# Experimental Evaluation of Control Policies for Segway Robot in Dynamic Environments

Ian Buckley, Niharika Arora, Varun Murali

ECE 6552: Nonlinear System Project

26 April 2015

# Motivation

- Navigation is a fundamental objective in mobile robotics

# Motivation

- Navigation is a fundamental objective in mobile robotics
- Deliberative path planning suffers from unexpected conditions (i.e. Probabilistic Roadmaps [1])

# Motivation

- Navigation is a fundamental objective in mobile robotics
- Deliberative path planning suffers from unexpected conditions (i.e. Probabilistic Roadmaps [1])
- Reactive methods suffer from local minima (i.e. Potential Fields [2])

# Motivation

- Navigation is a fundamental objective in mobile robotics
- Deliberative path planning suffers from unexpected conditions (i.e. Probabilistic Roadmaps [1])
- Reactive methods suffer from local minima (i.e. Potential Fields [2])
- A hybrid approach applies the benefits and mitigates the shortcomings of both

# Motivation

- Navigation is a fundamental objective in mobile robotics
- Deliberative path planning suffers from unexpected conditions (i.e. Probabilistic Roadmaps [1])
- Reactive methods suffer from local minima (i.e. Potential Fields [2])
- A hybrid approach applies the benefits and mitigates the shortcomings of both
- A nonlinear navigation control protocol can achieve navigational objectives while reacting to unexpected obstacles

# Robotic Platform

Figure : Jeeves - A modified Segway based robot mounted with a Primesense, Microsfot Kinect, Roboteye RE05, Hokuyo LTM-30x laser and the UR5. Currently modified to include the casters for static stability

# Unicycle Dynamics

$$\begin{pmatrix} \dot{r} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} -v \cos(\delta) \\ \frac{v}{r} \sin(\delta) \end{pmatrix} \quad (1)$$

$$\dot{\delta} = \frac{v}{r} \sin(\delta) + \omega \qquad (2)$$

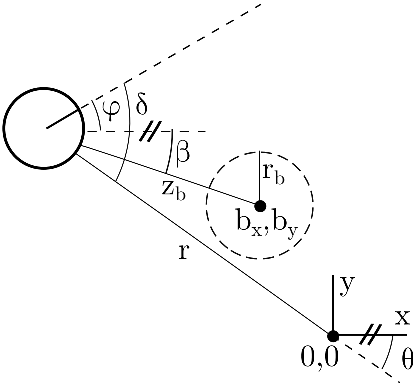


Figure : Egocentric Polar Unicycle Dynamics with Obstacle

# Objectives

**Soft Constraints**

- Navigate to target location
- Drive the state of the robot to zero (e.g. $\dot{x} = (\dot{r}, \dot{\theta}, \dot{delta})^T \rightarrow 0$)

**Hard Constraints**

- Avoid obstacles
- The distance to the obstacle must not exceed a minimum distance (e.g. $z_b > z_{safe}$)

**System Constraints**

- Inputs should not exceed threshold values
- $0 < v < v_{max}$ and $0 < \omega < \omega_{max}$

# Approach Overview

A switched mode *quadratic programming controller* that **simultaneously** satisfies the soft constraints, hard constraints, and system constraints was designed.

# Soft Constraints as CLFs

To establish soft constraints as CFLs:

1. Design a steering control that drives the state to zero [3]

# Soft Constraints as CLFs

To establish soft constraints as CFLs:

1. Design a steering control that drives the state to zero [3]
2. Demonstrate that the steering control satisfies the control Lyapunov function

# Soft Constraints as CLFs

To establish soft constraints as CFLs:

1. Design a steering control that drives the state to zero [3]
2. Demonstrate that the steering control satisfies the control Lyapunov function
3. Design a second steering control that drives the state *away* from zero

# Soft Constraints as CLFs

1. **Design a steering control that drives the state to zero**

$$z_1 = \delta - \arctan(-k_1\theta)$$

# Soft Constraints as CLFs

1. **Design a steering control that drives the state to zero**

$$z_1 = \delta - \arctan(-k_1\theta)$$

$$\dot{z}_1 = \left(1 + \frac{k_1}{1 + (k_1\theta)^2}\right)\frac{v}{r}\sin(z_1 + \arctan(-k_1\theta)) + \omega$$

