INL/DNL analysis of current steering DAC

Relative matching ("coefficient of variation")

$$\begin{cases}\sigma_u \overset{\text{def}}{=} \dfrac{\sigma}{\mu} = stdev\left(\dfrac{\Delta I}{I}\right) \\ I_j \overset{\text{def}}{=} I + \Delta I\end{cases}$$

1.1 INL of thermometer DAC

$$\begin{cases}INL(k) = \dfrac{I_{out}(k) - I_{out,uniform}(k)}{Step_{avg}} = \dfrac{\sum_{j=1}^{k} I_j - \frac{k}{N}\sum_{j=1}^{N} I_j}{\frac{1}{N}\sum_{j=1}^{N} I_j} = N \cdot \dfrac{\sum_{j=1}^{k} I_j}{\sum_{j=1}^{N} I_j} - k = N \cdot \dfrac{\sum_{j=1}^{k} I_j}{\sum_{j=1}^{k} I_j + \sum_{j=k+1}^{N} I_j} - k \\ A \overset{\text{def}}{=} \displaystyle\sum_{j=1}^{k} I_j \, , B \overset{\text{def}}{=} \sum_{j=k+1}^{N} I_j\end{cases}$$

$$\xrightarrow{yields} INL(k) = N \cdot \frac{A}{A+B}$$

$$Var(INL(k) = Var\left(N \cdot \frac{A}{A+B}\right) = N^2 \cdot Var\left(\frac{A}{A+B}\right) \xRightarrow{X=A,\ \ Y=A+B} N^2 \cdot Var\left(\frac{X}{Y}\right)$$

Note A and B is independent, but X and Y is dependent.

Because

$$Var\left(\frac{X}{Y}\right) = \left(\frac{\mu_X}{\mu_Y}\right)^2 \left(\frac{{\sigma_X}^2}{{\mu_X}^2} + \frac{{\sigma_Y}^2}{{\mu_Y}^2} - 2\frac{COV(X,Y)}{\mu_X . \mu_Y}\right)$$

$$\begin{cases}\left(\dfrac{\mu_X}{\mu_Y}\right)^2 = \dfrac{k^2}{N^2} \\ \dfrac{{\sigma_X}^2}{{\mu_X}^2} = \dfrac{1}{k}\dfrac{\sigma^2}{I^2}, \dfrac{{\sigma_Y}^2}{{\mu_Y}^2} = \dfrac{1}{N}\dfrac{\sigma^2}{I^2} \\ COV(X,Y) = E(XY) - E(X)E(Y) = E\big(A(A+B)\big) - kI \cdot NI = E(A^2) + E(AB) - Nk \cdot I^2 \\ = Var(A) + E(A)^2 + E(A)E(B) - Nk \cdot I^2 \\ = k \cdot \sigma^2 + k^2 I^2 + k(N-k)I^2 - Nk \cdot I^2 \\ = k \cdot \sigma^2\end{cases}$$

$$\xrightarrow{yields} Var\left(\frac{X}{Y}\right) = \frac{k^2}{N^2}\left(\frac{1}{k} - \frac{1}{N}\right)\frac{\sigma^2}{I^2}$$

Then,

$$Var(INL(k) = N^2 \cdot \frac{k^2}{N^2}\left(\frac{1}{k} - \frac{1}{N}\right)\frac{\sigma^2}{I^2} = k\left(1 - \frac{k}{N}\right)\frac{\sigma^2}{I^2}$$

