Locating magnetic contacts: a comparison of the horizontal gradient, analytic signal, and local wavenumber methods

Jeffrey D. Phillips, U.S. Geological Survey*

Summary

Three methods for locating isolated magnetic contacts from magnetic anomaly data are examined. All methods are similar in that they involve passing a small window over a derivative profile or grid, searching for local maxima within the window, estimating the strike (for grids) and horizontal position of the contact from the local maxima, then estimating the depth of the contact and other parameters by fitting the derivative data within the window to a theoretical curve. The methods differ in their complexity, accuracy, and sensitivity to noise and anomaly interference. The horizontal gradient method requires first-order horizontal derivatives and a reduction-to-the-pole or pseudogravity transformation. It is the method least susceptible to noise, but results are accurate only where the magnetization is induced and the sources are of very specific types. The analytic signal method requires first-order horizontal and vertical derivatives of the magnetic field or of the first vertical integral of the magnetic field. Horizontal location of isolated sources is generally accurate, but vertical position is accurate only for specific source types. Both the horizontal gradient method and the analytic signal method can be used to estimate minimum and maximum limits on source depths. The local wavenumber method requires first- and second-order horizontal and vertical derivatives, and is the most susceptible to noise and interference effects. In the absence of these problems, it provides accurate horizontal and vertical locations of isolated sources along with structural indices for the sources. The analytic signal and local wavenumber methods can be extended to estimate the geologic dip and magnetic susceptibility contrast across isolated contacts under the assumption of induced magnetization.

Introduction

Blakely and Simpson (1986) published the first automated method for locating the horizontal positions of magnetic contacts from gridded magnetic data. Their approach was based on the observations of Cordell and Grauch (1985) that the magnitude of the horizontal gradient of the pseudogravity transformation of a magnetic field peaks over vertical magnetic contacts. In the automated method, a 3x3 window is passed over the horizontal gradient magnitude (HGM) grid, and an attempt is made to fit parabolic peaks to the four 3-point scans passing through the center of the window. If a sufficient number of peaks is found (usually 2 to 4), the location of the largest peak is taken as the contact location.

The use of the HGM for depth estimation was first proposed by Roest and Pilkington (1993). Other functions that peak over magnetic contacts and can be used to estimate their depths include the squared magnitude of the analytic signal (Roest and others, 1992) and the local wavenumber (Thurston and Smith, 1997; Smith and others, 1998). Analysis methods corresponding to these three functions will be compared in terms of their accuracy in the horizontal and vertical location of simple sources, and their susceptibility to noise and interference effects.

A Common Approach

All three methods use a common approach to determine the horizontal locations and depths of contacts. For profile data, a 3-point window is moved along the derivative profile, and an attempt is made to fit a parabolic peak to each data triplet. If a peak is found, the horizontal location of the peak is chosen as the horizontal location of the contact. For each method, the derivative profile near the peak has the theoretical form

$$Z(h) = \frac{K_Z}{(h-h_0)^2 + {z_0}^2} \tag{1}$$

where Z is the amplitude of the derivative profile, K_Z is a method-dependent constant, z_0 is the depth, h_0 is the horizontal coordinate of the peak, and $|h\text{-}h_0|$ is the horizontal distance away from the peak. A solution for the squared depth, given by

$${z_0}^2 = \frac{Z(h)(h-h_0)^2}{Z(h_0) - Z(h)} \tag{2}$$

is obtained for each member of the data triplet contributing to the peak. Only the shallowest depth solution is retained.

For gridded data, the approach of Blakely and Simpson (1986) is used to locate local maxima internal to a 3x3 window. If atleast two internal maxima are found, the local strike direction is estimated by searching for additional local maxima around the periphery of the 3x3 window or within a larger 5x5 window, then fitting a least-squares line to the internal and peripheral maxima. The data in the window (including any estimated maxima) are extracted within a belt perpendicular to the strike direction, and the squared depth and numerator K_Z are estimated by a least-squares fit to equation (1).

