Declarative Knowledge/Technical Skills, 2011-2012

Outcomes

1 Modern Analysis

1.1 Fall 2011

The following four questions were embedded into the final exam.

1. Let A, B be two subsets of $\mathbb{R}$ that are bounded above and non-empty. Prove: The set $A+B$ is bounded above, non empty, and
$$\sup(A+B) = \sup A + \sup B.$$
2. Let $\{a_n\}$ be a sequence of real numbers. Prove: $\{a_n\}$ converges to L if both subsequences $\{a_{2n}\}$ and $\{a_{2n+1}\}$ converge to L.
3. Let I be an open interval in $\mathbb{R}$, $a \in I$; assume that $f : I \to \mathbb{R}$ is continuous and f is differentiable at all points $x \in I$ such that $x \neq a$. Assume $\lim_{x\to a} f'(x) = L$ exists. Prove: f is differentiable at a and $f'(a) = L$.
4. Let $f : [a, b] \to \mathbb{R}$ be integrable. Prove: $\int_a^b f > 0$ if and only if there exists a positive lower sum. In symbols:
$$\int_a^b f > 0 \quad \Leftrightarrow \quad \exists\ P \in \mathcal{P}[a, b] \quad \text{such that} \quad L(f, P) > 0.$$

Twenty Four students students took the final exam. The distribution by major was as follows:

BS Math: 8.

BA Math: 4.

BA Math Ed: 4.

Non Degree: 4.

Other: 4.

The non-degree students consisted of three teachers coming back for certification and an older student who may or may not continue as math major. "Other" consisted of two graduate students from electrical engineering, a student in the BBA Management Program of the College of Business, and a graduate student in the College of Education going for some form of certification.

The table below summarizes the results for the math degree seeking students. The four problems were graded on the scale 0-10. Rows BS1-BS8 contain the results for the 8 BS students, BA1-BA4 the results for the 4 BA in mathematics students, BAE1-BAE4 for the BA in math education. The average for each student is again on a scale of 0-10

	Quest. 1	Quest. 2	Quest. 3	Quest. 4	Average
BS1	4	0	0	0	1
BS2	7	3	9	1	5
BS3	7	7	0	0	3.5
BS4	4	7	0	5	4
BS5	7	7	0	8	5.5
BS6	7	3	0	4	3.5
BS7	7	1	1	1	2.5
BS8	10	7	0	9	6.5
BA1	3	1	1	5	2.5
BA2	7	8	1	8	6
BA3	6	0	0	0	1.5
BA4	2	3	0	0	1.25
BAE1	1	1	0	0	0.5
BAE2	6	0	0	0	0.5
BAE3	4	0	0	5	2.25
BAE4	0	0	0	1	1.25

1.2 Spring 2012

There were ten students that took the final, seven of these are math majors, four are BS and three are BA.

The instructor picked three questions.

Question one. This falls under the topic of sequences and series.

(15 pts.) If the infinite series $\sum a_n$ converges, then $\lim_{n\to\infty} a_n = 0$. Proof:

percent correct: 71

BA percent correct: 49

BS percent correct: 85

Question two. This falls under the topic of diffentiation.

(15 pts.) If f is differentiable at $x = a$, then f is continuous at $x = a$. Proof:

percent correct: 91

BA percent correct: 100

BS percent correct: 100

Question three. This falls under the topic of integration.

(20 pts.) Let $\mathcal{P} = \{x_0, \ldots, x_n\}$ be an arbitrary partition of $[0, 1]$. Answer the following for the function

$$f(x) = \begin{cases} x & \text{if } x \in \mathbb{Q} \\ 0 & \text{otherwise.} \end{cases}$$

1. Prove that $1/2$ is a lower bound for $\mathrm{US}(f, \mathcal{P})$, the upper sum for $\mathcal{P}$.
2. Prove that 0 is an upper bound for $\mathrm{LS}(f, \mathcal{P})$, the lower sum for $\mathcal{P}$.
3. Prove that f is not Riemann integrable on $[0, 1]$.

percent correct: 36

BA percent correct: 37.5

BS percent correct: 18

2 Modern Algebra

2.1 Fall 2011

14 students took the final exam which had 10 problems. As only the best 8 answers counted, the instructor picked three questions which were answered well by most students. In this sense, the averages are not representative for the final exam.

Problem 1: Average 5.5/8 = 69% Standard deviation 2.3
Problem 3: Average 6.9/8 = 86% Standard deviation 0.8
Problem 7: Average 7.1/8 = 89% Standard deviation 1.6

1. (a) Give the definition of a group. When is a group called abelian?
   (b) Give two examples of abelian groups, and two examples of non-abelian groups.

3. (a) When is a permutation called even?
   (b) Which of the following permutations are even:
   $$(12), (123), (1234), (12)(34)?$$
   (c) State a result about even and odd permutations
   (d) Show that any product of two odd permutations is even.

7. (a) State Lagrange's Theorem.
   (b) Is there a group G with a subgroup H such that G has 9 elements and H has 4 elements? If so, give an example. If not, explain why not!
   (c) Find all right cosets of the subgroup $\langle 3 \rangle$ in the group $\mathbb{Z}_9$.

2.2 Spring 2012

A total of 6 students took the final; classified below as S1 to S6. Of these $S1$ is a non-admitted graduate student, and $S6$ is a graduate student in Education. The embedded questions and the scores are listed below.

