Question 3

Part A: Flight to Oklahoma

Since the plane is built to be able to fly on one engine, the only way it can fail to complete a four-hour flight is if no engine were working, or equivalently both the engines failed in the given flight. This condition is both necessary and sufficient.

$$P(\text{plane fails to complete the flight}) = P(\text{engine 1 fails} \cap \text{engine 2 fails}) \tag{1}$$

Since the two engines operate independently, their failures (an event in their operation cycle) too are independent of each other. Thus, we get

$$P(\text{plane fails to complete the flight}) = P(\text{engine 1 fails}) \cdot P(\text{engine 2 fails}) \tag{2}$$

Substituting $P(\text{engine 1 fails}) = P(\text{engine 2 fails}) = \frac{1}{100}$, we get

$$P(\text{plane fails to complete the flight}) = \frac{1}{10000} = 0.01\% \tag{3}$$

Part B: Birthday Paradox

$$\begin{aligned}
&P(\text{atleast two people have same birthday}) \\
&= 1 - P(\text{no two people have the same birthday}) \\
&= 1 - P(\text{all 30 people have distinct birthday})
\end{aligned} \tag{4}$$

Since the birthdays are uniformly distributed over 365 days,

$$\begin{aligned}
&= 1 - \frac{\#\text{permutations of distinct 30 birthdays}}{\#\text{permutations of 30 birthdays}} \\
&= 1 - \frac{\frac{365!}{(365-30)!}}{365^{30}} \\
&\approx 1 - 0.294 \\
&= 0.706
\end{aligned} \tag{5}$$