Horizontal Gradient Method

The horizontal gradient method uses the magnitude of the horizontal gradient defined as

$$|H(h)| = \left| \frac{\partial M}{\partial h} \right| \tag{3}$$

for profile data, and as

$$|H(x,y)| = \sqrt{\left(\frac{\partial M}{\partial x}\right)^2 + \left(\frac{\partial M}{\partial y}\right)^2} \tag{4}$$

for gridded data, where M is either the reduced-to-pole magnetic field or the pseudogravity transformation of the magnetic field. Roest and Pilkington (1993) show that if M is the pseudogravity field of the isolated edge of a thin horizontal sheet, the HGM is given by

$$|H(h)| = \frac{K_H}{(h-h_0)^2 + z_0^{\;2}} \tag{5}$$

Here $K_H = 2kFz_0$, where k is the magnetic susceptibility contrast and F is the geomagnetic field strength. The same equation results if M is the reduced-to-pole magnetic field of a vertical contact of large depth extent. Thus the horizontal gradient method can be used to accurately locate the tops of isolated vertical contacts from the reduced-to-pole magnetic field (HGM-RTP, Figure 1a) or the isolated edges of horizontal sheet-like magnetic bodies from the pseudogravity field (HGM-PG, Figure 1d).

When the method is applied to the reduced-to-pole magnetic field of a thin horizontal body, the depth estimate will be too shallow, and fictitious, overly deep contacts may appear parallel to the actual contact due to dipolar effects (HGM-RTP, Figure 1d). When the method is applied to the pseudogravity field of a contact, the depth estimate will be too deep (HGM-PG, Figure 1a). Contacts with non-vertical dips (Figure 1b) will result in horizontal locations that are down dip from the true locations (Grauch and Cordell, 1987). Non-horizontal sheet-like bodies and horizontal line sources (eg. pipelines) will always produce double-peaked HGM curves that are inconsistent with equation (5) (Figure 1c,e,f).

Depth estimates from the reduced-to-pole magnetic field (excluding dipolar effects) represent minimum source depths, and are accurate only for contacts with large depth extent. Depth estimates from the pseudogravity field represent maximum source depths, and are accurate only for thin horizontal sheet sources. Thus, the horizontal gradient method is primarily useful for the approximate horizontal location of edges and for estimating minimum and maximum source depths.

Analytic Signal Method

The analytic signal method uses the square of the analytic signal amplitude defined as

$$|A(h)|^2 = \left(\frac{\partial M}{\partial h}\right)^2 + \left(\frac{\partial M}{\partial z}\right)^2 \tag{6}$$

for profile data, and as

$$|A(x,y)|^2 = \left(\frac{\partial M}{\partial x}\right)^2 + \left(\frac{\partial M}{\partial y}\right)^2 + \left(\frac{\partial M}{\partial z}\right)^2 \tag{7}$$

for gridded data, where M is the anomalous magnetic field. Typically, the horizontal derivatives are computed in the space domain using differences or splines, and the vertical derivative is computed in the wavenumber domain using fast Fourier transforms. For profile data, the vertical derivative can be computed as the Hilbert transform of the horizontal derivative.

If we assume M represents the field of an isolated magnetic contact of large depth extent, then

$$|A(h)|^2 = \frac{K_A}{(h-h_0)^2 + z_0^{\;2}} \tag{8}$$

(Nabighian, 1972). For total field anomaly data, $K_A = [\, 2kF\, (1\text{-}\cos^2 i \sin^2 a)\, \sin d\,]^2$, where d is the dip of the contact, i is the inclination of the geomagnetic field, and a is the angle between the profile direction and magnetic north (Nabighian, 1972). When applied to the observed magnetic field, the analytic signal method generally produces good horizontal locations for contacts and sheet sources regardless of their geologic dip or the geomagnetic latitude (AS-MAG, Figure 1). Depths are accurate for contacts (Figure 1a,b), and are too shallow for most other source types (Figure 1c,d,f). Dipolar effects are absent. Analogous to the horizontal gradient method, the analytic signal method can be applied to the pseudogravity field, or to the first vertical integral of the magnetic field, for more accurate depths to sheet sources (AS-FVI, Figure 1c,d).

