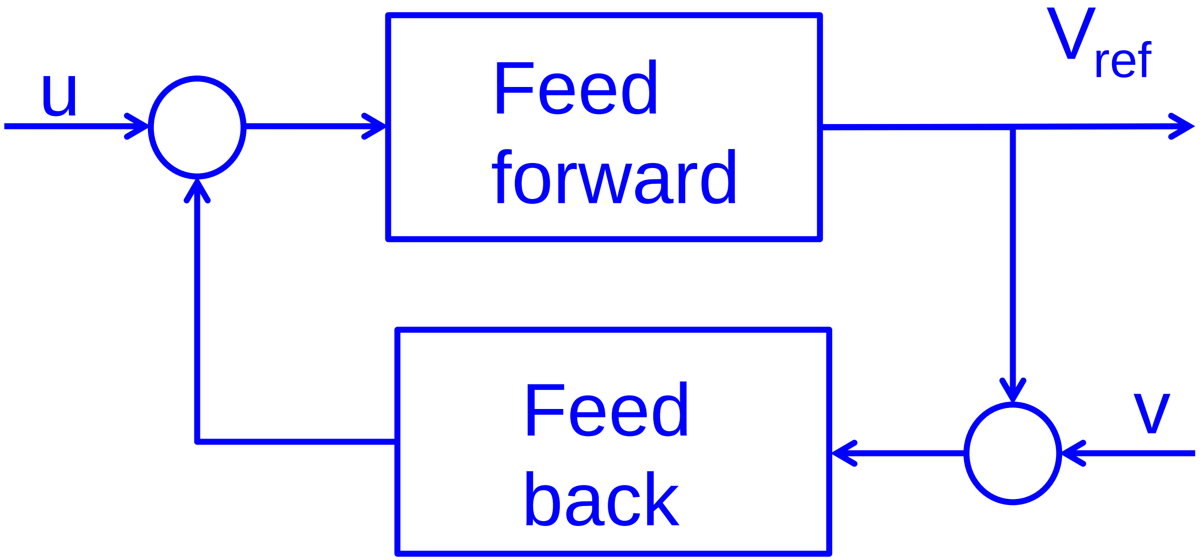
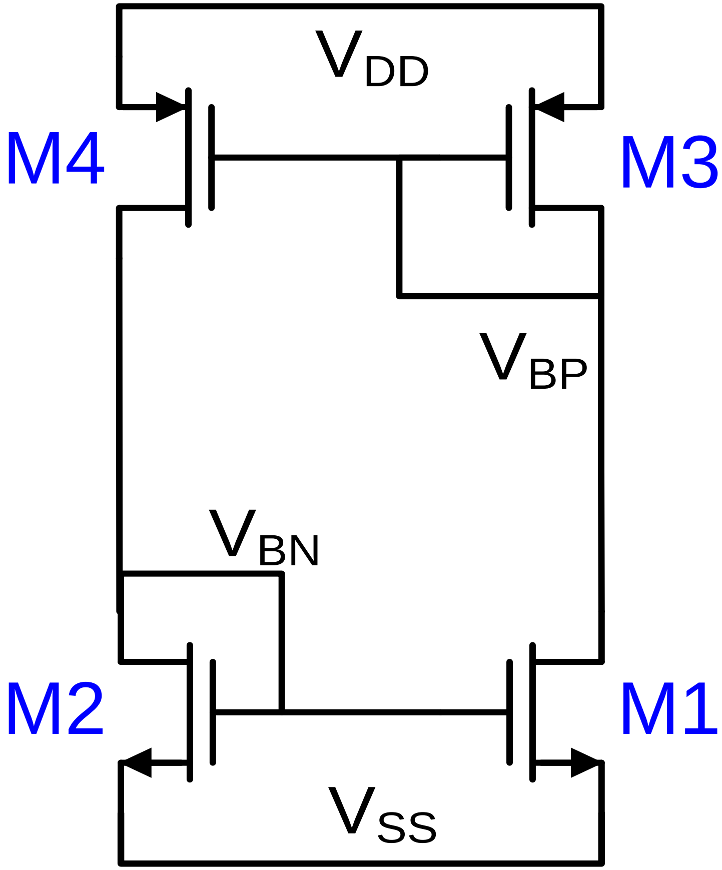
Well-posed-ness of feed back loops



The feed back system should not contain an algebraic loop with gain = 1.

