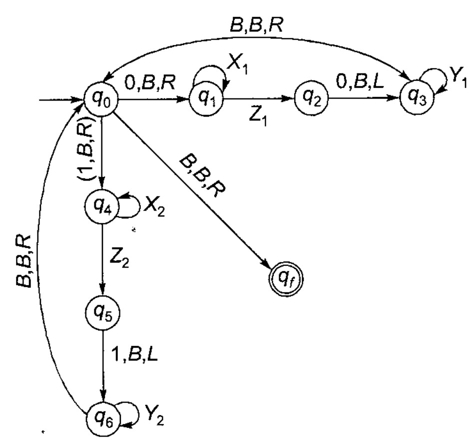
Q.4 Consider the following turing machine.



Identify the language accepted by the above turing machine:

(a) $\{a^m b^n | m > n\}$ (b) $\{a^m b^n | m = 2n\}$
(c) $\{a^m b^n | n = 2m\}$ (d) $\{a^m b^n | m \geq n\}$

Q.5 Consider the following TM.



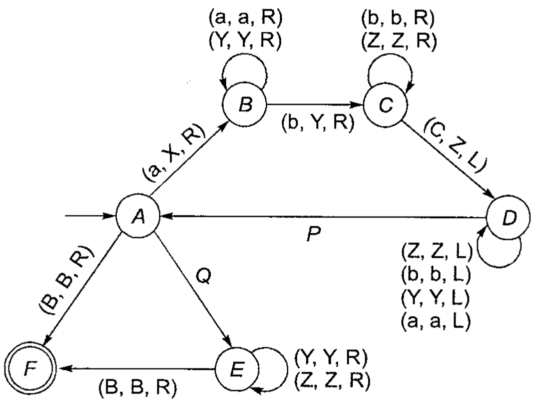
If the above TM accepts ww^R then find the missing transitions at X_1, X_2, Y_1, Y_2, Z_1 and Z_2.

(a) $X_1 = Y_1 = \{(0, 0, R), (1, 1, R)\}$
$X_2 = Y_2 = \{(0, 0, L), (1, 1, L)\}$
$Z_1 = \{(B, B, L)\}$
$Z_2 = \{(B, B, L)\}$

(b) $X_1 = X_2 = \{(0, 0, L), (1, 1, L)\}$
$Y_1 = Y_2 = \{(0, 0, R), (1, 1, R)\}$
$Z_1 = \{(B, B, L)\}$
$Z_2 = \{(B, B, L)\}$

(c) Both (a) and (b)
(d) None of these

Q.6 Consider the following turing machine.



If the above TM accepts a language $L = \{a^n b^n c^n \mid n \geq 0\}$, then what are the missing transitions at P and Q respectively

(a) (Y, Y, R) and (X, X, R)
(b) (X, X, R) and (Y, Y, R)
(c) (X, X, L) and (Y, Y, R)
(d) (Y, Y, R) and (X, X, L)

Q.7 Consider the following transition table of turing machine.

	a	b	B
$\rightarrow A$	(B,a,R)	–	–
B	(B,a,R)	(C,b,R)	–
C	–	(C,b,R)	(D,B,R)
Final State D	–	–	–

What is the language accepted by the above turing machine?

(a) $\{a^n b^n \mid n \geq 1\}$ (b) $\{a^m b^n \mid m, n \geq 1\}$
(c) $\{ab^n \mid n \geq 1\}$ (d) None of these

Q.8 The regular expression that describe the language generated by the grammar:
$G = (\{T, Z\}, \{a, b\}, Z, \{Z \rightarrow aZ \mid \lambda \mid, Z \rightarrow bT, T \rightarrow aZ\})$

(a) ab^*a^* (b) a^*ba^*b
(c) $ab^*a\,a$ (d) $(a + ba)^*$

Q.9 A PDM behaves like a TM when the number of auxiliary memory it has, is

(a) 0 (b) 1 or more
(c) 2 or more (d) none of these

Q.10 Regarding the power of recognizing the languages, which of the following statements is false?

(a) The NDFA are equivalent to DFA
(b) NDPDA are equivalent to DPDA
(c) NDTMs are equivalent to DTMs
(d) Mutiple tape TMs are equivalent to single tape TMs

Q.11 Which of the following languages can't be accepted by a deterministic PDA?

