MDL Assignment 3
Part 1 : Probability Theory

Deadline : 15th April, 2024 , 11:59 PM

Question 1
Consider a roulette-style decision game which involves a spinning wheel with equal probabilities for landing on red, black, or green, where red and black each take up x% of the wheel, and green takes up (100-2x)%. The player's objective is to maximise their returns over a series of bets.

Strategies to Evaluate:

1. **Constant Bet on Red/Black:** Always bet a fixed amount on either red or black.
2. **Doubling Bet on Red/Black:** After a loss, double the bet on the same colour. If you win, return to the initial bet.
3. **Constant Bet on Green:** Always bet a fixed amount on green.
4. **Random Betting:** Randomly choose a colour and bet a fixed amount on it each time.

Evaluate the different betting strategies using Monte Carlo simulations for different values of x. Determine the expected return and risk of ruin (the percentage of simulations where the bankroll reaches $0) for each strategy and identify which strategy minimises losses over the long term. Include a detailed analysis.

Details for the setup:

- Each simulation will run for 1000 rounds of the game.
- Evaluate each strategy over 1000 simulations to smooth out randomness.
- All probabilities should be non-zero.
- Initial bankroll for each strategy is set to a fixed amount of $1000.
- The fixed bet amount should be set at $10 for strategies that use a constant bet.

- For the doubling strategy, start with a $10 bet, doubling after each loss but ensuring the total betting amount does not exceed the bankroll.
- Payouts are as follows: Winning a bet on red or black doubles the bet; winning on green multiplies the bet by 18.

Question 2

Consider a coin that comes up heads with probability p and tails with probability 1-p. Let q_n be the probability that after n independent tosses, there have been an even number of heads. Derive a recursion that relates q_n to q_{n-1}, and solve this recursion to establish the formula $q_n = (1+(1-2p)^n)/2$.

Question 3

a) An aeroplane is built to be able to fly on one engine. If the plane' s two engines operate independently, and each has a 1% chance of failing in any given four-hour flight, what is the chance the plane will fail to complete a four-hour flight to Oklahoma due to engine failure?

b) In a roomful of 30 people, what is the probability that at least two people have the same birthday? Assume birthdays are uniformly distributed and there is no leap year complication

Distribution of marks:
Question 1 - 40
Question 2 - 30
Question 3 - 30
Total marks obtainable is out of 100

Submission Format:

Create a folder with your Roll No. as its name containing the three files (q1.ipynb, q2.pdf, q3.pdf). Zip it and name it Roll No.zip Example- 202101086.zip

Please start the assignment early to prevent clash with end sem preparation. **Plagiarism** is **strictly prohibited** and a straight 0 will be awarded in such cases