Ministry of Higher Education and Scientific Research

ESAT UNIVERSITY

University of Aviation & Technology

end of year project

Analysis of truss structure:
Howe-truss type

Elaborated by

Kadri Iheb **Farhat Mustapha**

Academic supervisor

Mr. Karoui Amine

Academic year: 2020/2021

Table of Contents

List of figures:

List of tables:

General and problematic context

As time passed, engineers became increasingly ambitious with building projects. Larger projects proved difficult. Tremendous pressure was placed on materials that were unable to bear significant weight. Consequently, engineers developed the truss. Trusses are web-shaped structures used to bear tremendous weight. Used in buildings and bridges of all sizes, trusses allow builders to extend the dimensions of structures and create interesting shapes. Because of their light weight and high strength, trusses are widely used, and their applications range from supporting bridges and roofs of buildings to being support structures in space stations. Thus, it is common to use truss in the aeronautical industry.

Furthermore, truss analysis has become mandatory either to size or to study its comportment due to compression and tension in different situations, shapes and uses. However, sizing a truss can be presented in 3 different approaches theoretically, experimentally, and digitally. These different methods provide not only the difficulty for engineers to choose which approach is more relevant and efficient. But also, the difference between these 3 approaches of analysis.

Based on this major problem, we decided to size a howe-truss using these 3 different approaches. Then, compare the results of each methods to stimulate the difference between each and choose which one is more efficient in truss analysis.

In order to achieve our goals, we will firstly make a bibliographic summary to provide the basic of truss analysis such as concepts and assumptions for static analysis of trusses. Secondly, we need to Size a howe-truss structure theoretically using both the method of joints and section method. Thirdly, we will analyze the same truss experimentally using a howe – truss Gunt. Then, we will also simulate the truss using digital systems based on the finite element methods such as RDM 6 and Abaqus software. Finally, we need to compare the different results by each approach and study the efficiency of these methods.

Chapter 1: Bibliographic summary

Chapter 1. Bibliographic summary

I. Truss structure

I.1 Definition of truss structure

A truss is an assembly of beams or other elements that creates a rigid structure. It is a simple structure whose members are subject to axial compression and tension only and but not bending moment.[2]

In engineering, a truss is a structure that consists of two-force members only, where the members are organized so that the assemblage behaves as a single object. A two-force member is a structural component where force is applied to only two points. Although this rigorous definition allows the members to have any shape connected in any stable configuration, trusses typically comprise five or more triangular units constructed with straight members whose ends are connected at joints referred to as nodes.

In this typical context, external forces and reactions to those forces are considered to act only at the nodes and result in forces in the members that are either tensile or compressive. For straight members, moments (torques) are explicitly excluded because, and only because, all the joints in a truss are treated as revolute, as is necessary for the links to be two-force members.

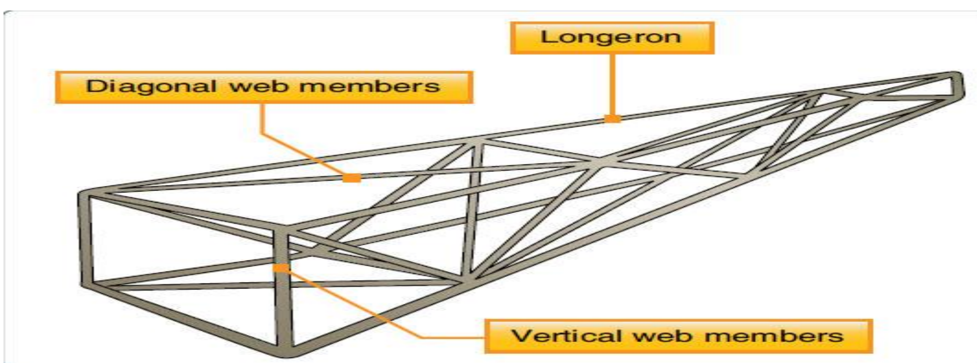


Figure 1: Truss structure of airplane fuselage

Source: Aeronautics guide

I.2 The efficiency of truss use

Structures frequently make use of trusses because of the additional support and strength that they provide. Triangular shapes are particularly stable because the length of the sides of the legs and the measure of the angles must correlate, making it difficult for triangles to lose their shape.

Trusses can resist the force of a great deal of weight compared to rectangular structures made from the same materials because a downward force applied to the corner of a triangle can be easily diffused throughout the shape. These features of triangles make the truss structure common in engineering and architecture.

Not only but also, they are also an extremely strong, well-accepted, cost-effective option for the construction of various structures. To maximize the efficiency of the structure.

In fact, the main principle of the truss structure system is the arrangement of the rod forming a triangular configuration, because the triangular shape (triangulation) is a good or stable form in making a balance to direct the load bearing into a normal truss force, namely compressive and tensile loads. Whereas the load acting on the truss shape which is not triangular with the joint laying system causes the structure to become unstable and will result in relatively large deformations as shown in Figure below.[1]

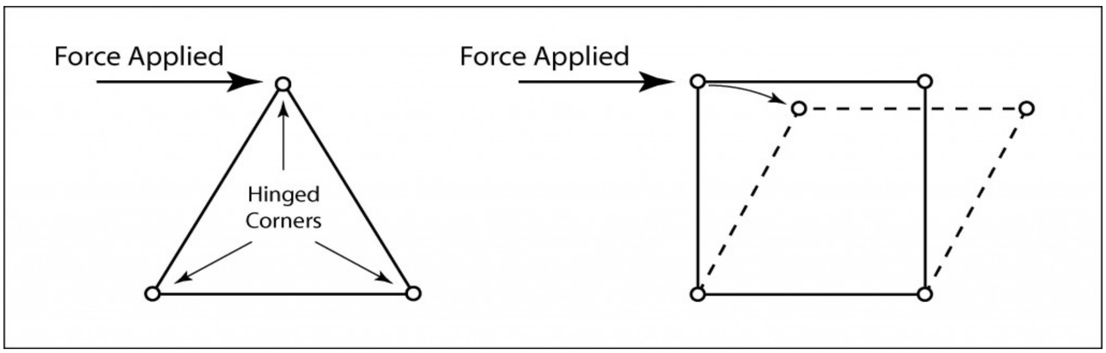


Figure 2: Difference of force application on triangles and rectangles shapes

Source: Civil engineering: compression members in steel structure

I.3 Truss members in Tension and Compression

every physical object which is in contact applies some force on one another. the tension force is the force that is transmitted through the member when it is pulled tight by forces acting from opposite ends. it is directed along the length of the cable and pulls equally on the objects on the opposite ends. tension may also be described as the action-reaction pair of forces acting at each end of the said elements. tension could be the opposite of compression.

However, Compression force (or compressive force) occurs when a physical force presses inward on an object, causing it to become compacted. In this process, the relative positions of atoms and molecules of the object change. This change can be temporary or permanent depending on the type of material receiving the compressive force. There can also be different results depending on the direction or position on the object that the compressive force is applied. [1]

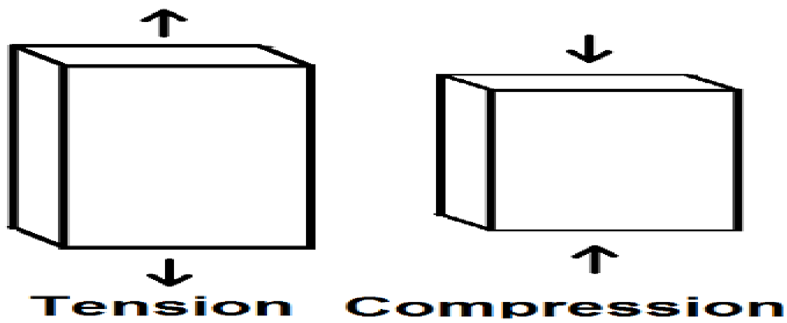


Figure 3: The difference between tension and compression

Source: Civil engineering: compression members in steel structure

The forces acting on the trunks of the truss structure are distributed in each of the forming rod to the fulcrum, so that there is a rod that accepts the tensile force and compressive force. The method used to determine the forces acting on the truss is based on a review of balance at the connecting point. In a simple trunk configuration, the nature of the force of the tensile or compressive can be determined by giving an idea of how the truss carry the load.

I.4 Types of truss

The truss structure system has many variations in shape. Determining of rod configuration is the initial stage in designing the frame structure before the rod style analysis process and determining the size of each structural element so that the trunk configuration will be used in accordance with the shape of the designed structure. Some forms of system configuration for truss structures are commonly used such as.

Pratt truss

Pratt truss The Pratt truss was patented in 1844 by two Boston railway engineers, Caleb Pratt, and his son Thomas Willis Pratt. The design uses vertical members for compression and diagonal members to respond to tension. it reduces the cost of the structure due to more efficient members, reduces the self-weight and eases the constructability of the structure. This type of truss is most appropriate for horizontal spans, where the force is predominantly in the vertical direction.[2]

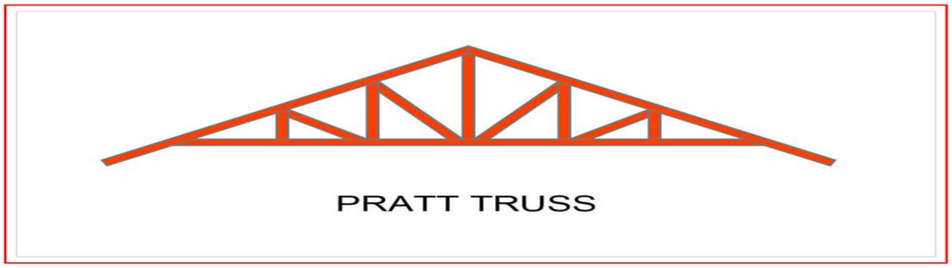


Figure 4: Pratt truss structure

Source: Trusses and Types of trusses - Basic Civil Engineering

Pratt truss is characterized by its simple, well accepted and used design which facilitates the awareness of members behavior (diagonal members are in tension, vertical members in compression). Unfortunately, it is not as advantageous if the load is not vertical. So, engineers often use it where a mix of loads are applied, and a simple structure is required.

