Declarative Knowledge/Technical Skills

Outcomes and Analysis

1 Modern Algebra

1.1 Fall 2010

The following five questions were embedded into the final exam:

1. Define: "cyclic subgroup."
2. Prove: If a divides bc and a and b are relatively prime, then a divides c. Use Bezout's Identity.
3. Give an example of a field with characteristic zero, or say that none exists.
4. Give an example of a field with nine elements, or say that none exists.
5. Prove: If F is a field and f a polynomial in $F[x]$, then f factors into a product of irreducibles. That is, for some $n \geq 1$, there are irreducible polynomials $p_1, \ldots, p_n$ in $F[x]$ such that $f = p_1 p_2 \cdots p_n$.

The total number of students taking the final exam was 6, out of which of which 3 were BS students, 1 was BA in Math Education, 1 was MST, and 1 was non-degree. The following table shows the percentage of correct responses for each question, and the percentage for the BS students, as prepared by the instructor of the course.

Item	% Correct	BS % Correct
1	23.3	33.3
2	48.3	53.3
3	66.7	66.7
4	16.7	33.3
5	0.0	0.0

1.2 Spring 2011

Professor Paul Yiu taught the course in the Spring semester. He is without doubt one of the best and most conscientious teachers in our department. He gave me a very comprehensive analysis of the outcome in his course and, rather than present here a butchered version, I add it in full.

MAS 4301 Modern Algebra (Yiu) Spring 2011
Report on Final Examination.

The comprehensive Final Exam consists of 9 problems. Each of problems 1-8 carries 10 points, and Problem 9 carries 5 points. Some of the problems are designed to test students' ability of writing proofs, deciding on validity of given statements with explanations, and familiarity with axioms of algebraic structures.

19 students took the Final Exam. Here are the results on problems of these types.

Problem 4 consists of two independent parts, each asking for a proof. The key to part (a) is proof by contradiction. 10 students gave perfect proof, 4 students did not get any point, and 5 students earned partial credits.

The key to Part (b) is the use of Bezout's identity. 17 students got this correctly, and 2 students earned partial credits.

Problem 5 consists of five independent statements. The students are asked to decide if each statement is true or false, and to give an explanation or a counterexample as appropriate. The first four statements are easy for most students. Only 6 students answered Part (e) correctly, but none could give a valid reason. (There is no field of 6 elements because the cardinality of a finite field must be a prime power). For the other four parts,
11 students answered 4 correctly with valid reasoning;
6 students answered 3 correctly with valid reasoning;
2 students answered 2 correctly with valid reasoning.

Each of **Problems 2 and 3** asks the students to decide if a given algebraic structure forms a group, and to verify the group axioms in each case.

	Problem 2	Problem 3
perfect answers	7	12
minor deductions	11	3
major deductions	1	4

In both cases, all students knew that the structure in question is a group, although some did not justify the closure property in Problem 3. A few calculated the inverse incorrectly. In Problem 2, a few students forgot to mention the associativity.

Problem 9 asks for the factorization of a polynomial over $\mathbb{F}_2$ of degree 6, given that it has some factor of multiplicity > 1. It aims at testing the students' reasoning ability to arrive at the factorization without extensive calculations, the key being the fact that $x^2 + x + 1$ is the only irreducible quadratic over $\mathbb{F}_2$. 10 students got this correctly.

Here are the subjects of the remaining 4 problems, and their results.

Problem	subject
1	order of an element and Chinese remainder theorem
6	factorization of polynomial over $\mathbb{F}_3$
7	order of an element in a finite field $\mathbb{F}_2[x]/(m(x))$
8	factorization of integer polynomial; Gauss' lemma

Problem	full marks	partial credits	zero mark
1	15	4	0
6	9	9	0
7	9	9	0
8	6	11	2

Distribution of final scores

Percentage	below 60	61-70	71-80	81-90	91-100
Students	0	3	6	7	3

mean = 80; median = 80; highest = 99, lowest = 61.

MAS 4301 Modern Algebra (Yiu) Spring 2011
Final Examination (2 hours). April 29.

1. For a positive integer m, let $\mathbb{Z}_m^\bullet$ denote the group of units of the ring $\mathbb{Z}_m$.

   (a) What is the order of the element $[2]_7 \in \mathbb{Z}_7^\bullet$?

   (b) What is the order of the element $[5]_8 \in \mathbb{Z}_8^\bullet$?

   (c) What is the element $[c]_{56}$ which corresponds to $([2]_7, [5]_8)$ under the natural isomorphism $\mathbb{Z}_{56} \to \mathbb{Z}_7 \times \mathbb{Z}_8$?

   (d) Find the order of the element $[c]_{56}$ in (c).

