Problem Solving

Master Syllabus

MAT 4937

Room: _____
Days: _____
Times: _____
Instructor: _____
Office Hours: _____

Prerequistes:	MAC 2312	Calculus and Analytic Geometry 2
	MAD 2104	Discrete Mathematics
	MAS 2103	Matrix Theory
Suggested:	MAD 2502	Introduction to Mathematical Computation

Course Description. This course will focus on some interesting theorems and problems in mathematics. Most of these problems will require very little in the way of background knowledge of mathematics. Some of the questions may involve some Geometry, Number Theory. This does not mean that you must have taken a course in either of these areas. In general, you may need to find inventive ways to use mathematics you already know.

Students should enjoy solving mathematical problems and be willing to tackle new problems and learn new techniques. The student should not be afraid to cross boundaries from one mathematical area to another in the solution of one problem. Some ability to program a computer or programmable calculator could be an asset for some problems.

Expectation for student performance in this course will be governed by the departmental Academic Learning Compact (ALC).

Academic Learning Compact. **COMMUNICATION**(Written and oral communication) and **CRITICAL THINKING**(Analytical skills): Graduates in mathematics will produce writing that is well-organized and grammatically correct, and they will express mathematical ideas clearly and concisely. Graduates in mathematics will analyze and solve a variety of mathematical problems.

Course Objectives. By the conclusion of this course, a student should be able to

- use a variety of heuristics in solving mathematics problems;
- solve problems that require a variety of mathematical skills and knowledge normally taught in the

lower divsion;specifically, analytic geometry, trigonometry, calculus, linear algebra, discrete mathematics;
- make reasonable attempts to solve problems which involve skills and techniques from more than one of the prerequisite courses.

Additionally, this course will be aimed at improving students' attitudes towards mathematical problem solving by building each individual's proficiency and confidence in approaching and possibly solving problems for which no solution is immediately known. The course will also aim to develop student's abilities to deal with mathematical concepts not previously seen.

Course Format. Classes will consist of a mixture of

- lectures by the instructor,
- interactive (group) problem-solving by the students (ALC Critical Thinking and Communicationcomponents),
- individual problem-solving by the students (ALC Critical Thinkingcomponent),
- student presentations of solutions (ALC Written and Oral Communicationcomponents),
- constructive student critiques of the work of other students (ALC Written and Oral Communicationcomponents).

Problems will be drawn from a variety of sources including, but not limited to, national and international high school competitions, problems sections of journals, and texts. All students are expected to keep a journal of solutions. That is, you will take down the solutions presented by the instructor or by the other students, and rewrite these solutions to make them presentable. It is stongly suggested that you start each solution on a new page, and that you copy the question onto the beginning of the page. The instructor may call for these journals at any time during the semester, to make sure that students are keeping up with the work and to assess the ALC Written Communicationcomponent.

Tests will generally be closed-book, but much of the content will be drawn from what was done in class. Some tests may consist entirely of previous-worked questions. Some tests may expect you to apply what you've learned (ALC Critcial Thinkingcomponent).

Students will also be expected to communicate their ideas, both written and orally. Finding a solution to a problem is an important step. Convincing others that the solution is valid is another. Elegance of the solution is also desirable, and we will frequently look for a better solution.

Overview. There are several steps that one should use to solve any mathematical problem. Many of these ideas apply to problems in other sciences. The following list is based on one in Larson's book, **Problem Solving through Problems**:

- Read definitions of any unknown terms.
- Look at known results regarding these terms.

- Do some examples. (Sometimes using a computer.)
- Draw pictures, if applicable.
- Try to find an equivalent problem.
- Modify the problem. See what happens if you leave out a hypothesis, or add a new one.
- Choose effective notation.
- Look for symmetry or parity.
- Divide into cases.
- Try working from conclusion to hypotheses.
- Try contradiction.
- Consider extreme cases.
- Simplify. Generalize. (Sometimes happens accidentally by misreading problem.)

Grading and Scoring Rubrics

Scoring rubrics are for assessing the class as a whole, and not for assessing individual individual students. These rubricswould not appear on the syllabus giving to the students.