Local Wavenumber Method

Thurston and Smith (1997) give the following expression for the local wavenumber along a profile over a 2-D source:

$$k(h) = \frac{1}{|A(h)|^2}\left(\frac{\partial^2 M}{\partial h \partial z}\frac{\partial M}{\partial h} - \frac{\partial^2 M}{\partial h^2}\frac{\partial M}{\partial z}\right) \tag{9}$$

where $M(h)$ is the anomalous magnetic field and $|A(h)|^2$ is given by equation (6). An equivalent expression for the local wavenumber of the gridded anomalous magnetic field $M(x,y)$ is

$$k(x,y) = \frac{1}{|A(x,y)|^2}\left(\frac{\partial^2 M}{\partial x \partial z}\frac{\partial M}{\partial x} + \frac{\partial^2 M}{\partial y \partial z}\frac{\partial M}{\partial y} + \frac{\partial^2 M}{\partial z^2}\frac{\partial M}{\partial z}\right) \tag{10}$$

where $|A(x,y)|^2$ is given by equation (7).

Smith and others (1998) show that, for isolated linear sources, the local wavenumber k is related to depth z_0 and structural index s by

$$k(h) = \frac{K_L}{(h-h_0)^2 + z_0^{\;2}} = \frac{(s-1)z_0}{(h-h_0)^2 + z_0^{\;2}} \tag{11}$$

The value of the structural index is 0 for the top of a contact or fault with large depth extent, 1 for the edge of a thin sheet, 2 for the axis of an extended pipe or cylinder, and 3 for the centroid of a sphere or dipole. Depth is estimated from equation (11), treating the numerator as a constant, then the structural index is calculated from the peak value

$$k(h_0) = (s-1)/z_0 \quad (12)$$

If the calculated structural index is less than 0 (or greater than 3), it is assigned to 0 (or 3) and the depth is recalculated from equation (12). Dip and magnetic susceptibility contrast are estimated as described by Thurston and Smith (1997). The local wavenumber method gives reliable depths for simple sources (LW-MAG, Figure 1a,b,c,d,f), but fails for the thick dike due to the interfering effects of the two corners, as do all the other methods.

Discussion

Although the two-dimensional models of Figure 1 provide a starting point for comparing the three methods, they do not address real-world issues of three-dimensional sources, noise in the data, and aliasing in the data caused by flight line spacings that are greater than the altitude above the sources (Reid, 1980). These issues are particularly important for the analysis of gridded aeromagnetic data.

The horizontal gradient method, because it only requires horizontal derivatives, is relatively insensitive to noise and aliasing. This method will produce apparent contacts that are linear and very continuous. Strike direction of the contacts can be estimated accurately within small windows. The apparent contacts will commonly be mislocated in the down-dip direction for dipping-contact sources. Non-horizontal sheet sources and horizontal pipe sources will produce paired apparent contacts off to either side. Some spurious contacts may be generated by dipole effects when the horizontal gradient of the reduced-to-pole magnetic field is used. Depth information is limited to minimum and maximum values. Reduction-to-the-pole and pseudogravity transformations are required. This can present problems at low magnetic latitudes and in areas of high remanent magnetization.

The analytic signal method is more sensitive than the horizontal gradient method to noise and aliasing in the data. Peaks of the analytic signal amplitude are generally less elongated and more circular than peaks of the HGM. Consequently, apparent contacts from the analytical signal method are less continuous, and strike directions estimated within small windows are less accurate. Horizontal locations of isolated contacts should be highly accurate. Depth information is limited to minimum and maximum values. If necessary, the effects of aliasing can be reduced by upward continuation of the data to an altitude above the sources that is equal to the flight line spacing. However, this may result in additional loss of resolution.