Example: not well-posed system



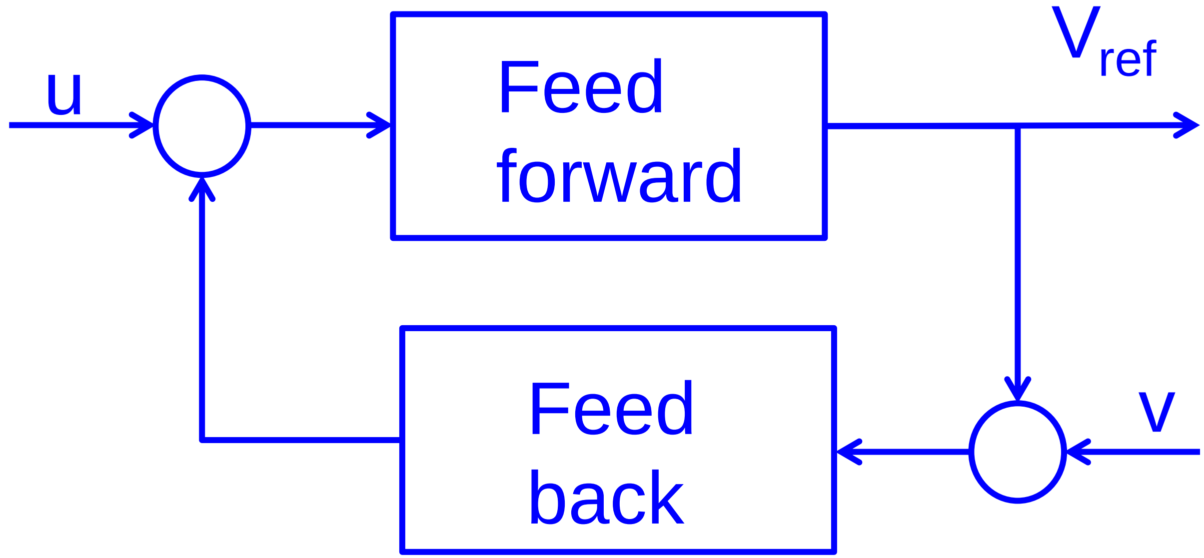
If M1 and M2 are matched, M3 and M4 are matched, and lambda effect can be ignored, then there is an algebraic loop with loop gain equal to 1.

➔ Circuit is not well posed.

To become well-posed:

- Include the lambda effect
  - Well defined solution at Vbn = Vbp
- Mismatch one of the transistor pairs
  - M3 and M4 to have different bulk connection
  - M3 and M4 to have different VT
  - Source degeneration on M3
    - With a resistor
    - With a transistor
  - Source degeneration on M4
  - M3 and M4 to have different sizes
    - Loop gain is either always >1 or always <1

Operating points



For any given constant values of u and v, the constant values of variables that solve the the feed back relationship are called the operating points, or equilibrium points.

Operating points can be either stable or unstable.

Stable operating point

An operating point is stable if all initial values near the operating point lead to convergence to that operating point.

An operating point is unstable if any or some small perturbation near it causes divergence away from that operating point.

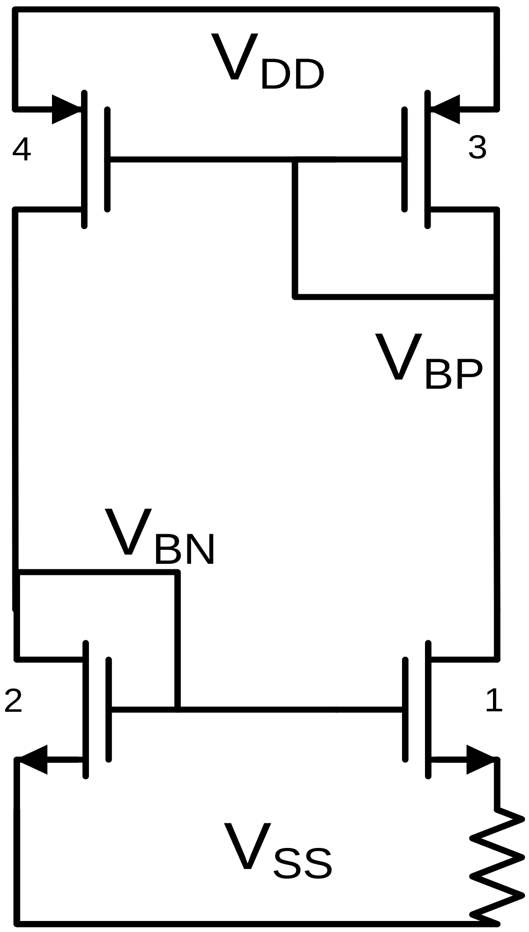
A circuit can have multiple operating points, both stable ones and unstable ones.

Small gain theorem

- If the loop gain evaluated at an operating point is less than one, that operating point is stable.

- This is a sufficient condition.

- If the loop gain is > 1, the operating point can still be stable.
  - Local linearization must have all poles in LHP
  - Small signal loop gain has PM>0 at UGF