Howe truss

A Howe truss is a truss bridge consisting of chords, verticals, and diagonals whose vertical members are in tension and whose diagonal members are in compression. The Howe truss was invented by William Howe in 1840 and was widely used as a bridge in the mid to late 1800s.

This type of truss was highly economical due to its ease of construction and simple cost-effective structure, which is widely used in wood buildings, particularly in providing roof support. in fact, the design of Howe truss is the opposite to that of Pratt truss. Both trusses found favor because they used far fewer members. [2]

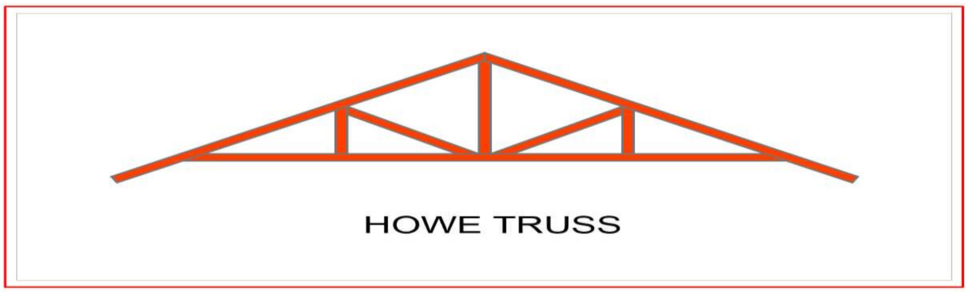


Figure 5: Howe truss structure

Source: Trusses and Types of trusses - Basic Civil Engineering

Warren truss

The Warren Truss is another very popular truss structure system and is easily identified by its construction from equilateral triangles. One of the main advantages of a Warren Truss is its ability to spread the load evenly across several different members; this is however generally for cases when the structure is undergoing a spanned load (a distributed load).

Warren truss contains a series of isosceles triangles or equilateral triangles. To increase the span length of the truss structure. The equilateral triangles minimize the forces to only compression and tension. Interestingly, as a load the forces for a member switch from compression to tension. This happens especially to the members near the center. [2]

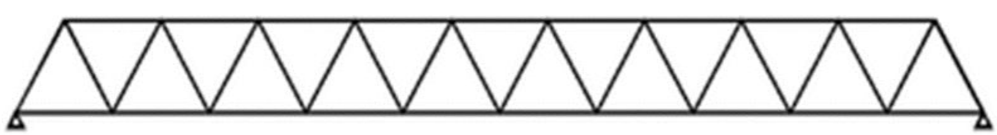


Figure 6: Warren truss structure (simple truss)

Source: Trusses and Types of trusses - Basic Civil Engineering

Its main advantage is also the cause of its disadvantage that the truss structure will undergo concentrated force under a point load. Under these concentrated load scenarios, the structure is not as good at distributing the load evenly across its members. Therefore, the Warren truss type is more advantageous for spanned loads, but not suitable where the load is concentrated at a single point or node. Thus, Warren truss is often used where an evenly distributed load is to be supported due to its poor performance under concentrated loads.

Fink Truss

The Fink truss in its most basic form has web members that follow a V-pattern which can be repeated several times. As the top chords are sloping downward from the center, the V pattern becomes noticeably smaller. As Fink trusses rely more on diagonal members, they can be very efficient at transmitting loads to the support. [2]

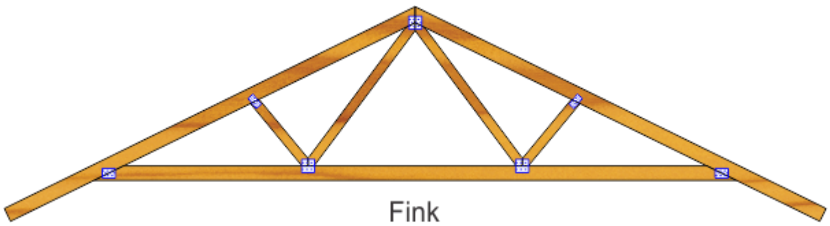


Figure 7: Fink truss structure

Source: Trusses and Types of trusses - Basic Civil Engineering

Derivatives of the Fink truss include the Double Fink and the Fan truss types. Double Fink trusses are essentially Fink trusses that repeat the pattern twice on either side. If the most basic Fink truss can be characterized by a double-V, then a double fink would look like a double-W. Fan trusses are essentially Fink trusses that have its web members 'fan out' from the joints at the bottom, usually the addition of vertical members. [2]

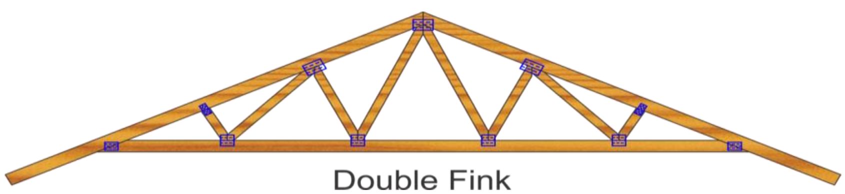


Figure 8: Double fin truss structure

Source: Trusses and Types of trusses - Basic Civil Engineering

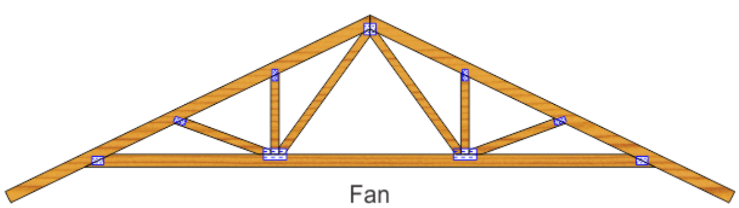


Figure 9: Fan truss structure

Source: Trusses and Types of trusses - Basic Civil Engineering

II. The use of truss in aerospace

The construction of aircraft fuselages evolved from the early wood truss structural arrangements to monocoque shell structures to the current semi monocoque shell structures.

Since the early aircraft construction up to nowadays, it is common to use trusses in aircraft construction more precisely in fuselage, wing, and tail structure.

The figure below shows the frame structure of Flyer 1903, Wright brothers, USA Take-off mass 283 kg, wingspan 12 m. [4]

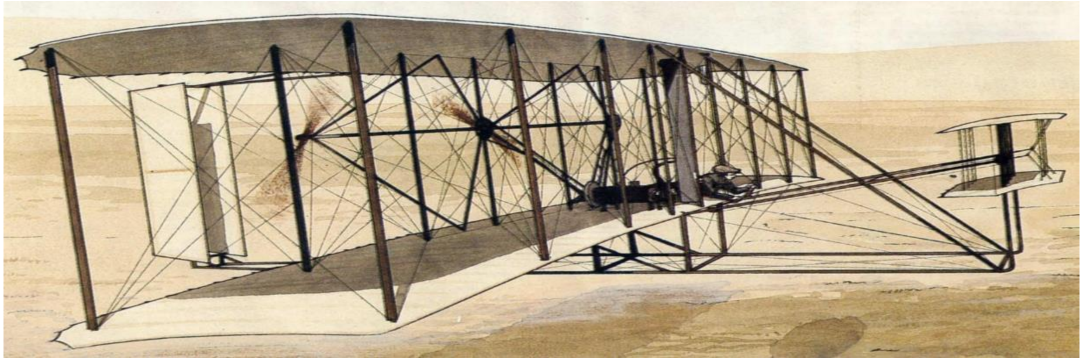


Figure 10: The frame structure of flyer 1903, wright brothers

Source: History of the Airplane and Flight: Orville and Wilbur Wright

In the other hand, this figure shows the frame airplane structure used in 1970 of Steen Sky-Bolt for an aerobatic bi-plane. [5]

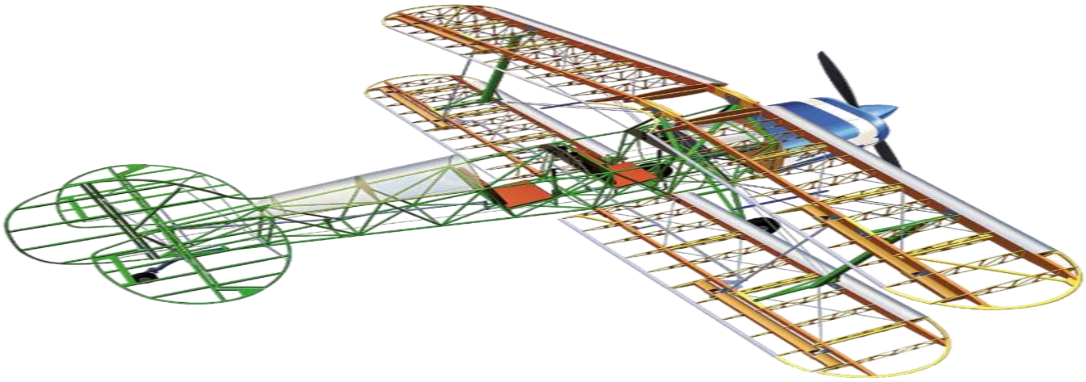


Figure 11: The frame of airplane structure in 1970

Source: Air Travel in the 1970s - Exploring the Seventies

II.1 Truss in airplane fuselage

Most early aircraft used this technique with wood and wire trusses and this type of structure is still in use in many lightweight aircraft using welded steel tube trusses. The truss type fuselage frame is assembled with members forming a rigid frame. Primary members of the truss are 4 longerons. There are two common types of truss structure used: Pratt and Warren trusses.

As for the Pratt truss, it was used in the early days throw wooden or metallic construction. Therefore, it presented not only an enormous weight but also a great difficulty to streamline as one of its main disadvantages. To strengthen the truss, Pratt structure is boxed with tubular longerons and vertical members.