# Soft Constraints as CLFs

1. **Design a steering control that drives the state to zero**

$$z_1 = \delta - \arctan(-k_1\theta)$$

$$\dot{z}_1 = \left(1 + \frac{k_1}{1 + (k_1\theta)^2}\right)\frac{v}{r}\sin(z_1 + \arctan(-k_1\theta)) + \omega$$

Feedback linearization of this objective by choosing the angular velocity:

$$\omega = -\frac{v}{r}\left[k_2 z_1 + \left(1 + \frac{k_1}{1 + (k_1\theta)^2}\right)\sin(z_1 + \arctan(-k_1\theta))\right]$$

# Soft Constraints as CLFs

1. **Design a steering control that drives the state to zero**

$$z_1 = \delta - \arctan(-k_1\theta)$$

$$\dot{z}_1 = \left(1 + \frac{k_1}{1+(k_1\theta)^2}\right)\frac{v}{r}\sin(z_1 + \arctan(-k_1\theta)) + \omega$$

Feedback linearization of this objective by choosing the angular velocity:

$$\omega = -\frac{v}{r}\left[k_2 z_1 + \left(1 + \frac{k_1}{1+(k_1\theta)^2}\right)\sin(z_1 + \arctan(-k_1\theta))\right]$$

$$\dot{z}_1 = -k_2\frac{v}{r}z_1$$

$\implies$ *Stabilization of the steering objective*

# Soft Constraints as CLFs

2. **Demonstrate that the steering control satisfies the control Lyapunov function**

$$\delta = \arctan(-k_1 \theta)$$

# Soft Constraints as CLFs

2. **Demonstrate that the steering control satisfies the control Lyapunov function**

$$\delta = \arctan(-k_1\theta)$$

$$V = \frac{1}{2}(r^2 + \theta^2)$$

# Soft Constraints as CLFs

2. **Demonstrate that the steering control satisfies the control Lyapunov function**

$$\delta = \arctan(-k_1\theta)$$

$$V = \frac{1}{2}(r^2 + \theta^2)$$

$$\dot{V} = -rv\,\cos(\delta) + \theta v\,\sin(\delta)$$

# Soft Constraints as CLFs

2. **Demonstrate that the steering control satisfies the control Lyapunov function**

$$\delta = \arctan(-k_1\theta)$$

$$V = \frac{1}{2}(r^2 + \theta^2)$$

$$\dot{V} = -rv\ \cos(\delta) + \theta v\ \sin(\delta)$$

$$\dot{V} = -rv\ \cos(\arctan(-k_1\theta)) + \theta v\ \sin(\arctan(-k_1\theta))$$

# Soft Constraints as CLFs

2. **Demonstrate that the steering control satisfies the control Lyapunov function**

$$\delta = \arctan(-k_1\theta)$$

$$V = \frac{1}{2}(r^2 + \theta^2)$$

$$\dot{V} = -rv\ \cos(\delta) + \theta v\ \sin(\delta)$$

$$\dot{V} = -rv\ \cos(\arctan(-k_1\theta)) + \theta v\ \sin(\arctan(-k_1\theta))$$

$$\dot{V} < 0$$

$\implies$ *Stabilization of the state to zero*

# Soft Constraints as CLFs

3. **Design a second steering control that drives the state away from zero**

$$z_2 = \delta - \arctan(-k_1\beta)$$

# Soft Constraints as CLFs

3. **Design a second steering control that drives the state away from zero**

$$z_2 = \delta - \arctan(-k_1 \beta)$$

Similar to $z_1$...

