CS&SS/STAT 563 — Statistical Demography — Spring 2020 - Homework no. 6

Due Monday May 18 at 2:00pm on the course Canvas website.

1. Obtain the values of life expectancy at birth for Honduras for 1950-2020 from the 2019 World Population Prospects and write them out in your homework.

   (a) Fit a version of the six-parameter double logistic gain model by numerical optimization to the gains in life expectancy, assuming that the error variance remains constant over time (this will just give one set of double logistic parameter values and an estimated error variance).

   (b) Plot the observed gains against their fitted values, and comment on the fit.

   (c) Find the predictive distribution of Honduras TFR for 2020-2025 conditional on this model, analytically or by simulation. Plot the distribution and give its median and a 95% prediction interval.

2. A fully converged simulation for Bayesian modeling and projection of female life expectancy at birth is available on the Homework web page. This consists of three MCMC chains, each of length 160,000, thinned by 50, and 1,000 project trajectories for both female and male life expectancy. After unpacking you will find a README file that contains the code used to generate the simulation.

   (a) Use the get.e0.mcmc and get.e0.prediction functions to obtain the MCMC and prediction objects, respectively. What are the contents of these objects?

   (b) Assess the double logistic fit for each of Algeria and Morocco.

   (c) Compare the fitted model for the two countries. In which one has life expectancy been rising faster, empirically, and according to the model?

   (d) Assuming conditional independence of life expectancy gain between countries given the model parameters (a reasonable assumption), compute the probability that the female e_0 of Algeria will be larger/smaller than the female e_0 of Morocco in each future period, and in all future periods to 2100. (Hint: the pmax [pmin] function gives element-wise maximum [minimum])

3. Using the converged TFR and e_0 simulations provided on the Homework web page, generate probabilistic population projections for all countries for 2020–2100.

   (a) For Canada, generate and plot probabilistic projections of the following quantities to 2100:

      i. total population

      ii. total male population

      iii. total population over 65

iv. the potential support ratio, defined as the number of people aged 20–64 divided by the number of people 65 and over. (This is sometimes roughly referred to as the number of workers per retiree.) Comment on the trend in this quantity.

(b) Generate and plot a probabilistic projection of the total population of North America from 2020 to 2100.

4. The probabilistic population projections in the previous question do not take account of uncerainty about future migration. This question involves taking a first approach to this.

   (a) Download the time series of net migration rates by five-year period for Canada from the UN's WPP website at
   https://population.un.org/wpp/Download/Standard/Migration/
   and write them out here in your homework.

   (b) Fit the first-order autoregressive (AR(1)) model given in the lecture to this series by finding estimates of the parameters in a frequentist manner (not using a Bayesian hierarchical model). What are your estimates of the parameters and their standard errors?

   (c) Find the analytic form of the predictive distribution of the net migration rate for Canada in 2020–2025 from this model.

   (d) Use the fitted AR(1) model to generate a sample from the predictive probability distribution of the net migration rate for Canada in 2015–2020, and show a histogram of the resulting values, with the analytic probability density function superimposed.

   (e) Specify an appropriate schedule of age-specific migration rates to break down your sampled all-ages net migration rates by age.

   (f) Using this and the population of Canada in 2020, generate a sample of vectors of projected age-specific net migration numbers for 2020-2025 and show histograms of the number for each age group.

   (g) By combining this probabilistic forecast of net migration in 2020–2025 with the probabilistic projections of fertility and mortality that you used in Question 3, generate a probabilistic projection of the Canadian population by age and sex (and total) in 2025. Give medians and 95% intervals for the number of people in each combination of sex and age group.