Week # 7

Discrete Structure

by

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Applications of Venn diagram

Exercise:

A number of computer users are surveyed to find out if they have a printer, modem or scanner. Draw separate Venn diagrams and shade the areas, which represent the following configurations.

1. modem and printer but no scanner
2. scanner but no printer and no modem
3. scanner or printer but no modem.
4. no modem and no printer.

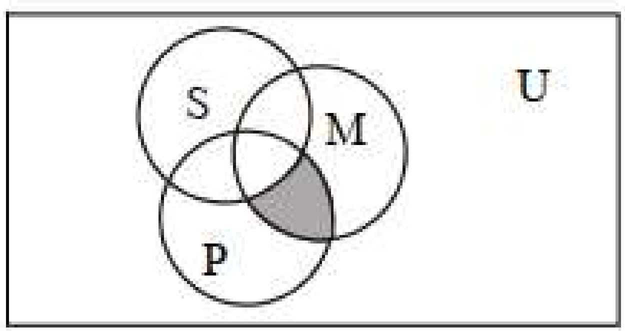
SOLUTION

Let

P represent the set of computer users having printer.
M represent the set of computer users having modem.
S represent the set of computer users having scanner.

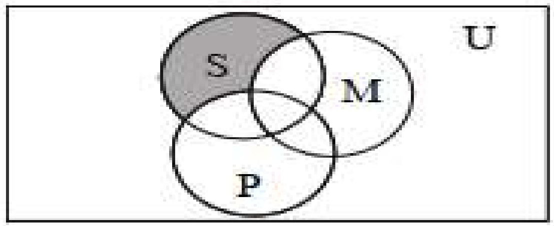
SOLUTION (i)

Modem and printer but no Scanner is shaded.



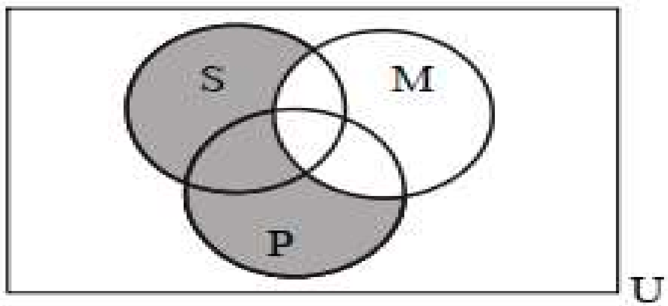
SOLUTION (ii)

Scanner but no printer and no modem is shaded.



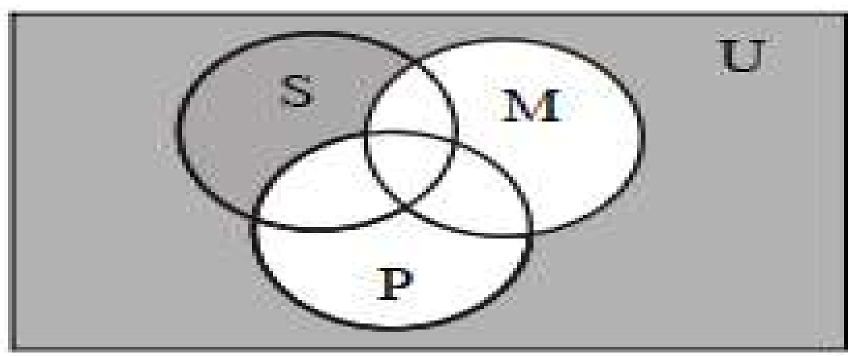
SOLUTION (iii)

Scanner or printer but no modem is shaded.



SOLUTION (iv)

No modem and no printer is shaded.



Relations

ORDERED PAIR:

An ordered pair (a, b) consists of two elements "a" and "b" in which "a" is the first element and "b" is the second element.

The ordered pairs (a, b) and (c, d) are equal if, and only if, a= c and b = d.

Note that (a, b) and (b, a) are not equal unless a = b.

EXERCISE:

Find x and y given $(2x, x + y) = (6, 2)$

SOLUTION:

Two ordered pairs are equal if and only if the corresponding components are equal. Hence, we obtain the equations:

$2x = 6$ ....(1)

and $x + y = 2$ ....(2)

Solving equation (1) we get $x = 3$ and when substituted in (2) we get $y = -1$.

ORDERED n-TUPLE:

The ordered n-tuple $(a_1, a_2, \ldots, a_n)$ consists of elements $a_1, a_2, \ldots, a_n$ together with the ordering: first a_1, second a_2, and so forth up to a_n. In particular, an ordered 2-tuple is called an ordered pair, and an ordered 3-tuple is called an ordered triple.

