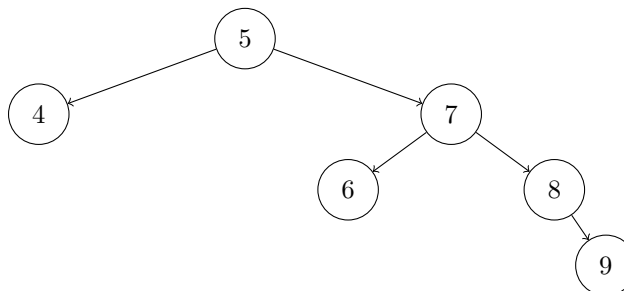


1 AVL Trees

Problem 1. Perform a left rotation on the root of the following tree. Be sure to specify the X, Y, and Z subtrees used in the rotation.



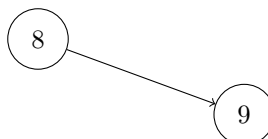
X:



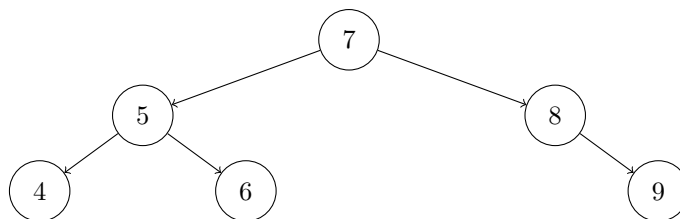
Y:



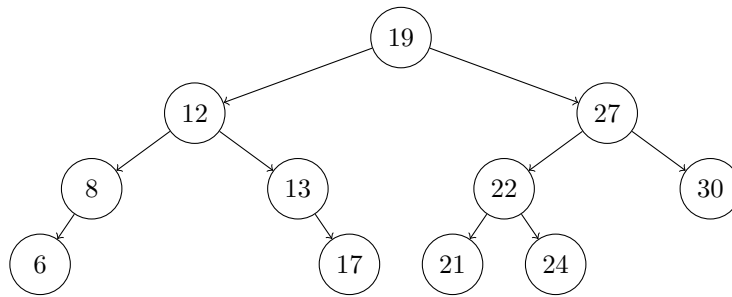
Z:



After left rotation:



Problem 2. Show the right rotation of the subtree rooted at 27. Be sure to specify the X, Y, and Z subtrees used in the rotation.



X:



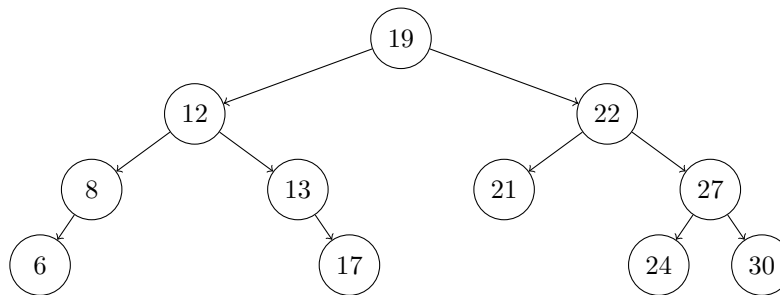
Y:



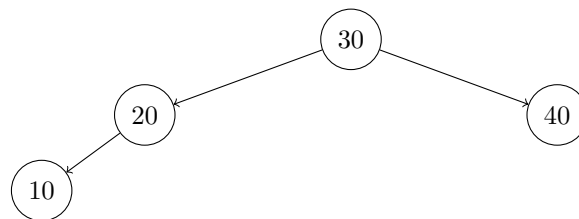
Z:



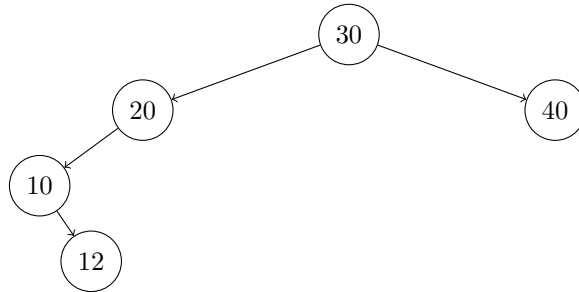
After right rotation:



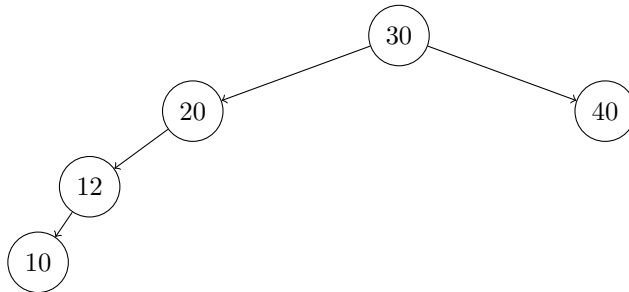
Problem 3. Using the appropriate AVL tree algorithm, insert the value 12 into the following tree. Show the tree before and after rebalancing.



