Robolab: Assignment 1
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Task 1

P_1	P_2	D_1	P_3	D_2	D_3	D_4	P_4
1	2	3	4	5	6	7	8
0001	0010	0011	0100	0101	0110	0111	1000

$$P_1 = D_1 \oplus D_2 \oplus D_4$$
$$P_2 = D_1 \oplus D_3 \oplus D_4$$
$$P_3 = D_2 \oplus D_3 \oplus D_4$$
$$P_4 = P_1 \oplus P_2 \oplus P_3 \oplus D_1 \oplus D_2 \oplus D_3 \oplus D_4$$

$$I_D = \begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \\ \end{array}$$
$$I_D = \left| \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right|$$

$$G'_{4,8} = \begin{array}{cccccccc|c} P_1 & P_2 & D_1 & P_3 & D_2 & D_3 & D_4 & P_4 & \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & R_1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & R_2 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & R_3 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & R_4 \end{array}$$

$$P_1 = D_1 \oplus D_2 \oplus D_4$$
$$P_1R_1 = 1 \oplus 0 \oplus 0$$
$$P_1R_2 = 0 \oplus 0 \oplus 0$$
$$P_1R_3 = 0 \oplus 0 \oplus 0$$
$$P_1R_4 = 0 \oplus 0 \oplus 1$$

$$P_2 = D_1 \oplus D_3 \oplus D_4$$
$$P_2R_1 = 1 \oplus 0 \oplus 0$$
$$P_2R_2 = 0 \oplus 0 \oplus 0$$
$$P_2R_3 = 0 \oplus 1 \oplus 0$$
$$P_2R_4 = 0 \oplus 0 \oplus 1$$

$$P_3 = D_2 \oplus D_3 \oplus D_4$$
$$P_3R_1 = 0 \oplus 0 \oplus 0$$
$$P_3R_2 = 1 \oplus 0 \oplus 0$$
$$P_3R_3 = 0 \oplus 1 \oplus 0$$
$$P_3R_4 = 0 \oplus 0 \oplus 1$$

$$P_4 = D_1 \oplus D_2 \oplus D_3 \oplus D_4 \oplus P_1 \oplus P_2 \oplus P_3$$

Parity column A from $G'_{4,8}$

$$A = \begin{array}{c} \begin{matrix} P_1 & P_2 & P_3 & P_4 \end{matrix} \\ \begin{vmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} \end{array}$$

transpose of $A = A^T$

$$A^T = \begin{array}{c} \begin{matrix} D_1 & D_2 & D_3 & D_4 \end{matrix} \\ \begin{vmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} \end{array}$$

Construct $H'_{4,8}$ matrix

$$H'_{4,8} = \begin{array}{c} \begin{matrix} P_1 & P_2 & D_1 & P_3 & D_2 & D_3 & D_4 & P_4 \end{matrix} \\ \begin{vmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \end{vmatrix} \end{array}$$

Replacing last Row with all '1'

$$H'_{4,8} = \begin{array}{c} \begin{matrix} P_1 & P_2 & D_1 & P_3 & D_2 & D_3 & D_4 & P_4 \end{matrix} \\ \begin{vmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{vmatrix} \end{array}$$

Task 2

$$G'_{4,8} = \begin{array}{cc} \begin{matrix} P_1 & P_2 & D_1 & P_3 & D_2 & D_3 & D_4 & P_4 \end{matrix} & \\ \begin{vmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \end{vmatrix} & \begin{matrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{matrix} \end{array}$$

RREF / Row Reduction echelon form.

$R_2 = R_2 + R_3$

$$G'_{4,8} = \begin{array}{cc} \begin{matrix} P_1 & P_2 & D_1 & P_3 & D_2 & D_3 & D_4 & P_4 \end{matrix} & \\ \begin{vmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \end{vmatrix} & \begin{matrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{matrix} \end{array}$$

$R_4 = R_1 + R_4$

$$G'_{4.8} = \begin{array}{c} \left| \begin{matrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \end{matrix} \right| \end{array} \begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array}$$

$R_1 = R_1 + R_3$

$$G'_{4.8} = \begin{array}{c} \left| \begin{matrix} 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \end{matrix} \right| \end{array} \begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array}$$