Two ordered n-tuples $(a_1, a_2, \ldots, a_n)$ and $(b_1, b_2, \ldots, b_n)$ are equal if and only if each corresponding pair of their elements is equal, i.e., $a_i = b_j$, for all $i, j = 1, 2, \ldots, n$.

CARTESIAN PRODUCT OF TWO SETS:

Let A and B be sets. The Cartesian product of A and B, denoted by $A \times B$ (read as "A cross B") is the set of all ordered pairs (a, b), where a is in A and b is in B.

Symbolically:

$$A \times B = \{(a, b) | a \in A \text{ and } b \in B\}$$

NOTE: If set A has *m* elements and set B has *n* elements then $A \times B$ has $m \times n$ elements.

EXAMPLE:

Let $A = \{1, 2\}$, $B = \{a, b, c\}$ then

$A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}$

$B \times A = \{(a, 1), (a, 2), (b, 1), (b, 2), (c, 1), (c, 2)\}$

$A \times A = \{(1, 1), (1,2), (2, 1), (2, 2)\}$

$B \times B = \{(a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b),(c, c)\}$

REMARK:

1. $A \times B \neq B \times A$ for non-empty and unequal sets A and B.
2. $A \times \phi = \phi \times A = \phi$

CARTESIAN PRODUCT OF MORE THAN TWO SETS:

The Cartesian product of sets $A_1, A_2, \ldots, A_n$, denoted $A_1 \times A_2 \times \ldots \times A_n$, is the set of all ordered n-tuples $(a_1, a_2, \ldots, a_n)$ where $a_1 \in A_1, a_2 \in A_2, \ldots, a_n \in A_n$.

Symbolically:

$$A_1 \times A_2 \times \ldots \times A_n = \{(a_1, a_2, \ldots, a_n) \mid a_i \in A_i, \text{ for } i = 1, 2, \ldots, n\}$$

BINARY RELATION:

Let A and B be sets. The binary relation R from A to B is a subset of $A \times B$.

When $(a, b) \in R$, we say 'a' is related to 'b' by R, written aRb.

Otherwise, if $(a, b) \notin R$, we write $a \not R\, b$.

EXAMPLE:

Let $A = \{1, 2\}$, $B = \{1, 2, 3\}$

Then $A \times B = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3)\}$

Let

$R_1 = \{(1,1), (1, 3), (2, 2)\}$

$R_2 = \{(1, 2), (2, 1), (2, 2), (2, 3)\}$

$R_3 = \{(1, 1)\}$

$R_4 = A \times B$

$R_5 = \varnothing$

All being subsets of $A \times B$ are relations from A to B.

DOMAIN OF A RELATION:

The domain of a relation R from A to B is the set of all first elements of the ordered pairs which belong to R denoted by Dom(R).

Symbolically,

$$\text{Dom}(R) = \{a \in A \mid (a, b) \in R\}$$

RANGE OF A RELATION:

The range of a relation R from A to B is the set of all second elements of the ordered pairs which belong to R denoted Ran(R).

Symbolically,

$$\text{Ran}(R) = \{b \in B \mid (a, b) \in R\}$$

EXERCISE:

Let $A = \{1, 2\}$, $B = \{1, 2, 3\}$,

Define a binary relation R from A to B as follows:

$$R = \{(a, b) \in A \times B \mid a < b\}$$

Then

a. Find the ordered pairs in R.
b. Find the Domain and Range of R.
c. Is 1R3, 2R2?

SOLUTION:

Given $A = \{1, 2\}$, $B = \{1, 2, 3\}$,

$A \times B = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3)\}$

a. $R = \{(a, b) \in A \times B \mid a < b\}$

$R = \{(1,2), (1,3), (2,3)\}$

b. $\text{Dom}(R) = \{1,2\}$ and $\text{Ran}(R) = \{2, 3\}$

c. Since $(1, 3) \in R$ so 1R3

But $(2, 2) \notin R$ so 2 is not related with3 or $2 \not{R} 2$

EXAMPLE:

Let A = {eggs, milk, corn} and B = {cows, goats, hens}

Define a relation R from A to B by (a, b) $\in$R *iff* a is produced by b.

Then R = {(eggs, hens), (milk, cows), (milk, goats)}

Thus, with respect to this relation eggs R hens , milk R cows, etc.