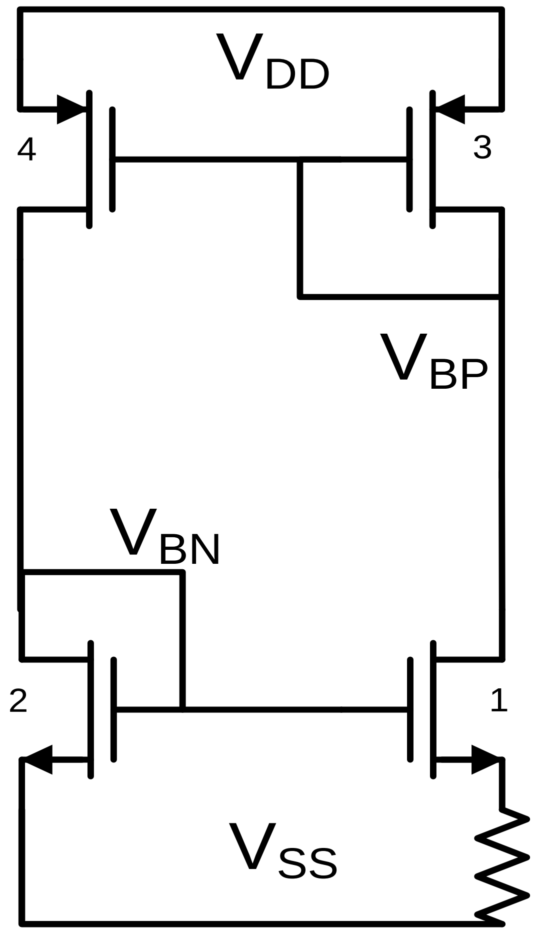


Example

Suppose there is an operating point at which all transistors are in saturation.
At this operating point, we can obtain gm and gds for all transistors.

$$\text{Loop Gain} \approx \frac{-g_{m4}}{g_{m2}+g_{ds2}+g_{ds4}} \frac{-g_{m1}/\left(1+g_{m1}R\right)}{g_{m3}+g_{ds3}}$$

$$\approx \frac{1}{1+g_{m1}R} \frac{g_{m1}}{g_{m2}} = \frac{1}{\dfrac{g_{m2}/I_D}{g_{m1}/I_D}+g_{m2}R} < 1$$



The operating point with saturation operation is the desired one.

It can be obtained by solving the constraints imposed by saturation equations.

$$V_{eff\,2} = V_{eff\,1} + V_R$$

$$\sqrt{\frac{2I}{\beta_2}} = \sqrt{\frac{2I}{\beta_1}} + IR, \quad \sqrt{\frac{2}{\beta_2}} = \sqrt{\frac{2}{\beta_1}} + \sqrt{I}R$$

$$\sqrt{I} = \frac{\sqrt{2\beta_1} - \sqrt{2\beta_2}}{R\sqrt{\beta_1\beta_2}}$$

$$I = \frac{2\beta_1 + 2\beta_2 - 4\sqrt{\beta_1\beta_2}}{R^2\beta_1\beta_2}$$

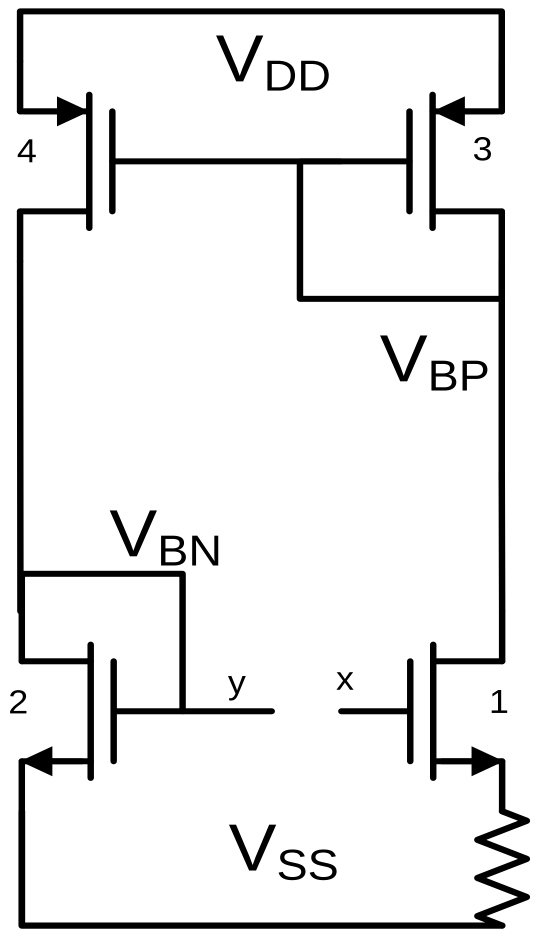
$$I = \frac{2\beta_1 + 2\beta_2 - 4\sqrt{\beta_1\beta_2}}{R^2\beta_1\beta_2} = \frac{\beta_2}{2}(V_{BN} - V_{T2})^2$$

$$4 + 4\frac{\beta_2}{\beta_1} - 8\sqrt{\frac{\beta_2}{\beta_1}} = R^2\beta_2^2(V_{BN} - V_{T2})^2$$

$$R\beta_2(V_{BN} - V_{T2}) = \sqrt{4 + 4\frac{\beta_2}{\beta_1} - 8\sqrt{\frac{\beta_2}{\beta_1}}} = 2(1 - \sqrt{\frac{\beta_2}{\beta_1}})$$

$$V_{BN} = V_{T2} + \frac{2}{R\beta_2}(1 - \sqrt{\frac{\beta_2}{\beta_1}})$$

$$IR = \frac{2}{R}(\sqrt{\frac{1}{\beta_2}} - \sqrt{\frac{1}{\beta_1}})^2$$



Suppose current mirror is ideal.