<table>
<tr><th>Question</th><th>Grades(%)</th><th>Avge (%)</th></tr>
<tr><td>.If H is the subgroup of S_7 generated by $(123)(456), (23456), (13)(45)$, could H have order 2525? Explain.</td><td><table><tr><td>S1</td><td>S2</td><td>S3</td><td>S4</td><td>S5</td><td>S6</td></tr><tr><td>0</td><td>37.50</td><td>0</td><td>31.25</td><td>50.00</td><td>31.25</td></tr></table></td><td>25</td></tr>
<tr><td>(a) Find the greatest common divisor of 265 and 752, and write it as an integer linear combination of 265 and 752. Use the Euclidean Algorithm. Show your work. (b) Find the multiplicative inverse of 265 in $Z_7$52.</td><td><table><tr><td>S1</td><td>S2</td><td>S3</td><td>S4</td><td>S5</td><td>S6</td></tr><tr><td>70.00</td><td>80.00</td><td>100.00</td><td>100.00</td><td>100.00</td><td>80.00</td></tr></table></td><td>88.33</td></tr>
<tr><td>1. If Q_u is the ring of elements of the form $a+bi+cj+dk$, where a, b, c, d are rational, and i, j, and k satisfy $i^2 = j^2 = k^2 = -1$, ij =-ji = k, jk = -kj = i, ki = -ik = j, then Q_u is a ring such that every non-zero element has a multiplicative inverse. Is Q_u a field?</td><td><table><tr><td>S1</td><td>S2</td><td>S3</td><td>S4</td><td>S5</td><td>S6</td></tr><tr><td>0.00</td><td>31.25</td><td>25.00</td><td>0.00</td><td>0.00</td><td>0</td></tr></table></td><td>9.38</td></tr>
<tr><td>1. Describe(classify) all the abelian groups of order 936, up to isomorphism.</td><td><table><tr><td>S1</td><td>S2</td><td>S3</td><td>S4</td><td>S5</td><td>S6</td></tr><tr><td>0.00</td><td>18.75</td><td>75.00</td><td>100.00</td><td>0.00</td><td>100.00</td></tr></table></td><td>48.96</td></tr>
<tr><td>1. Let H be a subgroup of the group G. Prove: If $|G|/|H| = 2$, then H is a normal subgroup of G.</td><td><table><tr><td>S1</td><td>S2</td><td>S3</td><td>S4</td><td>S5</td><td>S6</td></tr><tr><td>0.00</td><td>0.00</td><td>25.00</td><td>100.00</td><td>0.00</td><td>100.00</td></tr></table></td><td>37.50</td></tr>
</table>

3 Probability and Statistics

3.1 Fall 2011

Twelve (12) students took the final exam. The following three problems, were embedded in the final, where they appeared as Problems 1, 7, and 8.

1. A doctor is studying the relationship between blood pressure and heartbeat abnormalities in her patients. She tests a random sample of her patients and notes their blood pressures (high, low, or normal) and their heartbeats (regular or irregular). She finds that:

   (i) 14% have high blood pressure,

   (ii) 22% have low blood pressure,

   (iii) 18% have an irregular heartbeat.

   (iv) Of those with an irregular heartbeat, one-quarter have high blood pressure.

   (v) Of those with normal blood pressure, one-eighth have an irregular heartbeat.

   What portion of the patients selected have a regular heartbeat and low blood pressure?

7. Let X and Y denote the values of two stocks at the end of a five-year period. X is uniformly distributed on the interval $(0, 100)$. Given $X = x$, Y is uniformly distributed on the interval $(0, x)$. Find

   (a) $f_X(x)$, and $h(y|x)$. Include the supports.

   (b) $f(x, y)$. Are X and Y independent?

   (c) $E[Y|x]$. Is this expectation a linear function in x?

   (d) $E[Y]$ (use that $E[Y] = E[E[Y|X]]$).

   (e) $\text{Cov}(X, Y)$.

8. The time, T, that a manufacturing system is out of operation has cumulative distribution function $F(t) = 1 - ((3/t))^2$ for $t > 3$, and 0 elsewhere. The resulting cost to the company is $Y = T^2$.

   (a) Determine the density function (p.d.f) of Y, for $y > 9$.

   (b) Compare $E[T^2]$ to $E[T]^2$

Grades, scale 0-100			
	1	7	8
Average	68.70	62.10	33.00
Median	70	60	20

Instructor's Comments: The outcomes for problem 1 and 7 are within the normal range for a Probability and Statistics 1 class. The outcome for problem 8 is so low, because many students don't pay any attention to the questions at the end of the test (4 papers had a zero score). This type of question should not present any difficulty to the student; however, nobody achieved a full score for it.

3.2 Spring 2012

The following three questions were embedded in the final exam. Of these #2 and #3 are from the sample first actuarial exam, while #1 is chosen as being one of the fundamental though simple questions from Probability Theory.

1. Let X and Y be independent random variables, both distributed according to Poisson(λ) law, and let $Z = X + Y$. Prove that Z is also distributed according to a Poisson law.

2. An urn contains 10 balls: 4 red and 6 blue. A second urn contains 16 red balls and an unknown number of blue balls. A single ball is drawn from each urn. The probability that both balls are the same color is 0.44. Calculate the number of blue balls in second urn.

3. The time X, that a manufacturing system is out of operation has a probability density function

$$f(x) = \begin{cases} \frac{8}{x^3} & \text{if } x > 2, \\ 0 & \text{otherwise.} \end{cases}$$

The resulting cost to the company is $Y = X^2$. Find the cumulative probability distribution and density function of Y.

Twenty one (21) students took the final exam. The distribution of right and wrong answers turned out as follows.

Problem	# right	#wrong
1	13	8
2	9	12
3	3	18