12.11.19г. **Алгебра 1**

$$U: \left| \begin{aligned} x_1 + x_2 \quad - x_4 &= 0 \\ 2x_2 + x_3 + x_4 &= 0 \end{aligned} \right.$$

$$\left.\begin{aligned} a_1 &= (1, 2, 0, -1) \\ a_2 &= (1, 0, 1, -1) \\ a_3 &= (2, 2, 1, -2) \\ a_4 &= (0, 2, -1, 0) \end{aligned}\right\} W:$$

$W = \ell(a_1, a_2, a_3, a_4)$

Да се намерят бази на U, W, $U+W$ и $U \cap W$.

1) Да се намерят U и W като линейна обвивка и хомог. система. „хубави"

Реш:
$$\begin{array}{c} \begin{array}{cccc} x & p & x & q \end{array} \\ \begin{pmatrix} \boxed{1} & 1 & 0 & -1 \\ 0 & 2 & \boxed{1} & +1 \end{pmatrix} \end{array} \begin{array}{c} x \\ x \end{array}$$

$\cancel{x_1 + x_2 - x_4}$

$$\left.\begin{aligned} x_2 &= p \\ x_4 &= q \end{aligned}\right\} \begin{aligned} x_1 &= q - p \\ x_3 &= -q - 2p \end{aligned}$$

$$\left.\begin{aligned} (p,q)\ \mathbb{R}^2 &= (1,0) \longrightarrow (-1, 1, -2, 0) \\ (p,q) &= (0,1) \longrightarrow (1, 0, -1, 1) \end{aligned}\right\}$$ ФСР за системата, задаваща U

$b_1 = (-1, 1, -2, 0)$

$b_2 = (1, 0, -1, 1)$

$$U: \left| \begin{aligned} x_1 + x_2 - x_4 &= 0 \\ 2x_2 + x_3 + x_4 &= 0 \end{aligned} \right.$$

$$W: \begin{array}{c} a_1 \\ a_2 \\ a_3 \\ a_4 \end{array} \begin{pmatrix} 1 & 2 & 0 & -1 \\ 1 & 0 & \boxed{1} & -1 \\ 2 & 2 & 1 & -2 \\ 0 & 2 & -1 & 0 \end{pmatrix} \underset{\sim}{\begin{array}{c} R_3 - R_2 \\ R_4 + R_2 \end{array}} \begin{pmatrix} 1 & 2 & 0 & -1 \\ 1 & 0 & 1 & -1 \\ \cancel{1} & \cancel{2} & \cancel{0} & \cancel{-1} \\ \cancel{1} & \cancel{2} & \cancel{0} & \cancel{-1} \end{pmatrix}$$

*от ∀ лз винати оставаме 1

$$\begin{array}{c|cccc} & s & \times & \times & t \\ \times & 1 & \boxed{2} & 0 & -1 \\ \times & 1 & 0 & \boxed{1} & -1 \\ & \cancel{1} & \cancel{2} & \cancel{0} & \cancel{-1} \\ & \cancel{1} & \cancel{2} & \cancel{0} & \cancel{-1} \end{array}$$

$$\left.\begin{matrix}(1, 2, 0, -1) \\ (1, 0, +1, -1)\end{matrix}\right\} \text{ЛНЗ}$$

$W = c_1 = (1, 2, 0, -1)$
$c_2 = (1, 0, 1, -1)$

$$\ell(a_1, a_2, a_3, a_4) = \ell(c_1, c_2)$$

$$\begin{cases} x_1 = s \\ x_4 = t \\ x_2 = -\frac{s+t}{2} \\ x_3 = t - s \end{cases}$$

$(s, t) = (1, 0) \longrightarrow (1, -\frac{1}{2}, -1, 0) \cdot 2 = (2, -1, -2, 0)$

$(s, t) = (0, 1) \longrightarrow (0, \frac{1}{2}, 1, 1) \cdot 2 = (0, 1, 2, 2)$

$$W: \begin{cases} 2x_1 - x_2 - 2x_3 = 0 \\ x_2 + 2x_3 + 2x_4 = 0 \end{cases}$$

*образуване система

$$U \cap W: \begin{array}{c} U: \\ \\ W: \\ \\ \end{array} \begin{pmatrix} \boxed{1} & 1 & 0 & -1 \\ 0 & 2 & 1 & 1 \\ 2 & -1 & -2 & 0 \\ 0 & 1 & 2 & 2 \end{pmatrix} \begin{matrix} R_3 - 2R_1 \end{matrix} \sim \begin{array}{c|cccc} & \times & & & \\ \times & 1 & 1 & 0 & -1 \\ & 0 & 2 & \boxed{1} & 1 \\ & 0 & -3 & -2 & 2 \\ & 0 & 1 & 2 & 2 \end{array} \begin{matrix} R_3 + 2R_2 \\ R_4 - 2R_2 \end{matrix} \sim$$

$$\begin{array}{c|cccc} & \times & \cancel{\times} & \times & \\ \times & 1 & 1 & 0 & -1 \\ \times & 0 & 2 & 1 & 1 \\ & 0 & 1 & 0 & 4 \\ & 0 & -3 & 0 & 0 \end{array} \Big/ \cdot \left(-\frac{1}{3}\right) R_4 \sim \begin{pmatrix} 1 & 1 & 0 & -1 \\ 0 & 2 & 1 & 1 \\ 0 & 1 & 0 & 4 \\ 0 & \boxed{1} & 0 & 0 \end{pmatrix} \begin{matrix} R_3 - R_4 \\ R_2 - 2R_4 \\ R_1 - R_4 \end{matrix} \sim$$

$$\sim \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 \end{pmatrix} \cdot \left(-\frac{1}{4}\right) R_3 \sim \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & \boxed{1} \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{matrix} R_1 - R_3 \\ R_2 - R_3 \end{matrix} \sim$$

$$\begin{array}{c|cccc} & \times & \times & \times & \times \\ \hline \times & & & & \\ \times & & & & \\ \times & & & & \\ \times & & & & \end{array}\quad \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad \left| \begin{array}{l} x_1 = 0 \\ x_2 = 0 \\ x_3 = 0 \\ x_4 = 0 \end{array} \right.$$

$U \cap W = \{(0,0,0,0)\}$ – нулево ЛП — няма базис

$U \cap W$ — няма базис

$U \cap W = \ell(\emptyset) = \ell(\theta)$

$\Downarrow$

$U \oplus W =$

$U \leq \mathbb{R}^4$

$W \leq \mathbb{R}^4$

$\dim(U+W) = \dim U + \dim W - \dim(U \cap W) = 2+2-0=4$

$U + W \leq \mathbb{R}^4$

$\dim(U + W) = 4 = \dim(\mathbb{R}^4)$

$\Rightarrow U+W = \mathbb{R}^4$

$U+W = \ell(e_1, e_2, e_3, e_4)$

$$U+W: \begin{array}{c} b_1 \\ b_2 \\ c_1 \\ c_2 \end{array} \begin{pmatrix} -1 & 1 & -2 & 0 \\ 1 & 0 & -1 & -1 \\ 1 & 2 & 0 & -1 \\ 1 & 0 & 1 & -1 \end{pmatrix} \longrightarrow \cdots \longrightarrow \begin{array}{c} \times \\ \times \\ \times \\ \times \\ \end{array} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \times & \times & \times & \times \end{pmatrix} \begin{array}{c} e_1 \\ e_2 \\ e_3 \\ e_4 \\ \end{array}$$