Because it uses the highest-order derivatives, the local wavenumber method is very sensitive to noise and aliasing in the data. Discontinuous contacts and poor control of strike direction are common. Without upward continuation, aliased data will produce many apparent contacts along the flight-line direction. Interference effects can be severe, resulting in inappropriate depth solutions. For isolated magnetic sources of all types, the local wavenumber method provides accurate horizontal locations and good control on depth and structural index.

Conclusions

The above discussion implies that using a combination of the three methods is generally the best approach. Horizontal gradient contacts that overlie analytic signal or local wavenumber contacts indicate the edges of horizontal sheets or vertically dipping contacts. Horizontal gradient contacts that are offset from analytic signal or local wavenumber contacts indicate dipping contacts, with the true location close to the analytic signal or local wavenumber solution and the dip in the direction of the horizontal gradient solution. Paired horizontal gradient contacts can be resolved as single analytic signal or local wavenumber contacts over non-horizontal sheet or pipeline sources.

The structural index estimated by the local wavenumber method can be used to resolve conflicting interpretations. Source depths should be close to the local wavenumber depths unless the local wavenumber depth falls outside the minimum/maximum ranges of the horizontal gradient depths and the analytic signal depths.

References

Blakely, R.J., and Simpson, R.W., 1986, Approximating edges of source bodies from magnetic or gravity anomalies: Geophysics, v.51, no.7, p.1494-1498.

Cordell, Lindrith, and Grauch, V.J.S., 1985, Mapping basement magnetization zones from aeromagnetic data in the San Juan basin, New Mexico *in* Hinze, W.J., ed., The utility of regional gravity and magnetic anomaly maps: Society of Exploration Geophysicists, p.181-197.

Grauch, V.J.S., and Cordell, Lindrith, 1987, Limitations on determining density or magnetic boundaries from the horizontal gradient of gravity or pseudogravity data: Geophysics, v.52, no.1, p.118-121.

Nabighian, M.N., 1972, The analytic signal of two-dimensional magnetic bodies with polygonal cross-section: its properties and use for automated anomaly interpretation: Geophysics, v.37, no.3, p.507-517.

Phillips, J.D., 1997, Potential-field geophysical software for the PC, version 2.2: U.S. Geological Survey Open-File Report 97-725, 34p. (online at ftp://greenwood.cr.usgs.gov/pub/open-file-reports/ofr-97-0725/pfofr.htm).

Reid, A.B., 1980, Aeromagnetic survey design: Geophysics, v.45, no.5, p.973-976.

Roest, W.R., and Pilkington, Mark, 1993, Identifying remanant magnetization effects in magnetic data: Geophysics, v.58, no.5, p.653-659.

Roest, W.R., Verhoef, Jacob, and Pilkington, Mark, 1992, Magnetic interpretation using the 3-D analytic signal: Geophysics, v.57, no.1, p.116-125.

Smith, R.S., Thurston, J.B., Dai, Ting-Fan, and MacLeod, I.N., 1998, iSPI™ - the improved source parameter imaging method: Geophysical Prospecting, v.46, p.141-151.

Thurston, J.B., and Smith, R.S., 1997, Automatic conversion of magnetic data to depth, dip, and susceptibility contrast using the SPI™ method: Geophysics, v.62, no.3, p.807-813.