$$\frac{\beta_2}{2}(y-V_{T2})^2 = I = \frac{\beta_1}{2}(x-IR-V_{T1})^2$$

$$\sqrt{\beta_2}(y-V_{T2}) = \sqrt{\beta_1}\left(x-\frac{R\beta_2}{2}(y-V_{T2})^2-V_{T1}\right)$$

$$x = \frac{R\beta_2}{2}y^2 + \left(\sqrt{\frac{\beta_2}{\beta_1}}-R\beta_2 V_{T2}\right)y$$

$$+V_{T1}-V_{T2}\sqrt{\frac{\beta_2}{\beta_1}}+\frac{R\beta_2}{2}V_{T2}^2$$

$$\frac{dx}{dy} = R\beta_2 y + \sqrt{\frac{\beta_2}{\beta_1}} - R\beta_2 V_{T2}$$

$$= \sqrt{\frac{\beta_2}{\beta_1}} + R\beta_2(y-V_{T2})$$

At the ideal operating point:

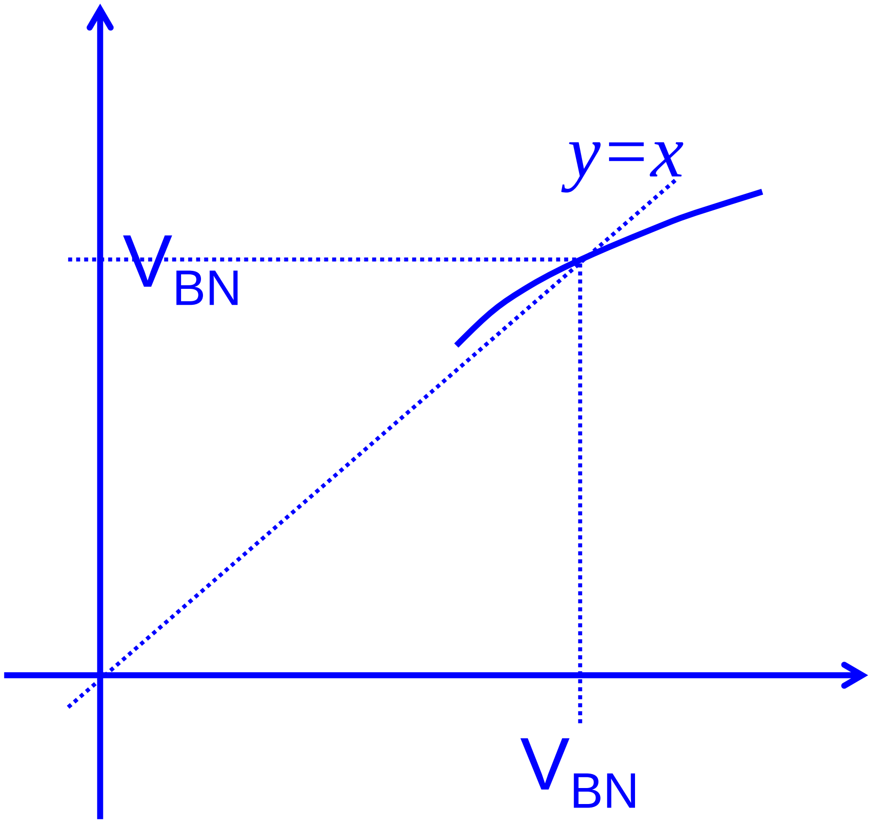
$$y = V_{BN} = V_{T2} + \frac{2}{R\beta_2}\left(1 - \sqrt{\frac{\beta_2}{\beta_1}}\right)$$

$$\frac{dx}{dy} = \sqrt{\frac{\beta_2}{\beta_1}} + R\beta_2 (y - V_{T2})$$

$$= \sqrt{\frac{\beta_2}{\beta_1}} + 2\left(1 - \sqrt{\frac{\beta_2}{\beta_1}}\right) = 2 - \sqrt{\frac{\beta_2}{\beta_1}}$$

If $\beta_1 = 4\beta_2$, $\frac{dx}{dy} = 2 - \frac{1}{2} = 1.5$

$$\frac{dy}{dx} = \frac{2}{3}$$



One quick design strategy

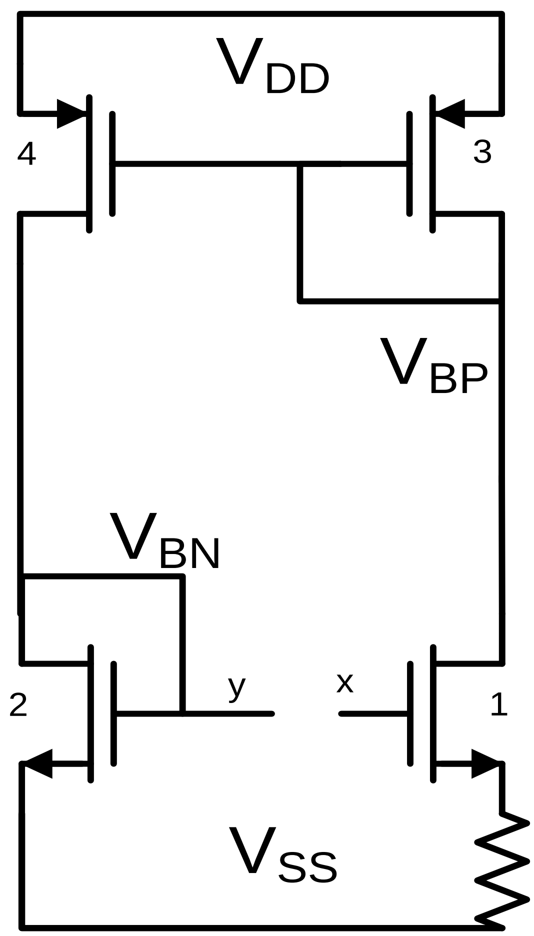
Make $IR = V_{EB1} = \frac{1}{2} V_{EB2}$ by making $\beta_1 = 4\beta_2$. Then