# Soft Constraints as CLFs

3. **Design a second steering control that drives the state away from zero**

$$z_2 = \delta - \arctan(-k_1\beta)$$

Similar to $z_1$...

$$\dot{z}_2 = -k_2 \frac{v}{r} z_2$$

$\Longrightarrow$ *Stabilization of the second steering objective*

# Soft Constraints as CLFs

3. **Design a second steering control that drives the state away from zero**

$$z_2 = \delta - \arctan(-k_1 \beta)$$

Similar to $z_1$...

$$\dot{z}_2 = -k_2 \frac{v}{r} z_2$$

$\implies$ *Stabilization of the second steering objective*

$$\dot{V} = -rv \cos(\arctan(k_1 \beta)) + \theta v \sin(\arctan(-k_1 \beta))$$

# Soft Constraints as CLFs

3. **Design a second steering control that drives the state away from zero**

$$z_2 = \delta - \arctan(-k_1\beta)$$

Similar to $z_1$...

$$\dot{z}_2 = -k_2 \frac{v}{r} z_2$$

$\implies$ *Stabilization of the second steering objective*

$$\dot{V} = -rv\cos(\arctan(k_1\beta)) + \theta v \sin(\arctan(-k_1\beta))$$

$$\dot{V} \nleq 0$$

$\implies$ *The second steering control drives the state away from zero*

# Hard Constraint as ZBF [4],[5],[6]

$$\dot{h}(x) <= \gamma h(x)$$

# Hard Constraint as ZBF [4],[5],[6]

$$\dot{h}(x) <= \gamma h(x)$$

$$C = x \in \mathbb{R}^3 | z_b \geq z_{safe}$$
$$\partial C = x \in \mathbb{R}^3 | z_b = z_{safe}$$
$$\mathrm{Int}(C) = x \in \mathbb{R}^3 | z_b > z_{safe}$$

# Hard Constraint as ZBF [4],[5],[6]

$$\dot{h}(x) <= \gamma h(x)$$

$$\begin{aligned} C &= x \in \mathbb{R}^3 | z_b \geq z_{safe} \\ \partial C &= x \in \mathbb{R}^3 | z_b = z_{safe} \\ \mathrm{Int}(C) &= x \in \mathbb{R}^3 | z_b > z_{safe} \end{aligned}$$

$$h(x) = \sqrt{(-v\ \cos(\theta) - b_x)^2 + (v\ \sin(\theta) - b_y)^2}$$

# Hard Constraint as ZBF [4],[5],[6]

$$\dot{h}(x) <= \gamma h(x)$$

$$C = x \in \mathbb{R}^3 | z_b \geq z_{safe}$$
$$\partial C = x \in \mathbb{R}^3 | z_b = z_{safe}$$
$$\mathrm{Int}(C) = x \in \mathbb{R}^3 | z_b > z_{safe}$$

$$h(x) = \sqrt{(-v\ \cos(\theta) - b_x)^2 + (v\ \sin(\theta) - b_y)^2}$$

$$\dot{h}(x) =$$

# Switched Mode QP Controller

# Simulation Results

needs octo plots!

# Experimental Results

needs robots!

# References I


L. E. Kavraki, P. Svestka, J. C. Latombe, and M. H. Overmars, "Probabilistic roadmaps for path planning in high-dimensional configuration spaces," in *RIEEE Transactions on Robotics and Automation (Volume:12 , Issue: 4 )*. IEEE, 1996, pp. 566–580.

O. Khatib, "Real-time obstacle avoidance for manipulators and mobile robots," in *Robotics and Automation. Proceedings. 1985 IEEE International Conference on (Volume:2 )*. IEEE, 1985, pp. 500–505.

J. J. Park and B. Kuipers, "A smooth control law for graceful motion of differential wheeled mobile robots in 2d environment," in *Robotics and Automation (ICRA), 2011 IEEE International Conference on*. IEEE, 2011, pp. 4896–4902.

A. D. Ames, J. W. Grizzles, and P. Tabuada, "Control barrier function based quadratic programs with application to adaptive cruise control," in *53rd IEEE Conference on Decision and Control*. IEEE, 2014, pp. 6271–6278.

X. Xu, P. Tabuada, J. W. Grizzle, and A. D. Ames, "Robustness of control barrier functions for safety critical control," in *Analysis and Design of Hybrid Systems(Volume 48)*, 2015, pp. 54–61.

# References II

A. D. Ames, K. Galloway, K. Sreenath, and J. W. Grizzle, "Rapidly exponentially stabilizing control lyapunov functions and hybrid zero dynamics," in *IEEE Transactions on Automatic Control (Volume:59 , Issue: 4 )*. IEEE, 2014, pp. 876–891.

Questions?