# Fully Bayesian Benchmarking of Small Area Estimation Models

STAT 563 - Term Paper Presentation

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# Background:

## Overview

- Obtaining estimates for small geographic/social/demographic areas presents challenges when there are few observations per area
    - Direct methods (i.e. counting events and exposure) lack credibility
    - Modeled estimates are not consistent with aggregate area estimates (found using direct methods)
- Discrepancies present issues, political and otherwise, so small area estimates are adjusted to match aggregate estimates
- Current adjustment methods focus on providing point estimates, not distributions

# Background:

## Terms

- Benchmarks:
    - Aggregate area estimates, generally obtained using direct methods
    - What you compare your small area estimates to
        - **Internal benchmarks:** calculated from small area data sources
        - **External benchmarks:** calculated from other data sources
- Benchmarking:
    - Techniques for forcing small area estimates to agree with benchmarks
    - How you reconcile discrepancies between small area estimates and aggregate estimates
        - **Exact benchmarking:** Making estimates exactly agree with benchmarks
        - **Inexact benchmarking:** Allowing discrepancy between estimates and benchmarks

# Methods:
## Summary

- Published in 2020 by Junni L. Zhang and John Bryant
- Present a fully Bayesian approach to small area benchmarking
  - Benchmarks are estimates for underlying aggregate parameters
  - Benchmark agreement is a distribution conditional on the aggregate parameters
  - The original likelihood is multiplied by the probability distribution of the benchmarks
    - A "compromise" between the original likelihood and requirement to meet the benchmark
  - The revised likelihood and prior together yield the benchmarked posterior distribution

# Methods:
## Advantages

- Include multiple benchmarks
- Benchmarks can be nonlinearly related to small area estimates (i.e. age specific mortality benchmarked against life expectancy)
- Specify allowable discrepancy

# Methods:

## Bayesian estimation of area-level models

Given:

- $n$ areas (e.g. defined by age, sex, region)
- Area-level parameters: $\boldsymbol{\gamma} = \{\gamma_1, \dots, \gamma_n\}^T$
- Area-level data: $\boldsymbol{y} = \{y_1, \dots, y_n\}^T$
- Vector of hyperparameters $\boldsymbol{\phi}$

$$p(\boldsymbol{\gamma},\ \boldsymbol{\phi} \mid \boldsymbol{y}) \propto p(\boldsymbol{\phi})\, p(\boldsymbol{\gamma} \mid \boldsymbol{\phi})\, p(\boldsymbol{y} \mid \boldsymbol{\gamma})$$

# Methods:

## Bayesian estimation of area-level models with benchmarked posterior

- Benchmarks: $\boldsymbol{m} = \{m_1, \dots, m_d\}^T$ with $d \ll n$
- Underlying benchmark parameters: $\boldsymbol{\psi} = \{\psi_1, \dots, \psi_d\}^T$
- $\boldsymbol{\psi} = \boldsymbol{f}(\boldsymbol{\gamma})$ through deterministic benchmarking function $\boldsymbol{f}$

$$p(\boldsymbol{\gamma},\ \boldsymbol{\phi} \mid \boldsymbol{y},\ \boldsymbol{m}) \propto p(\boldsymbol{\phi})\, p(\boldsymbol{\gamma} \mid \boldsymbol{\phi})\, p(\boldsymbol{y} \mid \boldsymbol{\gamma})^{\left[\frac{\boldsymbol{m}}{\boldsymbol{\psi}}\right]}(\boldsymbol{m} \mid \boldsymbol{f}(\boldsymbol{\gamma}))$$

Where $p(\boldsymbol{y} \mid \boldsymbol{\gamma})^{\left[\frac{\boldsymbol{m}}{\boldsymbol{\psi}}\right]}(\boldsymbol{m} \mid \boldsymbol{f}(\boldsymbol{\gamma}))$ is a probability distribution of the benchmarks conditional on the aggregate parameters

# Proposal

- Recreate the authors' application of their methods to age-sex specific mortality rates in local authority districts of England and Wales
- Apply these method in the Philippines -- another location with small subnational units and available data

*Note: the authors have released code alongside their article, which was used to re-evaluate their methods*

