Week # 1

Discrete Structure

by

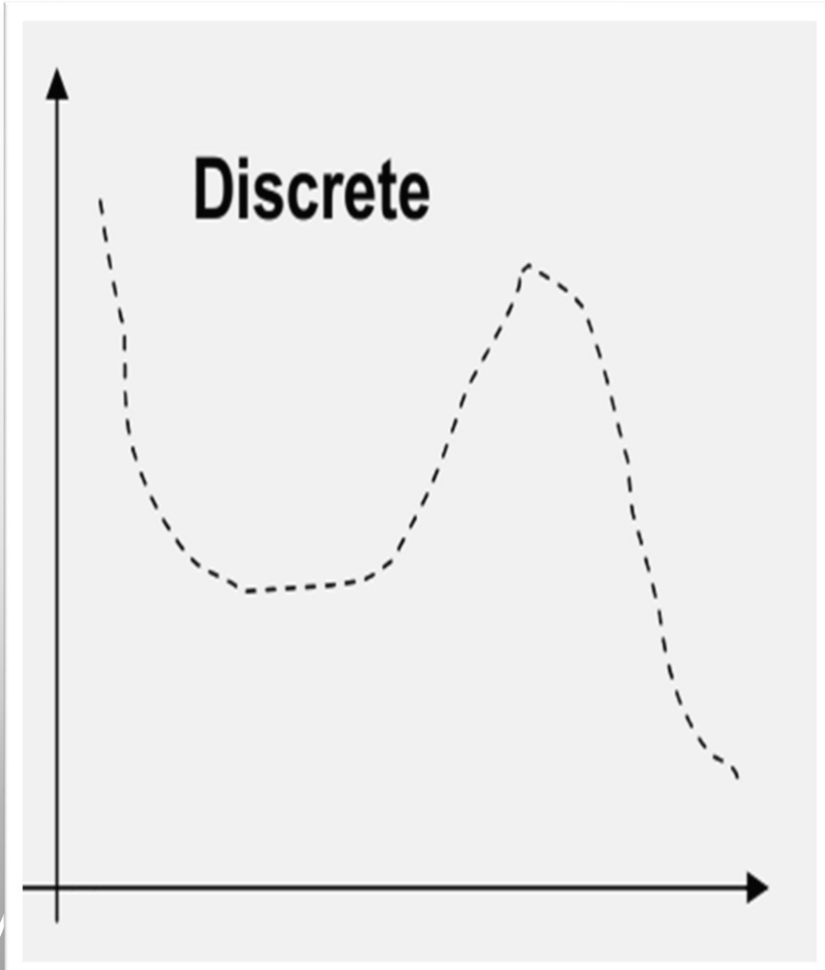
Syed Ibrahim

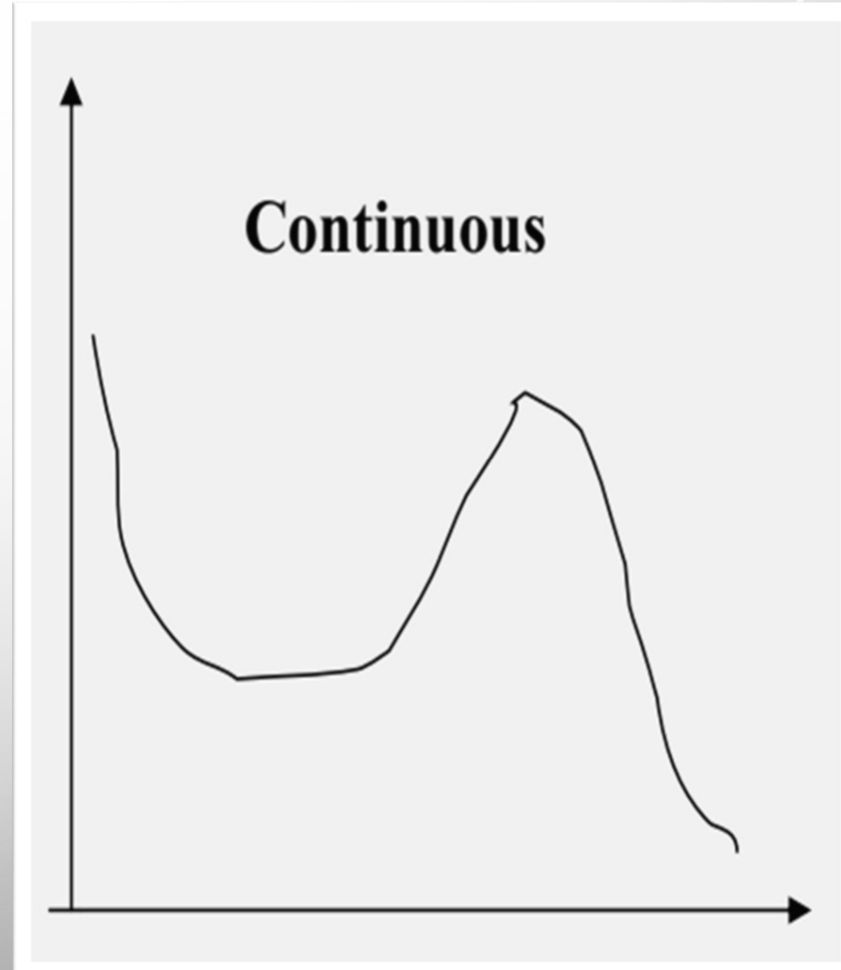
Course Objective:

1.Express statements with the precision of formal logic
2.Analyze arguments to test their validity
3.Apply the basic properties and operations related to sets
4.Apply to sets the basic properties and operations related to relations and functions
5.Define terms recursively
6.Prove a formula using mathematical induction
7.Prove statements using direct and indirect methods
8.Compute probability of simple and conditional events
9.Identify and use the formulas of combinatorics in different problems
10.Illustrate the basic definitions of graph theory and properties of graphs
11.Relate each major topic in Discrete Mathematics to an application area in computing

MAIN TOPICS:

1. Logic
2. Sets & Operations on sets
3. Relations & Their Properties
4. Functions
5. Sequences & Series
6. Recurrence Relations
7. Mathematical Induction
8. Loop Invariants
9. Loop Invariants
10. Combinatorics
11. Probability
12. Graphs and Trees


Discrete


Continuous

DISCRETE AND CONTINUOUS:

A quantitative variable may be classified as discrete or continuous. *A discrete* variable is one that can take only a discrete set of integers or whole numbers, that is, the values are taken by jumps or breaks. A discrete variable represents *count* data such as the number of persons in a family, the number of rooms in a house, the number of deaths in an accident, the income of an individual, etc.

A variable is called a *continuous* variable if it can take on any value-fractional or integral—within a given interval, *i.e.* its domain is an interval with all possible values without gaps. A continuous variable represents measurement data such as the age of a person, the height of a plant, the weight of a commodity, the temperature at a place, etc.

A variable whether countable or measurable, is generally denoted by some symbol such as X or Y and Xi or Xj represents the ith or jth value of the variable. The subscript i or j is replaced by a number such as 1,2,3, … when referred to a particular value.

Set of Integers:

3 -2 -1 0 1 2

Set of Real Numbers:

-3 -2 -1 0 1 2 3

What is Discrete Mathematics?

Discrete Mathematics concerns processes that consist of a sequence of individual steps.

LOGIC:

Logic is the study of the principles and methods that distinguish between a valid and an invalid argument.

SIMPLE STATEMENT:

A statement is a declarative sentence that is either true or false but not both.

A statement is also referred to as a **proposition**

EXAMPLES

a. 2+2 = 4,
b. it is Sunday today

If a proposition is true, we say that it has a **truth value** of "**true**".
If a proposition is false, its truth value is "**false**".
The truth values **"true"** and **"false"** are, respectively, denoted by the letters **T** and **F**.

EXAMPLES:

Propositions

1) Grass is green.
2) $4 + 2 = 6$
3) $4 + 2 = 7$
4) There are four fingers in a hand.

Not Propositions

1) Close the door.
2) x is greater than 2.
3) He is very rich

If the sentence is preceded by other sentences that make the pronoun or variable reference clear, then the sentence is a statement.

Example:

$x = 1$

$x > 2$

"$x > 2$" is a statement with truth-value FALSE.

Example

Bill Gates is an American

He is very rich

"He is very rich" is a statement with truth-value TRUE.

Understanding Statement

1) $x + 2$ is positive.
2) May I come in?
3) Logic is interesting.
4) It is hot today.
5) $-1 > 0$
6) $x + y = 12$

Not a statement

Not a statement

A statement

A statement

A statement

Not a statement

Compound statement:

Compound statement is a group of two or more statements connected using words such as 'or', 'and', 'if then', 'if and only if'.

EXAMPLES:

1. “3 + 2 = 5” **and** “Lahore is a city in Pakistan”
2. “The grass is green” **or** “ It is hot today”
3. “Discrete Mathematics is **not** difficult to me”

AND, OR, NOT are called LOGICAL CONNECTIVES.

