# Week # 4

# Discrete Structure

## by

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# Argument

Before we discuss in detail about the argument, we first consider the following argument:

An interesting teacher keeps me awake. I stay awake in Discrete Mathematics class. Therefore, my Discrete Mathematics teacher is interesting.

Is the above argument valid?

## ARGUMENT:

An **argument** is a list of statements called **premises** (or **assumptions** or **hypotheses**) followed by a statement called the **conclusion.**

$P_1$    Premise

$P_2$    Premise

$P_3$    Premise

. . . . .. . . . .

$P_n$    Premise

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$\therefore C$    Conclusion

**NOTE:** The symbol $\therefore$ read "therefore" is normally placed just before the conclusion.

## VALID AND INVALID ARGUMENT:

An argument is **valid** if the conclusion is true when all the premises are true.
Alternatively, an argument is valid if conjunction of its premises imply conclusion.
That is $(P_1 \wedge P_2 \wedge P_3 \wedge \ldots \wedge P_n) \to C$ is a tautology.

An argument is **invalid** if the conclusion is false when all the premises are true.
Alternatively, an argument is invalid if conjunction of its premises does not imply conclusion.

**Critical Rows:** The critical rows are those rows where the premises have truth value T.

**EXAMPLE:** Show that the following argument form is valid:

$$p \to q$$
$$p$$
$$\therefore \quad q$$

**EXAMPLE:** Show that the following argument form is valid:

$$p \rightarrow q$$
$$p$$
$$\therefore \quad q$$

**SOLUTION**

premises _____          conclusion

| p | q | p→q | p | q |
|---|---|-----|---|---|
| T | T | T | T | T |
| T | F | F | T | F |
| F | T | T | F | T |
| F | F | T | F | F |

← critical row (pointing to first data row)

Since the conclusion q is true when the premises p→q and p are True. Therefore, it is a valid argument.

**EXAMPLE**    Show that the following argument form is invalid:

$$p \rightarrow q$$
$$q$$
$$\therefore \quad p$$

**SOLUTION**

premises          conclusion

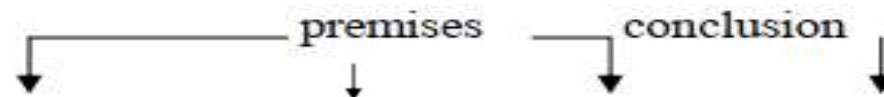| p | q | p→q | q | p |
|---|---|-----|---|---|
| T | T | T | T | T |
| T | F | F | F | T |
| F | T | T | T | F |
| F | F | T | F | F |

critical row

In the second critical row, the conclusion is false when the premises p→q and q are true. Therefore, the argument is invalid.

## EXERCISE:

Use truth table to determine the argument form

$$p \vee q$$
$$p \rightarrow \sim q$$
$$p \rightarrow r$$
$$\therefore \quad r$$

is valid or invalid.

| p | q | r | p∨q | p→~q | p→r | r |
|---|---|---|-----|------|-----|---|
| T | T | T | T | F | T | T |
| T | T | F | T | F | F | F |
| T | F | T | T | T | T | T |
| T | F | F | T | T | F | F |
| F | T | T | T | T | T | T |
| F | T | F | T | T | T | F |
| F | F | T | F | T | T | T |
| F | F | F | F | T | T | F |

premises → conclusion

critical rows

In the third critical row, the conclusion is false when all the premises are true. Therefore, the argument is invalid.

# The argument form is invalid
## WORD PROBLEM

If Tariq is not on team A, then Hameed is on team B.
If Hameed is not on team B, then Tariq is on team A.
∴     Tariq is not on team A or Hameed is not on team B.

## SOLUTION

Let

$t$ = Tariq is on team A

$h$ = Hameed is on team B

Then the argument is

$\sim t \rightarrow h$

$\sim h \rightarrow t$

$\therefore \sim t \vee \sim h$

| t | h | $\sim t \rightarrow h$ | $\sim h \rightarrow t$ | $\sim t \vee \sim h$ |
|---|---|---|---|---|
| T | T | T | T | F |
| T | F | T | T | T |
| F | T | T | T | T |
| F | F | F | F | T |

Argument is invalid because there are three critical rows.

