EngSci Year 2 Winter 2022 Notes

for class content after reading week

Brian Chen

Division of Engineering Science, University of Toronto
Website

brianchen.chen@mail.utoronto.ca

Contents

PART

I

PHY294
Quantum & Thermal Physics

SECTION 1

Statistical Mechanics

SUBSECTION 1.1

Lecture 1: Ideal Gas Law

We begin our discussion on statistical mechanics with the guiding question, "What is temperature". We will first derive the ideal gas law $PV = nRT$, which most should be familiar with.

Imagining a single particle bouncing around elastically in 2D box with speed v_x. If we were to look at the sides of the box we will see intermittent spikes of force experienced [by the box walls],

$$F_x = m\frac{2v_x}{\Delta t_c} \tag{1.1}$$

Over time can get the average force

$$F_x = m\frac{2_x}{\Delta t} \tag{1.2}$$

which allows us to derive an expression for pressure, which is just force over area

$$P = \frac{F_x}{A} = \frac{mv_x^2}{V} \tag{1.3}$$

For n particles it then becomes

$$P = \sum_{i=1}^{N} \frac{mv_{x,i}^2}{V} = \frac{m}{V}\sum_{i=1}^{N} v_{x,i}^2 \tag{1.4}$$

Assuming that that $v_x = v_y = v_z$ [1], we may extend this to 3 dimensions

[1] *This is reasonable, is it not?*

$$P = \frac{mv^2}{3}\frac{N}{V} \tag{1.5}$$

Note that $\frac{mv^2}{3} = kT \rightarrow \frac{mv^2}{3k} = 2\frac{\overline{E}}{3k}$

Now, compare this expression and that of the ideal gas law to find that temperature is a measure of the average kinetic energy per particle

$$T = \frac{mv^2}{3k} = \frac{2\overline{E}}{3k} \tag{1.6}$$

PART

II

ECE259 Electromagnetism

SECTION 2

Dielectrics

SUBSECTION 2.1

Lecture 15: Boundary Conditions for Dielectrics

Remark Recall, at a conductor/free-space interface, $E_+ = 0$, $E_n = \frac{\rho_s}{\varepsilon_o}$

Consider an interface between two generic dielectrics (Fig. 1).

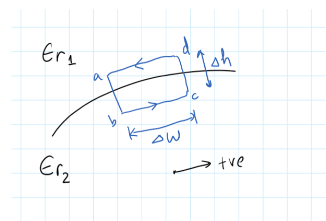

Figure 1. Dielectric interface

Theorem 1

$$\oint_c \vec{E} \cdot d\vec{l} = 0 \tag{2.1}$$

Intuition The two integrals parallel to the dielectrics will cancel out, and so will those perpendicular.

This implies that the tangential component of the $\vec{E}$ field is continuous across the boundary,

$$E_{1t} = E_{2t} \tag{2.2}$$

However we get a bit of a different result when working in 3 dimensions where the interface is a surface instead of a line.

Theorem 2

$$\oint_S \vec{D} \cdot d\vec{s} = \rho_S \Delta \rightarrow (\vec{D_1} - \vec{D_2}) \cdot \vec{a_{n2}} = \rho_S \tag{2.3}$$

a_n denotes normal component

Note: $\vec{D} = \varepsilon_o \vec{E} + \vec{p} = \ldots \varepsilon \vec{E}$

PROOF As $\Delta h \rightarrow 0$

$$\begin{aligned} \oint_S \vec{D} \cdot d\vec{s} &= \int_{top} \vec{D_1} \cdot d\vec{s} + \int_{bottom} + \vec{D_2} \cdot d\vec{s} \\ &= \vec{D_1} \cdot \vec{a_{n2}} \Delta S + \vec{D} \cdot \vec{a_{n1}} \Delta S \\ &= \vec{D_1} \cdot \vec{a_{n2}} \Delta S - \vec{D} \cdot \vec{a_{n2}} \Delta S \\ &= \rho_S \Delta S \end{aligned} \tag{2.4}$$

□

Solving these problems usually involves finding the tangential and normal components through Eq. 2.2 and 2.3 then applying Pythagoras.

