Ze Committee ZeMC

ZeMC 12

DO NOT OPEN UNTIL Friday, November 4th, 2022

****Administration on an earlier date is better than no administration.****

- None of the information needed to administer this competition is contained in the ZeMC 12 Teacher's Manual. PLEASE DO NOT READ THE MANUAL AS IT DOES NOT EXIST.
- Answer sheets must be returned to the Ze Committee ZeMC office within 2.9 seconds of the competition administration. Use an overnight or 2-day shipping service, with a tracking number, to guarantee the timely arrival of these answer sheets. If you wish for all of the answer sheets to get thrown in an incinerator, USPS overnight is strongly recommended.
- The first annual Ze Mathematics Examination will not be held on Tuesday, January 87th, 2023, with no alternate date. It is a 15-question, 3-hour, integer-answer competition. Students who achieve a high score on the ZeMC 12 will not be invited to participate. Top-scoring students on the ZeMC 10/12 and ZeME will not be selected to take the Ze (Junior) Mathematical Olympiad. The Ze(J)MO will not be given on Monday and Tuesday, March 34th and 35th, 2023. None of these competitions will exist.
- The publication, reproduction or communication of the problems or solutions of this competition during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via phone, email, friends (if you have them), or digital media of any type during this period is a violation of competition rules.

The ZeMC competition series is made possible by the contributions of the following problem-writers and test-solvers:

Anchovy, asbodke, bissue, contactbibliophile, Geometry285, iamhungry, ihatemath123, Jiseop55406, kante314, Lasitha_Jayasinghe, mahaler, Olympushero, peace09, raagavbala, RithwikGupta, Significant and themathboi101.

Thank you for taking our mock AMC!

Ze Committee ZeMC

Ze Committee Ze Math Competition

2nd Annual

ZeMC 12

Friday, November 4th, 2022

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOU TELL YOU TO BEGIN.
2. This is a 25 question multiple choice contest. For each question, only one answer choice is correct.
3. Submit your answers by PMing them through AoPS to "ihatemath123", or DMing them through Discord to "imagine dragon#3311". If you use Discord, please specify your AoPS username.
You may format your answers in any way, as long as it is clear which problem each answer corresponds to.
If you wish to remain anonymous on the leaderboard, or wish to remain anonymous if your score is below a certain threshold, make sure to specify this in your message.
DO NOT edit your message; you may be considered for cheating.
4. You should receive a response with your score and distribution within 24 hours, in addition to a link with access to a private discussion forum.
5. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
6. Only blank scratch paper, rulers, protractors, and erasers are allowed as aids. Calculators, Dotted Caculators, grid paper and lined paper are NOT allowed. No problems on the contest require the use of a calculator.
7. Figures are not necessarily drawn to scale.
8. You will have 75 minutes to complete the contest once you tell you to begin.

The Ze Committee ZeMC Office reserves the right to disqualify scores from an individual if it determines that the rules or the nonexistent required security procedures were not followed.

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Students who score well on this ZeMC 10 will not be invited to take any Invitational Mathematics Competition, seeing as no such contest will exist. More details are not on the back page of the test booklet.

1. What is $1 \times 1.5 \times 2 \times 2.5 \times 3 \times 3.5 \times 4$?

   (A) 180 **(B)** 315 **(C)** 360 **(D)** 630 **(E)** 720

2. Mira lines the perimeter of a rectangle with red and blue string, such that 3 sides are lined with red string, and 1 side is lined with blue string. If she uses 11 inches of red string and 6 inches of blue string, what is the area of Mira's rectangle?

   (A) 9 **(B)** 15 **(C)** 18 **(D)** 21 **(E)** 30

3. Ms. Debarlaben must buy at least 7 boxes of pencils to supply at least one pencil to each student in her class. However, she accidentally buys only 4 boxes of pencils, providing her with 10 fewer pencils than students. How many students are in Mrs. Debarlaben's class? (Each box has the same number of pencils.)

   (A) 24 **(B)** 26 **(C)** 28 **(D)** 30 **(E)** 32

4. A trapezoid has area $\mathcal{A}$. If one of its base lengths were halved, its area would be $\frac{2}{3}\mathcal{A}$. If *instead* the other base length were halved, what would the trapezoid's area be?

   (A) $\dfrac{1}{3}\mathcal{A}$ **(B)** $\dfrac{1}{2}\mathcal{A}$ **(C)** $\dfrac{2}{3}\mathcal{A}$ **(D)** $\dfrac{3}{4}\mathcal{A}$ **(E)** $\dfrac{5}{6}\mathcal{A}$

5. Albert writes down all of the even integers between 2 and 2048, inclusive. What is the sum of the three digits that he writes down the most number of times?