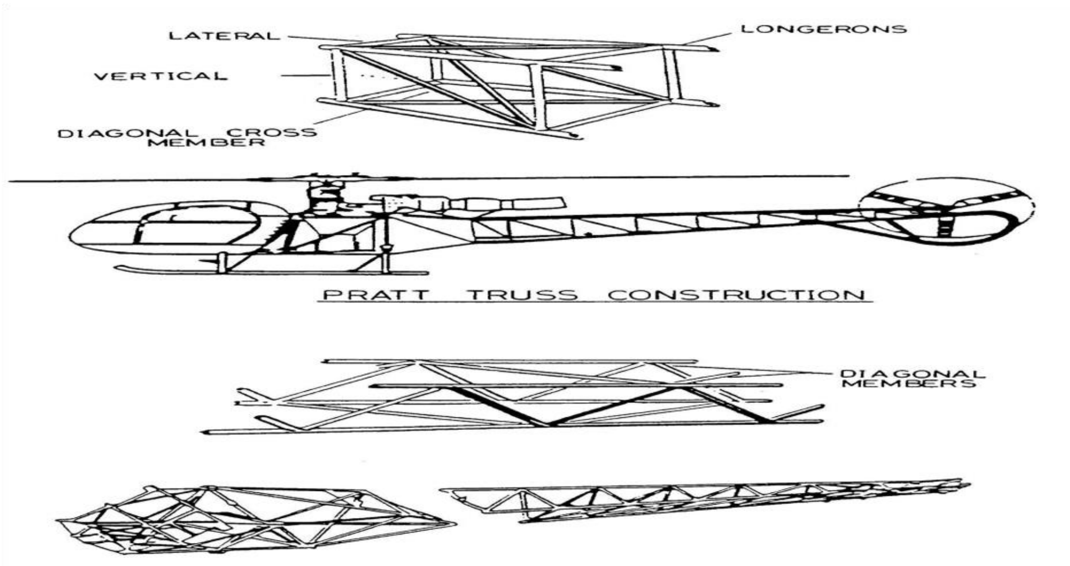


Figure 12: Pratt truss structure in airplane fuselage

Source: AIRFRAME STRUCTURAL DESIGN (INSTITUTE OF AERONAUTICAL ENGINEERING)

As for the Warren truss, it is more used nowadays because they are capable to transfer the force to every other structure. Furthermore, they present more space, strength, rigidity, and less weight because of the reduction of the amount of webs work. It is characterized by a better streamline because the structure is boxed with longerons plus only diagonal members. [6]

The Warren truss-type fuselage frame is usually constructed of steel tubing welded together in such a manner that all members of the truss can carry both tension and compression loads. In some aircraft, principally the light, single engine models, truss fuselage frames may be constructed of aluminum alloy and may be riveted or bolted into one piece, with cross-bracing achieved by using solid rods or tubes. This figure shows a Warren truss structure of an airplane.

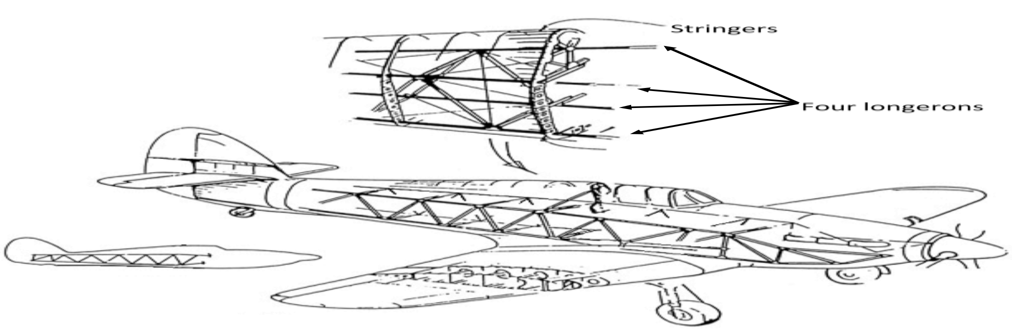


Figure 13: Airplane fuselage construction

Source: AIRFRAME STRUCTURAL DESIGN (INSTITUTE OF AERONAUTICAL ENGINEERING)

II.2 Truss in airplane wings

Wings are airfoils that, when moved rapidly through the air, create lift. They are built in many shapes and sizes. Wing design can vary to provide certain desirable flight characteristics. Control at various operating speeds. Often wings are of full cantilever design. This means they are built so that no external bracing is needed. They are supported internally by structural members assisted by the skin of the aircraft.

The internal structures of most wings are made up of spars and stringers running spanwise and ribs and formers or bulkheads running chordwise. The spars are the principle structural members of a wing. They support all distributed loads, as well as concentrated weights. The skin, which is attached to the wing structure, carries part of the loads imposed during flight. It also transfers the stresses to the wing ribs. The ribs, in turn, transfer the loads to the wing spars. [6]

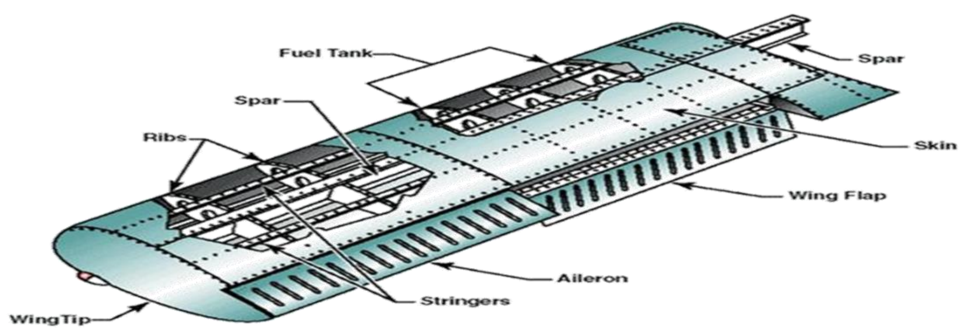


Figure 14: Wing structure and composition

Source: AIRFRAME STRUCTURAL DESIGN (INSTITUTE OF AERONAUTICAL ENGINEERING)

More precisely, Ribs are the structural crosspieces that combine with spars and stringers to make up the framework of the wing. They give the wing its cambered shape and transmit the load from the skin and stringers to the spars. Similar ribs are also used in ailerons, elevators, rudders, and stabilizers. Wing ribs are usually manufactured from either wood or metal. Wood ribs are usually manufactured from spruce. The three most common types of wooden ribs are the plywood web, the lightened plywood web, and the truss types. Of these three, the truss type is the most efficient because it is strong and lightweight, but it is also the most complex to construct.

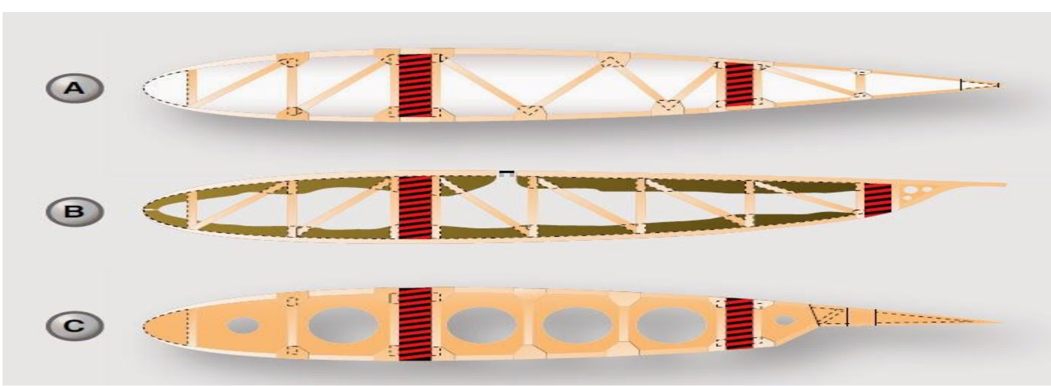


Figure 15: Examples of wing ribs constructed of wood.

Source: Aeronautics guide

These figure shows examples of different wing ribs constructed in woods.

As for type A, the cross-section of a wing rib with a truss-type web is illustrated. The dark rectangular sections are the front and rear wing spars.

As for type B, the truss web rib is shown with a continuous gusset. It provides greater support throughout the entire rib with very little additional weight. This aids in preventing buckling and helps to obtain better rib-skin joints and can resist the driving force of nails better than the other types.

As for type C, it shows a rib with a lighten plywood web. It also contains gussets to support the web/cap strip interface. The cap strip is usually laminated to the web, especially at the leading edge.

III. Concepts for static analysis of truss

As shown previously, trusses are composed from members and joints (or nodes). In the truss analysis, it is fundamental that the sum of forces at each joint, or node, must equal zero. Furthermore, joints are pinned and frictionless which means that pins will not support a moment. Additionally, each element is a two-force member which provides that the direction of the force is along the axis; If an element is in tension, it will pull on both joints. However, if an element is in compression, it will push on both joints. In fact, a force cannot be applied at any point, just at the ends which guaranties no bending for the members. Finally, the external reactions are statically determinant, and the supports are frictionless.

III.1 The method of joints

The analysis of Trusses by the Method of Joints is based on 5 fundamental steps.

Firstly, if the support reactions are not given, draw a FBD of the entire truss and determine all the support reactions using the equations of equilibrium. Secondly, locate a joint with only two members, and draw the FBD of that pin. Determine the unknown forces at that joint. thirdly, locate a member where the forces in only two members are still unknown. Then, apply the scalar equations of equilibrium, $\sum FX = 0$ and $\sum FY = 0$, to determine the unknown(s). If the answer is positive, then it is tension, otherwise it is compression. Finally, repeat this process until all member forces are known. [7]

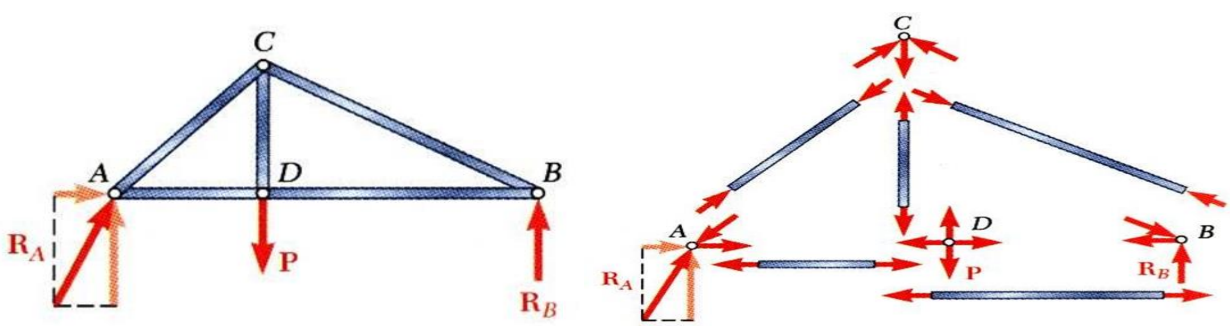


Figure 16: Truss members reaction to forces and reactions applied using joints methods.