(a) The set of palindromes over alphabet {a, b}
(b) The set of all strings of balanced parenthesis
(c) $L = \{WcW^R \mid W \text{ in } (0 + 1)^*\}$
(d) $L = \{0^n 1^n \mid n \geq 0\}$

Q.12 A TM $M = (\{q_0, q_1, \ldots, q_6\}, \{0,1\}, \{0,1,B\}, \delta q_0; B, \phi\}$.
The transition are

$(q_0, 0) = q_1, B, R$
$(q_1, 0) = q_1, 0, R$
$(q_0, 1) = q_2, 1, R$
$(q_1, 1) = q_2, 1, R$
$(q_2, 0) = q_2, 1, R$
$(q_2, 1) = q_3, 1, L$
$(q_3, 0) = q_3, 0, L$
$(q_3, 1) = q_3, 1, L$
$(q_3, B) = q_0, B, R$
$(q_2, B) = q_4, B, L$
$(q_4, 1) = q_4, B, L$
$(q_4, 0) = q_4, 0, L$
$(q_4, B) = q_6, 0, R$
$(q_0, 1) = q_5, B, R$
$(q_5, 1) = q_5, B, R$
$(q_5, B) = q_6, B, R$

What is the laugnage accepted by TM?

(a) 0^*10^* (b) $0^{(m-n)} \mid m < n$
(c) $(0 + 1)^*$ (d) $0^n 10^m \mid m \leq n$

Q.13 Consider the transition table of a TM given below. Here "b" represents the blank symbol.

δ	0	1	b
q_0	$q_0 0R$	$q_1 0R$	$q_2 0R$
q_1	$q_1 0R$	$q_0 0R$	–
q_2	–	–	–

The given turing machine accepts

(a) set of all even palindromes over {0, 1}
(b) strings over {0, 1} containing even number of 1's
(c) strings over {0, 1} containing even number of 1's and odd no. of 0's
(d) string over {0, 1} starting with zero

Q.14 Consider the following transition table for turing machine.

δ	0	1	2	□
q_0	q_0, 0, R	q, 1, R	q_2, 2, R	q_f, □, R
q_1			q_2, 2, R	q_f, □, R
q_2			q_2, 2, R	q_f, □, R
q_F	–	–	–	–

Find the language accepted by following turing machine?

(a) $(0 + 1 + 2)^*$ (b) $(012)^*$
(c) $(0^* 1^* 2^*)$ (d) $(0^+ 1^+ 2^+)$

Q.15 If L is a turing recognisable language, but not a decidable language, then which of the following is true regarding L^C?

(a) L^C is turing recognisable
(b) L^C may be turing recognisable
(c) L^C is not turing recognisable
(d) L^C is decidable

Q.16 Let $L = \{a^{m^n} | m, n > 0\}$. Then L is

(a) Regular (b) CFL
(c) Recursive (d) REL

Q.17 $L = (\overline{L}_3 \cap L_2) \cup L_1$. Let L_1 be a regular, L_2 be a CFL and L_3 be a recursive language. Then L is

(a) Regular (b) CFL
(c) Recursive (d) None of these

Q.18 Which of the following is false?

(a) Every language on Σ^* is countable.
(b) The set of all recursively enumerable languages is countable
(c) Every subset of a given regular language is countable
(d) The set of all not recursively enumerable language is countable

Q.19 If L be a recursive language. Then finite subset of L is.

(a) Regular (b) CFL
(c) CSL (d) Recursive

Q.20 Recursive languages are not closed under _______ operation.

(a) Union (b) Intersection
(c) Reversal (d) Substitution

Q.21 Let L_1 be a recursive language and L_2 be a recursive enumerable language. Then $L_2 - L_1$ is

(a) Recursive language
(b) Recursive enumerable language
(c) Non-recursive enumerable language
(d) None of these

Q.22 Consider the following statements.

1. Is FA accepts a given string?
2. Is TM accepts regular?
3. Is CFG generates regular?
4. Is Halting TM accepts a given string?

Find which of the above are decidable.

(a) 1 and 2 (b) 1 and 3
(c) 1 and 4 (d) 1, 2, 3 and 4

Q.23 Let $L = \{(a^P)^* | P \text{ is prime}\}$ is

(a) Regular (b) CFL
(c) Recursive (d) REL

Q.24 If turing machine restricts its head to read only then what is the language accepted by such TM?

(a) Regular language
(b) Context free language
(c) Recursive language
(d) Recursive enumerable language

Q.25 Halting problem/language is

(a) RE as well as recursive
(b) Recursive and NP
(c) RE but not recursive
(d) Neither recursive nor RE

Q.26 If L_1 and L_2 are a pair of complementary languages. Which of the following statement is not possible?

