Book Information

Name: Discrete Mathematics

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Contents

Chapter 1

Set Theory (Draft)

Set theory is a branch of mathematics where sets are studied. In general a set is a group of objects (that can have any type).

Sequences

Definitions

Sequence: A function with domain $\mathbb{W}$ or $\mathbb{N}$ and none-empty range is called a sequence. Elements that appear on the range are called terms of the sequence. We represent the i'th term by $a_i = f(i)$, which simply means the image of integer i is a_i.

We can have repetitions in a sequence and order matters, so a sequence with $a_1 = 1, a_2 = 2$ is not the same as a sequence with $a_1 = 2, a_2 = 1$

A. Sequence a_n is called strictly increasing if for every $n \in \mathbb{N}$ the below property holds:

$$a_{n+1} > a_n$$

B. Sequence a_n is called increasing if for every $n \in \mathbb{N}$ the below property holds:

$$a_{n+1} \geq a_n$$

C. Sequence a_n is called strictly decreasing if for every $n \in \mathbb{N}$ the below property holds:

$$a_{n+1} < a_n$$

D. Sequence a_n is called decreasing if for every $n \in \mathbb{N}$ the below property holds:

$$a_{n+1} \leq a_n$$

E. All of these mentioned sequences are called monotonic sequences.

F. A sequence is called bounded above if there exists an integer M such that for every $n \in \mathbb{N}$ we have:

$$a_n \leq M$$

G. A sequence is called bounded below if there exists an integer M such that for every $n \in \mathbb{N}$ we have:

$$a_n \geq M$$

H. A sequence is called boundeded if it's both bounded below and bounded above.

I. A sequence that's not bounded is called an unbounded sequence.

Subsequence: A sequence which some elements are removed from is called a subsequence [of that sequence]

We can see that $\{A, B, F\}$ is a subsequence of $\{A, B, C, D, E, F\}$, but not a subsequence of $\{B, A, C, D, E, F\}$ (since order is important).

Since subsequences are sequences themselves, it's no surprise that a subsequence can be increasing, decreasing, monotonic, etc.

Puzzle

Suppose we have a sequence $A = \{a_1, ..., a_n\}$ and our goal is to find an increasing subsequence of A which is as big as possible.

- One possible way to solve this problem is by simply writing every possible subsequence of A. Then we remove those subsequences which aren't increasing. Finally we find the subsequence with the biggest size. This approach is called a **bruteforce** algorithm

- We can use a greedy approach instead. Take the first element of A (which is a_1), Then find the least possible $i > 1$ such that $a_i \geq a_1$. After that, find the least $j > i$ such that $a_j \geq a_i$. Continue this algorithm to the point we can't add any more element (in each step if the least possible term is the i'th term, we append a_i to our answer). Unfortunately, this algorithm doesn't work ☹ (can you say why?)

The above question is called the Longest Increasing Subsequence (LIS for short). You can take a look at wikipedia for more information if you are interested.

Some Specific Sequences

Recursive Sequences

Generating Functions