Source: Engineering code courses (introduction to plane truss)

Furthermore, truss analysis can be sampled under Special Loading Conditions. In fact, forces in opposite members intersecting in two straight lines at a joint are equal, forces in two members connected at a joint are equal if the members are aligned or zero otherwise, and forces in two opposite members are equal when a load is aligned with a third member.

Clearly, recognition of joints under special loading conditions simplifies a truss analysis.

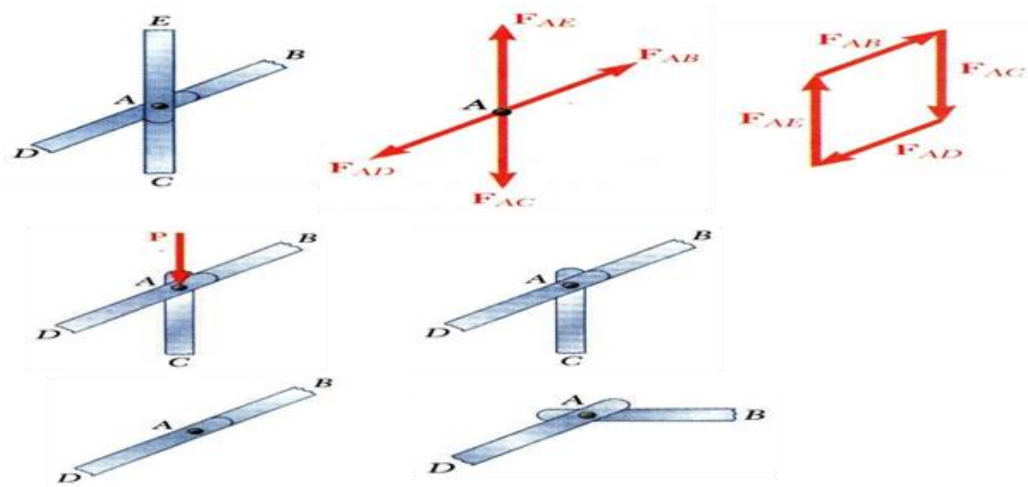


Figure 17: Truss members reaction to forces and reactions applied using joints methods.

Source: Engineering code courses (introduction to plane truss)

III.2 The method of sections

The analysis of Trusses by the Method of sections is based on 4 fundamental steps.

First, decide how you need to cut the truss. This is based on where you need to determine forces, and where the total number of unknowns does not exceed three. Second, decide which side (left or right) of the cut truss will be easier to work with by minimizing the number of reactions you have to find. Third, if required determine the necessary support reactions by drawing a FBD of the entire truss and applying the equations of equilibrium. Finally, apply the equations of equilibrium to the selected cut section of the truss to solve for the unknown member forces. [7]

Chapter 2: Truss analysis

Chapter 2. Truss analysis

I. Introduction:

In this analysis, we will focus on a howe-truss problematic structure which is constructed of 13 bars interconnected by 8 joints. Furthermore, the structure has two forms of connectivity in both A and E. additionally, it is applied to a down-pulling force represented by the symbol F in C. This pulling force is directed down with a variable vector quantity; in fact, its magnitude can change from 100N up to 400N.

The 13 bars are the mix of:

- 7 members (AB, BC, CH, CF, CD, DE, and CG) lengthened 140mm
- 4 members (AH, HG, GF, and FE) lengthened 121mm
- 2 members (HB and DF) lengthened 40mm

All 13 members are cylindric bars composed of steel characterized of Young modulus equals to E=6N/m^2 and a diameter equals to D=5.98mm.

Moreover, this figure below presents schematically the structure of howe-truss used in this analysis.

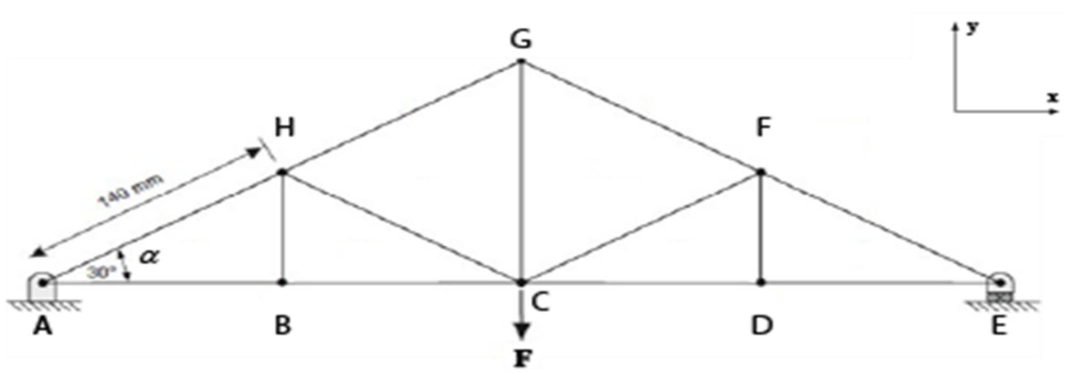


Figure 18: The howe-truss structure used in this study.

Source: Elaborated by the authors

II. Theoretical study

II.1 Determination of the unknowns and the type of structure

$X_A, Y_A, Y_E, \mu_{AB}, \mu_{BC}, \mu_{CD}, \mu_{DE}, \mu_{EF}, \mu_{FG}, \mu_{AH}, \mu_{HG}, \mu_{BH}, \mu_{CH}, \mu_{CF}, \mu_{DF},$
μ_{CG} = 16 Unknown

Equations numbers = 2 * 8 =16 $\Rightarrow$ Isostatic Structure

II.2 Truss balance

In A $\begin{Bmatrix} X_A & 0 \\ Y_A & 0 \\ 0 & 0 \end{Bmatrix}_A$, In C $\begin{Bmatrix} 0 & 0 \\ -F & 0 \\ 0 & 0 \end{Bmatrix}_C \xrightarrow{A} \begin{Bmatrix} 0 & 0 \\ -F & 0 \\ 0 & \dfrac{-FL}{2} \end{Bmatrix}_A$,

In E $\begin{Bmatrix} 0 & 0 \\ Y_E & 0 \\ 0 & 0 \end{Bmatrix}_E \xrightarrow{A} \begin{Bmatrix} 0 & 0 \\ Y_E & 0 \\ 0 & Y_E L \end{Bmatrix}_A$

By establishing the PFS we obtain,

$$\begin{Bmatrix} X_A = 0 \\ Y_A - F + Y_E = 0 \\ \dfrac{-FL}{2} + Y_E L = 0 \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} Y_E = \dfrac{FL}{2L} \\ Y_A = F - Y_E \end{Bmatrix} \Leftrightarrow \boxed{X_A = 0} \boxed{Y_E = \frac{F}{2}} \boxed{Y_A = \frac{F}{2}}$$

II.3 Method of joints

Using this method, we need to divide the truss structure to its composing joint. Thus, we will study each joint one by one. To different the joints, we will use the appointment shown previously.

The method of joints analyzes the force in each member of a truss by breaking the truss down and calculating the forces at each individual joint. Newton's Third Law indicates that the forces of action and reaction between a member and a pin are equal and opposite. Therefore, the forces exerted by a member on the two pins it connects must be directed along that member. This will be more clearly seen in the next few steps.

The analysis of the truss reduces to computing the forces in the various members, which are either in tension or compression.

In joint A

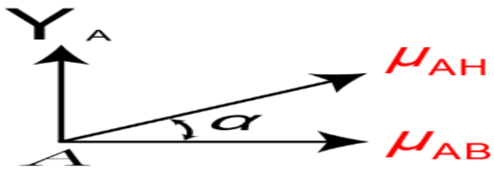


Figure 19: Forces and reactions in joint A

Source: Elaborated by the authors

$$\begin{Bmatrix} \sum F_x = 0 \\ \sum F_y = 0 \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} \mu_{AB} + \mu_{AH}\cos(\alpha) = 0 \\ Y_A + \mu_{AH}\sin(\alpha) = 0 \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} \mu_{AH} = \dfrac{-Y_A}{\sin(\alpha)} = \dfrac{-F}{2\sin(\alpha)} \\ \mu_{AB} = -\mu_{AH}\cos(\alpha) = \dfrac{F}{2\tan(\alpha)} \end{Bmatrix} \Leftrightarrow \begin{matrix} \boxed{\mu_{AH} = \dfrac{-F}{2\sin(\alpha)}} \\ \boxed{\mu_{AB} = \dfrac{F}{2\tan(\alpha)}} \end{matrix}$$

In joint E

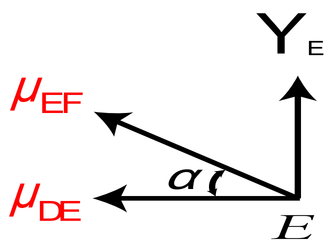
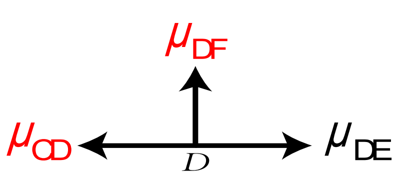


