

# CS1.305: Introduction to Algorithms Engineering

## t-Spanner Construction

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### 1 3-Spanner Construction

#### 1.1 Algorithm

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**Algorithm 1:** 3-Spanner Algorithm

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```
1 Initialization:  $E_S = R = \phi$ 
2 for  $v \in V$  do
3   | add  $v$  to  $\mathcal{R}$  with probability  $\frac{1}{\sqrt{n}}$ 
4 end
5 for  $v \in V - \mathcal{R}$  do
6   | if  $v$  is not adjacent to  $x \in \mathcal{R}$  then
7     | add all edges incident on  $v$  to  $E_S$ 
8   | else
9     |  $N(v, \mathcal{R}) \leftarrow$  nearest neighbor in  $\mathcal{R}$ 
10    | add  $(v, N(v, \mathcal{R}))$  and all lighter edges  $(v, *)$  to  $E_S$ 
11  | end
12 end
13 for  $v$  belonging to a cluster do
14   | for each adjacent cluster  $c$  do
15     | add the least weight edge in  $E(v, c)$  to  $E_S$ 
16   | end
17 end
```

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## 1.2 Results

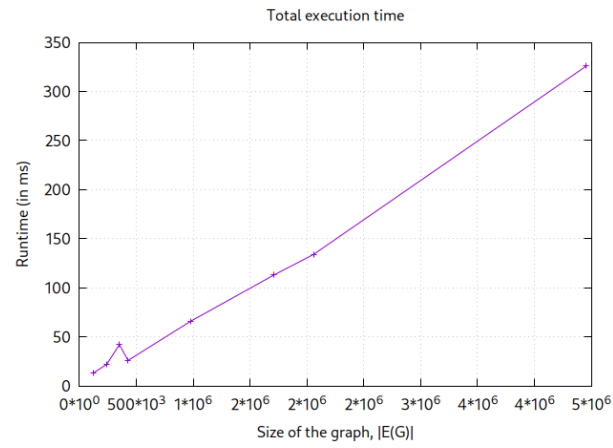


Figure 1: Total execution time

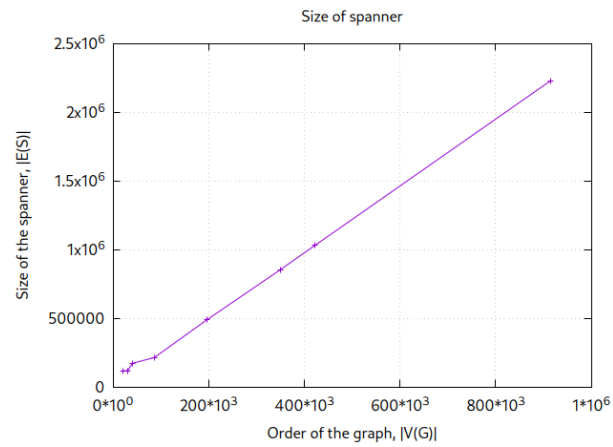


Figure 2: Spanner size

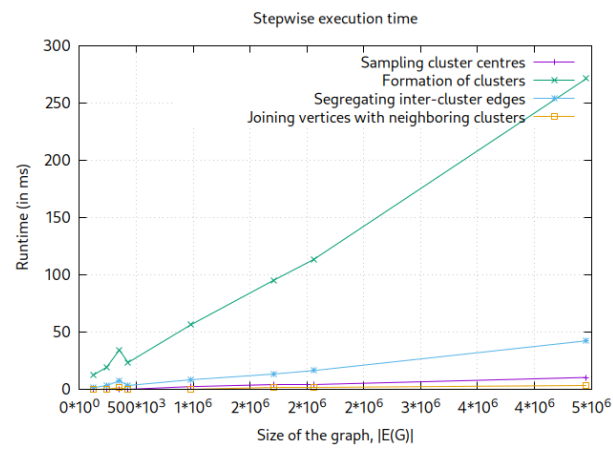


Figure 3: Stepwise execution time

## 2 (2k - 1)-Spanner Construction

### 2.1 Algorithm

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**Algorithm 2:** (2k - 1)-Spanner Algorithm: Cluster Formation

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```

1 Initialization:  $E' = E, V' := V(E' \cup \mathcal{E}_{i-1}), E_S = \mathcal{E}_0 = \phi, C_0 = \{\{v\} \mid v \in V\}$ 
2 for  $i \in \{1, 2, \dots, num\_iterations\}$  do
3   sample clusters  $\mathcal{C}_i$  from  $\mathcal{C}_{i-1}$  independently with probability  $n^{-\frac{1}{k}}$ 
4   for  $v \in V' \setminus \cup \mathcal{C}_i$  do
5     if  $v$  is not adjacent to  $c \in \mathcal{C}_i$  then
6       for  $c' \in \mathcal{C}_{i-1}$  do
7         add the least weight edge  $E'(v, c')$  to  $E_S$ 
8         remove the edges  $E'(v, c')$  from  $E'$ 
9       end
10    else
11       $c, e_v \leftarrow$  nearest cluster in  $\mathcal{C}_i$  and leading edge
12      add the edge  $e_v$  to  $E_S$  and  $\mathcal{E}_i$ 
13      remove the edges  $E'(v, c)$  from  $E'$ 
14      for  $c' \in \mathcal{C}_{i-1}$  do
15        if  $c'$  is reachable with edge lighter than  $e_v$  then
16          add the least weight edge  $E'(v, c')$  to  $E_S$ 
17          remove the edges  $E'(v, c')$  from  $E'$ 
18        end
19      end
20    end
21  end
22  remove all intra-cluster edges of  $\mathcal{C}_i$  from  $E'$ 
23 end
24 for  $v \in V', c \in \mathcal{C}_{k-1}$  do
25   add the least weight edge in  $E'(v, c)$  to  $E_S$ 
26   remove the edges  $E'(v, c)$  in  $E'$ 
27 end

