single-pole filter model in systemverilog

A linear system having a pole ω_1 can be expresed in Laplace s-domain as follows:

$$T(s) = \frac{Y(s)}{X(s)} = \frac{1}{1 + s/\omega_1}$$

A ramp input X(s) in Laplace s-domain is:

$$X(s) = \frac{a}{s} + \frac{b}{s^2}$$

The complete response is composed with zero-state response and zero-input response

1) zero-state response

$$Y(s) = T(s) \bullet X(s) = (\frac{a}{s} + \frac{b}{s^2})\frac{1}{1 + s/\omega_1}$$

```
>> syms s a b w1;
>> Y = (a/s+b/s^2)/(1+s/w1);
>> ilaplace(Y)
ans =
b*t - (b - a*w1)/w1 + (exp(-t*w1)*(b - a*w1))/w1
```

2) zero-input response

$$\frac{Y(s)}{X(s)} = \frac{1}{1 + s/\omega_1}$$

with X(s) equals zero, we get

$$Y \bullet (1 + s/\omega_1) = 0$$

$$Y + \frac{1}{\omega_1}sY = 0$$

The differential equation is

$$y(t) + \frac{1}{\omega_1}\dot{y}(t) = 0$$

The Laplace Transform with initial conditon is

$$Y + \frac{1}{\omega_1}(sY - y_0) = 0$$

$$Y = y_0\frac{1}{s + \omega_1}$$

```
>> syms y0
>> Y = y0/(s+w1);
>> ilaplace(Y)
ans =
y0*exp(-t*w1)
```

So the complete response is

b*t - (b - a*w1)/w1 + exp(-t*w1)*(b/w1 - a+y0)

Appendix: Transient Response from Transfer Function Representation

1) zero-state response

Key Concept: The Zero State Response and the Transfer Function

Given the transfer function of a system:

$$H(s)=\frac{Y(s)}{X(s)}=\frac{\text{Output}}{\text{Input}}$$

The zero state response is easily found

$$Y_{zs}(s)=H(s)\cdot X(s)$$

Note: h(t)=system impulse response and in the time domain $y_{zs}(t)=h(t)*x(t)$.

2) zero-input response

Key Concept: The Zero Input Response and the Transfer Function

Given the transfer function of a system:

$$H(s)=\frac{Y(s)}{X(s)}=\frac{\text{Output}}{\text{Input}}$$

The zero input response is found by first finding the system differential equation (with the input equal to zero), and then applying initial conditions.

For example if the transfer function is

$$H(s)=\frac{Y(s)}{X(s)}=\frac{b_0 s+b_1}{s^2+a_1 s+a_2}$$

then the system differential equation (with zero input) is

$$\ddot{y}_{zi}+a_1\dot{y}_{zi}+a_2 y_{zi}=0$$

and the Laplace Transform (with initial conditions) is

$$\left(s^2Y_{zi}(s)-sy_{zi}\left(0^-\right)-\dot{y}_{zi}\left(0^-\right)\right)+a_1\left(sY_{zi}(s)-y_{zi}\left(0^-\right)\right)+a_2Y_{zi}(s)=0$$

or

$$Y_{zi}(s)=\frac{sy_{zi}\left(0^-\right)+\dot{y}_{zi}\left(0^-\right)+a_1y_{zi}\left(0^-\right)}{s^2+a_1s+a_2}$$

3) complete response

Key concept: Determining the complete response using the transfer function

To find the complete response of a system from its transfer function: $H(s)=\frac{Y(s)}{X(s)}=\frac{\text{Output}}{\text{Input}}$

1. Find the zero state response by multiplying the transfer function by the input in the Laplace Domain.
   $Y_{zs}(s)=H(s)\cdot X(s)$
2. Find the zero input response by using the transfer function to find the zero input differential equation. Take the Laplace Transform of that equation (including initial conditions), and solve.
3. The complete response is the sum of the zero-state and zero input response
   $y_c(t)=y_{zs}(t)+y_{zi}(t)$

complex conjugate poles

pole's real part and imagiary part is ω_r and ω_i, then transfer function is

$$\mathrm{T(s)} = \frac{1}{(1+\frac{\mathrm{s}}{\omega_r + j\omega_i})(1+\frac{\mathrm{s}}{\omega_r - j\omega_i})} = \frac{\omega_r^2 + \omega_i^2}{(\mathrm{s}+\omega_r)^2 + \omega_i^2}$$

Zero input response

$$\left[(\mathrm{s}+\omega_r)^2 + \omega_i^2\right] \bullet Y = 0$$

$$\ddot{y}(t) + \dot{y}(t) \bullet 2\omega_r + y(t)(\omega_r^2 + \omega_i^2) = 0$$

with initial state

$$s^2 Y - s \bullet y_0 - \dot{y}_0 + (sY - y_0) \bullet 2\omega_r + Y(\omega_r^2 + \omega_i^2) = 0$$

Then

$$\mathrm{Y} = y_0 \bullet \frac{\mathrm{s}+\omega_r}{(\mathrm{s}+\omega_r)^2 + \omega_i^2} + \frac{y_0\omega_r + \dot{y}_0}{\omega_i} \bullet \frac{\omega_i}{(\mathrm{s}+\omega_r)^2 + \omega_i^2}$$

inverse laplace transform, with $e^{-at} f(t) \overset{L}{\leftrightarrow} F(s+a)$

$$\mathrm{y(t)} = e^{-\omega_r t}\left[y_0 cos(\omega_i t) + \frac{y_0\omega_r + \dot{y}_0}{\omega_i}\sin(\omega_i t)\right]$$

Zero state response

$$\left[(\mathrm{s}+\omega_r)^2 + \omega_i^2\right] \bullet Y = (\omega_r^2 + \omega_i^2) \bullet X$$

$$Y = \frac{(\omega_r^2 + \omega_i^2)}{\left[(\mathrm{s}+\omega_r)^2 + \omega_i^2\right]} \bullet X$$