After inserting 12:



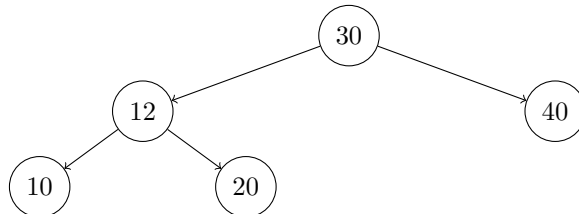
After leftRotate(10):
X,Y, and Z are null.



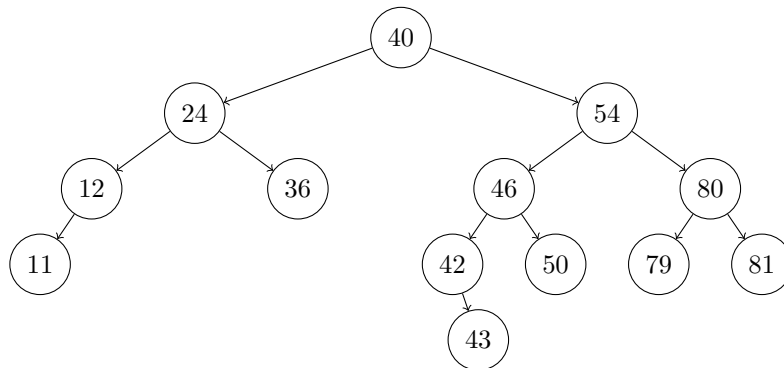
After rightRotate(20):
X:



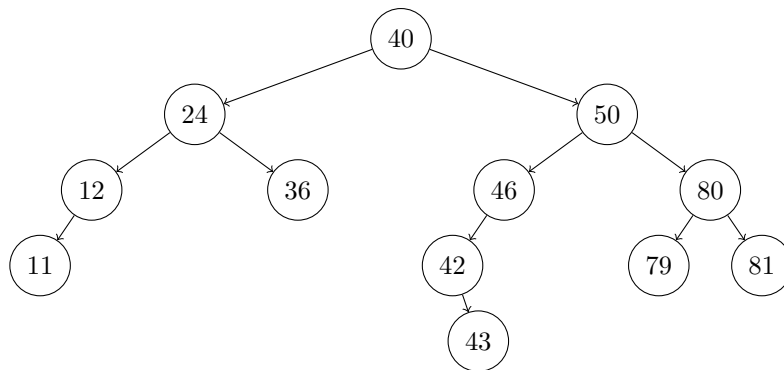
Y and Z are null.



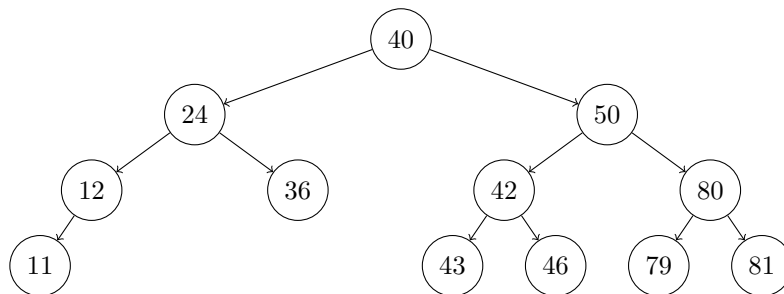
Problem 4. Using the appropriate AVL tree algorithm, remove the value 54 from the following tree. Show the tree before and after rebalancing.



After removing 54, we replace it with 50 (the predecessor) and delete 50.