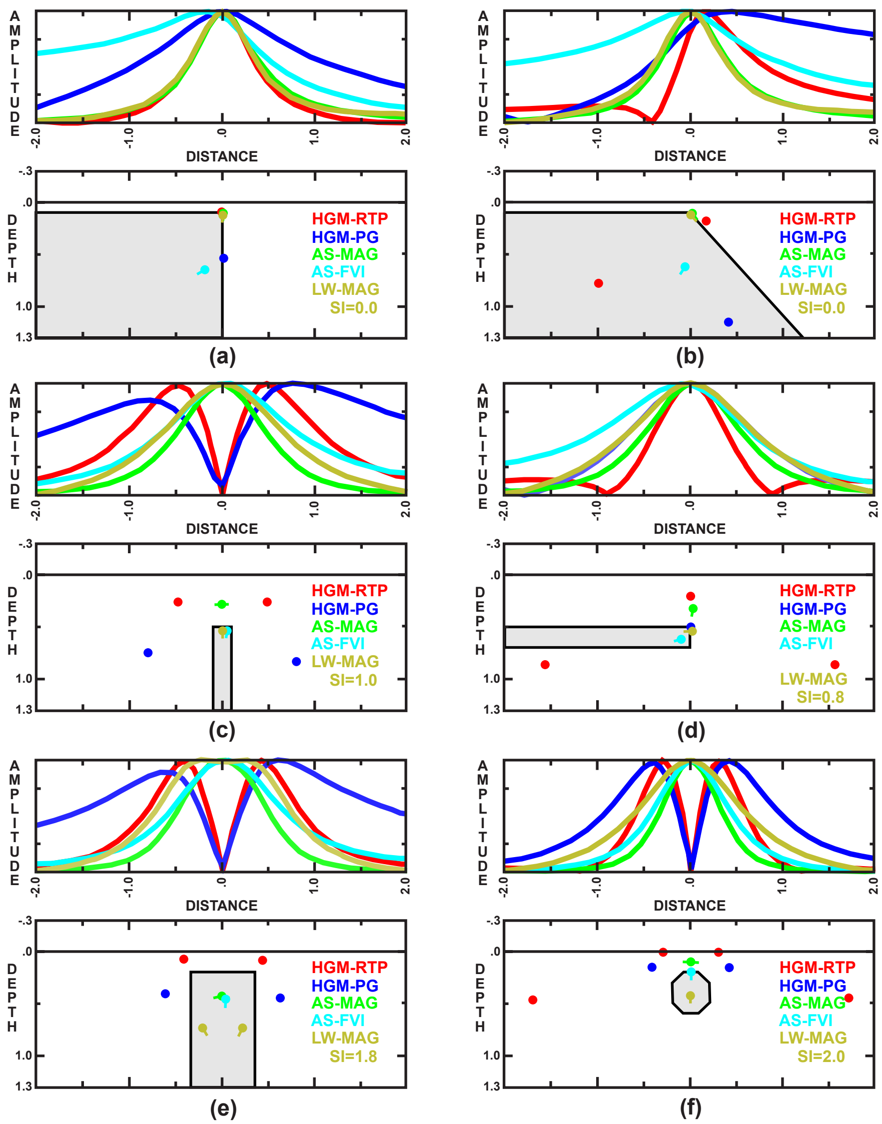

Figure 1 – Derivative curves and depth estimates for several two-dimensional models, including (a) a vertical contact, (b) a dipping contact, (c) a vertical sheet, (d) a horizontal sheet, (e) a thick dike, and (f) a horizontal pipe. Distance and depth are measured in the same arbitrary units. The observation level is at –0.3 units in each case. Normalized curves and depth estimates are color coded as (HGM-RTP) horizontal gradient magnitude of reduced-to-pole magnetic field, (HGM-PG) horizontal gradient magnitude of pseudogravity, (AS-MAG) analytic signal of observed magnetic field, (AS-FVI) analytic signal of the first vertical integral of the magnetic field, and (LW-MAG) local wavenumber of the observed magnetic field. Dip angles estimated from the analytic signal and local wavenumber are indicated by tails on the depth estimates. The structural index (SI) estimated from the local wavenumber is indicated in each case. Models and depth estimates are from a current version of program PDEPTH (Phillips, 1997).