(a) Both L_1 and L_2 are recursive
(b) L_1 is recursive and L_2 is recursively enumerable but not a recursive
(c) Neither L_1 nor L_2 is recursively enumerable
(d) One is recursively enumerable but not recursive, the other is is not recursively enumberable

Q.27 If the strings of a language L can be effectively enumerated in lexicographic (i,e, alphabetic) order which of the following statements is true?
(a) L is necessarily finite
(b) L is regular but not necessarily finite
(c) L is context free but not necessarily regular
(d) L is recursive but not necessarily context free

Q.28 If A is reducible to B and B is finite language then A is
(a) finite
(b) infinite
(c) can not determine
(d) may be finite

Q.29 Let X, Y and Z are three problems, X is reducible to Y, and Y is reducible to Z. If Z is NP-problem, then X is
(a) P-problem (b) NP-problem
(c) NP-complete (d) NP-hard

Q.30 If PCP (post-correspondence problem) is reducible to a problem L, then what can be said about L?
(a) L is decidable
(b) L may be decidable
(c) L is undecidable
(d) L has to be the halting problem

Q.31 Which of the following statements are true?
(i) The complement of a language is always regular.
(ii) The intersection of regular languages is regular.
(iii) The complement of a regular language is regular.
(a) (i) and (ii) only (b) (ii) and (iii) only
(c) (i) and (iii) only (d) All of the above

Q.32 Which of the following is not true?
(a) CFLs are closed under union and concatenation.
(b) Regular languages are closed under union and intersection.
(c) CFLs are not closed under intersection and complementation.
(d) If L is a CFL and R is a regular set then $L \cap R$ is not a CFL.

Q.33 Let L_1 and L_2 are regular sets defined over alphabet Σ^*. Mark the false statement
(a) $L_1 \cup L_2$ is regular
(b) $L_1 \cap L_2$ is not regular
(c) $\Sigma^* - L_1$ is regular
(d) $L_1{}^*$ is regular

Q.34 CFLs are not closed under
(a) Union (b) Concatenation
(c) Closure (d) Intersection

Q.35 The statement $P \subseteq NP$ is
(a) true
(b) false
(c) still open for argument
(d) none of these

Q.36 If $\forall w \in \Sigma^*$, it can be determined in finite time, whether or not $w \in L$, then L is
(a) decidable
(b) undecidable
(c) non-deterministic
(d) intractable

Q.37 Ram and shyam have been asked to show that a cartain problem Π is NP-complete. Ram shows a polynomial time reduction from the 3-SAT problem to Π, and shyam shows a polynomial time reduction from Π to 3-SAT. Which of the following can be inferred from these reductions?
(a) Π is NP-hard but not NP-complete
(b) Π is NP-complete
(c) Π is in NP, but is not NP-complete
(d) Π is neither NP-hard, nor in NP

Q.38 P, Q, R are three languages. If P and R are regular and if $PQ = R$, then
(a) Q has to be regular
(b) Q can not be regular
(c) Q need not be regular
(d) Q has to be a CFL

Q.39 The Travelling Salesman Problem (TSP) is
(a) NP but not NP complete
(b) NP-complete
(c) Neither NP nor NP-complete
(d) None of these

Q.40 Which of the following is false?
(a) PATH is a P class problem
(b) Djikstra's algorithm is a problem in P
(c) CLIQUE is a NP class problem
(d) RELPRIME is a NP class problem

Q.41 If there is an NP-complete language L whose complement is in NP, then the complement of any language in NP is in
(a) NP (b) P
(c) Both (a) and (b) (d) None of these

Answers REC, RE Languages & Turing Machines, Decidability

1.	(d)	2.	(c)	3.	(c)	4.	(b)	5.	(a)	6.	(b)	7.	(b)	8.	(d)	9.	(c)
10.	(b)	11.	(a)	12.	(a)	13.	(b)	14.	(c)	15.	(c)	16.	(a)	17.	(c)	18.	(d)
19.	(a)	20.	(d)	21.	(b)	22.	(c)	23.	(a)	24.	(a)	25.	(c)	26.	(b)	27.	(d)
28.	(a)	29.	(b)	30.	(c)	31.	(b)	32.	(d)	33.	(b)	34.	(d)	35.	(c)	36.	(a)
37.	(b)	38.	(c)	39.	(b)	40.	(d)	41.	(c)								

Explanations REC, RE Languages & Turing Machines, Decidability

1. (d)
TM is more powerful than any other machine (FA, PDA and LBA). TM is equivalent to finite length program.
There exist a program to solve any problem if and only if TM exist.
(a), (b) and (c) are possible to construct by TM.

