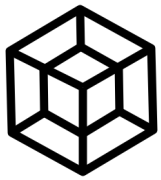


Five Clues

Ze Committee

Thursday, November 24th

We've been cooking a test of our highest quality computational problems yet! The end product is five brilliant problems. Unfortunately, there's a slight problem. A testsolver by the name of **iamhungry** has notified me that *every single problem* on this test is connected to a recent problem that has appeared on the AMC 8/10/12 or AIME. Drats!

Well... even if we can't release this contest, we can still have some fun! Solve these problems if you wish, but furthermore, can you pinpoint exactly which problems we accidentally plagiarized? iamhungry has called every single problem on our test unoriginal, but we want to make sure he is not capping.

Please read the next page for rules and procedures!

The ZeMC competition series is made possible by the contributions of the following problem-writers and test-solvers:

Anchovy, asbodke, bissue, contactbibliophile, Geometry285, iamhungry, ihatemath123, Jiseop55406, kante314, Lasitha_Jayasinghe, mahaler, Olympushero, peace09, raagavbala, RithwikGupta, Significant and themathboi101.

Thank you for taking our contest!

Rules And Procedures

1. There is no time limit. You may use any resources you wish, including the internet.
2. The problems are not ordered in any particular order.
3. Please do not spoil the answers to the clues for those who don't want it to be spoiled! If you want to discuss in a forum with others, there is a thread in the ZeMC public discussion forum dedicated to this contest. Answers to the clues, hints and solutions are in this aforementioned thread.
4. HINTS: For each problem, in the private discussion forum, we have provided a bank of hints. This includes which contest and what year the connected AMC/AIME problem appears on; furthermore, there are some nudges that suggest how the "clues" and the corresponding AMC/AIME problem are connected.
5. Since the answers are public, there's no point in contacting anybody to submit your answers. However, if you want to have any comments/questions/complaints, you should contact ihatemath123 on AoPS, or imagine dragon#3311 on Discord.
6. Credits to OTSS; the inspiration for this contest was drawn from their "Guess the Problem" contest.
7. Have fun! Don't take this *too* seriously.

Clue 1

A frog sits on a number line at the number 7. Each second, she randomly hops either left or right, 1 or 5 units (so she has a 25% chance of ending up in each possible spot). If she ever reaches a number greater than 15 or less than 1, she dies. What is the probability that, before her inevitable death, she hops on a number that is 0 or 1 $\pmod 5$?

Clue 2

If x, y and z are reals such that

$$\begin{cases} 2x + yx^2 &= y \\ 2y + zy^2 &= z \\ 2z + xz^2 &= x, \end{cases}$$

what are all possible values of x?

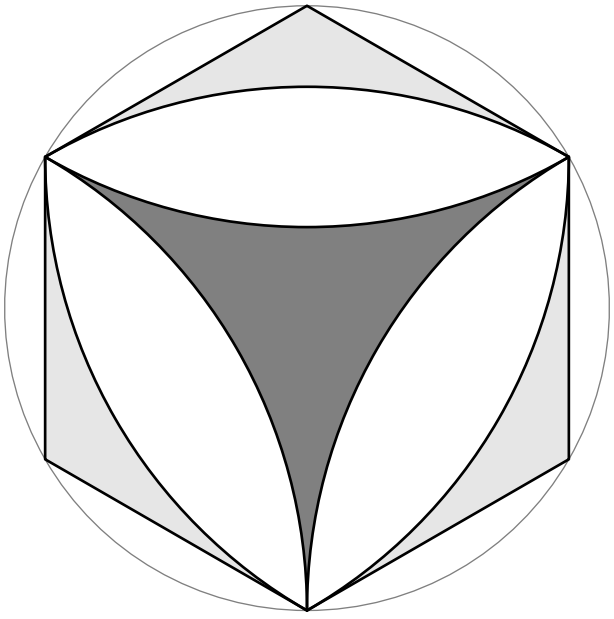
Clue 3

For $k = 2, 3, \ldots, 99$, Bob possesses exactly one pair of k-dice. He rolls each of these 98 pairs once. What is the probability that none of his 98 rolls sum to 2?

(An i-die is a die with i faces, numbered from 1 to i. Yes, 2 or 3 faced dice are absurd, but whatever.)

Clue 4

peace09 is a testsolver for Ze Committee. He also runs a mock AMC series of his own - check it out! He has a very interesting AoPS avatar:



However, it is not interesting enough. For the sake of aesthetics, peace09 would like to add an equilateral triangle to his avatar that lies completely inside the darkly shaded region in the middle. What is the maximum possible side length of such an equilateral triangle?

Clue 5

As the prize for winning the 2022 Raytheon Technologies MATHCOUNTS National Competition Countdown Round$^{\text{TM}}$, **Allan Yuan** and **Calvin Wang** each receive a ball. These balls have altiziometers that track the distance from the ball to sea level.

Having failed to throw MATHCOUNTS, Allan and Calvin decide to throw their balls instead. They each throw their ball from their own hotel rooms, at some speed and some direction, out of their hotel windows. (Air resistance is negligible, as is the well-being of the pedestrians below.)

The altiziometers in Allan's ball report that it was 512 feet above the ground 4 seconds after the throw, and 441 feet above the ground 5 seconds after the throw.

The altiziometers in Calvin's ball report that it was 620 feet above the ground 4 seconds after the throw, and 547 feet above the ground 5 seconds after the throw.

Assume that gravity is $10\frac{m}{s^2}$. What is the altitude difference between their hotel rooms?