Picker's Calculus

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```
[1]: from math import *
  import matplotlib.pyplot as plt
  import numpy as np
```

1 Mathematical Model

Picker Calculus:

Symbol	Label	Units
\overline{t}	Universe time	s
n	Picker operations per second	${ m s}^{-1}$
T	Time per Picker operation	\mathbf{s}
γ	Time conversion factor	
H	Cumulative Harolds' Time	\mathbf{s}
\mathcal{R}	Relative Picker time	\mathbf{s}
\mathcal{A}	Absolute Picker time	\mathbf{s}

Population model:

Symbol	Label	Units
\overline{P}	Population of Universe	indiv.
P_{civ}	Population of Universe	civs.
P_0	Initial population of Universe	indiv.
K	Carrying capacity of Universe	indiv.
K_{world}	Carrying capacity of civilizations	indiv.
K_{civ}	Carrying capacity of civs. in Universe	civ.
b	Rate of births (for planetary populations)	indiv. s $^{-1}$
d	Rate of deaths (for planetary populations)	indiv. s $^{-1}$
b_T	Rate of Transcendentalist-caused births	civ. s $^{-1}$
d_T	Rate of Transcendentalist-caused deaths	civ. s $^{-1}$
r	Rate of natural increase (for civs. in Universe)	civ. s $^{-1}$

1.1 Absolute Picker Time

The average absolute Picker time, meaning the time they are Picker in relation to the Universe and not their subjective experience, between two times t_0 and t_1 is simply,

$$\mathcal{A} = \frac{(t_1 - t_0) - (H(t_1) - H(t_0))}{D(t_1) - D(t_0)}$$

where the total time (minus the time taken up in the period t_0 to t_1 by the Harolds) divided by the number of deaths gives the amount of time per death, or in between deaths. This is the minimum time each person must be Picker for all moments in time to have a Picker.

To calculate the "instantenous" Picker time, the following limit is set up,

$$\mathcal{A}(t) = \lim_{t_1 \rightarrow t} \frac{t_1 - t - H(t_1) - H(t)}{D(t_1) - D(t)}$$

Solving the limit using the definition of the derivative,

$$\mathcal{A}(t) = \frac{1 - H'(t)}{D'(t)}$$

But due to the low number of Harolds,

$$\boxed{\mathcal{A}(t) \approx \frac{1}{D'(t)}}$$

All units here are in seconds, but for the Python calculations will often be done in Ma (millions of years).

1.2 Relative Picker Time

To convert from absolute Picker time to relative Picker time (the time actually experienced by the Picker themselves), the equation

$$\mathcal{R}(t) = \gamma \mathcal{A}(t)$$

is used where γ is the time factor. This means that, for every second in the real world, the Picker experiences γ seconds. (The units of time don't matter as long as they cancel out). Due to the nature of the Picker's job, this time factor is massive, leading to the tongue-in-cheek definition of the Picker's job as Hell. The approximation (ignoring relativity) for γ is given by,

$$\gamma = nT$$

where n is the number of required Picker operations per unit time and T is the time per Picker operation.

1.3 Population Model

From here on, t = 0 is exactly -5000 Ma. At t = 0, the population, P_0 , of the universe is defined as exactly the population of one civilized world (10 billion). This is necessary for the equations, but really whatever value this is has no effect 5 billion years later on the population.

```
[2]: init_pop = 10**10 # world
init_time = -5000 # Ma
end_time = 100 # Ma
```

Modelling the population of the Universe is the first step to calculating Picker times and other things. The standard equation for population dynamics is this logistic differential equation,

$$\frac{\mathrm{d}P}{\mathrm{d}t} = rP\left(1 - \frac{P}{K}\right)$$

$$r = b - d$$

And in non-differential form,

$$P(t) = \frac{P_0 K e^{rt}}{(K - P_0) + P_0 e^{rt}}$$

When finding the population at all times in a range, it is easier to numerically integrate the preivous equation, as will be done in Python.

```
[3]: # Normal model
     def pop_change(b, d, pop, carry):
         limiter = (1 - pop/carry)
         diff = (b - d) * pop * limiter
         return diff
     def make_pop_data(b, d, init_pop, carry, init_time, end_time, iterator=1):
         time = np.arange(init_time, end_time, iterator)
         pop = np.array([init_pop])
         d_pop = np.array([])
         for i in time:
             change = pop_change(b, d, pop[-1], carry)
             # Edge case where change goes over carry
             if pop[-1] + change > carry:
                 pop_fill = np.ones(size(time) - size(pop))*carry
                 pop = np.append(pop, pop_fill)
                 d_pop_fill = np.zeros(size(time) - size(d_pop))
                 d_pop = np.append(pop, d_pop_fill)
                 # This may go one over or under for the array sizes, but it is
                 # unused so it doesn't really need to work correctly
                 break
             d_pop = np.append(d_pop, change)
             pop = np.append(pop, pop[-1] + change)
```

```
pop = pop[0:-1] # pop goes 1 over size(time)
return time, pop, d_pop
```

However, that assumes reproduction occurs between members of the population. While this is true for individual civilizations, the population of civilizations in the universe is only dependent on the number of planets which *can* become civilized. Thus, a better equation is,

$$\frac{\mathrm{d}P}{\mathrm{d}t} = r(K - P)$$

Non-differential form,

$$P(t) = K - (P_0 - K)e^{-rt}$$

This seems to make sense. At any moment in time, the change in the number of civilizations is equal to the remaining possible planets were civilizations may emerge times the rate at which any single planet gains a civilization. At the time scales in question, the thousands of years it takes to develop civilization are ignored. Interstellar colonization is also ignored, as the rate of growth for this, given the energy requirements to transform worlds were civilizations can't occur to ones where civilization is possible, is far to low.

It also doesn't make sense to define r as b-d since civilizations don't die (or are assumed not to for now, ignoring the Transcendentalists' War). Instead, r is simply the rate of formation of civilizations.