EXERCISE :

Find all binary relations from {0,1} to {1}

SOLUTION:

Let A = {0,1} & B = {1}

Then $A \times B = \{(0,1), (1,1)\}$

All binary relations from A to B are in fact all subsets of

A $\times$B, which are:

$R_1 = \varnothing$

$R_2 = \{(0,1)\}$

$R_3 = \{(1,1)\}$

$R_4 = \{(0,1), (1,1)\} = A \times B$

REMARK:

If $|A| = m$ and $|B| = n$

Then as we know that the number of elements in $A \times B$ are $m \times n$. Now as we know that the total number of and the total number of relations from A to B are $2^{m \times n}$.

RELATION ON A SET:

A relation on the set A is a relation from A to A.

In other words, a relation on a set A is a subset of $A \times A$.

EXAMPLE:

Let $A = \{1, 2, 3, 4\}$

Define a relation R on A as

$(a,b) \in R$ *iff* a divides b {symbolically written as $a \mid b$}

Then $R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)\}$

REMARK:

For any set A

1. $A \times A$ is known as the universal relation.
2. $\varnothing$ is known as the empty relation.

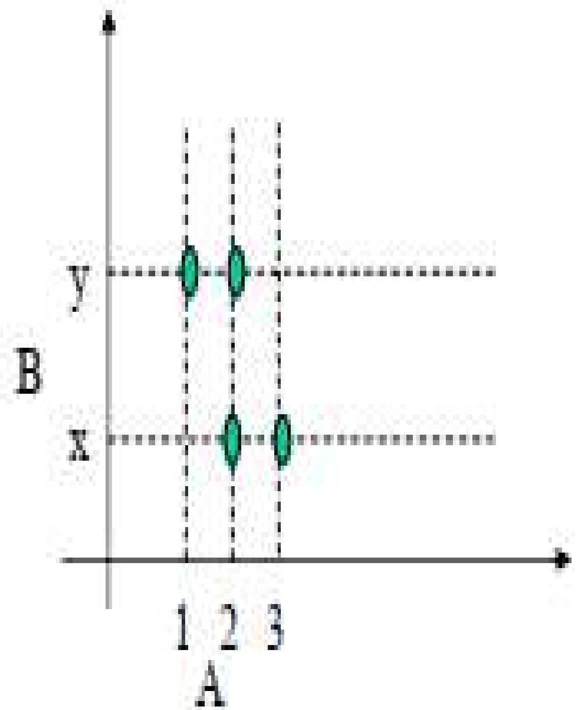
COORDINATE DIAGRAM (GRAPH) OF A RELATION:

Let $A = \{1, 2, 3\}$ and $B = \{x, y\}$

Let R be a relation from A to B defined as

$$R = \{(1, y), (2, x), (2, y), (3, x)\}$$

The relation may be represented in a coordinate diagram as follows:



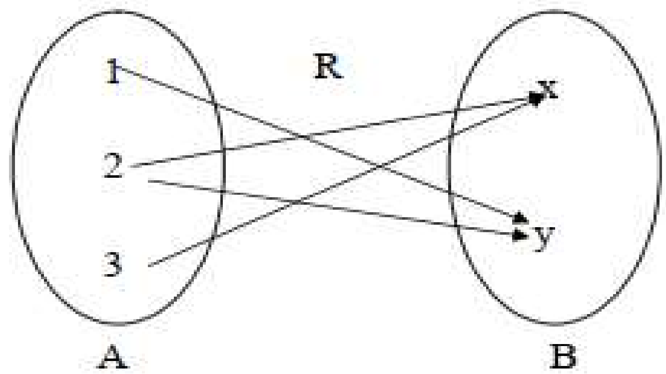
ARROW DIAGRAM OF A RELATION:

Let

$A = \{1, 2, 3\}$, $B = \{x, y\}$ and
$R = \{(1,y), (2,x), (2,y), (3,x)\}$

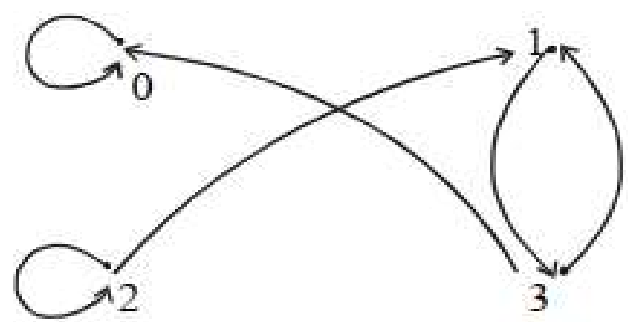
be a relation from A to B.

The arrow diagram of R is:



DIRECTED GRAPH OF A RELATION:

Let $A = \{0, 1, 2, 3\}$
and $R = \{(0,0), (1,3), (2,1), (2,2), (3,0), (3,1)\}$
be a binary relation on A.



DIRECTED GRAPH