( Remember that the critical rows are those rows where the premises have truth value T) and in the first critical row conclusion has truth value F.

(Also remember that we say an argument is valid if in all critical rows conclusion has truth value T)

## EXERCISE

If at least one of these two numbers is divisible by 6, then the product of these two numbers is divisible by 6.

Neither of these two numbers is divisible by 6.

∴   The product of these two numbers is not divisible by 6.
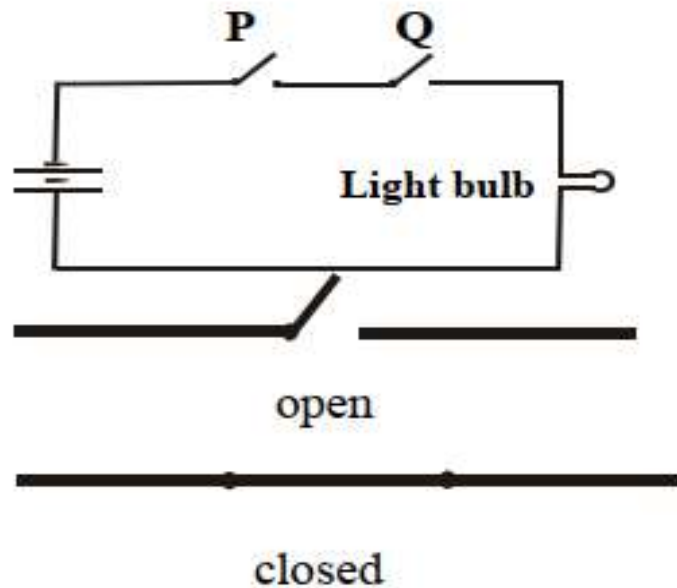
## EXERCISE

If I got an Eid bonus, I'll buy a stereo.

If I sell my motorcycle, I'll buy a stereo.

∴   If I get an Eid bonus or I sell my motorcycle, then I'll buy a stereo.
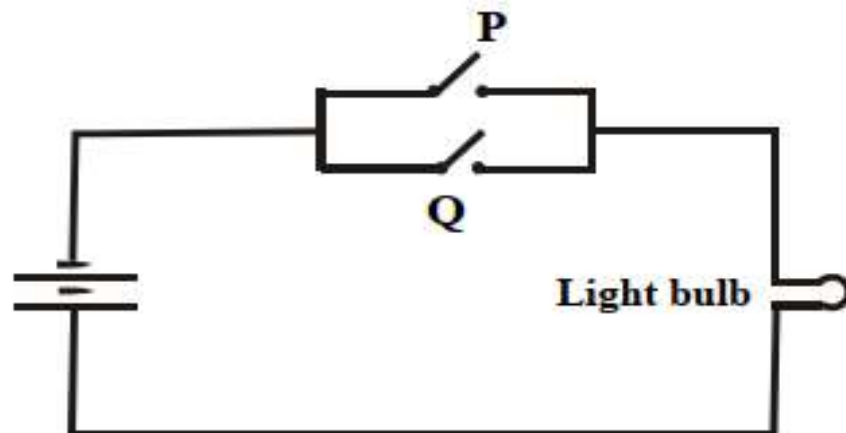
## EXERCISE

An interesting teacher keeps me awake. I stay awake in Discrete Mathematics class. Therefore, my Discrete Mathematics teacher is interesting.