2. (c)
$L = \{a^m b^n c^k \mid m, n, k > 0 \text{ and } m = k\}$
Here, a's are replaced by x and c's are replaced by y in every scan from $q_0 \to q_1 \to q_2$
To reach final state, atleast one b should appear and atleast one y (y represents c hence a also must appear) should appear.
$\therefore$ $L = \{a^i b^j c^i \mid i, j > 0\}$ is accepted by TM

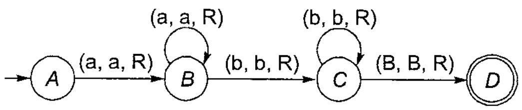
3. (c)
(a) For a given input string, particular state may or may not be reached. Finding the reachability for a particular string is undecidable. [State entry problem is undecidable]
(b) Writing a particular symbol 'x' on tape is undecidable.
(c) It is easy to check moving head to the left with finite steps using a UTM, for given string. Hence it is decidable.
(d) Language accepted by M is finite or not, is undecidable. [Finiteness problem is undecidable, this problem is non-trivial by Rice theorem].

4. (b)
The language accepted by TM = $\{a^{2i} b^i \mid i \geq 0\}$ = $\{a^m b^n \mid m = 2n\}$

5. (a)
X_1 and X_2 are used to skip all symbols and moves to the right direction. [Where as Y_1 and Y_2 moves to left direction]. Z_1 and Z_2 are used to find B and moves to the left direction
$\therefore$ $X_1 = X_2 = \{(0, 0, R), (1, 1, R\}$
$Z_1 = Z_2 = \{(B, B, L)\}$
$Y_1 = Y_2 = \{(0, 0\ L), (1, 1, L)\}$

6. (b)
$P = (X, X, R)$; To find the left most 'a', state B will move left until X appears. When X appears, it moves to the right position to read symbol 'a' if exists.
Q = (Y, Y, R); If all a's are written by X's then A will find Y that indicates all b's and all c's are written by Y's and Z's respectively.
$A \Rightarrow E \Rightarrow F$ path to read all Y's then all Zs, finally goes to final state.

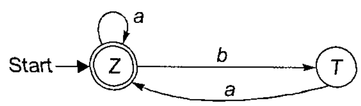
7. (b)



TM accepts $L = \{a^+ b^+\} = \{a^m b^n \mid m, n \geq 1\}$

8. (d)

Since the grammar is right linear grammar and hence regular we can design a DFA for it



The language corresponding to the above DFA is

$L = \{\lambda, a, aa, aaa, \ldots\ldots, ba, baba, baba, \ldots\ldots\}$

$\therefore\ L = (a + ba)^*$

9. (c)

Auxilliary memory = memory used by the machine for calculations or swappings.

PDA with 2 or more stacks is equivalent to TM.

Hence correct option is (c).

10. (b)

Regarding the power of recognizing of languages

(a) NDFA has same power as DFA

(c) NDTMs are equivalent to DTMs

(d) Multiple tape TMs are equivalent to single tape TMs

Only DPDA $\subseteq$ NPDA in the power.

Hence correct option is (b)

11. (a)

Deterministic PDA can accept

(b) The set of all strings of balanced parenthesis.

(c) $L = \{WcW^R / W \text{ in } (0 + 1)^*\}$

(d) $L = \{0^n 1^n \mid n \geq 0\}$

in each of the above the PDA can deterministically separate the push and pop operations. But it cannot do the same in case (a), since it has to guess middle of palindrome, In DPDA we can't do guesing, NPDA is required for this.

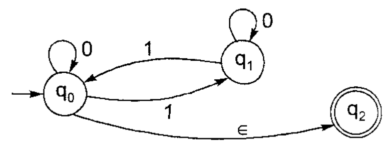
Hence option (a) is correct.

12. (a)

The analysis of transition of TM. M shows that it accepts language 0^*10^*.