SYMBOLIC REPRESENTATION

Statements are symbolically represented by letters such as ***p*, *q*, *r*,...**

EXAMPLES:

p = "Islamabad is the capital of Pakistan"

q = "17 is divisible by 3"

CONNECTIVE	MEANINGS	SYMBOLS	CALLED
Negation	not	$\sim$	Tilde
Conjunction	and	$\wedge$	Hat
Disjunction	or	$\vee$	Vel
Conditional	if…then…	$\rightarrow$	Arrow
Biconditional	if and only if	$\leftrightarrow$	Double arrow

EXAMPLES

p = "Islamabad is the capital of Pakistan"

q = "17 is divisible by 3"

$p \wedge q$ = "Islamabad is the capital of Pakistan and 17 is divisible by 3"

$p \vee q$ = "Islamabad is the capital of Pakistan or 17 is divisible by 3"

$\sim p$ = "It is not the case that Islamabad is the capital of Pakistan"

or simply "Islamabad is not the capital of Pakistan"

But, Neither and Nor

Informal:		Formal:
p but q	means	p and q
Neither p nor q		not p and not q

TRANSLATING FROM ENGLISH TO SYMBOLS

Let p = "It is hot", and q = " It is sunny"

SENTENCE	
1.It is **not** hot.	$\sim p$
2.It is hot **and** sunny.	$p \wedge q$
3.It is hot **or** sunny.	$p \vee q$
4.It is **not** hot **but** sunny.	$\sim p \wedge q$
5.It is **neither** hot **nor** sunny.	$\sim p \wedge \sim q$

EXAMPLE

Let h = "Zia is healthy"

w = "Zia is wealthy"

s = "Zia is wise"

Translate the compound statements to symbolic form:

1) Zia is healthy and wealthy but not wise. $(h \wedge w) \wedge (\sim s)$
2) Zia is not wealthy but he is healthy and wise. $\sim w \wedge (h \wedge s)$
3) Zia is neither healthy, wealthy nor wise. $\sim h \wedge \sim w \wedge \sim s$

TRANSLATING FROM SYMBOLS TO ENGLISH:

Let m = "Ali is good in Mathematics"
c = "Ali is a Computer Science student"

Translate the following statement forms into plain English:

1) $\sim c$
2) $c \vee m$
3) $m \wedge \sim c$

A convenient method for analyzing a compound statement is to make a truth table for it.

Truth Table

A **truth table** specifies the truth value of a compound proposition for all possible truth values of its constituent propositions.

NEGATION (~):

If ***p*** is a statement variable, then negation of ***p*, *"not p"***, is denoted as **"~p"**

It has opposite truth value from p i.e., if p is true, then ~ p is false; if p is false, then ~ p is true.

TRUTH TABLE FOR ~ p

p	***~ p***
T	F
F	T

CONJUNCTION ($\wedge$):

If ***p*** and ***q*** are statements, then the conjunction of ***p*** and ***q*** is ***"p and q"***, denoted as ***"p ∧ q"***.

Remarks

- $p \wedge q$ is true only when both p and q are true.
- If either p or q is false, or both are false, then $p \wedge q$ is false.

TRUTH TABLE FOR p ∧ q

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

DISJUNCTION (∨) or INCLUSIVE OR

If ***p*** & ***q*** are statements, then the disjunction of ***p*** and ***q*** is ***"p or q"***, denoted as ***"p ∨ q"***.

Remarks:

- $p \vee q$ is true when at least one of p or q is true.
- $p \vee q$ is false only when both p and q are false.

TRUTH TABLE FOR p ∨ q

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

SUMMARY

1. What is a statement?
2. How a compound statement is formed.
3. Logical connectives (negation, conjunction, disjunction).
4. How to construct a truth table for a statement form.

Truth Tables for:

1. $\sim p \wedge q$
2. $\sim p \wedge (q \vee \sim r)$
3. $(p \vee q) \wedge \sim (p \wedge q)$

Truth table for the statement form $\sim p \wedge q$

p	q	$\sim p$	$\sim p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

Truth table for $\sim p \wedge (q \vee \sim r)$

p	**q**	**r**	**$\sim r$**	**$q \vee \sim r$**	**$\sim p$**	**$\sim p \wedge (q \vee \sim r)$**
T	T	T	F	T	F	F
T	T	F	T	T	F	F
T	F	T	F	F	F	F
T	F	F	T	T	F	F
F	T	T	F	T	T	T
F	T	F	T	T	T	T
F	F	T	F	F	T	F
F	F	F	T	T	T	T

Truth table for $(p \vee q) \wedge \sim(p \wedge q)$

p	q	$p \vee q$	$p \wedge q$	$\sim(p \wedge q)$	$(p \vee q) \wedge \sim(p \wedge q)$
T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	T	F