$R_3 = R_2 + R_3$

$$G'_{4.8} = \begin{array}{c} \left| \begin{matrix} 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \end{matrix} \right| \end{array} \begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array}$$

$R_1 = R_1 + R_4$

$$G'_{4.8} = \begin{array}{c} \left| \begin{matrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \end{matrix} \right| \end{array} \begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array}$$

$R_3 = R_3 + R_4$

$$G'_{4.8} = \begin{array}{c} \left| \begin{matrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \end{matrix} \right| \end{array} \begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array}$$

$R_4 = R_3 + R_4$

$$G'_{4.8} = \begin{array}{c} \left| \begin{matrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{matrix} \right| \end{array} \begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array}$$

$$\therefore \quad G_{4,8} = \begin{array}{cccccccc|c} D_1 & D_2 & D_3 & D_4 & P_1 & P_2 & P_3 & P_4 & \\ \hline 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & R_1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & R_2 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & R_3 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & R_4 \end{array}$$

step 1 : cut the identity matrix.

step 2 : transpose the A part left.

$$A = \begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} \qquad \therefore A^T = \begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix}$$

Step 3 : Add the identity matrix at the end to form H

$$H_{4,8} = \begin{array}{cccccccc} P_1 & P_2 & P_3 & P_4 & D_1 & D_2 & D_3 & D_4 \\ \hline 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{array}$$

Task 3

formular $\vec{x} = (\vec{a} \cdot G')_{mod 2}$

a. $\vec{a} = (0100)$

$$\vec{x} = \left(\begin{vmatrix} 0 & 1 & 0 & 0 \end{vmatrix} \cdot \begin{array}{cccccccc} P_1 & P_2 & D_1 & P_3 & D_2 & D_3 & D_4 & P_4 \\ \hline 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \end{array} \right)_{mod 2}$$

$$\vec{x} = \begin{vmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{vmatrix}_{mod 2}$$

$$\vec{x} = \begin{vmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{vmatrix}$$

here, input word was '0100', in our $\vec{x}$ positions of data bits correctly represent input word.

b. $\vec{a} = (1001)$

$$\vec{x} = \left(\begin{vmatrix} 1 & 0 & 0 & 1 \end{vmatrix} \cdot \begin{array}{cccccccc} P_1 & P_2 & D_1 & P_3 & D_2 & D_3 & D_4 & P_4 \\ \hline 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \end{array} \right)_{mod 2}$$

$$\vec{x} = \begin{vmatrix} 2 & 2 & 1 & 1 & 0 & 0 & 1 & 1 \end{vmatrix}_{mod 2}$$

$$\vec{x} = \begin{vmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \end{vmatrix}$$

here, input word was '1001', in our $\vec{x}$ positions of data bits correctly represent input word.

c. $\vec{a} = (0011)$

$$\vec{x} = \left(\begin{vmatrix} 0 & 0 & 1 & 1 \end{vmatrix} \cdot \begin{array}{cccccccc} P_1 & P_2 & D_1 & P_3 & D_2 & D_3 & D_4 & P_4 \\ \hline 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \end{array} \right)_{\bmod 2}$$

$$\vec{x} = \begin{vmatrix} 1 & 2 & 0 & 2 & 0 & 1 & 1 & 1 \end{vmatrix}_{\bmod 2}$$

$$\begin{array}{cc} & \begin{array}{cccccccc} P_1 & P_2 & D_1 & P_3 & D_2 & D_3 & D_4 & P_4 \end{array} \\ \vec{x} = & \begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{vmatrix} \end{array}$$

here, input word was '0011', in our $\vec{x}$ positions of data bits correcty represent input word.

d. $\vec{a} = (1101)$

$$\vec{x} = \left(\begin{vmatrix} 1 & 1 & 0 & 1 \end{vmatrix} \cdot \begin{array}{cccccccc} P_1 & P_2 & D_1 & P_3 & D_2 & D_3 & D_4 & P_4 \\ \hline 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \end{array} \right)_{\bmod 2}$$

$$\vec{x} = \begin{vmatrix} 3 & 2 & 1 & 2 & 1 & 0 & 1 & 2 \end{vmatrix}_{\bmod 2}$$

$$\begin{array}{cc} & \begin{array}{cccccccc} P_1 & P_2 & D_1 & P_3 & D_2 & D_3 & D_4 & P_4 \end{array} \\ \vec{x} = & \begin{vmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{vmatrix} \end{array}$$

here, input word was '1101', in our $\vec{x}$, positions of data bits correcty represent input word.

