Tutorial - 4

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1.) $T(n) = 3T(n/2) + n^2$

$T(n) = aT(n/b) + f(n^2)$

$a \geq 1,\ b \geq 1$

On comparing,

$a = 3,\ b = 2,\ f(n) = n^2$

Now., $C = \log_b a = \log_2 3 = 1.584$

$n^c = n^{1.584} < n^2$

$\therefore f(n) > n^c$

$\therefore T(n) = O(n^2)$

2.) $T(n) = 4T(n/2) + n^2$

$a \geq 1,\ b > 1$

$a = 4,\ b = 2,\ f(n) = n^2$

$C = \log_2 4 = 2$

$n^c = n^2 \Rightarrow f(n) = n^2$

$\therefore T(n) = O(n^2 \cdot \log_2 n)$

3.) $T(n) = T(n/2) + 2^n$

$a = 1,\ b = 2$

$f(n) = 2^n$

$C = \log_b a = \log_2 0 = 0$

$n^c = n^0 = 1.$

$f(n) > n^c$

$T(n) = O(2^n)$

4. $T(n) = 2^n T(n/2) + n^n$

$a = 2^n$

$b = 2$, $f(n) = n^2$

$c = \log_b a = \log_2 2^n$

$= n$

$n^c \Rightarrow n^n$

$f(n) = n^c$

$f(n) = \theta(n^2 . \log_2 n)$

5. $T(n) = 16T(n/a) + n$

$a = 16$, $b = 4$

$f(n) = n$

$c = \log_4 16 = \log_4 (4^2) = 2 \log_4 4$

$= 2$

$n^c = n^2$

$f(n) < n^c$

$\therefore T(n) = \theta(n^2)$

6. $T(n) = 2T(n/2) + n \log n$

$a = 2$, $b = 2$

$f(n) = n \log n$

$c = \log_2 2 = 1$

$n^c = n^1 = n$

$n \log n > n^c$

$f(n) > n^c$

$T(n) = \theta(n \log n)$

7. $T(n) = 2T(n/2) + n\log n$

$a = 2,\ b = 2,\ f(n) = n\log n$

$c = \log_2 2 = 1$

$n^c = n^1 = n$

$\therefore \frac{n}{\log n} < n$

$\therefore f(n) < n^c$

$\therefore T(n) = O(n)$

8.) $T(n) = 2T(n/4) + n^{0.51}$

$a = 2,\ b = 4,\ f(n) = n^{0.51}$

$c = \log_b a = \log_4 2 = 0.5$

$n^c \leq n^{0.51}$

$f(n) > n^c$

$\therefore T(n) = \Theta(n^{0.51})$

9.) $T(n) = 0.5T(n/2) + 1/n$

$\rightarrow a = 0.5,\ b = 2$

$a \geq 1$ but here a is 0.5

So, we cannot apply Master's Theorem.

10. $T(n) = 16T(n/4) + n!$

$a = 16, b = 4, f(n) = n!$

$\therefore c = \log_b a = \log_4 16 = 2$

$n^c = n^2$

As $n! > n^2$

$\therefore T(n) = \Theta(n!)$

11. $4T(n/2) + \log n$

$a = 4, b = 2, f(n) = \log n$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$f(n) = \log n$

$\because \log n < n^2$

$f(n) < n^c$

$T(n) = \Theta(n^c)$

$T(n) = \Theta(n^c)$

12. $T(n) = \text{sqrt}(n) + T(n/2) + \log n$

$\rightarrow a = \sqrt{n}, b = 2$

$\therefore c = \log_b a = \log_2 \sqrt{n} = \frac{1}{2} \log_2 n$

$\therefore \frac{1}{2} \log_2 n \leq \log(n)$

$\therefore f(n) > n^c$

$T(n) = \Theta(f(n))$

$T(n) = \Theta(\log(n)))$

13. $T(n) = 3T(n/2) + n$

$\rightarrow a = 3, b = 2, f(n) = n$

$c = \log_b a = \log_2 3 = 1.5849$

$n^c = n^{1.5489}$

$n < n^{1.5489}$

$\Rightarrow f(n) < n^c$

$T(n) = \theta(n^{1.5849})$

14. $T(n) = 3T(n/3) + \text{sqrt}(n)$

$\rightarrow a = 3, b = 3$

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

As, $\text{sqrt}(n) < n$

$f(n) < n^c$

$f(n) = \theta(n)$

15. $T(n) = 4T(n/2) + n$

$\rightarrow a = 4, b = 2$

$c = \log_b a = \log_2 4 = 2$

$n < n^2$ (for any constant)

$f(n) < n^2$

$f(n) = \theta(n^2)$

16. $T(n) = 3T(n/4) + n\log n$

$a = 3,\ b = 4,\ f(n) = n\log n$

$c = \log_b a = \log_4 3 = 0.792$

$n^c = n^{0.792}$

$n^{0.792} < n\log n$

$T(n) = \theta(n\log n)$

17. $T(n) = 3T(n/3) + n/2$

$a = 3,\ b = 3$

$c = \log_b a = \log_3 3 = 1$

$f(n) = n/2$

$\therefore n^c = n^1 = n$

As $n/2 < n$

$f(n) < n^c$

$\therefore T(n) = \theta(n)$

18. $T(n) = 6T(n/3) + n^2 \cdot \log n$

$a = 6,\ b = 3$

$c = \log_b a = \log_3 6 = 1.6309$

$n^c = n^{1.6309}$

As, $n^{1.6309} < n^2\log n$

$\therefore T(n) = \theta(n^2\log n)$

19. $T(n) = 4T(n/2) + n/\log n$

$a = 4, b = 2, f(n) = \frac{n}{\log n}$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$\frac{n}{\log n} < n^2$

$T(n) = \Theta(n^2)$

20. $T(n) = 64T(n/8) - n^2 \log n$

$a = 64, b = 8$

$c = \log_b a = \log_8 64 = \log_8 (8^2)$

$c = 2$

$n^c = n^2$

$n^2 \log n > n^2$

$T(n) = \Theta(n^2 \log n)$

21. $T(n) = 7T(n/3) + n^2$

$a = 7, b = 3, f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.7712$

$n^c = n^{1.7712}$

$n^{1.7712} < n^2$

$T(n) = \Theta(n^2)$

22. $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1, \; b = 2$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$n((2 - \cos n)) > n^c$

$T(n) = \Theta(n(2 - \cos n))$

Ans