$$I = \frac{2\beta_1 + 2\beta_2 - 4\sqrt{\beta_1\beta_2}}{R^2\beta_1\beta_2} = \frac{1}{2R^2\beta_2}$$

$$V_{EB1} = IR = \frac{1}{2R\beta_2}$$

Design steps:

- Select desired $IR = V_{EB1}$, and desired I,
- Compute R=$\frac{V_{EB1}}{I}$, $\beta_2 = \frac{1}{2RV_{EB1}}$, and $\beta_1 = 4\beta_2$,
- Size M3 and M4.



If both M1 and M2 are in weak inversion,

IR will be very small due to very small I.

$$\beta_1 I_s e^{\frac{V_{GS1}}{kT/q}} = I_1 = I = I_2 = \beta_2 I_s e^{\frac{V_{GS2}}{kT/q}}$$

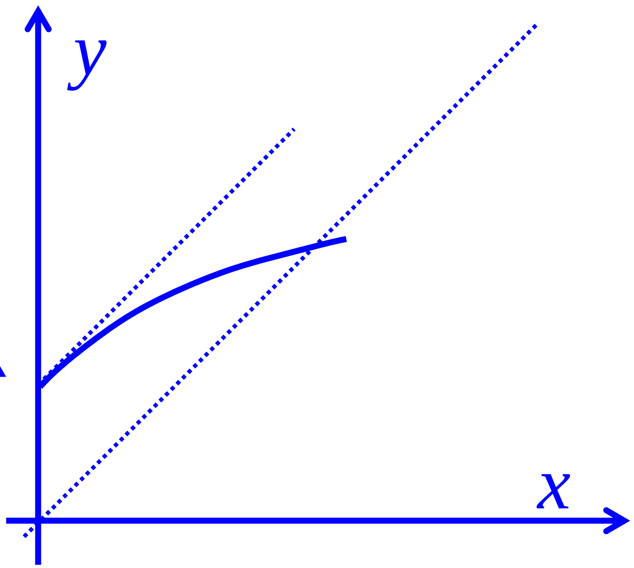
$$\beta_1 I_s e^{\frac{x-IR}{kT/q}} = I_1 = I = I_2 = \beta_2 I_s e^{\frac{y}{kT/q}}$$

$$y - x + IR = \frac{kT}{q}\ln(\frac{\beta_1}{\beta_2})$$

Under feedback constraint $y = x$:

$$I = \frac{kT}{Rq}\ln(\frac{\beta_1}{\beta_2}) = \frac{26mV}{R}\ln(\frac{\beta_1}{\beta_2})$$