Figure 20: Forces and reactions in joint E

Source: Elaborated by the authors

$$\begin{Bmatrix} \sum F_x = 0 \\ \sum F_y = 0 \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} -\mu_{DE} - \mu_{EF}\cos(\alpha) = 0 \\ Y_E + \mu_{EF}\sin(\alpha) = 0 \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} \mu_{EF} = \dfrac{-Y_E}{\sin(\alpha)} \\ \mu_{AB} = -\mu_{EF}\cos(\alpha) \end{Bmatrix} \Leftrightarrow \begin{matrix} \boxed{\mu_{EF} = \dfrac{-F}{2\sin(\alpha)}} \\ \boxed{\mu_{DE} = \dfrac{F}{2\tan(\alpha)}} \end{matrix}$$

In joint D

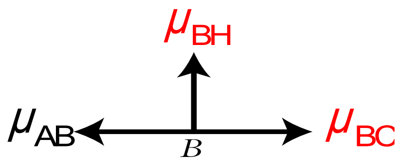


$$\begin{Bmatrix} \sum F_x = 0 \\ \sum F_y = 0 \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} \mu_{CD} = \mu_{DE} \\ \mu_{DF} = 0 \end{Bmatrix} \Leftrightarrow \begin{matrix} \boxed{\mu_{DF} = 0} \\ \boxed{\mu_{CD} = \dfrac{F}{2\tan(\alpha)}} \end{matrix}$$

Figure 21: Forces and reactions in joint D

Source: Elaborated by the authors

In joint B



$$\begin{cases}\sum F_x = 0\\ \sum F_y = 0\end{cases} \Leftrightarrow \begin{cases}\mu_{BC} = \mu_{AB}\\ \mu_{BH} = 0\end{cases} \Leftrightarrow \begin{matrix}\boxed{\mu_{BH} = 0}\\ \boxed{\mu_{BC} = \dfrac{F}{2\tan(\alpha)}}\end{matrix}$$

Figure 22: Forces and reactions in joint B

Source: Elaborated by the authors

In joint F

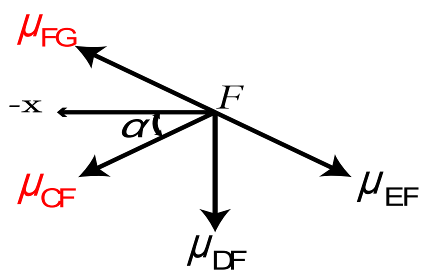


Figure 23: Forces and reactions in joint F

Source: Elaborated by the authors

$$\begin{cases}\sum F_x = 0\\ \sum F_y = 0\end{cases} \Leftrightarrow \begin{cases}-\mu_{CF}\cos(\alpha) - \mu_{FG}\cos(\alpha) + \mu_{EF}\cos(\alpha) = 0\\ -\mu_{CF}\sin(\alpha) + \mu_{FG}\sin(\alpha) + \mu_{EF}\sin(\alpha) = 0\end{cases} \Leftrightarrow \begin{cases}-2\mu_{CF} = 0\\ \mu_{FG} = \mu_{EF}\end{cases} \Leftrightarrow \begin{matrix}\boxed{\mu_{FG} = \dfrac{-F}{2\sin(\alpha)}}\\ \boxed{\mu_{CF} = 0}\end{matrix}$$

In joint H

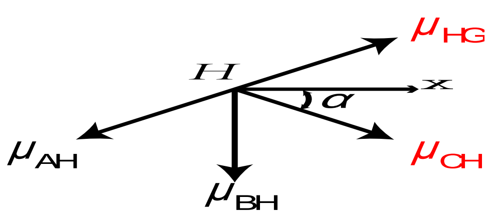


Figure 24: Forces and reactions in joint H

Source: Elaborated by the authors

$$\begin{cases}\sum F_x = 0\\ \sum F_y = 0\end{cases} \Leftrightarrow \begin{cases}\mu_{CH}\cos(\alpha) - \mu_{HG}\cos(\alpha) - \mu_{AH}\cos(\alpha) = 0\\ -\mu_{CH}\sin(\alpha) + \mu_{HG}\sin(\alpha) - \mu_{AH}\sin(\alpha) = 0\end{cases} \Leftrightarrow \begin{cases}2\mu_{CH} = 0\\ \mu_{HG} = \mu_{AH}\end{cases} \Leftrightarrow \begin{matrix}\boxed{\mu_{HG} = \dfrac{-F}{2\sin(\alpha)}}\\ \boxed{\mu_{CH} = 0}\end{matrix}$$

In joint G

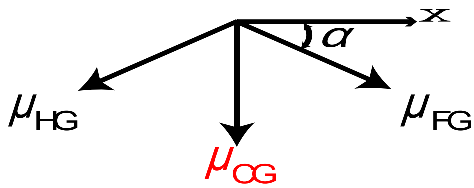


Figure 25: Forces and reactions in joint G

Source: Elaborated by the authors

$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \end{cases} \Leftrightarrow \begin{cases} \mu_{FG}\cos(\alpha) - \mu_{HG}\cos(\alpha) = 0 \\ -\mu_{CG} - \mu_{FG}\sin(\alpha) - \mu_{HG}\sin(\alpha) = 0 \end{cases} \Leftrightarrow \left\{ \mu_{CG} = \frac{2F}{2} \right\} \Leftrightarrow \boxed{\mu_{CG} = F}$$

In joint C

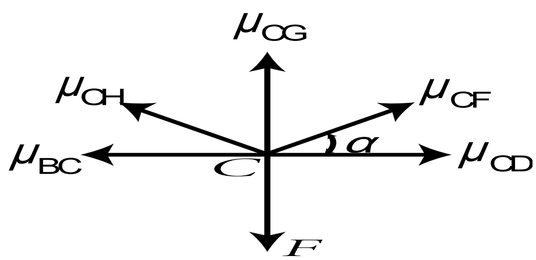


Figure 26: Forces and reactions in joint C

Source: Elaborated by the authors

$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \end{cases} \Leftrightarrow \begin{cases} \mu_{CD} - \mu_{BC} + \mu_{CF}\cos(\alpha) - \mu_{CH}\cos(\alpha) \overset{?}{=} 0 \\ \mu_{CG} - F + \mu_{CF}\sin(\alpha) - \mu_{CH}\sin(\alpha) \overset{?}{=} 0 \end{cases} \Leftrightarrow \begin{cases} \dfrac{F}{2\tan(\alpha)} - \dfrac{F}{2\tan(\alpha)} + \sin\alpha(0+0) = 0 \\ F - F + \sin\alpha(0+0) = 0 \end{cases}$$

II.4 Section method

to use this approach, we need to cut the structure into different sections. Thus, we will enumerate each section following the figure bellow.

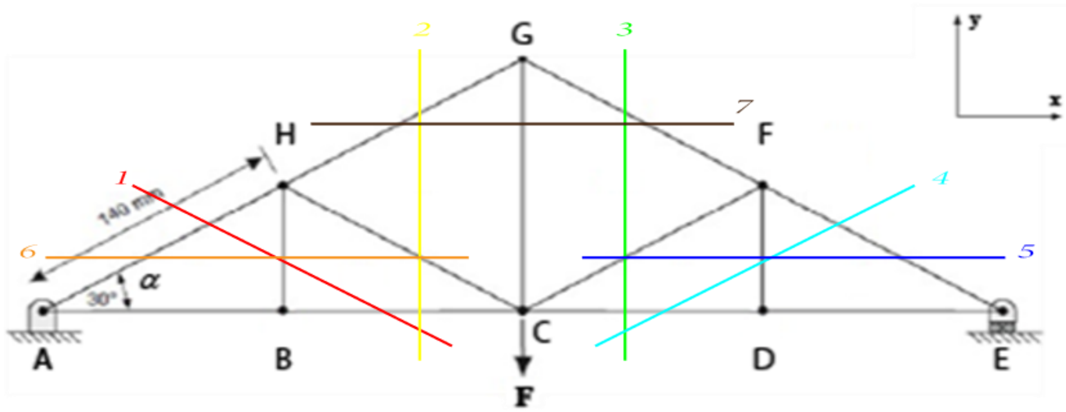


Figure 27: Sections applied on the structure using the section method.

Source: Elaborated by the authors

Section 1

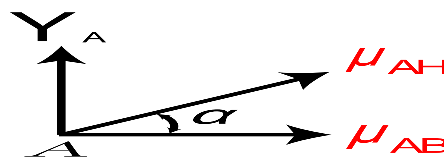


Figure 28: Forces and reactions remaining after section 1.

Source: Elaborated by the authors

$$\begin{Bmatrix} \mu_{AB}+\mu_{AH}\cos(\alpha) & 0\\ Y_A+\mu_{AH}\sin(\alpha) & 0 \\ 0 & 0\end{Bmatrix}_A \Rightarrow \begin{Bmatrix}\sum F_x=0\\ \sum F_y=0\\ \sum M_z=0\end{Bmatrix} \Leftrightarrow \begin{Bmatrix}\mu_{AB}+\mu_{AH}\cos(\alpha)=0\\ Y_A+\mu_{AH}\sin(\alpha)=0\\ 0\end{Bmatrix}\Leftrightarrow\begin{Bmatrix}\mu_{AH}=\frac{-Y_A}{\sin(\alpha)}\\ \mu_{AB}=-\mu_{AH}\cos(\alpha)\\ 0\end{Bmatrix}\Leftrightarrow \begin{matrix}\boxed{\mu_{AH}=\frac{-F}{2\sin(\alpha)}}\\ \boxed{\mu_{AH}=\frac{F}{2\tan(\alpha)}}\end{matrix}$$

Section 2

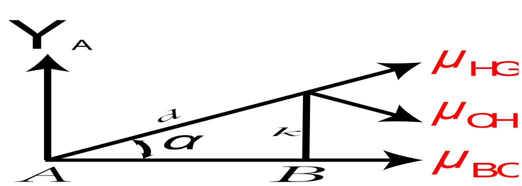


Figure 29: Forces and reactions remaining after section 2.

