Levels	ProblemSolving	Reasoning and Proof	Communication	Connections	Representation
Novice	No strategy is chosen, or a strategy is chosen that will not lead to a solution. Little or no evidence of engagement in the task is present.	Arguments are made with no mathematical basis. No correct reasoning nor justification for reasoning is present.	No awareness of audience or purpose is communicated. Little or no communication of an approach is evident. Everyday, familiar language is used to communicate ideas.	No connections are made.	No attempt is made to construct mathematical representations.
Apprentice	A partially correct strategy is chosen, or a correct strategy for only solving part of the task is chosen. Evidence of drawing on some relevant previous knowledge is present, showing some relevant engagement in the task.	Arguments are made with some mathematical basis. Some correct reasoning or justification for reasoning is present with trial and error, or unsystematic trying of several cases.	Some awareness of audience or purpose is communicated, and may take place in the form of paraphrasing of the task. Some communication of an approach is evident through verbal/written accounts and explanations, use of diagrams or objects, writing, and using mathematical symbols. Some formal math language is used, and examples are provided to communicate ideas.	Some attempt to relate the task to other subjects or to own interests and experiences is made.	An attempt is made to construct mathematical representations to record and communicate problem solving.

Levels	ProblemSolving	Reasoning and Proof	Communication	Connections	Representation
Practitioner Note: The practitioner must achieve a correct answer.	A correct strategy is chosen based on the mathematical situation in the task. Planning or monitoring of strategy is evident. Evidence of solidifying prior knowledge and applying it to the problem-solving situation is present.	Arguments are constructed with adequate mathematical basis. A systematic approach and/or justification of correct reasoning is present. This may lead to: 1. Clarification of the task. 2. Exploration of mathematical phenomenon. 3. Noting patterns, structures and regularities.	A sense of audience or purpose is communicated. Communication of an approach is evident through a methodical, organized, coherent, sequenced, and labeled response. Formal math language is used throughout the solution to share and clarify ideas.	Mathematical connections or observations are recognized.	Appropriate and accurate mathematical representations are constructed and refined to solve problems or portray solutions.
Expert Note: The practitioner must achieve a correct answer.	An efficient strategy is chosen and progress toward a solution is evaluated. Adjustments in strategy, if necessary, are made along the way, and/or alternative strategies are considered. Evidence of analyzing the situation in mathematical terms, and extending prior knowledge is present.	Deductive arguments are used to justify decisions and may result in more formal proofs. Evidence is used to justify and support decisions made and conclusions reached. This may lead to: 1. Testing and accepting or rejecting of a hypothesis or conjecture. 2. Explanation of phenomenon. 3. Generalizing and extending the solution to other cases.	A sense of audience and purpose is communicated. Communication at the practitioner level is achieved, and communication of arguments is supported by mathematical properties used. Precise math language and symbolic notation are used to consolidate math thinking and to communicate ideas.	Mathematical connections or observations are used to extend the solution.	Abstract or symbolic mathematical representations are constructed to analyze relationships, extend thinking, and clarify or interpret phenomenon.