# Experimental Evaluation of Control Policies for Segway Robot in Dynamic Environments

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ECE 6552: Nonlinear System Project

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- A hybrid approach applies the benefits and mitigates the shortcomings of both
- A nonlinear navigation control protocol can achieve navigational objectives while reacting to unexpected obstacles

# System Description

# Robotic Platform


**Figure 1:** Jeeves - A modified Segway based robot mounted with a Primesense, Microsfot Kinect, Roboteye RE05, Hokuyo LTM-30x laser and the UR5. Currently modified to include the casters for static stability

# Unicycle Dynamics

$$\begin{pmatrix} \dot{r} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} -v\ \cos(\delta) \\ \frac{v}{r}\ \sin(\delta) \end{pmatrix} \quad (1)$$

$$\dot{\delta} = \frac{v}{r}\ \sin(\delta) + \omega \qquad (2)$$

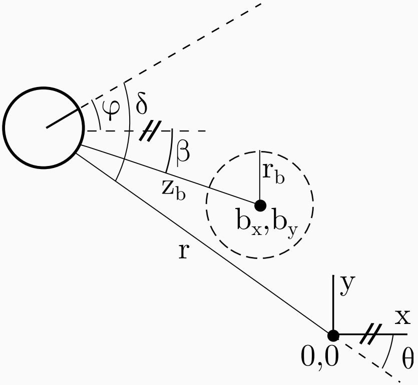


**Figure 2:** Egocentric Polar Unicycle Dynamics with Obstacle

# Approach

# Objectives

**Soft Constraints**

- Navigate to target location
- Drive the state of the robot to zero (e.g. $\dot{x} = (\dot{r}, \dot{\theta}, \dot{delta})^T \rightarrow 0$)

**Hard Constraints**

- Avoid obstacles
- The distance to the obstacle must not exceed a minimum distance (e.g. $z_b > z_{safe}$)

**System Constraints**

- Inputs should not exceed threshold values
- $0 < v < v_{max}$ and $0 < \omega < \omega_{max}$

# Approach Overview

A switched mode *quadratic programming controller* that **simultaneously** satisfies the soft constraints, hard constraints, and system constraints was designed.

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Feedback linearization of this objective by choosing the angular velocity:

$$\omega = -\frac{v}{r}\left[k_2 z_1 + \left(1 + \frac{k_1}{1 + (k_1\theta)^2}\right)\sin(z_1 + \arctan(-k_1\theta))\right]$$

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$$\dot{z}_1 = -k_2\frac{v}{r}z_1$$

$\implies$ *Stabilization of the steering objective*

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$$\dot{V} < 0$$

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$$\dot{V} = -rv\,\cos(\arctan(k_1\beta)) + \theta v\,\sin(\arctan(-k_1\beta))$$

$$\dot{V} \nleq 0$$

$\implies$ *The second steering control drives the state away from zero*

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$$\dot{h}(x) =$$

# Switched Mode QP Controller

# Results

# Simulation Results

needs octo plots!

## Experimental Results

needs robots!

# Conclusions

# References I


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**Questions?**