Source: Elaborated by the authors

$$Pt\ \mathrm{A}\begin{Bmatrix} 0 & 0 \\ Y_A & 0 \\ 0 & 0 \end{Bmatrix}_A \xrightarrow{H} \begin{Bmatrix} 0 & 0 \\ Y_A & 0 \\ 0 & -dY_A\cos(\alpha) \end{Bmatrix}_H \quad \mathrm{Pt\ H}\begin{Bmatrix} \mu_{HG}\cos(\alpha)+\mu_{CH}\cos(\alpha) & 0 \\ \mu_{HG}\sin(\alpha)-\mu_{CH}\sin(\alpha) & 0 \\ 0 & 0 \end{Bmatrix}_H \quad \mathrm{Pt\ B}\begin{Bmatrix} \mu_{CB} & 0 \\ 0 & 0 \\ 0 & 0 \end{Bmatrix}_B \xrightarrow{H} \begin{Bmatrix} \mu_{CB} & 0 \\ 0 & 0 \\ 0 & k\mu_{CB} \end{Bmatrix}_H$$

$$\begin{Bmatrix} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_z = 0 \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} \mu_{CB}+\mu_{CH}\cos(\alpha)+\mu_{HG}\cos(\alpha)=0 \\ Y_A-\mu_{CH}\sin(\alpha)+\mu_{HG}\sin(\alpha)=0 \\ -dY_A\cos(\alpha)+k\mu_{AB}=0 \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} \mu_{CB}=\dfrac{d}{k}Y_A\cos(\alpha) \\ \mu_{CH}=\dfrac{-\mu_{CB}-\mu_{HG}\cos(\alpha)}{\cos(\alpha)} \\ \dfrac{d}{k}=\dfrac{1}{\sin(\alpha)} \end{Bmatrix} \Leftrightarrow \begin{matrix} \boxed{\mu_{CB}=\dfrac{F}{2\tan(\alpha)}} \\ \boxed{\mu_{HG}=\dfrac{-F}{2\sin(\alpha)}} \\ \boxed{\mu_{CH}=0} \end{matrix}$$

Section 3

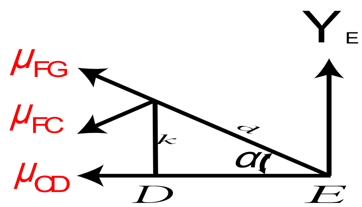


Figure 30: Forces and reactions remaining after section 3.

Source: Elaborated by the authors

$$Pt\ \mathrm{E}\begin{Bmatrix} 0 & 0 \\ Y_E & 0 \\ 0 & -dY_E\cos(\alpha) \end{Bmatrix}_F \quad \mathrm{Pt\ F}\begin{Bmatrix} -\mu_{FG}\cos(\alpha)-\mu_{CF}\cos(\alpha) & 0 \\ \mu_{FG}\sin(\alpha)-\mu_{CF}\sin(\alpha) & 0 \\ 0 & 0 \end{Bmatrix}_F \quad \mathrm{Pt\ D}\begin{Bmatrix} -\mu_{CD} & 0 \\ 0 & 0 \\ 0 & -k\mu_{CD} \end{Bmatrix}_F$$

$$\begin{Bmatrix} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_z = 0 \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} -\mu_{CD}-\mu_{CF}\cos(\alpha)-\mu_{FG}\cos(\alpha)=0 \\ Y_E-\mu_{CF}\sin(\alpha)+\mu_{FG}\sin(\alpha)=0 \\ dY_E\cos(\alpha)-k\mu_{CD}=0 \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} \mu_{CD}=\dfrac{d}{k}Y_E\cos(\alpha) \\ \mu_{FG}=\dfrac{-Y_E+\mu_{CF}\sin(\alpha)}{\sin(\alpha)} \\ \dfrac{d}{k}=\dfrac{1}{\sin(\alpha)} \end{Bmatrix} \Leftrightarrow \begin{matrix} \boxed{\mu_{CD}=\dfrac{F}{2\tan(\alpha)}} \\ \boxed{\mu_{FG}=\dfrac{-F}{2\sin(\alpha)}} \\ \boxed{\mu_{CF}=0} \end{matrix}$$

Section 4

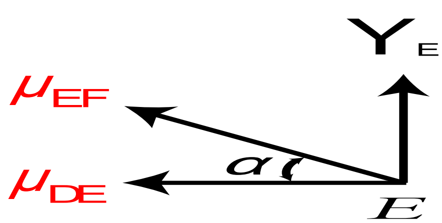


Figure 31: Forces and reactions remaining after section 4.

Source: Elaborated by the authors

$$\begin{Bmatrix} -\mu_{DE}-\mu_{FE}\cos(\alpha) & 0 \\ Y_E+\mu_{FE}\sin(\alpha) & 0 \\ 0 & 0 \end{Bmatrix}_E \Rightarrow \begin{Bmatrix} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_z = 0 \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} -\mu_{DE}-\mu_{FE}\cos(\alpha)=0 \\ Y_E+\mu_{FE}\sin(\alpha)=0 \\ 0 \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} \mu_{FE}=\dfrac{-Y_E}{\sin(\alpha)} \\ \mu_{DE}=-\mu_{FE}\cos(\alpha) \\ 0 \end{Bmatrix} \Leftrightarrow \begin{matrix} \boxed{\mu_{FE}=\dfrac{-F}{2\sin(\alpha)}} \\ \boxed{\mu_{DE}=\dfrac{F}{2\tan(\alpha)}} \end{matrix}$$

Section 5

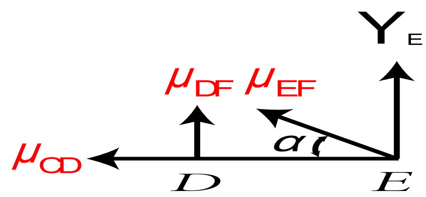


Figure 32: Forces and reactions remaining after section 5.

Source: Elaborated by the authors

$$\text{Pt D}\begin{Bmatrix} 0 & 0 \\ \mu_{DF} & 0 \\ 0 & \dfrac{-L}{4}\mu_{DF} \end{Bmatrix}_E \quad \text{Pt E}\begin{Bmatrix} -\mu_{EF}\cos(\alpha) & 0 \\ \mu_{EF}\sin(\alpha)+Y_E & 0 \\ 0 & 0 \end{Bmatrix}_E \quad \text{Pt C}\begin{Bmatrix} -\mu_{CD} & 0 \\ 0 & 0 \\ 0 & 0 \end{Bmatrix}_E$$

$$\begin{Bmatrix} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_z = 0 \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} -\mu_{CD}-\mu_{FE}\cos(\alpha)=0 \\ Y_E+\mu_{FE}\sin(\alpha)+\mu_{DF}=0 \\ \dfrac{-L}{4}\mu_{DF}=0 \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} \mu_{FE}=\dfrac{-Y_E}{\sin(\alpha)} \\ \mu_{CD}=-\mu_{FE}\cos(\alpha) \\ \dfrac{-L}{4}\mu_{DF}=0 \end{Bmatrix} \Leftrightarrow \begin{matrix} \boxed{\mu_{DF}=0} \\ \boxed{\mu_{DF}=\dfrac{-F}{2\sin(\alpha)}} \\ \boxed{\mu_{CD}=\dfrac{F}{2\tan(\alpha)}} \end{matrix}$$

Section 6

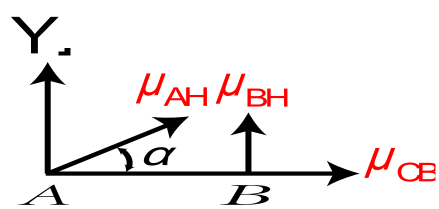


Figure 33: Forces and reactions remaining after section 6.

Source: Elaborated by the authors

$$Pt\ B\begin{Bmatrix} 0 & 0 \\ \mu_{BH} & 0 \\ 0 & \frac{L}{4}\mu_{DF} \end{Bmatrix}_A \quad Pt\ A\begin{Bmatrix} \mu_{AH}\cos(\alpha) & 0 \\ \mu_{AH}\sin(\alpha)+Y_A & 0 \\ 0 & 0 \end{Bmatrix}_A \quad Pt\ C\begin{Bmatrix} -\mu_{CB} & 0 \\ 0 & 0 \\ 0 & 0 \end{Bmatrix}_A$$

$$\begin{Bmatrix} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_z = 0 \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} \mu_{CB} + \mu_{AH}\cos(\alpha) = 0 \\ Y_A + \mu_{AH}\sin(\alpha) + \mu_{BH} = 0 \\ \frac{L}{4}\mu_{BH} = 0 \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} \mu_{AH} = \frac{-Y_A}{\sin(\alpha)} \\ \mu_{CB} = -\mu_{AH}\cos(\alpha) \\ \frac{L}{4}\mu_{DF} = 0 \end{Bmatrix} \Leftrightarrow \begin{matrix} \boxed{\mu_{BH} = 0} \\ \boxed{\mu_{AH} = \frac{-F}{2\sin(\alpha)}} \\ \boxed{\mu_{CB} = \frac{F}{2\tan(\alpha)}} \end{matrix}$$

Section 7

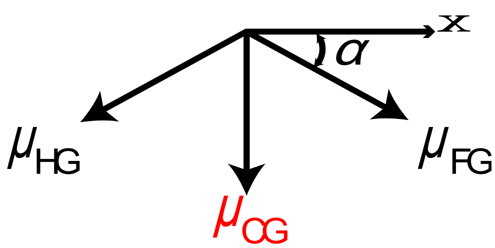


Figure 34: Forces and reactions remaining after section 7.

