Declarative Knowledge/Technical Skills

Outcomes and Analysis

1 Modern Algebra

1.1 Fall 2009

The following two questions were embedded into the final exam:

1. M is the matrix $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, and $H = \langle M \rangle$ is the cyclic group it generates. Find the elements of the cosets xH and Hx, where $x = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.
2. Find an isomorphism between the multiplicative group of complex numbers and the set of all matrices of the form $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$, excluding the all zero matrix, with matrix multiplication.

The total number of BA/BS students taking the final exam was 10 of which 6 were BA students, 4 were BS.

- Question 1 was answered correctly by 4 of the 6 BA students and by 3 of the 4 BS students.
- Question 2 was answered correctly by 1 of the 6 BA students and by 2 of the 4 BS students.

1.2 Spring 2010

The following two questions were embedded into the final exam.

1. Given the multiplicative group G consisting of 2×2 matrices with entries from $\mathbf{Z}$ and with determinant ± 1, the subgroup H generated by the element $h = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, and the element $x = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$, display the elements of the subgroup H, and determine the two cosets Hx and xH (clearly labelling which is which).
2. Suppose that G is a finite group and that H is a subgroup of G such that $o(G) = 2o(H)$. Prove that $Hx = xH$ for every $x \in G$.

Eighteen students took the final exam. Out of a possible 10 points per problem, the outcomes were as follows. In the tables, the top row indicates a point value, the number below it in the second row is the number of students scoring that value.

Question 1.

Points	10	9	8	7	6	5	4	3	2	1	0
Nr. Stds.	7	1	1	2	0	1	0	0	0	1	6

The mean grade is 5.89, the median 7.5.

Question 2.

Points	10	9	8	7	6	5	4	3	2	1	0
Nr. Stds.	0	0	0	0	1	0	2	0	2	0	13

The mean grade is 1, the median 0.

2 Modern Analysis

2.1 Fall 2009

The following three questions were embedded into the final exam.

1. Let A and B be bounded sets of real numbers. Assume $A \subset B$. Prove $\sup A \leq \sup B$.
2. Show that $x \mapsto \dfrac{1}{x} : [1, \infty) \to \mathbb{R}$ is uniformly continuous.
3. Show that the equation $xe^x = 1$ admits only one solution.

Seven students took the final exam. Out of a possible 10 points per problem, the outcomes were as follows. In the tables, the top row indicates a point value, the number below it in the second row is the number of students scoring that value.

Question 1.

Points	10	9	8	7	6	5	4	3	2	1	0
Nr. Stds.	2	0	0	0	0	0	2	0	0	0	3

The mean grade is 4.00, the median 4.00.

Question 2.

Points	10	9	8	7	6	5	4	3	2	1	0
Nr. Stds.	2	0	0	0	0	0	0	0	0	0	5

The mean grade is 2.86, the median 0.

Question 3.

Points	10	9	8	7	6	5	4	3	2	1	0
Nr. Stds.	2	0	2	0	0	0	2	0	0	0	3

The mean grade is 5.14, the median 8

2.2 Spring 2010

The following three questions, in multiple choice format, were embedded into the final exam.

1. If A is a nonempty set of real numbers that is bounded above, and $s = \sup A$, then
   - ◯ There exists $x \in A$ such that $s - 1 < x \leq s$.
   - ◯ There exists $x \in A$ such that $a \leq x$ for all $a \in A$.
   - ◯ There exists $x \in A$ such that $x > s$.
   - ◯ There exists $x \in A$ such that $s \leq x < s + 1$.
   - ◯ None of these.
2. If we know that $\lim_n a_n = 3$, then which of the following statements must be true:
   - ◯ There exists $N \in \mathbb{N}$ such that $a_n < 4$ for all $n \leq N$.
   - ◯ There exists $N \in \mathbb{N}$ such that $a_n \leq a_{n+1}$ for all $n \geq N$.
   - ◯ There exists $N \in \mathbb{N}$ such that $a_n > 1$ for all $n \geq N$.
   - ◯ There exists $N \in \mathbb{N}$ such that $a_n < 2$ for all $n \geq N$.
   - ◯ None of these.

