Quantum Channels: Transition from Classical to Quantum Channels

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Accessible Information

Holevo Bound

Schumacher's Quantum Noiseless Channel Coding Theorem

Classical Capacity of Quantum Channels

Private Capacity of Quantum Channels

Quantum Capacity of Quantum Channels

Super Additivity of Quantum Channels

Examples of Quantum Channels

Sample Frame

Sample Frame

Sample Frame

Sample Frame

Private Capacity of a Wiretap Channel

The private capacity $P(\mathcal{N})$ of a classical wiretap channel $\mathcal{N} = p_{Y,Z|X}$ is defined as follows.

$$P(\mathcal{N}) = \max_{p_{U,X}(u,x)} [I(U;Y) - I(U;Z)]$$

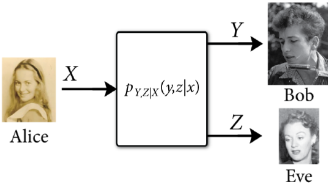

Figure: Classical wiretap channel.

Properties: Private Capacity of a Wiretap Channel

- Non-negativity

$$P(\mathcal{N}) \geq 0$$

- Additivity

$$P(\mathcal{N} \otimes \mathcal{M}) = P(\mathcal{N}) + P(\mathcal{M})$$

- Equivalence of asymptotic and single use capacity

$$\lim_{n \to \infty} \frac{1}{n} P(\mathcal{N}^{\otimes n}) = P(\mathcal{N})$$

Private Capacity of a Quantum Channel

Consider a classical-quantum state of the form given below.

$$\rho_{XA'} = \sum_x p_X(x)|x\rangle\langle x|_X \otimes \rho_{A'}^x$$

The private capacity $P(\mathcal{N})$ of a quantum channel $\mathcal{N}$ is defined as follows.

$$P(\mathcal{N}) = \max_{\rho_{XA'}} [I(X;B)_\rho - I(X;E)_\rho]$$

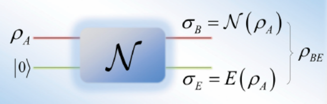

Figure: Private communication through a quantum channel.

Properties: Private Capacity of a Quantum Channel

- Non-negativity

$$P(\mathcal{N}) \geq 0$$

- Bounded by quantum capacity

$$P(\mathcal{N}) \leq Q(\mathcal{N})$$

- Non-additivity

$$P(\mathcal{N} \otimes \mathcal{M}) \neq P(\mathcal{N}) + P(\mathcal{M})$$

Quantum Capacity of a Quantum Channel

Sample Frame

Sample Frame

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