Source: Elaborated by the authors

$$\begin{Bmatrix} \mu_{FG}\cos(\alpha) - \mu_{HG}\cos(\alpha) & 0 \\ -\mu_{FG}\sin(\alpha) - \mu_{HG}\sin(\alpha) + \mu_{CG} & 0 \\ 0 & 0 \end{Bmatrix}_G \Rightarrow \begin{Bmatrix} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_z = 0 \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} \mu_{FG}\cos(\alpha) - \mu_{HG}\cos(\alpha) = 0 \\ -\mu_{FG}\sin(\alpha) - \mu_{HG}\sin(\alpha) + \mu_{CG} = 0 \\ 0 \end{Bmatrix} \Leftrightarrow \begin{matrix} \boxed{\mu_{FG} = \frac{-F}{2\sin(\alpha)}} \\ \boxed{\mu_{HG} = \frac{-F}{2\sin(\alpha)}} \\ \boxed{\mu_{CG} = F} \end{matrix}$$

II.5 study of results

The Results are identical by both methods. Furthermore, we not only found that,

$$\boxed{X_A = 0}\boxed{Y_E = \frac{F}{2}}\boxed{Y_A = \frac{F}{2}}$$

But also, μ_{EF} μ_{FG} μ_{AH} μ_{HG} are bars in Compression. On the other hand, μ_{AB} μ_{BC} μ_{CD} μ_{DE} μ_{CG} are bars in Tension. However, μ_{BH} μ_{CH} μ_{CF} μ_{DF} are Inactive bars.

III. Truss experimental study

III.1. Introduction

In this experimental study, we will use a howe truss-Gunt in order to obtain an analysis of the studied-truss comportment facing a variable down-pulling force. The direction of the applied force is fixed down as well as its location. However, its magnitude can change from 100N up to 400N.

III.2. Experiment objectives

- measurement of the bar forces in a single plane truss, Howe type.
- dependency of bar forces on the external force.
- magnitude, direction, point of application.
- Calculating averages measurements
- Comparing the measurements to theoretical results

III.3. The howe-gunt

The howe-Gunt setup provides experiments on single plane trusses with a high degree of measuring accuracy and computerized result read out based on software. The ready assembled truss is mounted horizontally on a frame. The influence of the dead weight is minimized by horizontal experimental setup. The bars are joined by a “hinged” connection using node disks. Consequently, our truss can be considered as an ideal truss. The external force is generated with the aid of a threaded spindle. The force can be applied in various directions and at various points.

The forces occurring on the truss bars are recorded by strain gauge measurement. All measuring points are housed together in a connection box. From there, they are connected to a measuring amplifier.

Figure 35: A Howe-gunt

Source: Elaborated by the authors

III.4. Experimental study

In the laboratory, we can create whatever truss shape we desire. Thus, we built the howe truss used in our analysis using the members and joints as shown below.

Figure 36: The built Howe truss

Source: Elaborated by the authors

Additionally, we wired node disks to the gunt and the display to post the obtained results by placing the structure on the gunt.

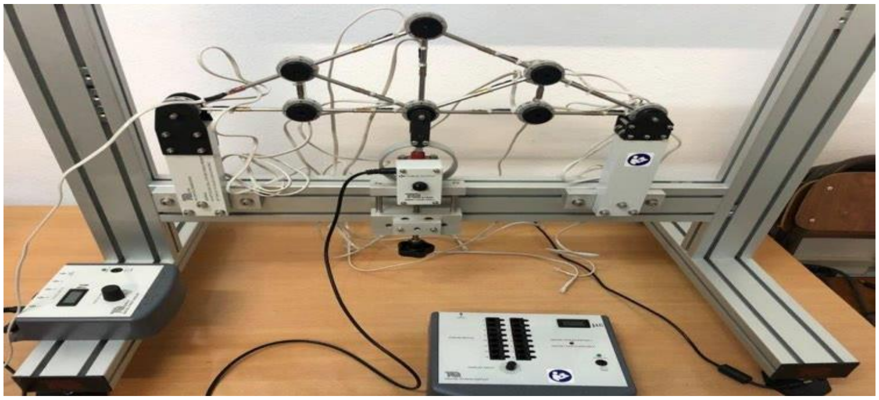


Figure 37: The howe truss placed on the gunt

Source: Elaborated by the authors

In order to guaranty the results, we repeated each measurement 6 times. First, each time we began by resetting the calculated measures. Second, we changed the puling-down force magnitude from 100N up to 400N manually.

Figure 38; Changing the magnitude of the force and taking measurements.

Source: Elaborated by the authors

Thirdly, we took the results posted on the Howe-gunt display and wrote them down the table below.

Table 1: Test1

	AH	HG	GF	AB	BC	CD	DE	FE	BH	HC	CG	CF	FD
100	-17	-17	-18	16	16	16	15	-17	0	1	20	0	1
200	-34	-32	-34	32	31	31	30	-34	0	1	37	0	1
300	-54	-51	-54	50	50	48	47	-54	0	1	58	0	1
400	-70	-66	-69	64	64	62	61	-69	0	1	74	1	1

Table 2: Test2

	AH	HG	GF	AB	BC	CD	DE	FE	BH	HC	CG	CF	FD
100	-17	-17	-17	16	17	15	16	-17	-1	1	18	0	1
200	-36	-35	-36	33	34	32	32	-42	-1	-1	38	0	0
300	-54	-52	-53	49	51	48	48	-54	-1	0	57	0	1
400	-72	-68	-71	66	67	64	64	-72	-1	0	75	0	1

Table 3: Test3

	AH	HG	GF	AB	BC	CD	DE	FE	BH	HC	CG	CF	FD
100	-18	-17	-18	16	18	16	15	-18	-1	-1	20	0	1
200	-35	-33	-35	32	34	32	30	-35	-1	-1	38	0	1
300	-54	-51	-54	49	51	48	46	-54	-1	1	57	0	0
400	-73	-69	-72	67	68	64	62	-73	-1	1	76	0	0

Table 4: Test4

	AH	HG	GF	AB	BC	CD	DE	FE	BH	HC	CG	CF	FD
100	-19	-18	-19	17	18	16	16	-19	1	0	21	0	1
200	-37	-34	-36	33	33	32	31	-37	0	2	39	-1	1
300	-57	-53	-56	51	51	49	48	-56	0	2	60	0	0
400	-72	-67	-71	65	65	62	61	-72	0	2	76	0	0

Table 5: Test5

	AH	HG	GF	AB	BC	CD	DE	FE	BH	HC	CG	CF	FD
100	-19	-17	-19	17	18	16	16	-19	0	0	21	0	1
200	-37	-34	-37	34	34	33	31	-37	0	2	39	0	0
300	-56	-52	-56	50	51	49	47	-57	0	2	59	0	1
400	-74	-68	-72	65	66	63	62	+72	0	-1	76	0	1

Table 6: Test6

	AH	HG	GF	AB	BC	CD	DE	FE	BH	HC	CG	CF	FD
100	-19	-18	-19	17	19	17	19	-19	0	1	21	-1	1
200	-37	-34	-36	32	34	32	34	-37	-1	2	39	-1	1
300	-56	-52	-56	50	52	50	51	-56	0	2	60	-1	1
400	-72	-67	-71	65	67	64	65	-72	0	2	76	-1	1

Furthermore, we calculated using Metrologic methods to find average measurements and put it down the table below.

Table 7: Average test

	AH	HG	GF	AB	BC	CD	DE	FE	BH	HC	CG	CF	FD
100	-18.1	-17.3	-18.3	16.5	17.6	16	16.1	-18.1	-0.16	0.33	20.1	-0.16	0.83
200	-36	-33.6	-35.6	32.6	33.3	32	31.3	-37	-0.5	0.83	39.3	-0.33	0.83
300	-55.1	-51.8	-54.8	49.8	51	48.6	47.8	-55.1	-0.33	1.33	58.5	-0.16	0.83
400	-72.1	-67.5	-71	653	66.1	63.1	62.5	-71.6	-0.33	0.83	75.5	0	0.66

Table 8: Theoretical calculation

	AH	HG	GF	AB	BC	CD	DE	FE	BH	HC	CG	CF	FD
100	-100	-100	-100	86.6	86.6	86.6	86.6	-100	0	0	-100	0	0
20	-200	-200	-200	173.2	173.2	173.2	173.2	-200	0	0	-200	0	0
300	-300	-300	-300	259.8	259.8	259.8	259.8	-300	0	0	-300	0	0
400	-400	-400	-400	346.41	346.41	346.41	346.41	-400	0	0	-400	0	0

VI. Simulation study

VI.1 Introduction

In this section, we will simulate the studied truss using digital methods based on the finite element method (RDM6 and Abaqus).

VI.2 RDM 6

- We draw the truss shape by fixing joints coordination and connect them with bars fixing length of each bar used and the diameter of the section of each member.

we used the same structure configuration used previously.

- 7 members lengthened 140mm.
- 4 members lengthened 121mm2 members lengthened 40mm.
- All 13 members are cylindric bars composed of steel characterized of Young modulus equals to E=6N/m^2 and a diameter equals to D=5.98mm.

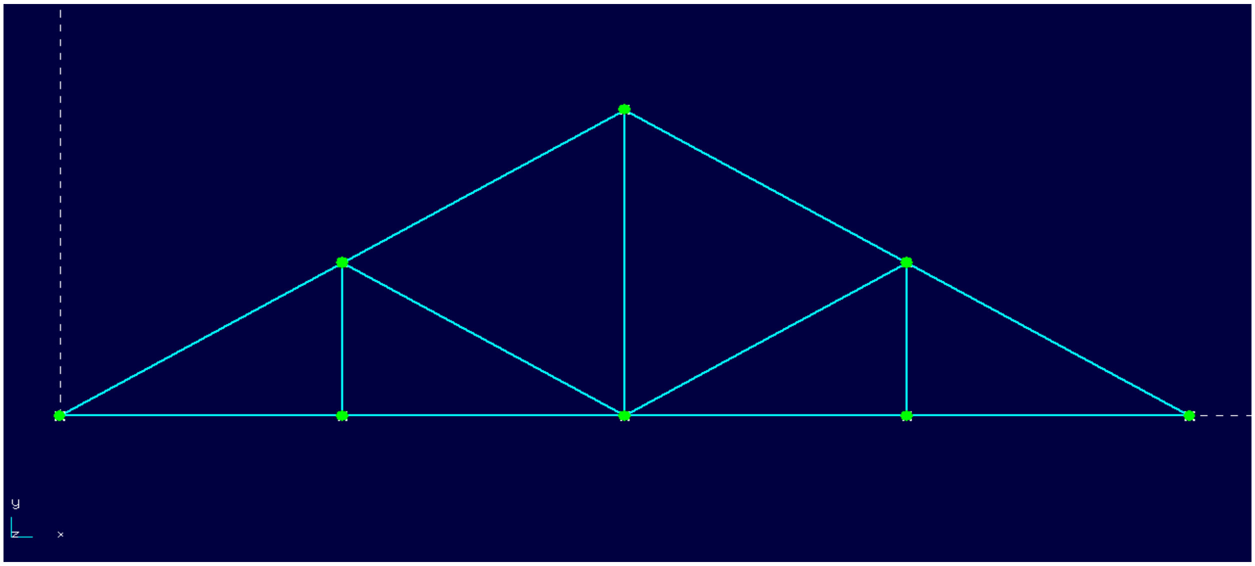


Figure 39: Truss RDM6

Source: Elaborated by the authors

- Using the software features. We made a simulation of the deformation of the used truss and captured it below.