3. Suppose $f(x) = \begin{cases} x^2 & \text{if } x \in \mathbb{Q} \\ x & \text{if } x \notin \mathbb{Q} \end{cases}$. Then

- ○ $\lim_{x \to a} f(x)$ exists for exactly one $a \in \mathbb{R}$.
- ○ $\lim_{x \to a} f(x)$ exists for no $a \in \mathbb{R}$.
- ○ $\lim_{x \to a} f(x)$ exists for all $a \in \mathbb{R}$.
- ○ $\lim_{x \to a} f(x)$ exists for exactly two elements $a \in \mathbb{R}$.
- ○ None of these.

Eleven students took the final exam; 7 BA students and 4 BS students. The outcome is summarized in the following table. The BA students are identified as BA1 to BA7, the BS students as BS1 to BS4. A plus symbol (+) indicates a correct answer. A blank entry indicates that the student did not answer the question correctly.

	BA1	BA2	BA3	BA4	BA5	BA6	BA7	BS1	BS2	BS3	BS4
#1		+				+	+	+	+	+	+
#2			+						+	+	+
#3				+						+	
	0	1	1	1	0	1	1	1	2	3	2

Percentage of correct answers for the BA students: 24%.

Percentage of correct answers for the BS students: 67%.

3 Probability and Statistics

3.1 Fall 2009

Because the instructor of the course, Professor Niederhausen, made such a careful analysis of the performance of the students in the course, the assessment data appears as a separate attachment.

3.2 Spring 2010

Following the lead of what was done in the previous semester, the embedded questions came from sample tests of the Actuary Society. The questions were:

1. You are given $P(A \cup B) = 0.7$ and $P(A \cup B') = 0.9$. Determine $P(A)$.
2. An actuary has discovered that policyholders are three times as likely to file two claims as to file four claims. If the number of claims filed has a Poisson distribution, what is the variance of the number of claims filed?
3. The lifetime of a printer costing \$200 is exponentially distributed with mean 2 years. The manufacturer agrees to pay a full refund to a buyer if the printer fails during the first year following its purchase, and a one-half refund to a buyer if it fails during the second year. If the manufacturer sells 100 printers, how much should it expect to pay in refunds?
4. A device runs until either of two components fails, at which point the device stops running. The joint density function of the lifetime of the two components, both measured in hours, is

$$f(x, y) = \frac{x+y}{8} \quad \text{for} \quad 0 < x < 2 \quad \text{and} \quad 0 < y < 2.$$

What is the probability that the device fails during the first hour of operation?

5. Claims filed under auto insurance policies follow a normal distribution with mean 19,400 and standard deviation 5,000. What is the probability that the average of 25 randomly selected claims exceeds 20,000? (For this problem, a table was attached to the end of the exam.)

The table below specifies the outcome. Twenty three students took the final exam.

Question	Average grade (out of 100)	# of students getting 100
1	60.10	11
2	34.80	2
3	16.8	0
4	9.80	0
5	43.80	6

4 Analysis

The samples consist of a very small number of students. It is difficult, probably impossible, to reach definite conclusions from these samples, except to say that the results for 2009-2010 were not as satisfactory as one might desire. One of the problems our average student encounters is that her or his basic lower division mathematics courses do not place enough emphasis on concepts, and emphasize functionality over understanding. For example, in calculus a lot of time is spent teaching students how to compute integrals, not enough in teaching what an integral really is. Improving the outcomes in the upper division mathematics courses may require a revision of the lower division courses taken by mathematics majors. Such revisions are being discussed in our department. How a student does in the three courses used for assessment purposes depends in very large part on the nature of the prerequisite courses that he or she took.