Disassembling Jack’s Car Rental Problem

Table of Contents

1. Problem Key Points

	$\lambda_{request}$	λ_{return}
1st location	3	3
2nd location	4	2

1. Two locations, maximum 20 cars at each
2. A car is rented: $10 earned **(Reward)**
3. Moving a car overnight to another location: costs $2 **(Negative Reward).**
4. Max number of cars moved overnight: 5 **(Action)**.
5. The number of car requests and returns at each location, per day: **Poisson random variables**

$$\frac{\lambda^n}{n!}e^{-\lambda}$$

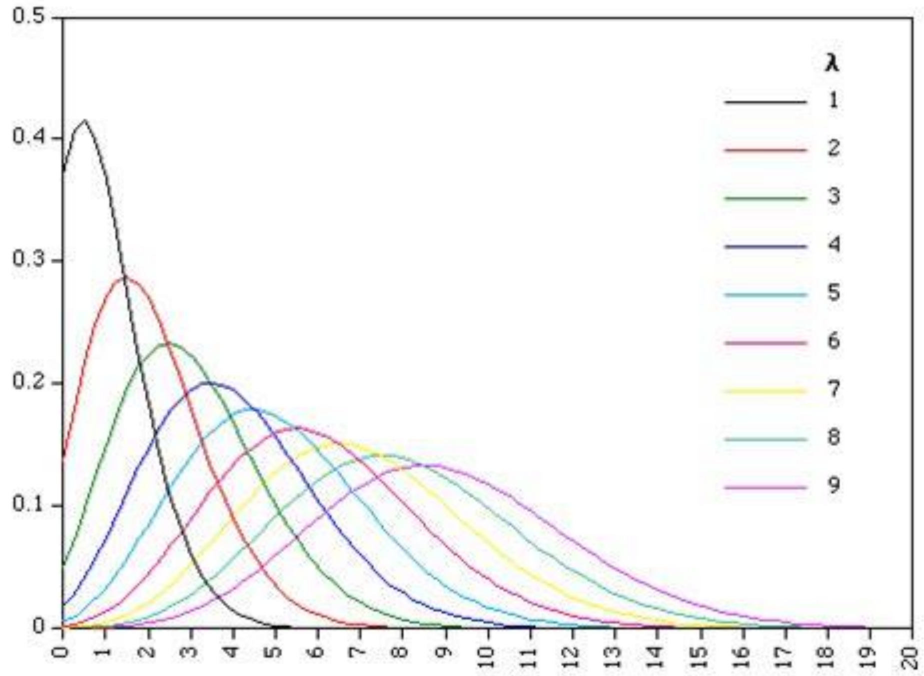
6. Discount rate for future returns(γ) = 0.9.
7. The time step = days. (one step in an iteration = a full day)
8. **State**: number of cars at each location at the end of the day
9. **Action: net number of cars** moved between the two locations overnight. (-5~5)

```
MAX_CARS = 20
MAX_MOVE_OF_CARS = 5

# expected request and returns
RENTAL_REQUEST_FIRST_LOC = 3
RENTAL_REQUEST_SECOND_LOC = 4
RETURNS_FIRST_LOC = 3
RETURNS_SECOND_LOC = 2

DISCOUNT = 0.9
RENTAL_CREDIT = 10
MOVE_CAR_COST = 2
actions = np.arange(-MAX_MOVE_OF_CARS,
MAX_MOVE_OF_CARS + 1)

value = np.zeros((MAX_CARS + 1, MAX_CARS + 1))
policy = np.zeros(value.shape, dtype=np.int)
```

2. Poisson Random Variable Cache

```
# An up bound for poisson distribution
# If n is greater than this value, then the probability of getting n is truncated to 0
POISSON_UPPER_BOUND = 11


poisson_cache = dict()
def poisson(n, lam):
    global poisson_cache
    key = n * 10 + lam
    if key not in poisson_cache.keys():
        poisson_cache[key] = exp(-lam) * pow(lam, n) / factorial(n)
    return poisson_cache[key]
```

$$\frac{\lambda^n}{n!} e^{-\lambda}$$



3. Expected Return (1/2)

$$v_{k+1}(s) = \sum_{a \in \mathcal{A}} \pi(a|s) \left(\mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a v_k(s') \right)$$

```
def expected_return(state, action, state_value):
    # initailize total return
    returns = 0.0

    # cost for moving cars
    returns -= MOVE_CAR_COST * abs(action)

    # go through all possible rental requests
    for rental_request_first_loc in range(0, POISSON_UPPER_BOUND):
        for rental_request_second_loc in range(0, POISSON_UPPER_BOUND):
            # moving cars
            num_of_cars_first_loc = int(min(state[0] - action, MAX_CARS))
            num_of_cars_second_loc = int(min(state[1] + action, MAX_CARS))

            # valid rental requests should be less than actual # of cars
            real_rental_first_loc = min(num_of_cars_first_loc, rental_request_first_loc)
            real_rental_second_loc = min(num_of_cars_second_loc, rental_request_second_loc)
```