2. Consider the subset $A := \{5,\ 15,\ 25,\ 35\}$ of $\mathbb{Z}_{40}$.

   (a) Complete the multiplication table of the elements in A, giving each product as an element of $\mathbb{Z}_{40}$.

	5	15	25	35
5				
15				
25				
35				

   (b) Does A form a group under multiplication?
   Show clearly how each of the group axioms is satisfied or violated.

3. Consider the set $G = \mathbb{R} \setminus \{-1\}$ with an operation $\star$ given by

$$a \star b = a + b + ab.$$

   (a) Show that $\star$ is a binary operation, *i.e.* $a \star b \in G$ if $a,\ b \in G$.

   (b) Show that $\star$ is associative.

   (c) Does there exist an identity element $e \in G$ for which $e \star a = a$ for every $a \in G$? If so, name the element and verify that it is an identity. If not, give a reason.

   (d) Does G form a group with binary operation $\star$? Justify your answer.

4. (a) Prove that if n is not a prime number, it must have a prime divisor $\leq \sqrt{n}$.
   (b) Let $a, b \in \mathbb{Z}$ with $\gcd(a, b) = d$. If $a = dx$ and $b = dy$, show that $\gcd(x, y) = 1$.

5. True or false? Give a simple proof or a counterexample as appropriate.

   (a) Suppose a is an odd integer and $ab \equiv ac \pmod{8}$.
   Then $b \equiv c \pmod{8}$.
   (b) Suppose a is an even integer and $ab \equiv ac \pmod{9}$.
   Then $b \equiv c \pmod{9}$.
   (c) If a cubic polynomial $f(x) \in F[x]$ satisfies $f(a) \neq 0$ for every $a \in F$, then $f(x)$ is irreducible.
   (d) If G is a finite abelian group with 3 elements of order 2, then it cannot be a cyclic group.
   (e) There is a field with exactly 6 elements.

6. A monic polynomial is one whose leading coefficient (of highest degree term) is 1.

   (a) How many monic polynomials of degree 2 are there in $\mathbb{F}_3[x]$?
   (b) Find all irreducible monic polynomials of degree 2 in $\mathbb{F}_3[x]$.
   Hint: There are 3 of them.
   (c) It is known that the polynomial $x^4 - x^3 - x - 1 \in \mathbb{F}_3[x]$ is reducible. Factorize the polynomial.

7. Let $K = \mathbb{F}_3[x]/(x^2 + x + 2)$.

   (a) Why is K a field?
   (b) How many elements does it have?
   (c) Show that $[1 + x]$ is an element of order 8 in K.

8. It is known that $x^4 - 3x^3 + 3x + 1 \in \mathbb{Z}[x]$ is reducible. Factorize it.

9. (Bonus problem) Factorize $f(x) = x^6 + x^5 + x^3 + x + 1$ over $\mathbb{F}_2$ given that it has a multiple factor.

1 Modern Analysis

1.1 Fall 2010

The following four questions were embedded into the final exam.

1. Let A be a non-empty set of real numbers that is bounded above. Let $\alpha = \sup A$ and let $b < \alpha$. Prove there exists $x \in A$ such that $b < x \le \alpha$.
2. Let $\{a_n\}$ be a sequence of real numbers, assume that $\lim_{n\to\infty} a_n = a > 0$. Prove there exists $N \in \mathbb{N}$ such that $a_n > a/2$ if $n \ge N$.
3. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2$. Prove using only the definition that $\lim_{x\to 2} f(x) = 4$.
4. Let
$$f(x) = \begin{cases} x^2, & x \in \mathbb{Q} \\ x, & x \in \mathbb{R}\backslash\mathbb{Q}. \end{cases}$$
Prove: f is continuous at 0, 1; discontinuous everywhere else.

Nineteen students took the final exam; of these five were BA students, the remaining fourteen BS students. The following table, where the BA students are listed as BA1, BA2, ..., BA5; the BS students as BS1, BS2, ..., BS 9, summarizes the results. The last column shows how each student did, on a percentage scale.

Problem	1	2	3	4	% right
Point Value	15	15	10	20	100%
BA1 scores	0	0	0	5	8.33%
BA2 scores	0	0	0	10	16.67%
BA3 scores	0	0	0	10	16.67%
BA4 scores	0	5	0	15	33.33%
BA5 scores	5	15	7	20	78.33%
BS1 scores	0	0	0	0	0%
BS2 scores	0	0	0	5	8.33%
BS3 scores	0	5	0	5	16.67%
BS4 scores	0	5	0	10	25%
BS5 scores	0	9	3	13	41.67%
BS6 scores	5	9	3	13	50%
BS7 scores	5	13	5	13	60%
BS8 scores	5	15	5	15	66.67%
BS9 scores	10	15	5	15	75%
BS10 scores	10	15	5	20	83.33%
BS11 scores	10	15	7	20	86.67%
BS12 scores	12	15	7	20	90%
BS13 scores	15	15	8	20	96.67%
BS14 scores	15	15	10	20	100%

1.2 Spring 2011

The data is for the math majors, does not include math education majors. There were eleven math majors in the course; of these, two were in the BA program, nine in the BS program. The instructor of the course used the same four embedded questions as in the Fall 2010 semester. The results are summarized in the following table. This time all problems was valued at a maximum of 5 points. I seem to have only been given data for eight of the math BS majors.