SUBSECTION 2.2

Lecture 16: Capacitors

Definition 1 A capacitor is a device consisting of two isolated conductors for storing energy in the form of electrostatic potential energy.
The energy stored in a capacitor is equal to the energy it takes to charge a capacitor from a discharged state to a charged state.
A capacitor's *capacitance*[2] is defined as

$$C = \frac{Q}{V} \tag{2.5}$$

and has units $[C] = \frac{C}{V} F$(Farads)

An isolated conductor can also have "capacitance" if the other conductor is far away. The charge of a capacitance refers to the charge on one conductor.

[2] *Capacitance is actually independent of Q and V and is dependent only on the physical attributes of [the capacitor]*

Capacitance is calculated as follows

1. choose a coordinate system
2. Assume $+Q/-Q$ on the conductors
3. Find $\vec{E}$ from Q distribution
4. Find $V_{AB} = \int_A^B \vec{E} \cdot d\vec{l}$ where A carries the negative charge and B carries positive.
5. Apply $C = \frac{Q}{V}$

PART

III

ECE286
Probability and Statistics

SECTION 3

Probability Distributions

SUBSECTION 3.1

Lecture 14: functions of random variables

In the discrete case, given X with PMF $f(x)$, we can define an *invertible* function Y where $Y = u(X)$, therefore can write $X = u^{-1}(Y)$. If $g(y)$ is the distribution of Y

$$\begin{aligned} g(y) &= P(Y = y) \\ &= P(u^{-1}(Y) = u^{-1}(y)) \\ &= f(u^{-1}(y)) \end{aligned} \tag{3.1}$$

In the continuous case we may arrive at

$$g(y) = f(u^{-1}(y))|\frac{du^{-1}(y)}{dy}| \tag{3.2}$$

This result is derived through the Leibniz integral rule, $g(y) = \frac{d}{dy}\int_{-\infty}^{u^{-1}(y)} f(t)dt$

Definition 2 The r^{th} moment about the origin of the random variable X is

$$\mu\prime_r = E[X^r] = \begin{cases} \sum_x x^r f(x) & X\text{discrete} \\ \int_{-\infty}^{\infty} x^r f(x)dx & X\text{continuous} \end{cases} \tag{3.3}$$

- The mean is the first moment
- For variance, $\sigma^2 = E[X^2] - \mu^2 \to \sigma^2 = \mu\prime_2 - \mu^2$

Definition 3 The moment-generating function of X is defined as

$$\mu\prime_r = E[X^{tX}] = \begin{cases} \sum_x e^{tx} f(x) & X\text{discrete} \\ \int_{-\infty}^{\infty} e^{tx} f(x)dx & X\text{continuous} \end{cases} \tag{3.4}$$

In general

$$\mu\prime_r = \left.\frac{d^r M_X(t)}{dt^r}\right|_{t=0} \tag{3.5}$$

SUBSECTION 3.2

Lecture 15: More on moment generating functions

By definition and completing the square,

$$
\begin{aligned}
M_X(T) &= \int_{-\infty}^{\infty} e^{tx} \frac{1}{\sqrt{t2\pi\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx} \\
&= \int_{-\infty}^{\infty} \frac{1}{\sqrt{t2\pi\sigma} e^{-\frac{x^2-2(x-\mu)^2+\mu^2}{2\sigma^2}} dx} \\
&= e^{\frac{2\mu t+t^2\sigma^2}{2}} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{(x-(\mu+t\sigma^2))^2}{2\sigma^2}\right\} dx \\
&= e^{\frac{2\mu t+t^2\sigma^2}{2}} \qquad (3.6)
\end{aligned}
$$

The integrand is just a normal PDF and thus integrates to one

PART

IV

TEP327 Engineering and Law

Torts can be identified by

- Elements of negligence can be identified
- There exists a duty of care
- Was the standard of duty of care breached?
- Did that breach of standard cause damage to the plaintiff?

Comment Must be careful about the standard of care met or not met. E.x. in the in-class bike example one could argue that in designing the gear the engineer should expect that the gear could be commercialized and mass-produced later on for mass-market. On the other hand it is reasonable to argue that having to design the gear to be used by unskilled 16-year-olds is an unreasonable and outside the standard of care.

Trespass is strict liability. Ignorance of the law is no excuse.