Course Activity	Percentage	ALC Written Communication	ALC Oral Communication	ALC Critical Thinking
Solution Attempts In-class and homework	20			Poor →1 reasonable, but unsuccessful, →3 successful →4 elegant →5
Contribution to Group Solutions	15		Poor →1 Understandable and Colleagial →3 Clear and Collegial →4 Clear, Informative, Collegial →5	Poor →1 some reasonable ideas →3 some useful ideas →4 key ideas →5

Presentation of Solutions (Blackboard, Overheads, Powerpoint, etc.)	15		Poor →1 Somewhat Understandable →2 Accurately presented, readable →3 Clear and Accurate, Good use of equipment →4 Clear, Accurate, *Good Illustrations, Good Motivation of Solution →5
Written Solutions or Journal	15	Poor →1 Somewhat Understandable →2 Accurately presented, readable →3 Clear and Accurate →4 Clear, Accurate, *Good Illustrations, Good Motivation of Steps in Solution →5	
ConstructiveCritiques of student presentations	10	Poor →1 Somewhat Understandable →2 Informative and constructive →3 Informative, Constructive, Clear, Grammatical →4 Informative, Constructive, Clear, Grammatical, Concise, and Genreally well-written →5	Poor →1 Somewhat Understandable →2 Informative and constructive →3 Informative, Constructive, Clear, Grammatical →4 Informative, Constructive, Clear, Grammatical, Concise, and Generally well-presented →5

Written Examinations	25	Poor →1 Somewhat Understandable →2 Informative and constructive →3 Informative, Constructive, Clear, Grammatical →4 Informative, Constructive, Clear, Grammatical, Concise, and Generally well-written →5		Poor →1 reasonable, but unsuccessful, →3 successful →4 elegant →5

*If Appropriate to Presentation.

Suggested Bibliography

Books focusing on Problem Solving Techniques:

1. G. Polya. **How to Solve It**. Princeton Science Libarary, Princeton University Press 1988. *(This classic book is written as if the reader is a teacher, but deserves to be read by every high school student.)*
2. L.C. Larson. **Problem Solving through Problems**. Problem Books in Mathematics, Springer-Verlag, 1983.

A Sampling of Source Books for Problems:

1. G.L. Alexanderson, L.F. Klosinski, L.C. Larson. **The William Lowell Putnam Mathematical Competition, Problems and Solutions: 1965-1984**. Mathematical Association of America 1985.
2. T. Andreescu, Z, Feng, G. Lee. **Mathematical Olympiads, Problems and Solutions From Around the World 1998-1999**. Mathematical Association of America Problem Books 2003.
3. T. Andreescu, Z, Feng, G. Lee. **Mathematical Olympiads, Problems and Solutions From Around the World 1999-2000**. Mathematical Association of America Problem Books 2003.
4. T. Andreescu, Z, Feng, G. Lee. **Mathematical Olympiads, Problems and Solutions From Around the World 2000-2001**. Mathematical Association of America Problem Books 2003.
5. N.N. Chentzov, D.O. Shklarsky, I.M. Yaglom. **The USSR Olympiad Problem Book: Selected Problems and Theorems of Elementary Mathematics**. Dover 1993.
6. H. Dörrie. **100 Great Problems of Elementary Mathematics**. Dover 1965.
7. H. Fukagawa, J.F. Rigby. **Traditional Japanese Mathematics Problems of the 18th and 19th Centuries**. SCT Publishing 2002.
8. A.M. Gleason, R.E. Greenwood, L.M. Kelly. **The William Lowell Putnam Mathematical**

Competition, Problems and Solutions: 1938-1964. Mathematical Association of America 1980.
9. S.L. Greitzer. **International Mathematical Olympiads 1959-1977**. New Mathematical Library 27, Mathematical Association of America, 1977.
10. K. Hardy and K. Williams. **The Green Book: 100 Practice Problems for Undergraduate Mathematics Competitions**. Integer Press 1985.
11. K. Hardy and K. Williams. **The Red Book: 100 Practice Problems for Undergraduate Mathematics Competitions**. Integer Press 1988.
12. K.S. Kedlaya, B. Poonen, R. Vakil. **The William Lowell Putnam Mathematical Competition 1985-2000: Problems, Solutions and Commentary**. Mathematical Association of America Problem Books 2002.
13. M.S. Klamkin. **International Mathematical Olympiads 1979-1985**. New Mathematical Library 31, Mathematical Association of America, 1986.
14. M.E. Kuczma. **International Mathematical Olympiads 1086-1999**. Mathematical Association of America Problem Books 2003.
15. E. Rapaport. **Hungarian Problem Book I: based on the Eötvös Competitions 1894-1904**. New Mathematical Library 11, L.W. Singer Company, 1963.
16. E. Rapaport. **Hungarian Problem Book II: based on the Eötvös Competitions 1906-1928**. New Mathematical Library 12, Mathematical Association of America, 1963.
17. H. Steinhaus. **One Hundred Problems in Elementary Mathematics**. Dover 1964.
18. A.M. Yaglom and I.M. Yaglom. **Challenging Mathematical Problems with Elementary Solutions. Volume I: Combinatorial Analysis and Probability Theory**. Dover 1964.
19. A.M. Yaglom and I.M. Yaglom. **Challenging Mathematical Problems with Elementary Solutions. Volume II: Problems from Various Branches of Mathematics**. Dover 1967.