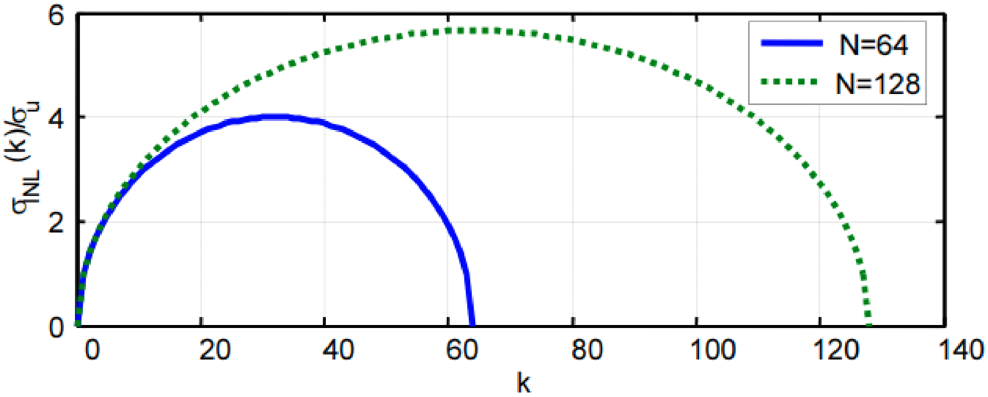
So,

$$\boldsymbol{\sigma_{INL,k} = \sqrt{k\left(1 - \frac{k}{N}\right)} \cdot \frac{\sigma}{I} = \sqrt{k\left(1 - \frac{k}{N}\right)} \cdot \sigma_u}$$

Worst case $k = N/2$

$$\boldsymbol{\sigma_{INL,max} \approx \frac{1}{2}\sigma_u\sqrt{2^B}}$$

$$B = 2 + 2log_2(\frac{\sigma_{INL}}{\sigma_u})$$



1.2 DNL of thermometer DAC

$$\begin{cases} DNL(k) = \dfrac{Step(k) - Step_{avg}}{Step_{avg}} \\ Step_{avg} = \dfrac{1}{N}\sum_{j=1}^{N} I_j \end{cases} \xrightarrow{yields} DNL(k) = \frac{I_k - \frac{1}{N}\sum_{j=1}^{N} I_j}{\frac{1}{N}\sum_{j=1}^{N} I_j}$$

$$DNL(k) = \frac{I + \Delta I_k - \frac{1}{N}\sum_{j=1}^{N}(I + \Delta I_j)}{\frac{1}{N}\sum_{j=1}^{N}(I + \Delta I_j)} = \frac{\left(1 - \frac{1}{N}\right)\Delta I_k - \frac{1}{N}\sum_{j=1;j\neq k}^{N}\Delta I_j}{I + \frac{1}{N}\sum_{j=1}^{N}\Delta I_j} \xRightarrow{ommit\ \frac{1}{N}\sum_{j=1}^{N}\Delta I_j}$$

$$DNL(k) = \frac{\left(1 - \frac{1}{N}\right)\Delta I_k - \frac{1}{N}\sum_{j=1;j\neq k}^{N}\Delta I_j}{I} = \left(1 - \frac{1}{N}\right)\frac{\Delta I_k}{I} - \frac{1}{N}\sum_{j=1;j\neq k}^{N}\frac{\Delta I_j}{I}$$

$$Var(DNL(k)) = \left[\left(1 - \frac{1}{N}\right)^2 + \frac{N-1}{N^2}\right]\frac{\sigma^2}{I^2} = \frac{N-1}{N}\frac{\sigma^2}{I^2} \xrightarrow{yields}$$

$$\boldsymbol{\sigma_{DNL,k} = \sqrt{1 - \frac{1}{N}} \cdot \frac{\sigma}{I} = \sqrt{1 - \frac{1}{N}}\sigma_u \xRightarrow{N\gg 1}}$$

$$\boldsymbol{\sigma_{DNL,k} = \sigma_u}$$

Another approximation method

$$DNL(k) = \frac{Step(k) - Step_{avg}}{Step_{avg}} = \frac{I + \Delta I - I}{I} = \frac{\Delta I}{I}$$

$$\boldsymbol{\sigma_{DNL,k} = stdev\left(\frac{\Delta I}{I}\right) = \sigma_u}$$

2.1 INL of Binary Weighted DAC

INL same as for thermometer DAC

2.2 DNL of Binary Weighted DAC

Assume $X_{[1,k]} \cap Y_{[1,k-1]} = \emptyset$

$$\begin{cases} DNL(k) = \dfrac{Step(k) - Step_{avg}}{Step_{avg}} \\ Step(k) = \displaystyle\sum_{i=1}^{k} X_i - \sum_{j=1}^{k-1} Y_j \end{cases} \xrightarrow{yields}$$

$$DNL(k) = \sum_{i=1}^{k} \frac{X_i - Step_{avg}}{Step_{avg}} - \sum_{j=1}^{k-1} \frac{Y_j - Step_{avg}}{Step_{avg}} = \sum_{i=1}^{k} DNL, th, i - \sum_{j=1}^{k-1} DNL, th, j$$