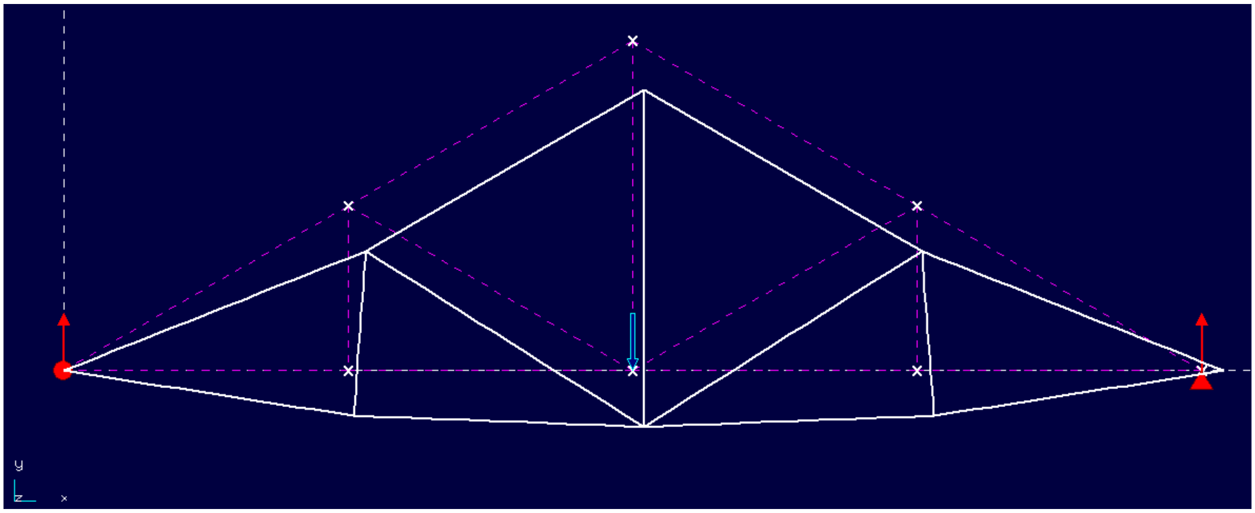


Figure 40: Deformation RDM6

Source: Elaborated by the authors

VI.3 Abaqus

- We also draw the truss shape by fixing nodes locations and connect them with fixed members.

we used the same structure configuration used in order to have the same simulation.

- 7 members lengthened 140mm.
- 4 members lengthened 121mm2 members lengthened 40mm.
- All 13 members are cylindric bars composed of steel characterized of Young modulus equals to E=6N/m^2 and a diameter equals to D=5.98mm.

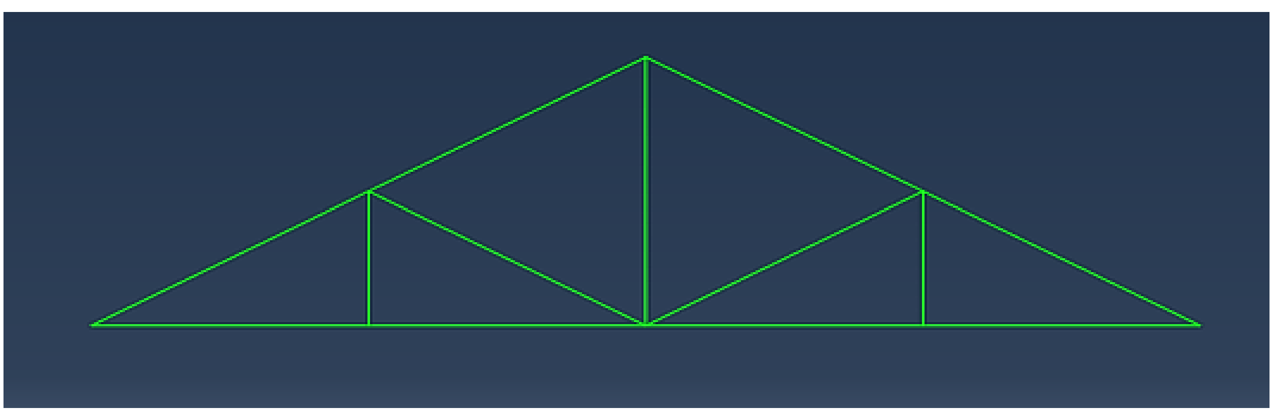


Figure 41: Truss Abaqus

Source: Elaborated by the authors

- Using the software features. We made a simulation of the deformation of the used truss and captured it below.

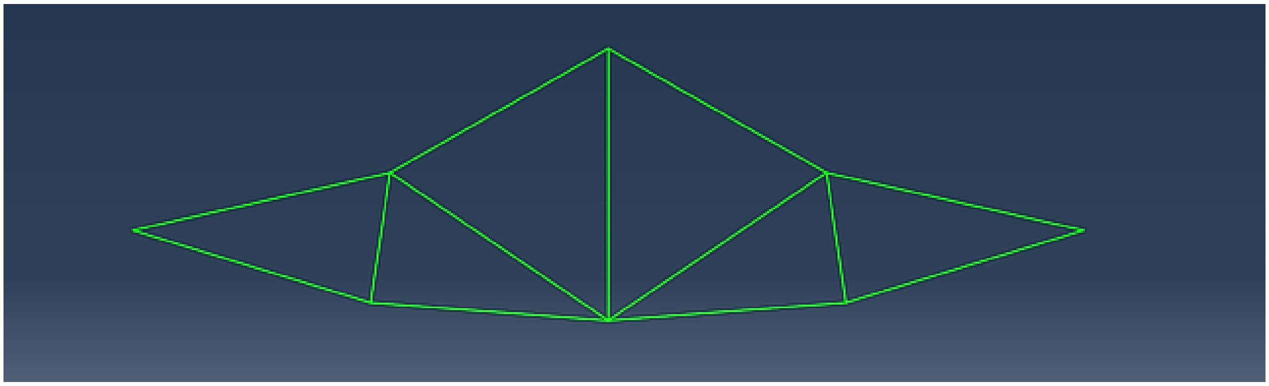


Figure 42: Deformation Abaqus

Source: Elaborated by the authors

- Finally, we draw the diagram of forces applied to each member and each force magnitude. We captured the results below.

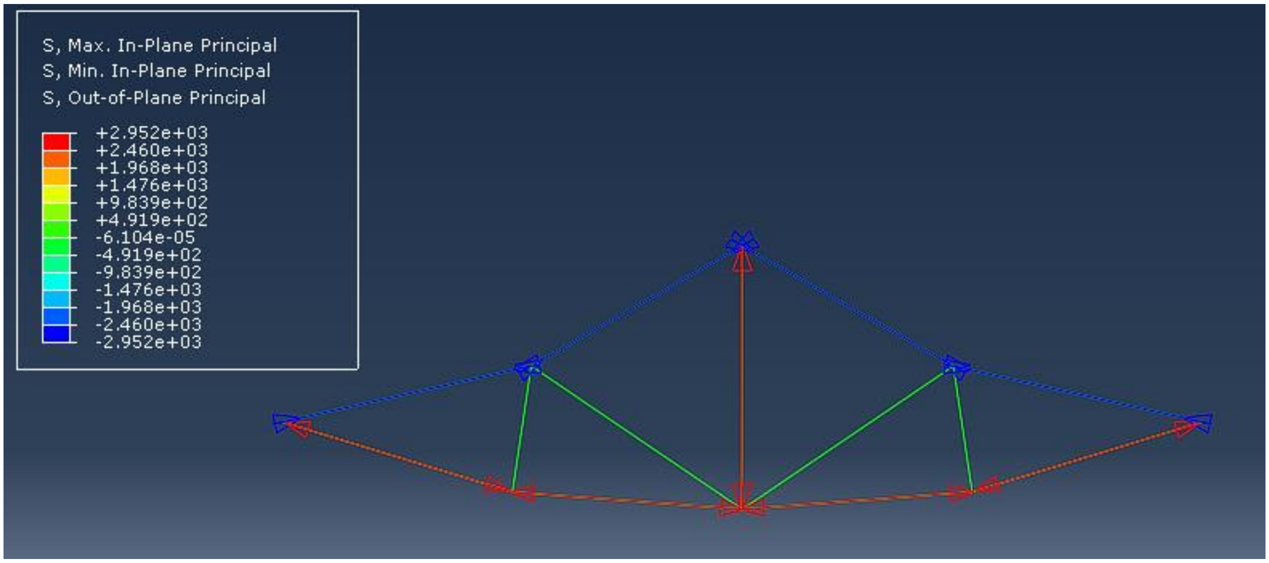


Figure 43: Forces applied on bars.

Source: Elaborated by the authors

Bibliographic Sources

Source1: civil engineering: compression members in steel structure

Source2: Trusses and Types of trusses - Basic Civil Engineering

Source3: aeronautics guide

Source4: History of the Airplane and Flight: Orville and Wilbur Wright

Source5: Air Travel in the 1970s - Exploring the Seventies

Source6: AIRFRAME STRUCTURAL DESIGN (INSTITUTE OF AERONAUTICAL ENGINEERING)

Source7: engineering code courses (introduction to plane truss)