Books for more recreational reading:

1. W. W. Rouse Ball and H. M. S. Coxeter, **Mathematical Recreations and Essays**, University of Toronto Press 1974.
2. Angela Dunn, **Mathematical Bafflers**, McGraw-Hill 1964.
3. Clifton Fadiman, **Fantasia Mathematica**, Simon and Schuster 1958.
4. Clifton Fadiman, **The Mathematical Magπ**, Simon and Schuster 1962.
5. George Gamow, **One Two Three ... Infinity**, Bantam 1971
6. Martin Gardner, **Further Mathematical Diversions**, George Allen and Unwin 1970.
7. Martin Gardner, **Knotted Doughnuts and Other Mathematical Entertainments**, Freeman 1986.
8. Martin Gardner, **Mathematical Carnival**, Mathematical Association of America 1989.
9. Martin Gardner, **Mathematical Circus**, Knopf 1979.
10. Martin Gardner, **Mathematical Magic Show**, Mathematical Association of America 1990.
11. Martin Gardner, **Mathematical Puzzles**, Crowell 1961.
12. Martin Gardner, **Mathematical Puzzle Tales**, MAA 2000. (Originally *Science Fiction Puzzle Tales*, 1981.)
13. Martin Gardner, **New Mathematical Diversions**, Mathematical Association of America 1995.

14. Martin Gardner, **Penrose Tiles to Trapdoor Ciphers ... and the Return of Dr. Matrix**, Freeman 1989.
15. Martin Gardner, **Riddles of the Sphinx**, New Mathematical Library 32, Mathematical Association of America 1987.
16. Martin Gardner, **Time Travel and Other Mathematical Bewilderments**, Freeman 1988.
17. Martin Gardner, **Wheels, Life, and Other Mathematical Amusements**, Freeman 1983.
18. Ross Honsberger, **Ingenuity in Mathematics**, New Mathematical Library 23, Random House/ Singer 1970.
19. Ross Honsberger, **In Pólya's Footsteps**, Dolciani Mathematical Expositions 19, Mathematical Association of America 1997.
20. Ross Honsberger, **Mathematical Chestnuts from Around the World**, Dolciani Mathematical Expositions 24, Mathematical Association of America 2001.
21. Ross Honsberger, **Mathematical Gems**, Dolciani Mathematical Expositions 1, Mathematical Association of America 1973.
22. Ross Honsberger, **Mathematical Gems II**, Dolciani Mathematical Expositions 2, Mathematical Association of America 1976.
23. Ross Honsberger, **Mathematical Gems III**, Dolciani Mathematical Expositions 9, Mathematical Association of America 1985.
24. Ross Honsberger, **Mathematical Plums**, Dolciani Mathematical Expositions 4, Mathematical Association of America 1979.
25. Ross Honsberger, **Mathematical Morsels**, Dolciani Mathematical Expositions 3, Mathematical Association of America 1978.
26. Ross Honsberger, **More Mathematical Morsels**, Dolciani Mathematical Expositions 10, Mathematical Association of America 1991.
27. T. H. O'Beirne, **Puzzles and Paradoxes: Fascinating Excursions in Recreational Mathematics**, Dover 1984.
28. Theoni Pappus, **The Joy of Mathematics**, Wide World Publishing/Tetra 1989.
29. Theoni Pappus, **More Joy of Mathematics**, Wide World Publishing/Tetra 1991.

Journals

1. **Fibonacci Quarterly**
2. MAA **American Mathematical Monthly**
3. MAA **Mathematics Magazine**
4. **Scientific American**(Mathematical Recreations column in older issues).
5. **Journal of Recreational Mathematics**.
6. **The IIMEJournal**.

URL: http://math.fau.edu/Locke/courses/ProblemSolving/MasterSyllabus.htm