

## CS&SS/STAT 563 — Statistical Demography — Spring 2020 - Homework no. 5

Due Monday April 27 at 2:00pm on the course Canvas website.

The purpose of this homework is to give you some experience with basic Bayesian statistics.

1. This question involves Bayesian estimation of a binomial mean. In a sample of 112 people living in Washington who married in 2005, 43 had divorced by 2015. Assume a uniform prior distribution for the proportion,  $\theta$ , of Washington residents who married in 2005 that had divorced by 2015.
  - (a) Find the posterior distribution of  $\theta$  in analytic form, i.e. as a mathematical expression. [Hint: A uniform distribution is the same as a Beta (1,1) distribution.]
  - (b) Simulate a sample of size 1,000 from the posterior distribution. Hence find the posterior mean and a 95% Bayesian confidence interval for  $\theta$ . [Hint: You can use the `rbeta` R function.]
  - (c) Compute and plot a nonparametric density estimate of the posterior density of  $\theta$ .
2. This question is about Bayesian estimation of a normal mean. A random sample of 25 students from a high school was taken, and the amount of time each student spent on studying (in hours) during an exam period was measured. The numbers were as follows:

2.1 9.8 13.9 11.3 8.9 15.7 16.4 4.5 8.9 11.9 12.5 11.1 11.6 14.5 9.6 7.4 3.3 9.1 9.4 6.0 7.4  
8.5 1.6 11.4 9.7

Assume that the true standard deviation of the number of hours is 4. Then find the posterior distribution of the true mean of hours spent studying by the students in the high school, using a normal prior distribution with mean 10 hours and standard deviation 3 hours. Hence find a posterior 95% Bayesian confidence interval for the true mean.