DNL, th, i is given by thermometer DNL formula, so

$$Var\big(DNL(k)\big) = (k + k - 1) \cdot \left(\sqrt{1 - \frac{1}{N}}\sigma_u\right)^2$$

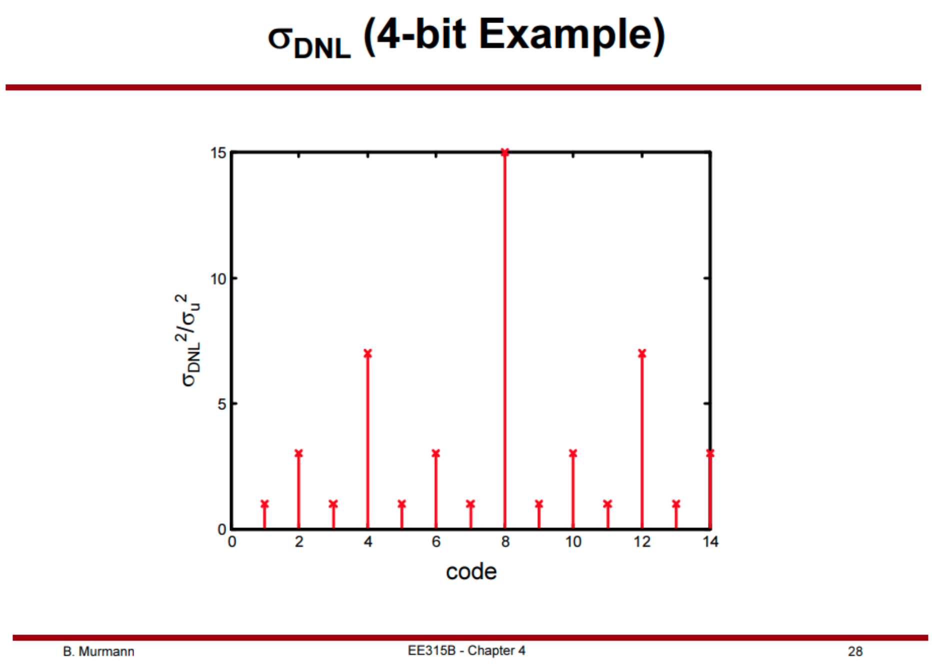
$$\sigma_{DNL,k} = \sqrt{2k-1}\sqrt{1 - \frac{1}{N}}\sigma_u \overset{N \gg 1}{\Longrightarrow} \sqrt{2k-1}\sigma_u$$

Worst case@ $k = 2^{B-1}, k - 1 = 2^{B-1} - 1$

$$\boldsymbol{\sigma_{DNL,k} = \sqrt{2^B - 1}\sqrt{1 - \frac{1}{N}}\sigma_u \overset{N \gg 1}{\Longrightarrow} \sqrt{2^B - 1}\sigma_u}$$

Generally (X, Y is not excluded)

$$\sigma_{DNL,k} = \sqrt{int(bin(k) \oplus bin(k-1))}\sqrt{1 - \frac{1}{N}}\sigma_u$$



3.1 INL of Segmented DAC

Same as in thermometer DAC

3.2 DNL of Segmented DAC

– Worst case occurs when LSB DAC turns off and one more MSB DAC element turns on

– Essentially same DNL as a binary weighted DAC with Bb+1 bits

$B = B_t + B_b$, $B_t=2$ $B_b=4$

4 DAC INL/DNL Summary

Comparison

	Thermometer	Segmented	Binary Weighted
σ_{INL} (worst case)	$\cong \frac{1}{2}\sigma_u\sqrt{2^B}$		
σ_{DNL} (worst case)	$\cong \sigma_u$	$\cong \sigma_u\sqrt{2^{B_b+1}-1}$	$\cong \sigma_u\sqrt{2^B-1}$
Number of Switched Elements	2^B-1	$B_b+2^{B_t}-1$	B