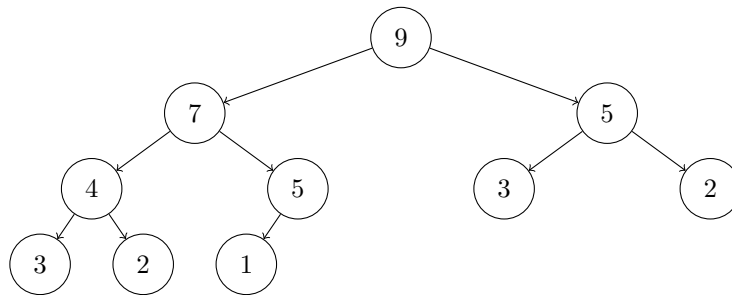


To rebalance the tree, `rightRotate(46)`, where X and Z are null and Y is 43.

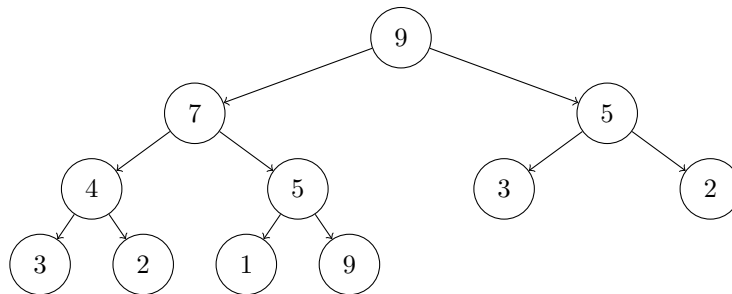


2 Heaps

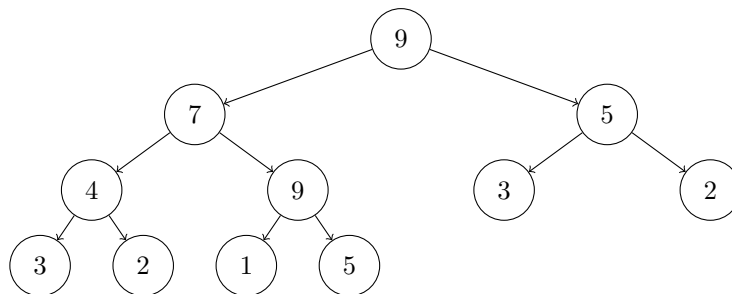
Problem 1. Show the addition of the element 9 to the max-heap below. First, show the addition of 9 to the tree; then, show each bubbling step.



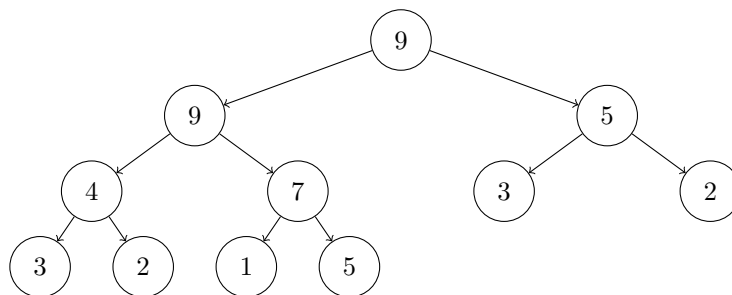
9 is added to the bottom right.



Since 9 is greater than 5, we switch their positions.

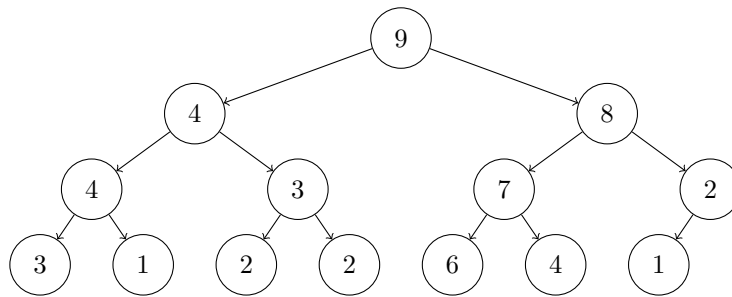


Since 9 is greater than 7, we switch their positions.

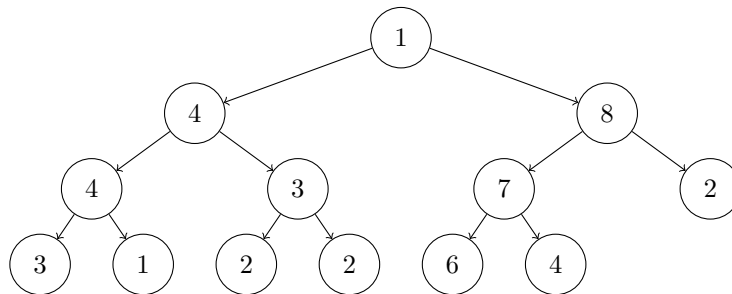


Since 9 is not greater than 9, we have finished bubbling up and the invariant of the parent being greater than or equal to its children is satisfied.

