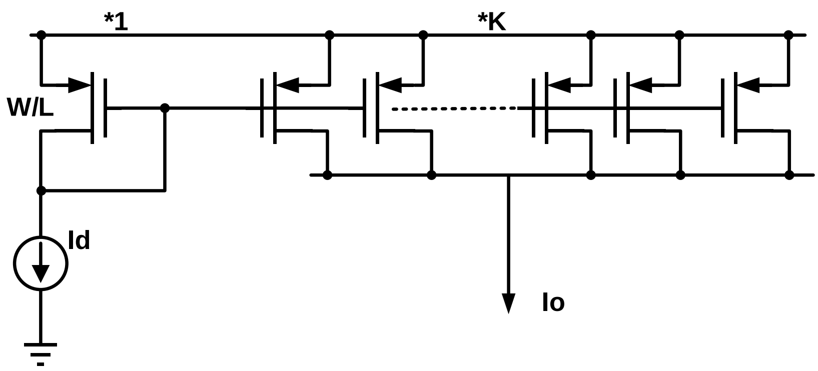
Current mirror mismatch analysis



1) step by step method

ΔV_T of current sink(Id) and mirror(Io) given as below, K is mirror ratio,

$$\Delta V_T = \frac{A_{VT}}{\sqrt{WL}\sqrt{2}}$$

$$\Delta V_{T,0} = \frac{A_{VT}}{\sqrt{WL}\sqrt{2}}$$

$$\ldots$$

$$\Delta V_{T,K-1} = \frac{A_{VT}}{\sqrt{WL}\sqrt{2}}$$

$\sqrt{2}$ come from definition of "self-mismatch" [1]

output current(Io) variation:

$$\Delta I_o = g_m\big(\Delta V_{T,0} - \Delta V_T\big) + g_m\big(\Delta V_{T,0} - \Delta V_T\big) + \cdots + g_m(\Delta V_{T,K-1} - \Delta V_T)$$

$$\Delta I_o = g_m\left(\sum_{n=0}^{K-1} \Delta V_{T,n} - K\cdot\Delta V_T\right) \xrightarrow{variance}$$

$$(\Delta I_o)^2 = (g_m)^2\big[K\cdot(\Delta V_{T,n})^2 + K^2\cdot(\Delta V_T)^2\big]$$

$$(\frac{\Delta I_o}{I_o})^2 = \left(\frac{g_m}{K\cdot I_d}\right)^2 [K\cdot(\Delta V_T)^2 + K^2\cdot(\Delta V_T)^2]$$

$$(\frac{\Delta I_o}{I_o})^2 = \left(\frac{g_m}{I_d}\right)^2 \left[\frac{1}{K}\cdot(\Delta V_T)^2 + (\Delta V_T)^2\right]$$

$$(\frac{\Delta I_o}{I_o})^2 = \left(\frac{g_m}{I_d}\right)^2 \left[(\frac{1}{K}+1)(\frac{A_{VT}}{\sqrt{WL}\sqrt{2}})^2\right]$$

2) quotient equation[1]

$$f = \frac{I_o}{I_d}$$

$$\begin{cases} \left(\frac{\delta_f}{f}\right)^2 = \left(\frac{\delta_{Io}}{Io}\right)^2 + \left(\frac{\delta_{Id}}{Id}\right)^2 \\ \delta_{Id} = g_m\cdot\Delta V_T \\ \delta_{Io} = K\cdot g_m\cdot\Delta V_T/\sqrt{K} \\ \Delta V_T = \frac{A_{VT}}{\sqrt{WL}\sqrt{2}} \end{cases} \xrightarrow{yields} \begin{cases} \frac{\delta_{Id}}{Id} = \frac{g_m}{I_d}\cdot\frac{A_{VT}}{\sqrt{WL}\sqrt{2}} \\ \frac{\delta_{Io}}{Io} = \frac{g_m}{I_d}\cdot\frac{A_{VT}}{\sqrt{WL}\sqrt{2}}\cdot\frac{1}{\sqrt{K}} \end{cases} \xrightarrow{yields} \left(\frac{\delta_f}{f}\right)^2 = \left(\frac{g_m}{I_d}\right)^2\left[(\frac{1}{K}+1)(\frac{A_{VT}}{\sqrt{WL}\sqrt{2}})^2\right]$$

which is consistent with method 1.

TEXT: mirror ratio related

[1] Random Offset in CMOS IC Design, https://designers-guide.org/forum/Attachments/mismatch_presentation.pdf