# Evaluation:

## Estimating age-sex-specific mortality rates in England and Wales

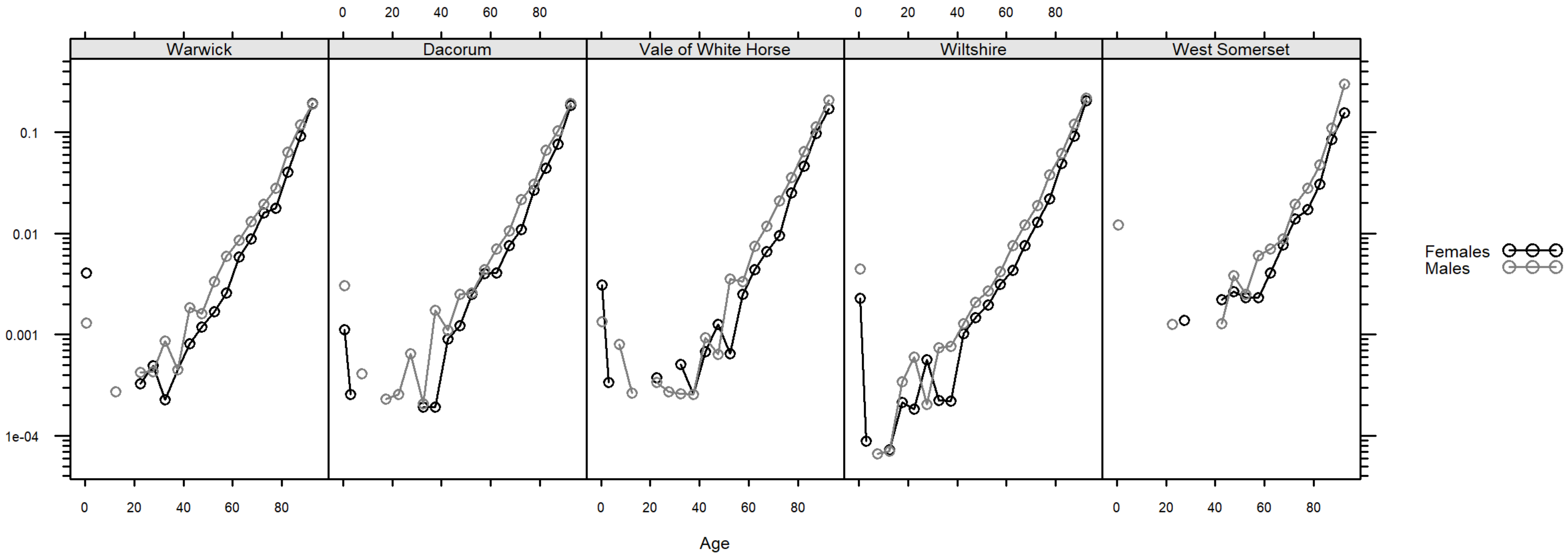
- Data: death counts and at-risk populations in 2014, for 20 age groups 0 to 90+, males and females, in 348 local authority districts (belonging to 10 regions)
  - Total deaths: 500,3134
  - Total population: 57,408,654

| Variable | Min | 25% | Median | Mean | 75% | Max |
|---|---|---|---|---|---|---|
| Deaths | 0.00 | 1.00 | 8.00 | 36.05 | 48.00 | 1004.00 |
| Population | 17 | 1985 | 3211 | 4136 | 5,033 | 50,381 |

# Evaluation:

## Estimating age-sex-specific mortality rates in England and Wales

### Sample of mortality rates using direct estimation:



# Evaluation:

## Estimating age-sex-specific mortality rates in England and Wales

- Method: MCMC Metropolis-Hastings algorithm
  - 4 chains each with 40,000 burn-in + 40,000 iteration and n-thinning = 80
- District-level estimates benchmarked against region-level life expectancy at birth
- Death counts modeled as:

$$y_{asd} \sim Poisson(w_{asd}\, \gamma_{asd})$$

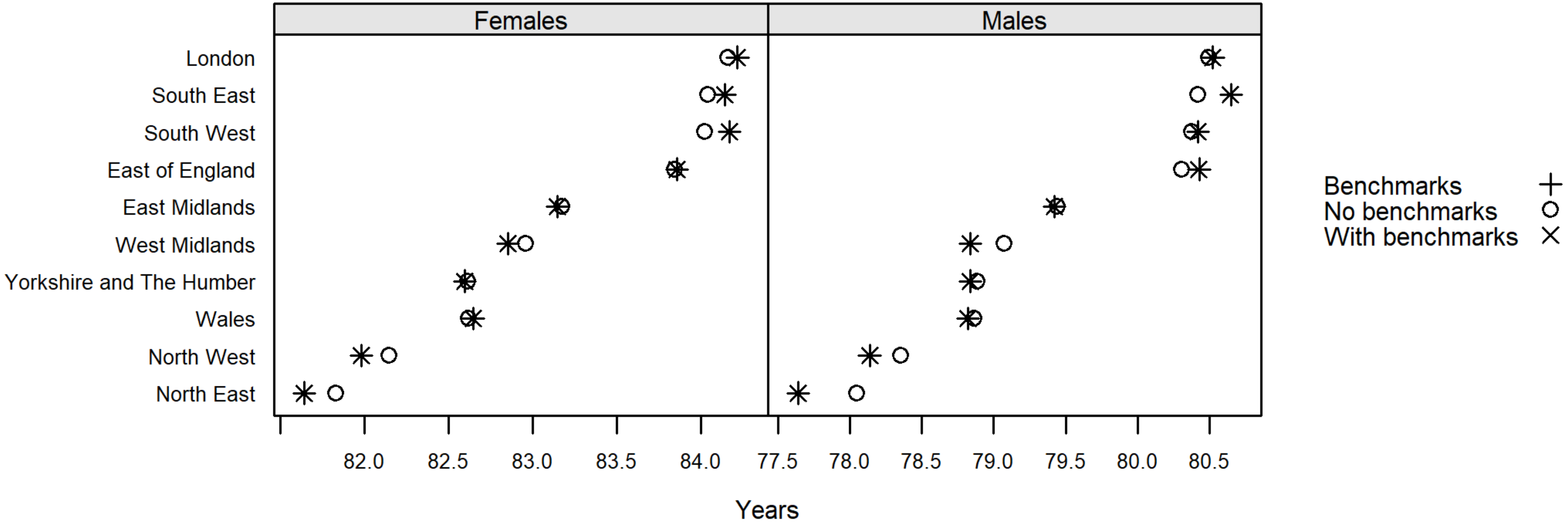
Where $w_{asd}$ is population and $\gamma_{asd}$ is mortality rate

(age effects are assumed to follow a random walk with drift)

# Evaluation:

## Estimating age-sex-specific mortality rates in England and Wales

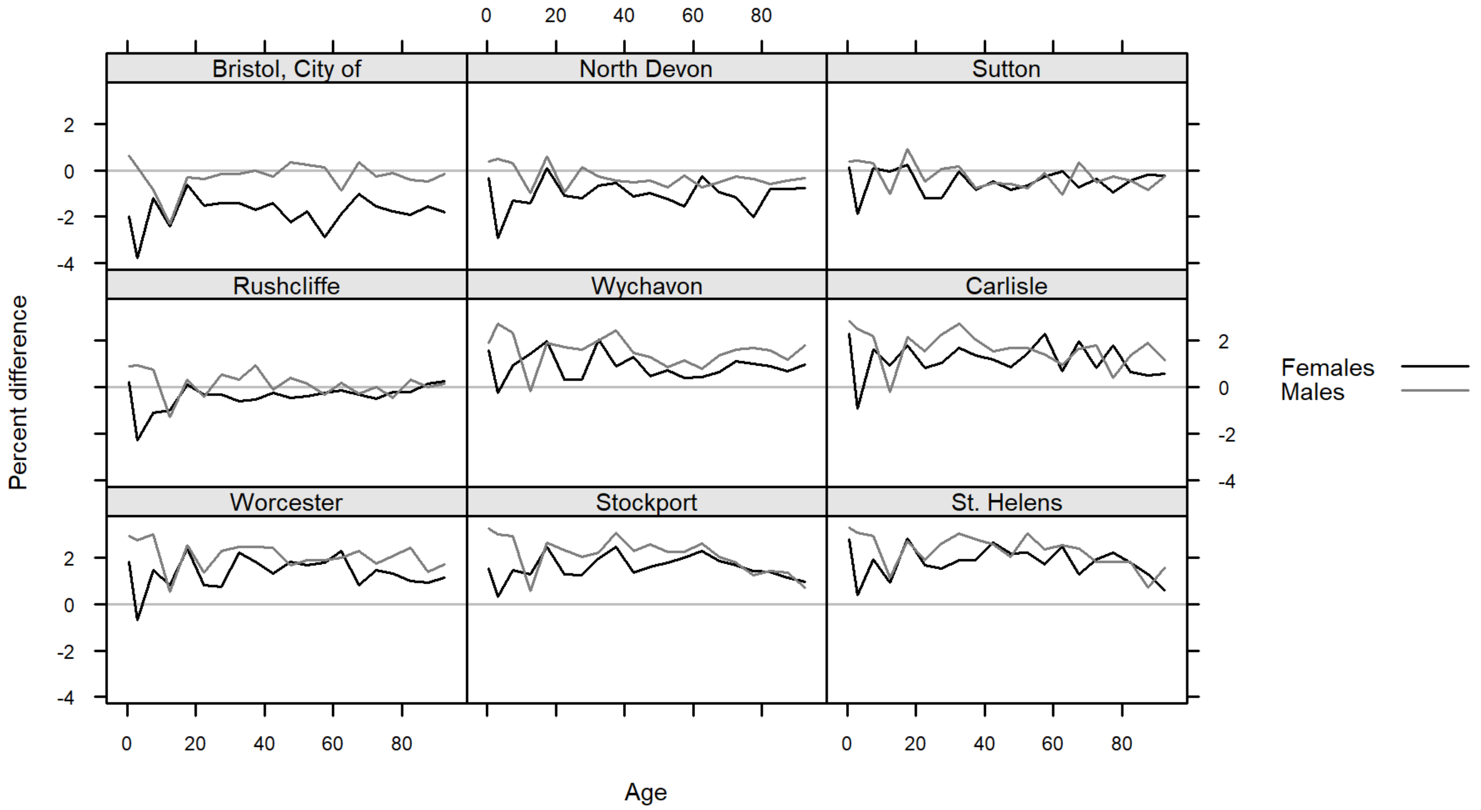
### Point estimates of life expectancy by region for benchmarked and non-benchmarked models