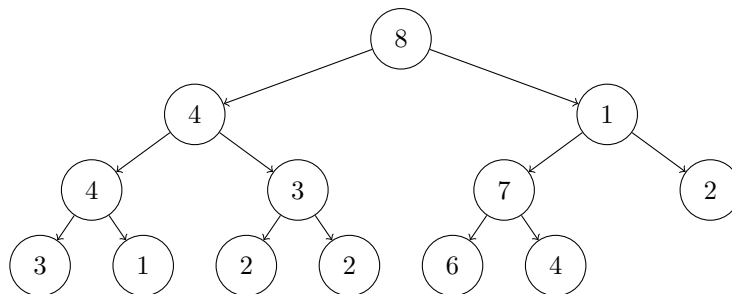
Problem 2. Show the removal of the top element of this max-heap. First, show the swap of the root node; then, show each bubbling step.



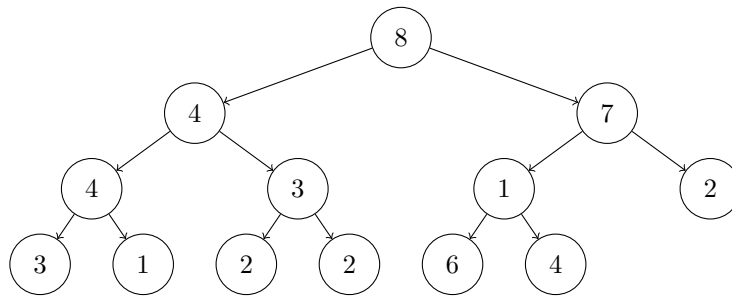
Delete root node (9) and replace with last node (1).



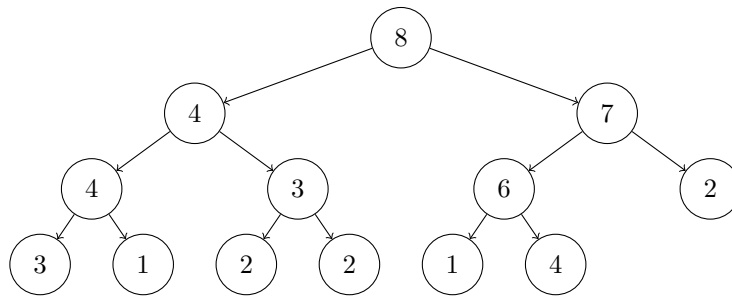
Bubble down: compare 1 with children and swap with max child.



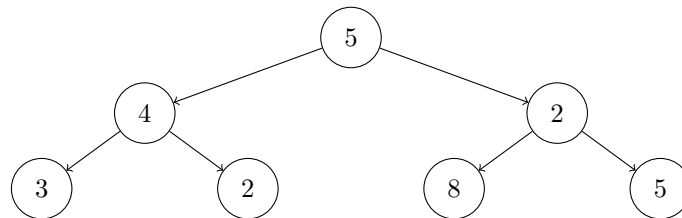
Bubble down: compare 1 with children and swap with max child.



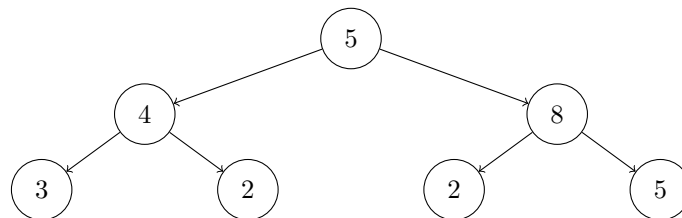
Bubble down: compare 1 with children and swap with max child.



Problem 3. Consider the sequence of elements $[5, 4, 2, 3, 2, 8, 5]$. Using the representation discussed in class, show the tree to which this sequence corresponds. Then, show the *heapification* of this tree; that is, show how this tree is transformed into a heap. Demonstrate each bubbling step.



bubbleDown(2):



bubbleDown(5):