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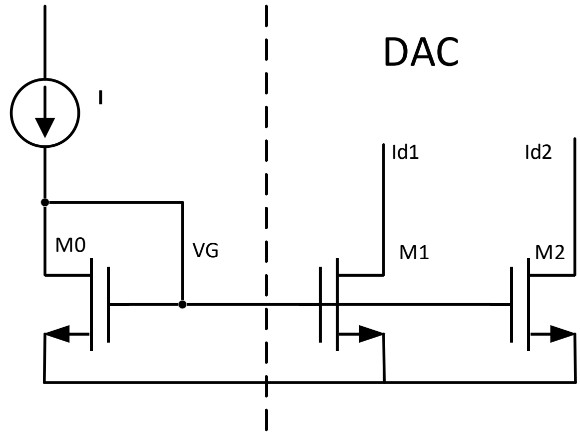
- DAC choice of architecture has significant impact on DNL
- INL is independent of DAC architecture and requires element matching commensurate with overall DAC precision
- Results assume uncorrelated random element variations
- Systematic errors and correlations are usually also important and may affect final DAC performance

Ref: Kuboki, S.; Kato, K.; Miyakawa, N.; Matsubara, K. Nonlinearity analysis of resistor string A/D converters. IEEE Transactions on Circuits and Systems, vol.CAS-29, (no.6), June 1982. p.383-9.

5 Demystifying current source variation effect



Since I is forced by external constant current, VG track the change of Vth,0, so

$$I_{D1} - I = g_m\left(\Delta V_{th,0} - \Delta V_{th,1}\right) - 0$$

$$\frac{\Delta I_{D1,0}}{I} = \frac{g_m}{I}\left(\Delta V_{th,0} - \Delta V_{th,1}\right) \xrightarrow{yields}$$

$$\boldsymbol{\sigma_{\frac{\Delta I_{D1,0}}{I}} = \frac{g_m}{I}\sqrt{2}\frac{A_{VT}}{\sqrt{2}\sqrt{WL}} = \frac{g_m}{I}\frac{A_{VT}}{\sqrt{WL}}}$$

For M1, M2, same $\Delta V_{th,0}$ is applied

$$I_{D2} - I_{D1} = g_m\left(\Delta V_{th,0} - \Delta V_{th,1}\right) - g_m\left(\Delta V_{th,0} - \Delta V_{th,2}\right)$$

$$\Delta I_{D1,2} = g_m\left(\Delta V_{th,2} - \Delta V_{th,1}\right)$$

$$\frac{\Delta I_{D1,2}}{I} = \frac{g_m}{I}\left(\Delta V_{th,2} - \Delta V_{th,1}\right) \xrightarrow{yields}$$

$$\boldsymbol{\sigma_{\frac{\Delta I_{D1,2}}{I}} = \frac{g_m}{I}\frac{A_{VT}}{\sqrt{WL}}}$$

For DAC part (right side of split line), VG is fixed, all branches are connected with same voltage level (ΔI_{root} due to M0 variation, $\Delta I_{Dj,self}$ due to M1/2 self-variation)

$$\begin{cases}\Delta I_{D1,tot} = \Delta I_{root} + \Delta I_{D1,self} \\ \Delta I_{D2,tot} = \Delta I_{root} + \Delta I_{D2,self}\end{cases}$$

$$\frac{\Delta I_{D1,tot}}{I} = \frac{\Delta I_{root}}{I} + \frac{\Delta I_{D1,self}}{I} \xrightarrow{yields} Var\left(\frac{\Delta I_{D1,tot}}{I}\right) = 2{\sigma_u}^2$$

$$\sigma_{\frac{\Delta I_{D1,tot}}{I}} = \sqrt{2}\sigma_u = \sqrt{2}\frac{g_m}{I}\frac{A_{VT}}{\sqrt{2}\sqrt{WL}} = \frac{g_m}{I}\frac{A_{VT}}{\sqrt{WL}}$$

Same with $\Delta I_{D2,tot}$

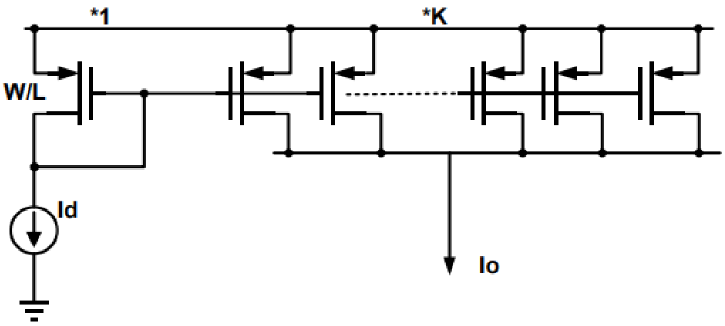
One question, how to get σ_u, substruction of $\Delta I_{D1,tot}$ and $\Delta I_{D2,tot}$ can filter ΔI_{root} due to current source variation.