At $x=0$, $y=\frac{kT}{q}\ln(\frac{\beta_1}{\beta_2})$



Near $x=0$, I inceases with x,

$$y=x-IR+\frac{kT}{q}\ln(\frac{\beta_1}{\beta_2})$$

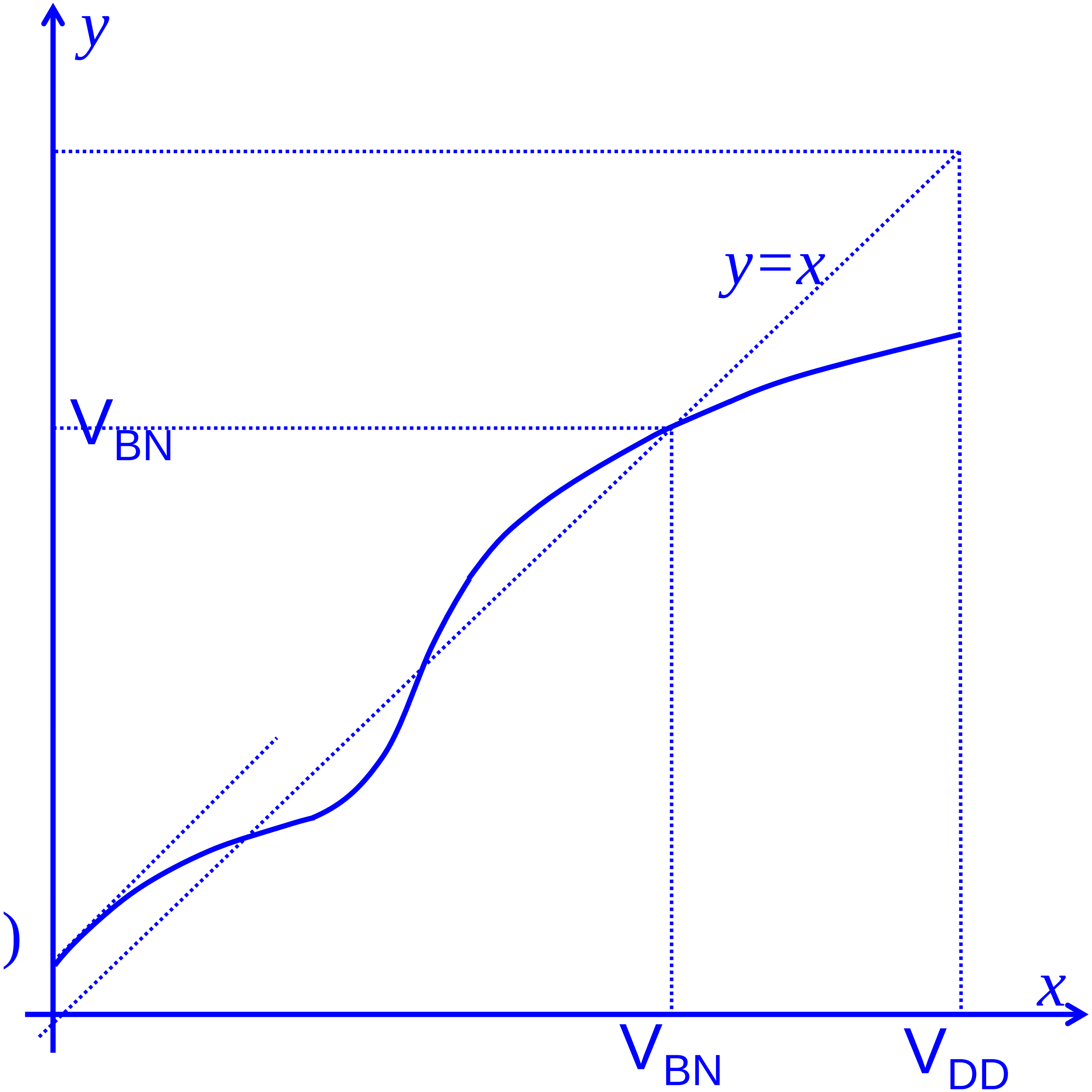
The slope starts at 1 and gradually reduces.

When it intersects with the line $y=x$, the slope is <1.

$$I=\frac{kT}{Rq}\ln(\frac{\beta_1}{\beta_2}),$$

$$y=\frac{kT}{q}\ln(\frac{I}{\beta_2 I_s})=\frac{kT}{q}\ln(\frac{kT}{Rq\beta_2 I_s}\ln(\frac{\beta_1}{\beta_2}))$$

We may do similar analysis near the other operating point. Putting together, we have:



$$\frac{kT}{q}\ln\left(\frac{\beta_1}{\beta_2}\right)$$

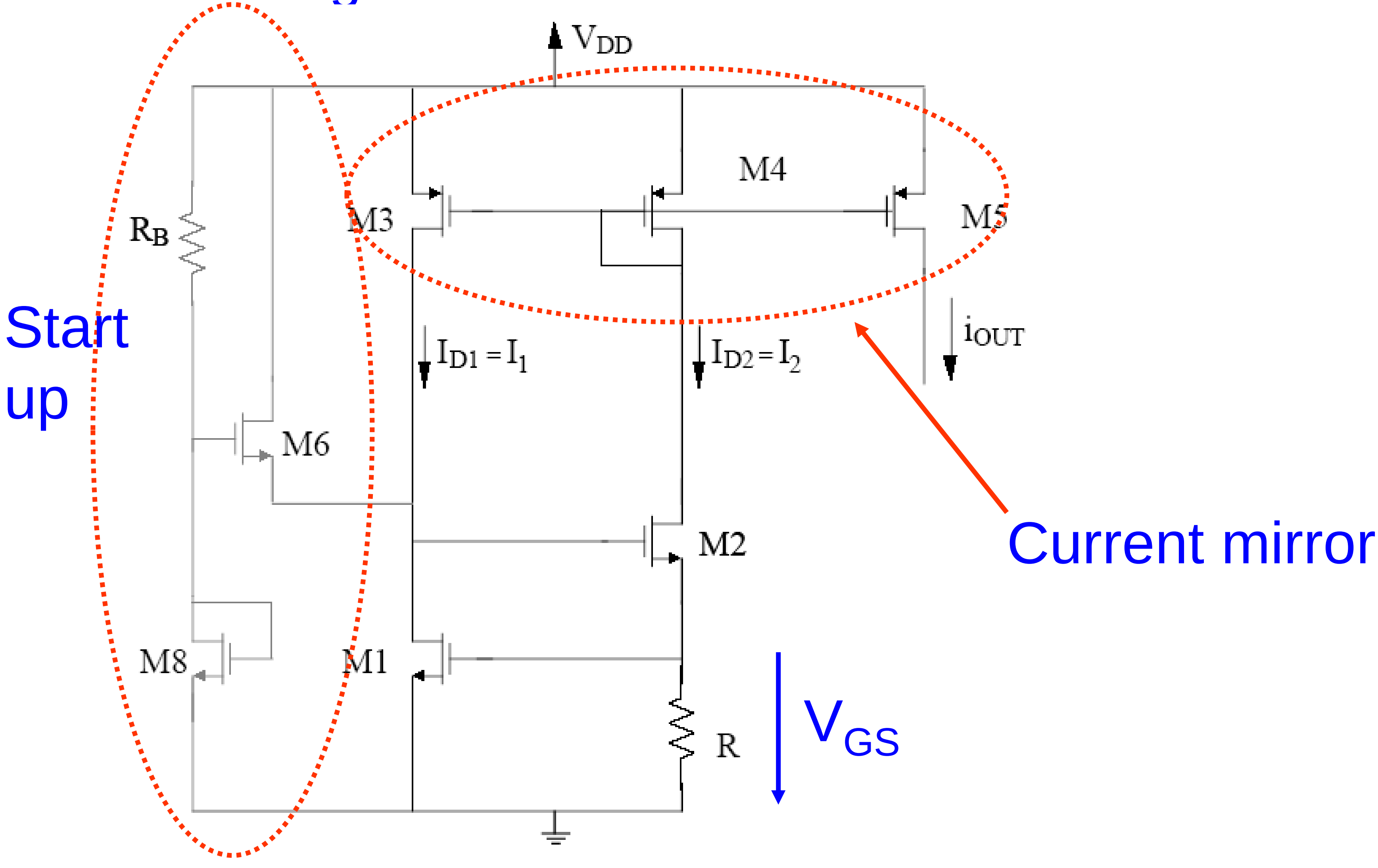
Multiple operating points

- For a simple circuit as the one above, it is possible to have multiple operating points.
- In the example, two are stable:
  - The desired one with x=y=VBN
  - The one in weaker inversion
- The third operating point is unstable.

- Need start-up circuit to prevent getting stuck at the undesired stable operating point.