# Evaluation:

## Estimating age-sex-specific mortality rates in England and Wales

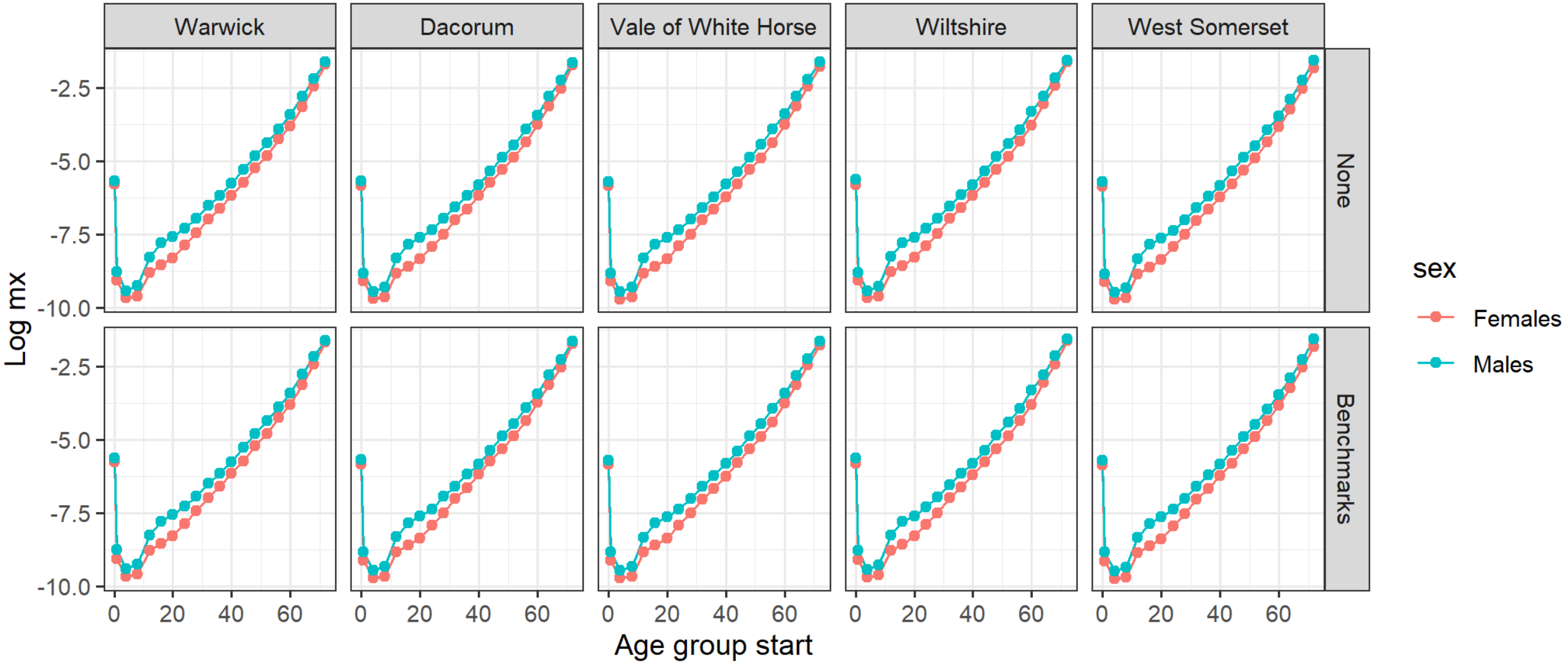
### Percent difference between district mortality rates for benchmarked and non-benchmarked models



# Evaluation:

## Estimating age-sex-specific mortality rates in England and Wales

### Percent difference between district mortality rates for benchmarked and non-benchmarked models



# Evaluation:

## Philippines

- Currently in the process of extending the authors’ methods to new data