$$\Delta I_{D1,tot} - \Delta I_{D2,tot} = \Delta I_{D1,self} - \Delta I_{D2,self} \xrightarrow{yields} \boldsymbol{\sigma_{\frac{\Delta I_{D1,2,tot}}{I}} = \sqrt{2}\frac{g_m}{I}\frac{A_{VT}}{\sqrt{2}\sqrt{WL}}}$$

Then

$$\sigma_u = \frac{g_m}{I}\frac{A_{VT}}{\sqrt{2}\sqrt{WL}} = \frac{\sigma_{\frac{\Delta I_{D1,2,tot}}{I}}}{\sqrt{2}}$$

6 Current mirror mismatch analysis – another method



$$\Delta I_o = k \cdot \Delta I_{root} + \sum_{j}^{k} \Delta I_{Dj,self} \xrightarrow{yields}$$

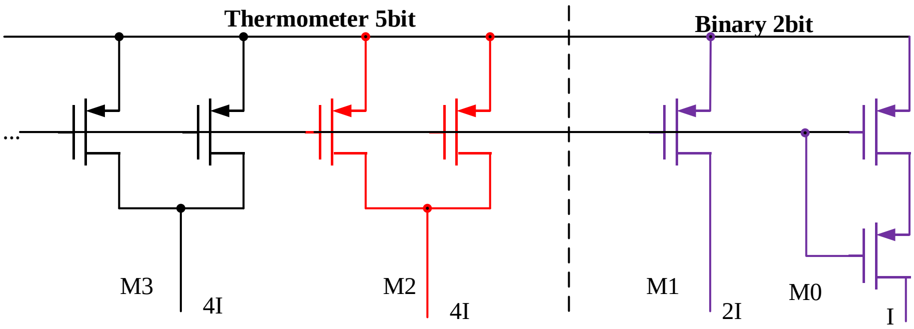
$$\frac{\Delta I_o}{I_o} = \frac{\Delta I_{root}}{I} + \frac{1}{k} \sum_{j}^{k} \frac{\Delta I_{Dj,self}}{I} \xrightarrow{yields}$$

$$Var\left(\frac{\Delta I_o}{I_o}\right) = Var\left(\frac{\Delta I_{root}}{I}\right) + \frac{1}{k^2} Var\left(\sum_{j}^{k} \frac{\Delta I_{Dj,self}}{I}\right) \xrightarrow{yields}$$

$$\boldsymbol{\sigma_{\frac{\Delta I_o}{I_o}} = \sqrt{\sigma_u{}^2 + \frac{1}{k^2} \cdot k\sigma_u{}^2} = \sqrt{1 + \frac{1}{k}}\sigma_u = \sqrt{1 + \frac{1}{k}} \frac{g_m}{I} \frac{A_{VT}}{\sqrt{2}\sqrt{WL}}}$$

7 practical example – IDAC

Thermometer: 5 bit; Binary: 2 bit



$$stdev\left(\frac{\Delta I}{I}\right) \propto \frac{1}{\sqrt{WL}}$$

$$\begin{cases} sdev\left(\frac{\Delta I_2}{I_2}\right) = \frac{A}{\sqrt{2W \cdot L}} \stackrel{\text{def}}{=} \sigma_u \\ stdev\left(\frac{\Delta I_1}{I_1}\right) = \frac{A}{\sqrt{W \cdot L}} = \sqrt{2}\sigma_u \\ stdev\left(\frac{\Delta I_0}{I_0}\right) = \frac{A}{\sqrt{W \cdot 2L}} = \sigma_u \end{cases}$$

@worst case

$$Var\left(\frac{\Delta I_2-\Delta I_1-\Delta I_0}{I}\right)=Var\left(4\frac{\Delta I_2}{4I}\right)+Var\left(2\frac{\Delta I_1}{2I}\right)+Var\left(\frac{\Delta I_0}{I}\right)$$