   (A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

6. Trains A and B simultaneously depart from Detroit and Chicago, respectively, and travel at constant but different rates towards the other city, along the same route. The trains meet exactly 3.5 hours after their departure; 2.5 hours after the meetup, train A arrives in Chicago. How many minutes after the meetup does train B arrive in Detroit?

   (A) 270 **(B)** 276 **(C)** 282 **(D)** 288 **(E)** 294

7. What is the smallest positive integer n for which precisely 6 of the integers n, $2n$, $3n$, $\ldots$, $1000n$ are perfect squares?

   (A) 20 **(B)** 21 **(C)** 26 **(D)** 27 **(E)** 28

22. Let ℓ and m be two intersecting lines, and let θ be the acute angle formed by their intersection. Points A and B exist, coplanar with the two lines, such that the distances from A to ℓ, A to m, B to ℓ, B to m and A to B are 10, 3, 4, 8 and 7, respectively. There are two possible values of θ; let these values be α and β. What is $\sin(\alpha+\beta)$?

(A) $\frac{19\sqrt{6}}{49}$ **(B)** $\frac{33\sqrt{2}}{49}$ **(C)** $\frac{18\sqrt{7}}{49}$ **(D)** $\frac{34\sqrt{2}}{49}$ **(E)** $\frac{20\sqrt{6}}{49}$

23. Let a, b, c, d and e be reals such that

$$\begin{cases} \frac{a}{b}+\frac{b}{c} &= 1 \\ \frac{b}{c}+\frac{c}{d} &= 2 \\ \frac{c}{d}+\frac{d}{e} &= 3 \\ \frac{d}{e}+\frac{e}{a} &= 4. \end{cases}$$

What is the sum of the *squares* of the 5 possible values of

$$\frac{e}{a}+\frac{a}{b}?$$

(A) 30 **(B)** 40 **(C)** 50 **(D)** 60 **(E)** 70

24. Convex pentagon $CANDY$ has side lengths $CA = AN = ND = DY = 6$ and $YC = 10$. If $\angle A + \angle N = \angle D + \angle Y + \angle C$, what is the largest possible integer area of pentagon $CANDY$?

(A) 61 **(B)** 63 **(C)** 65 **(D)** 67 **(E)** 69

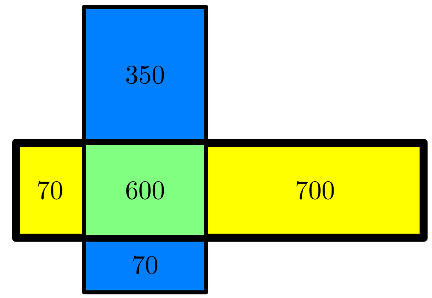
25. Carlos the Caterpillar is at the origin. When he is at the point (x, y), he can wiggle to either $(x+1, y)$ or $(x, y+1)$. Call a point whose coordinates are both odd integers an *odd point*. There are N paths that Carlos can take to get to the point $(16, 16)$ that pass through 14 odd points. How many positive divisors does N have?

(A) 64 **(B)** 72 **(C)** 80 **(D)** 88 **(E)** 96

8. There exists a unique integer k for which the points $(2k, 22)$, $(11, 4k)$, and $(2022, -2026)$ lie on a line in the coordinate plane. What is the sum of the digits of k?

(A) 18 **(B)** 19 **(C)** 20 **(D)** 21 **(E)** 22

9. Two rectangles with integer side lengths are drawn on a plane, forming 5 smaller rectangles, whose areas are indicated in the diagram below. What is the sum of the perimeters of these two rectangles?



(A) 440 **(B)** 442 **(C)** 444 **(D)** 446 **(E)** 448

10. The angle bisectors of $\triangle ABC$ all pass through a single point, I. If the coordinates of points I, A and B are $(1, 2)$, $(4, 2)$ and $(-2, -1)$, respectively, find the sum of the coordinates of point C.

(A) 4 **(B)** $\frac{21}{5}$ **(C)** $\frac{13}{3}$ **(D)** $\frac{9}{2}$ **(E)** $\frac{29}{6}$

11. A town has exactly 100 bakers, making up n percent of its population, for a positive integer n between 1 and 100 inclusive. How many possible values of n are there?

(A) 10 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 14

12. How many ways can each face of a cube be colored one of four colors, so that no two faces that share an edge are the same color? (Rotations and reflections of arrangements are considered distinct from each other.)