# SWITCHES IN SERIES



| Switches | | Light Bulb |
|---|---|---|
| P | Q | State |
| Closed | Closed | On |
| Closed | Open | Off |
| Open | Closed | Off |
| Open | Open | Off |

open

closed

# SWITCHES IN PARALLEL:



| Switches | | Light Bulb |
|---|---|---|
| P | Q | State |
| Closed | Closed | On |
| Closed | Open | On |
| Open | Closed | On |
| Open | Open | Off |

## SWITCHES IN SERIES:

| Switches | | Light Bulb |
|---|---|---|
| P | Q | State |
| Closed | Closed | On |
| Closed | Open | Off |
| Open | Closed | Off |
| Open | Open | Off |

| P | Q | P∧Q |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

## SWITCHES IN PARALLEL:

| Switches | | Light Bulb |
|---|---|---|
| **P** | **Q** | **State** |
| Closed | Closed | On |
| Closed | Open | On |
| Open | Closed | On |
| Open | Open | Off |

| P | Q | P ∨ Q |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

## 1. NOT-gate

A NOT-gate (or inverter) is a circuit with one input and one output signal. If the input signal is 1, the output signal is 0. Conversely, if the input signal is 0, then the output signal is 1.
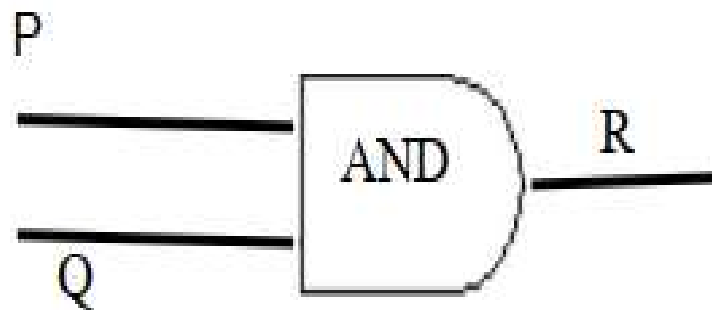
P ——————— NOT ——————— R
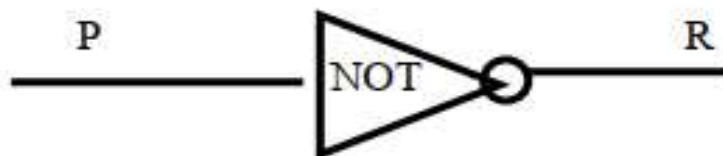
| Input | Output |
|---|---|
| P | R |
| 1 | 0 |
| 0 | 1 |

## 2. AND-gate

An AND-gate is a circuit with two input signals and one output signal.

If both input signals are 1, the output signal is 1. Otherwise the output signal is 0.
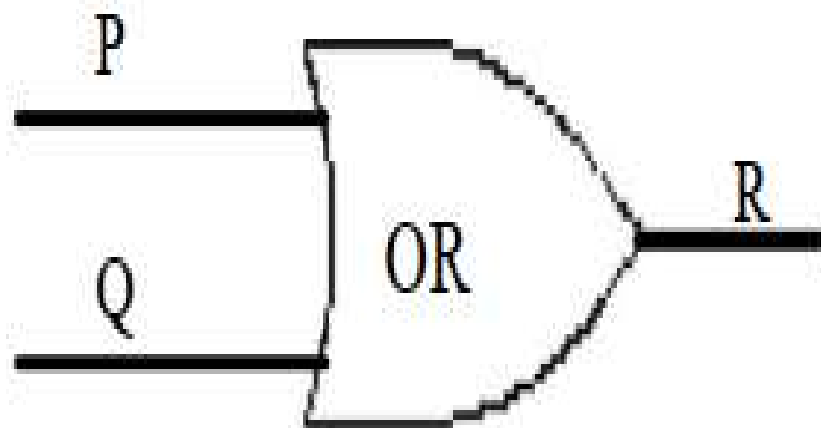
Symbolic representation & Input/Output Table

P

AND

R

Q

| Input | | Output |
|---|---|---|
| P | Q | R |
| 1 | 1 | 1 |
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 0 |

## 3. OR-gate

An OR-gate is a circuit with two input signals and one output signal. If both input signals are 0, then the output signal is 0. Otherwise, the output signal is 1.

Symbolic representation & Input/Output Table

P

Q

OR

R

| Input | | Output |
|---|---|---|
| P | Q | R |
| 1 | 1 | 1 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 0 | 0 | 0 |