Task 4

$$H_{4,8} = \begin{array}{cccccccc} P_1 & P_2 & P_3 & P_4 & D_1 & D_2 & D_3 & D_4 \end{array}$$
$$\begin{vmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

formular $\vec{z} = (H \cdot \vec{x}^T)_{mod2}$

a. $\vec{x} = \begin{vmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \end{vmatrix}$

$$\vec{z} = \left(\begin{vmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 1 \end{vmatrix} \right)_{mod2}$$

$$\vec{z} = \begin{vmatrix} 2 \\ 2 \\ 2 \\ 3 \end{vmatrix}_{mod2} \qquad \vec{z} = \begin{vmatrix} 0 \\ 0 \\ 0 \\ 1 \end{vmatrix}$$

$\because$ Syndrome $\vec{z}$ is not all '0' ; overall parity is 1 (odd), syndrome $!= 0$, single error Occured. the corresponding column is D_4 of H matrix. So position D_4 of $\vec{x}$ is needed to be fliped.

$\therefore$ Corrected word, $\vec{x} = \begin{array}{c} \begin{array}{cccccccc} P_1 & P_2 & P_3 & P_4 & D_1 & D_2 & D_3 & D_4 \end{array} \\ \begin{vmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \end{vmatrix} \end{array}$

$\therefore$ Source word, $\vec{a} = \begin{vmatrix} 1 & 1 & 0 & 0 \end{vmatrix}$

b. $\vec{x} = \begin{vmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{vmatrix}$

$$\vec{z} = \left(\begin{vmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 1 \end{vmatrix} \right)_{mod2}$$

$$\vec{z} = \begin{vmatrix} 2 \\ 2 \\ 2 \\ 2 \end{vmatrix}_{mod2} \qquad \vec{z} = \begin{vmatrix} 0 \\ 0 \\ 0 \\ 0 \end{vmatrix}$$

$\because$ syndrome $\vec{z}$ is all '0', no error occurred.

$\therefore$ Source word, $\vec{\alpha} = \begin{vmatrix} 1 & 0 & 0 & 1 \end{vmatrix}$

c. $\vec{x} = \begin{vmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \end{vmatrix}$

$$\vec{z} = \left(\begin{vmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{vmatrix} \right)_{mod2}$$

$$\vec{z} = \begin{vmatrix} 3 \\ 2 \\ 4 \\ 3 \end{vmatrix}_{mod2} \qquad \vec{z} = \begin{vmatrix} 1 \\ 0 \\ 0 \\ 1 \end{vmatrix}$$

$\because$ syndrome $\vec{z}$ is $!=0$, and no matching column is 'H', multiple error is occurred. code words cannot be corrected.

d. $\vec{x} = \begin{vmatrix} 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{vmatrix}$

$$\vec{z} = \left(\begin{vmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{vmatrix} \right)_{mod2}$$

$$\vec{z} = \begin{vmatrix} 2 \\ 3 \\ 3 \\ 3 \end{vmatrix}_{mod2} \qquad \vec{z} = \begin{vmatrix} 0 \\ 1 \\ 1 \\ 1 \end{vmatrix}$$

$\because$ Syndrome $\vec{z}$ is not all '0', overall parity 1 (odd), in position P_1 corresponds 'H', a bit needs to fliped.
So, single error occured.

$$\therefore \text{Corrected word, } \vec{x} = \begin{array}{cccccccc} P_1 & P_2 & P_3 & P_4 & D_1 & D_2 & D_3 & D_4 \end{array}$$
$$\begin{vmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{vmatrix}$$

$\therefore$ Source word, $\vec{a} = \begin{vmatrix} 0 & 1 & 0 & 1 \end{vmatrix}$