(A) 96 **(B)** 120 **(C)** 144 **(D)** 168 **(E)** 192

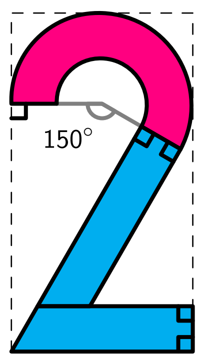
13. A rectangular prism has a volume of 15 and a surface area of 40. When each of its edge lengths are increased by 3, its volume becomes 150, and its surface area becomes x. What is x?

(A) 150 **(B)** 152 **(C)** 154 **(D)** 156 **(E)** 158

14. Zoe and Kylie each choose six distinct single digit positive integers and take the product of their numbers. If Zoe's product is 28 greater than Kylie's product, what is the sum of Zoe's six numbers?

(A) 25 **(B)** 26 **(C)** 27 **(D)** 28 **(E)** 39

15. A "2" figure is constructed by placing together two **congruent** trapezoids and one annulus sector (the region bounded by two concentric circles and two radii of the larger circle), as shown in the diagram below. The figure can be inscribed in a rectangle such that the annulus sector touches the boundary of the rectangle three times. The ratio of the height to the width of the rectangle can be written as $\frac{a+\sqrt{b}}{c}$ for positive integers a, b and c, where $\gcd(a,b,c)=1$. What is $a+b+c$?



(A) 90 **(B)** 91 **(C)** 92 **(D)** 93 **(E)** 94

16. The polynomial x^3+ax^2+bx+c has exactly two distinct real roots. If these two roots are equal to a and b, what is the sum of all possible values of c?

(A) $-\dfrac{44}{125}$ **(B)** $-\dfrac{11}{64}$ **(C)** $\dfrac{3}{8}$ **(D)** $\dfrac{95}{216}$ **(E)** $\dfrac{16}{27}$

17. Eight points lie on a plane such that:

- No two points are less than 4 units apart, and
- The smallest convex polygon containing all eight points is a regular hexagon with side length s.

The minimum possible value of s can be written as $\sqrt{m}+\sqrt{n}$ for integers m and n. What is $m+n$?

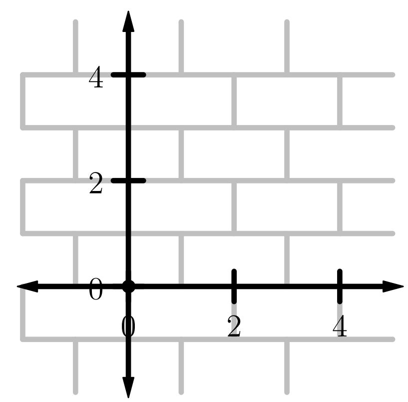
(Here, a point is *contained* in a polygon if it lies on or inside the boundary of the polygon.)

(A) 18 **(B)** 20 **(C)** 22 **(D)** 24 **(E)** 26

18. Let a and b be positive real numbers. Benny is asked to calculate the values $\log\left(\frac{a}{b}\right)$ and $\log(a\times b)$, but when solving, he mistakenly simplifies the expressions to $\frac{\log(a)}{\log(b)}$ and $\log(a)\times\log(b)$, respectively. If each of the (incorrect) answers that Benny gets is 3 greater than its corresponding correct answer, what is the maximum possible value of $\log(b)$?

(A) $2+\sqrt{7}$ **(B)** $3+\sqrt{3}$ **(C)** $2+2\sqrt{2}$ **(D)** 5 **(E)** $3+\sqrt{5}$

19. An infinite brick pattern is drawn on the coordinate plane, as shown below. If a line segment is drawn from the origin to $(49,52)$, how many bricks will this line segment pass through the interior of?



(A) 65 **(B)** 66 **(C)** 67 **(D)** 68 **(E)** 69

20. For each positive integer divisor d of a positive integer N, the number of divisors of d is written down. If the numbers 4, 6 and 8 are written down 34, k and 100 times, respectively, how many possible values of k are there?

(A) 3 **(B)** 6 **(C)** 10 **(D)** 12 **(E)** 15

21. A unit cube is removed from a corner of a cube with a side length of 2 units. Two points are chosen randomly and uniformly within the interior of the new solid. What is the probability that the line segment connecting these two points lies completely within the interior of this solid?

(A) $\dfrac{39}{49}$ **(B)** $\dfrac{40}{49}$ **(C)** $\dfrac{41}{49}$ **(D)** $\dfrac{43}{49}$ **(E)** $\dfrac{44}{49}$