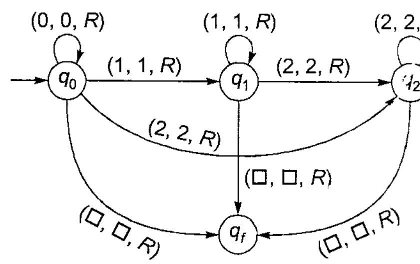
13. (b)

TM does not change any symbol and keeps moving to the right. We can treat it as an a FSM also. That is the given TM can be represented by an equivalent FSM transition diagram.



Regular expression : $0^* + 0^*(10^*1)^*$. clearly above FA accepts even number of 1's

14. (c)



The above turing machine accepts $0^* 1^* 2^*$.

15. (c)

Proof by contradiction: Let us assume that L^C is turing recognisable. Now, since it is given that L is turing recognisable, then by theorem on RE languages, we can say that both L and L^C will become recursive (decidable). But it is given that L is not decidable. So this is a contradiction. So our assumption that L^C is turing recognisable is false. So choice (a) and (b) are wrong and choice (c) is correct.

Choice (d) is also wrong since, if L^C is not turing recognisable, it can never be decidable.

16. (a)

$L = \{a^{m^n} \mid m,n > 0\}$ is regular language.

$\therefore$ option (a) is correct.

17. (c)

L_1 = reg, L_2 = CFL, L_3 = Recursive

$(\overline{L_3} \cap L_2) \cup L_1$

$= (\overline{\text{Recursive}} \cap CFL) \cup reg$

$= (\text{Recursive} \cap CFL) \cup \text{reg}$

$[\because \overline{\text{Recursive}} = \text{Recursive}]$

$= (\text{Recursive}) \cup \text{reg}$

[∵ Recursive languages are closed for union and intersection]

= Recursive

18. (d)

The set of all "not recursively enumerable" languages

$= 2^{\Sigma^*}$ – set of recursively enumerable languages

= Uncountable – Countable

= Uncountable

19. (a)

L is recursive language, finite subset of L is a finite set.

∴ L is regular language.

Note : Finite subset of any language is regular.

20. (d)

Recursive languages are not closed under Homomorphism and Substitution operations.

21. (b)

$L_2 - L_1 = L_2 \cap \overline{L_1}$

$= REL \cap \overline{RECURSIVE}$

$= REL \cap RECURSIVE = REL$

∴ $L_2 - L_1$ is Recursive Enumerable Language (REL).

22. (c)

1. Membership problem for FA is decidable
2. Regular acceptance of TM is undecidable
3. Is CFG generates regular? is undecidable
4. Membership problem for Halting is decidable

∴ 1 and 4 are valid statements.

23. (a)

$$L = \{(a^P)^* \mid P \text{ is prime}\}$$

a^p is not regular but $(a^p)^*$ is regular

$(a^p)^* = (a^2)^* \cup (a^3)^* \cup (a^5)^* \cup (a^7)^* ...$

$= \{\varepsilon, a^2, a^3, a^4, a^5, a^6, a^7, a^8, a^9, a^{10}, ...\}$

$= \{a^n \mid n \geq 2 \text{ or } n = 0\}$

$= \varepsilon + aa^+$

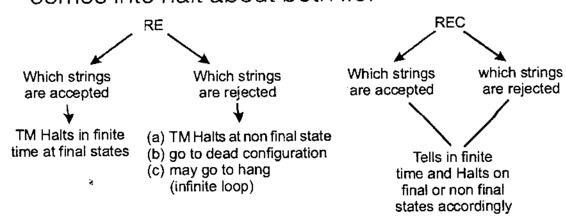
∴ $(a^p)^*$ is regular.

24. (a)

If turing machine has no writing capability on tape then turn around capability of head is not useful. So it accepts only regular language.

25. (c)

Halting problem belongs to the TM. The language in which halting problem belongs is recursively enumerable i.e. RE. In this TM can't tell in finite time the word is rejected or not. Although it can tell in finite time that the given word is accepted or not. Although REC can tell in finite time and comes into halt about both i.e.



26. (b)

The correct statement follows from following theorem:

Theorem: If L_1 and L_2 is a pair of complementary language then either

- Both L_1 and L_2 are recursive.
- Neither L_1 and L_2 are recursively enumerable.
- One is recursively enumerable but not recursive, the other is not RE.

Option (b) is not possible, since if L_1 is recursive, then $L_2 = L_1^c$ will also be recursive.

27. (d)

L is recursive iff L is enumerable in Lexicographic order (alphabetic).

It is also known that context free language is a subset of recursive languages.

Hence (d) is correct option