Problem	1	2	3	4	% right
Point Value	5	5	5	5	100%
BA1 scores	3	5	3	3	70%
BA2 scores	3	3	4	4	70%
BS1 scores	2	5	2	2	55%
BS1 scores	3	4	3	3	65%
BS3 scores	4	5	2	2	65%
BS4 scores	3	5	4	3	75%
BS5 scores	3	5	5	4	85%
BS6 scores	4	5	5	5	95%
BS7 scores	4	5	5	5	95%
BS8 scores	5	5	5	5	100%

Probability and Statistics (STA 4442)

Fall 2010

Three questions from the sample tests of the Actuary Society were embedded in the Final Exam of STA 4442 Probability and Statistics 1. Out of eighteen students enrolled to the course, sixteen took the final exam.

<u>The topics covered for the embedded questions</u>

1. The first question covered probability and conditional probability of sets.
2. The second question covered the sampling distribution of the sample mean.
3. The third question covered the function of a random variable and its summary statistics.

<u>Outcome</u>

The outcome is classified into four categories: Completely correct, Minor error, Major error and Complete failure. The following table shows the outcome:

Question	1	2	3
Completely correct	50%	56%	0%
Minor error	31%	6%	63%
Major error	19%	0%	13%
Complete failure	0%	38%	25%
Total	100%	100%	100%

From the table above, one observes that the majority of the students did well on the probability and conditional probability question, while only 62% of the students grasped the concept of the sampling distribution. The worst performance is indicated in the area of transformation of random variable and its summary statistics. For the last problem, the most of the mistakes were due to either failure to set up the question correctly or failure to do the integration by parts correctly.

<u>The three embedded questions</u>

1. In a sample space, events A and B have probabilities $P(A)=P(B)=1/2$, and $P(A\cup B)=2/3$. Calculate $P(A'\cap B)$ and $P(A'|B')$.

2. Claims filed under auto insurance policies follow a normal distribution with mean 19,500 and standard deviation 5,000. What is the probability that the average of 25 randomly selected claims exceeds 20,000?

3. An insurance policy reimburses a loss in excess of a 250 deductible. The policyholder's loss, X, follows an exponential distribution with mean 1,000. What is the expected value of the benefit paid under this insurance policy?

1 Spring 2011 (Probability and Statistics)

The following three questions were embedded in the final exam.

1. A group insurance policy covers the medical claims of the employees of a small company. The value, Y , of the claims made in one year is described by $Y = 100,000X$, where X is a random variable with probability density function

$$f(x) = \begin{cases} c(1-x)^4, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

   where c is a constant. Find $P(Y > 40,000|Y > 10,000)$.

2. The time, X, that a manufacturing system is out of operation has a probability density function

$$f(x) = \begin{cases} \frac{8}{x^3}, & x > 2 \\ 0, & \text{otherwise} \end{cases}$$

   The resulting cost to the company is $Y = X^2$. Find the probability density function of Y.

3. Let X represent the age of an insured automobile involved in an accident. Let Y represent the length of time the owner has insured the automobile at the time of the accident. X and Y have joint probability density function

$$f_{XY}(x, y) = \begin{cases} \frac{1}{64}(10 - xy^2), & 2 \le x \le 10, 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

   Find $E(X)$.

Twenty two students took the final exam. The following table summarizes the results, showing how many students gave correct answers.

Question	1	2	3
Correct Answers	6	14	11
Percentage	27%	63%	50%

2 Analysis

We may be getting to the point where we can look at the data over several semesters and reach some conclusions. However we should always keep in mind that the number of our majors is still very small and it is difficult to base statistical conclusions on such small samples. The results can vary dramatically from semester to semester due to the presence or absence of a few good students. There is no question that we can provide, and do provide, a first class education to qualified students, but a lot of our students come to us with serious deficiencies in their backgrounds and we must do more to help those students. In other words, while we can feel confident in the quality of our program, and are able to point out many successes, we still cannot be totally happy about the outcome. There is room for improvement; there always is room for improvement and this year our department will be reviewing very carefully our undergraduate program. This will involve (among other things) studying the latest recommendations of the Committee on the Undergraduate Program in Mathematics (CUPM), comparing our program to that of other institutions in Florida and in the nation, adding content to some of the lower division courses, so as to prepare students better for the more advanced mathematics courses.