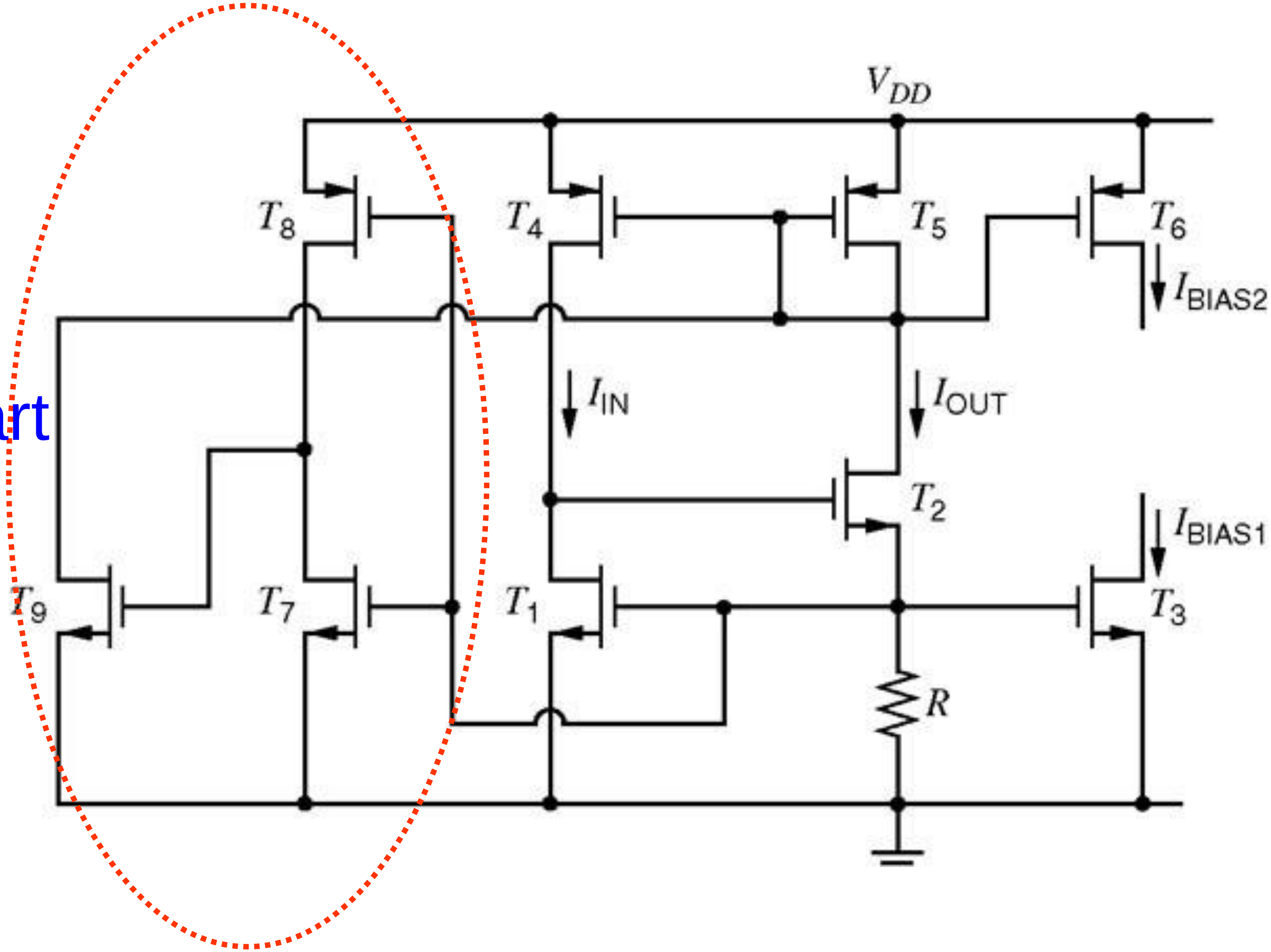
V_{GS} based Current reference

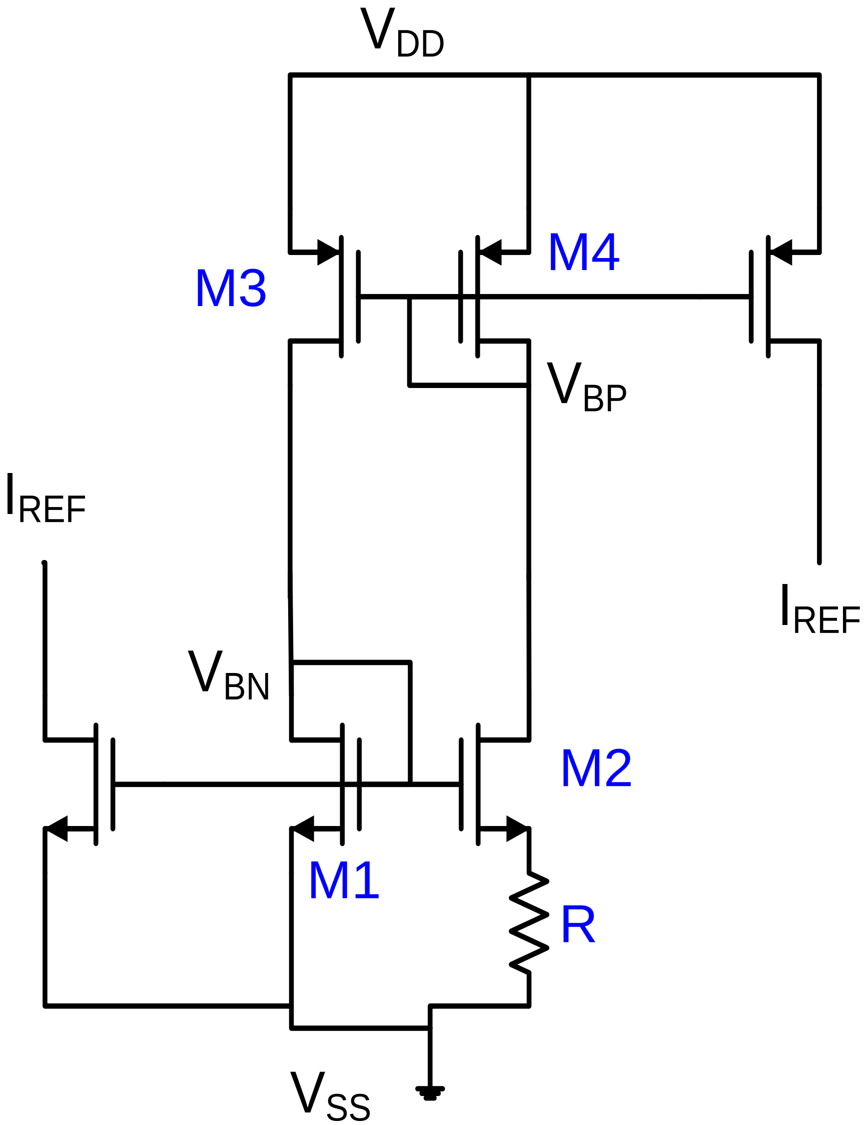
MOS version: use V_{GS} to generate a current and then use negative feed back stabilize i in MOS



Start up circuit consuming no static current

Start
up





1. Need to add start-up circuit
2. Add MOSCAPs between VBP and VDD, or between VBN and VSS
3. NMOS W ratio and R determines current value
4. Cascode to improve supply sensitivity
5. Or use a regulated amp
6. VBN and VBP may be directly used as biasing voltage for non-critical use

HW:

Assume M1~M4 in strong inversion and M2~M3 in saturation. Let M1 have V_{Tn}, M4 have V_{Tp}, M2 have V_{osn} and M2 have V_{osp}. Both M1 and M2 have μ_n, λ_n and both M3 and M4 have μ_p, λ_p. L1=L2=L3=L4, W2=kW1, W3=W4.

1. Write down the drain current equations for M1~M4.
2. Compute the sensitivity of I_2 with respect to V_{DD}. (You can set V_{os} to 0 for simplicity.)
3. Compute temp co of I_2. (You can set λ_n and λ_p to 0 for simplicity.) Note: V_{osp} is temperature dependent, V_{osn} is both temperature and I_2R dependent, and R is also temperature dependent.
4. Based on 2 and 3, comment on why this is good circuit and how performance can be improved by proper design.
5. Comment on why the Wilson current source is not good.