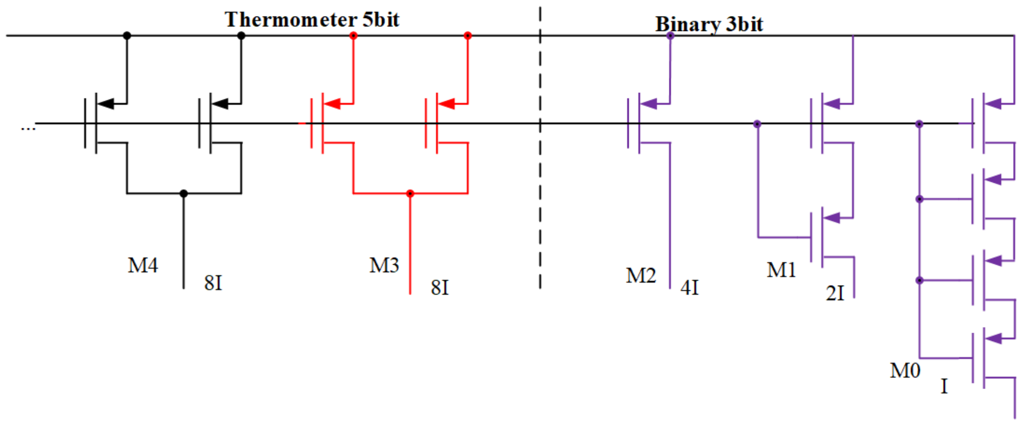
$$Var\left(\frac{\Delta I_2-\Delta I_1-\Delta I_0}{I}\right)=16{\sigma_u}^2+4\cdot 2{\sigma_u}^2+{\sigma_u}^2$$

$$stdev\left(\frac{\Delta I_2-\Delta I_1-\Delta I_0}{I}\right)=5\sigma_u \xrightarrow{yields} \boldsymbol{stdev(DNL)=5\sigma_u}$$

The current difference of 2 thermometer is $\sigma_{\Delta I}$ and equal $\sqrt{2}\sigma_u$
Simulation result $3\sigma_{\Delta I}$=110m, then $3stdev(DNL)$=0.389 LSB

8 practical example – VDAC

Thermometer: 5 bit; Binary: 3 bit



$$\begin{cases} stdev\left(\frac{\Delta I_3}{I_3}\right)=\frac{A}{\sqrt{2W\cdot L}}\stackrel{\text{def}}{=}\sigma_u \\ stdev\left(\frac{\Delta I_2}{I_2}\right)=\frac{A}{\sqrt{W\cdot L}}=\sqrt{2}\sigma_u \\ stdev\left(\frac{\Delta I_1}{I_1}\right)=\frac{A}{\sqrt{W\cdot 2L}}=\sigma_u \\ stdev\left(\frac{\Delta I_0}{I_0}\right)=\frac{A}{\sqrt{W\cdot 4L}}=\frac{\sigma_u}{\sqrt{2}} \end{cases}$$

@worst case

$$Var\left(\frac{\Delta I_3-\Delta I_2-\Delta I_1-\Delta I_0}{I}\right)=Var\left(8\frac{\Delta I_3}{8I}\right)+Var\left(4\frac{\Delta I_2}{4I}\right)+Var\left(2\frac{\Delta I_1}{2I}\right)+Var\left(\frac{\Delta I_0}{I}\right)$$

$$Var\left(\frac{\Delta I_3-\Delta I_2-\Delta I_1-\Delta I_0}{I}\right)=64{\sigma_u}^2+16\cdot 2{\sigma_u}^2+4{\sigma_u}^2+\frac{{\sigma_u}^2}{2}=100.5\cdot{\sigma_u}^2$$

$$stdev\left(\frac{\Delta I_3-\Delta I_2-\Delta I_1-\Delta I_0}{I}\right)\approx 10\sigma_u \xrightarrow{yields} \boldsymbol{stdev(DNL)=10\sigma_u}$$

The current difference of 2 thermometer is $\sigma_{\Delta I}$ and equal $\sqrt{2}\sigma_u$
Simulation result $3\sigma_{\Delta I}$=35m, then $3stdev(DNL)$=0.247 LSB

Reference:

ee315b_reader_2013.pdf B. Murmann
Mixed Analog and Digital Integrated Circuit Design 李强/黄乐年 2009–2010 学年下学期