```

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**Algorithm 3:** (2k - 1)-Spanner Algorithm: Vertex Cluster Joining

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```

1 execute cluster formation for  $k - 1$  iterations
2 for  $v \in V', c \in \mathcal{C}_{k-1}$  do
3   add the least weight edge in  $E'(v, c)$  to  $E_S$ 
4   remove the edges  $E'(v, c)$  in  $E'$ 
5 end

```

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**Algorithm 4:**  $(2k - 1)$ -Spanner Algorithm: Cluster Cluster Joining

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```
1 execute cluster formation for  $\lfloor \frac{k}{2} \rfloor$  iterations
2 if  $k$  is odd then
3   for  $c, c' \in \mathcal{C}_{\lfloor \frac{k}{2} \rfloor}$  do
4     | add the least weight edge in  $E'(c, c')$  to  $E_S$ 
5   end
6 else
7   for  $c \in \mathcal{C}_{\lfloor \frac{k}{2} \rfloor}, c' \in \mathcal{C}_{\lfloor \frac{k}{2} \rfloor - 1}$  do
8     | add the least weight edge in  $E'(c, c')$  to  $E_S$ 
9   end
10 end
```

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## 2.2 Results

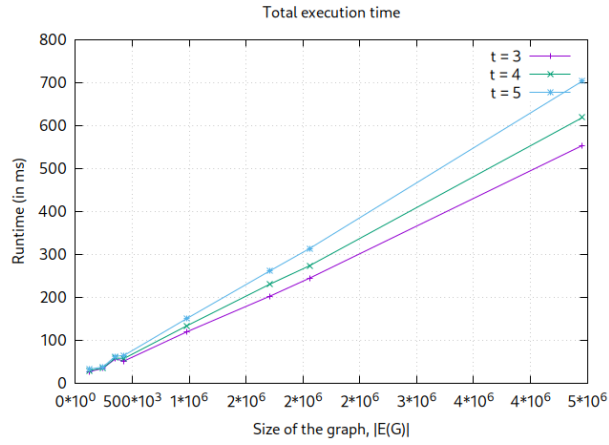


Figure 4: Total execution time

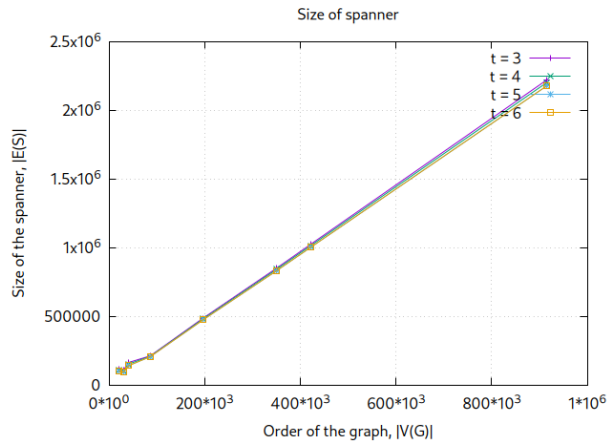


Figure 5: Spanner size

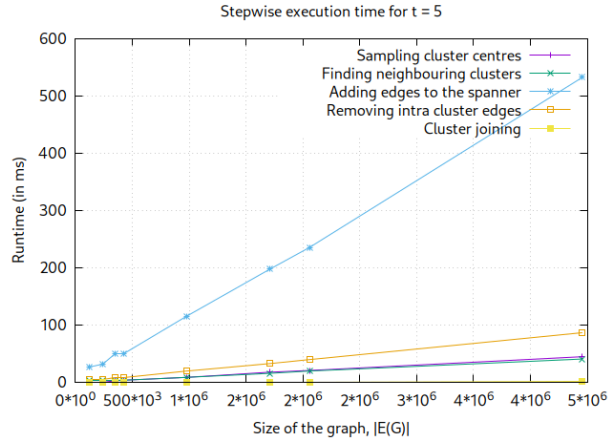


Figure 6: Stepwise execution time

## Bibliography

1. Baswana, Surender Sen, Sandeep. (2003). A Simple and Linear Time Randomized Algorithm for Computing Sparse Spanners in Weighted Graphs
2. Reyan Ahmed, Greg Bodwin, Faryad Darabi Sahneh, Keaton Hamm, Mohammad Javad Latifi Jebelli, Stephen Kobourov, Richard Spence (2019). Graph Spanners: A Tutorial Review.