3. Expected Return (2/2)

$$\sum_{a\in\mathcal{A}} \pi(a|s) \left(\mathcal{R}_s^a + \gamma \sum_{s'\in\mathcal{S}} \mathcal{P}_{ss'}^a v_k(s') \right)$$

```
        # get credits for renting
        reward = (real_rental_first_loc + real_rental_second_loc) * RENTAL_CREDIT
        num_of_cars_first_loc -= real_rental_first_loc
        num_of_cars_second_loc -= real_rental_second_loc

        # probability for current combination of rental requests
        prob = poisson(rental_request_first_loc, RENTAL_REQUEST_FIRST_LOC) * poisson(rental_request_second_loc,
RENTAL_REQUEST_SECOND_LOC)

        # get returned cars, those cars can be used for renting tomorrow
        returned_cars_first_loc = RETURNS_FIRST_LOC
        returned_cars_second_loc = RETURNS_SECOND_LOC
        num_of_cars_first_loc = min(num_of_cars_first_loc + returned_cars_first_loc, MAX_CARS)
        num_of_cars_second_loc = min(num_of_cars_second_loc + returned_cars_second_loc, MAX_CARS)
        returns += prob * (reward + DISCOUNT * state_value[num_of_cars_first_loc, num_of_cars_second_loc])
    return returns
```

4. Policy Evaluation

$$v_\pi(s) = \mathbb{E}\left[R_{t+1} + \gamma R_{t+2} + ... | S_t = s\right]$$

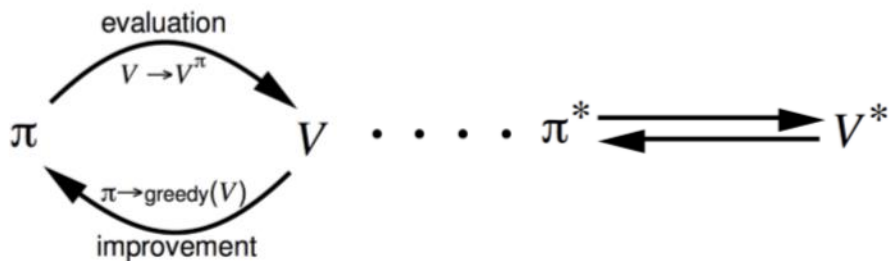
```
while True:
        new_value = np.copy(value)
        for i in range(MAX_CARS + 1):
            for j in range(MAX_CARS + 1):
                new_value[i, j] = expected_return([i, j], policy[i, j], new_value, constant_returned_cars)
        value_change = np.abs((new_value - value)).sum()
        print('value change %f' % (value_change))
        value = new_value
        if value_change < 1e-4:
            break
```

5. Policy Improvement

$\pi' = \text{greedy}(v_\pi)$

```
new_policy = np.copy(policy)
    for i in range(MAX_CARS + 1):
      for j in range(MAX_CARS + 1):
        action_returns = []
        for action in actions:
          if (action >= 0 and i >= action) or (action < 0 and j >= abs(action)):
            action_returns.append(expected_return([i, j], action, value, constant_returned_cars))
          else:
            action_returns.append(-float('inf'))
        new_policy[i, j] = actions[np.argmax(action_returns)]
```

6. Value/Policy Changes



value change 45727.511447

value change 37274.506706

value change 28991.916653

value change 22532.424045

value change 17731.181203

.

.

value change 0.000233

value change 0.000190

value change 0.000155

value change 0.000126

value change 0.000103

value change 0.000084

policy changed in 318 states

value change 4644.832146

value change 801.144919

value change 629.078073

value change 534.482508

value change 437.193226

.

.

value change 0.000272

value change 0.000221

value change 0.000180

value change 0.000146

value change 0.000119

value change 0.000097

policy changed in 260 states

. . .

value change 17.749357

value change 13.007169

value change 10.449136

value change 8.010925

value change 5.874832

.

.

value change 0.000264

value change 0.000215

value change 0.000175

value change 0.000142

value change 0.000116

value change 0.000094

policy changed in 10 states

. . .

value change 0.435993

value change 0.360778

value change 0.255713

value change 0.167818

value change 0.105331

.

.

value change 0.000286

value change 0.000229

value change 0.000185

value change 0.000149

value change 0.000121

value change 0.000098

policy changed in 0 states

7. Policy Result