```
[4]: # Civilizational model
     def civ_change(r, pop, carry):
         return r * (carry - pop)
     def make_civ_data(r, init_pop, carry, init_time, end_time, iterator=1):
         time = np.arange(init time, end time, iterator)
         pop = np.array([init_pop])
         d_pop = np.array([])
         for i in time:
             change = civ_change(r, pop[-1], carry)
             # Edge case where change goes over carry
             if pop[-1] + change > carry:
                 pop_fill = np.ones(size(time) - size(pop))*carry
                 pop = np.append(pop, pop_fill)
                 d pop fill = np.zeros(size(time) - size(d pop))
                 d_pop = np.append(pop, d_pop_fill)
                 # This may go one over or under for the array sizes, but it is
                 # unused so it doesn't really need to work correctly
```

```
break

d_pop = np.append(d_pop, change)
pop = np.append(pop, pop[-1] + change)

pop = pop[0:-1] # pop goes 1 over size(time)

return time, pop, d_pop

return 0
```

1.3.1 Carrying Capacity

Next, K must be calculated. Using results from Wikipedia, a high estimate (meaning a "maximum" possible value, or carrying capacity) of our galaxy is 15,600,000 civilizations. Since these civilizations have the technology to communicate into space, it must be assumed that they have similar technology to Earth, where the carrying capacity is around 10 billion (K_{world}). Expanding our galaxy's estimate to all stars in the Universe, 10e24; and calculating a Universal ratio of stars with civilizations (using the Milky Way, mw, as reference) by dividing the number of mw civilizations from the total number of stars in our galaxy, 10e11, K can be calculated as,

$$K = K_{world} \cdot n_{stars} \cdot \left(\frac{n_{civs}}{n_{stars}}\right)_{mw}$$

The carrying capcity in terms of civilizations (K_{civ}) and not individuals is calculated the same way ignoring the K_{world} term.

```
[5]: # Constants
k_world = 10**10
num_stars_uni = 10**24
num_civs_mw = 1.560 * 10**7
num_stars_mw = 10**11

k_civ = num_stars_uni * (num_civs_mw/num_stars_mw)
k = k_civ * k_world

print(f"K_civ: {k_civ}")
print(f"K: {k}")
```

K_civ: 1.56e+20
K: 1.56e+30

1.3.2 Rates of Increase

Finding a rate of natural increase for individual civilizations is easy. On Earth, there are many RNI values per year. This can be converted to seconds, as with all units.

```
[6]: # Using America as reference as an example

b_america = 12.3/1000 # per year
d_america = 8.4/1000 # per year

r_america_per_year = b_america - d_america
r_america_per_sec = r_america_per_year/(60*60*24*365) # per year to per second

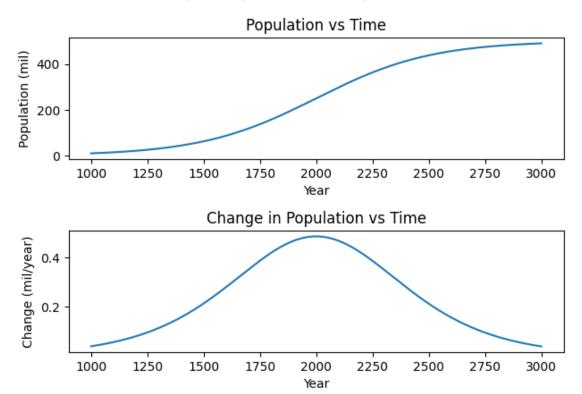
print(f"r per year: {r_america_per_year}")
print(f"r per second: {r_america_per_sec}")
```

r per year: 0.003899999999999999999999 r per second: 1.2366818873668184e-10

Now a sample graph can be made for America with example values.

```
[7]: # None of these values are intended to be historically correct, they are choosen
     # purely to make the graph look nice
    k america = 500 000 000 / 1 000 000 # in millions
    init_pop_america = 10_000_000 / 1_000_000 # in millions
    time_america, pop_america = \
        make_pop_data(b_america, d_america, init_pop_america, k_america, 1000, 3000)
    fig1, (fig1_ax1, fig1_ax2) = plt.subplots(2)
    fig1.suptitle("Sample Graph (American Population)")
    fig1_ax1.plot(time_america, pop_america)
    fig1_ax1.set_title("Population vs Time")
    fig1_ax1.set_xlabel("Year")
    fig1_ax1.set_ylabel("Population (mil)")
    fig1_ax2.plot(time_america, d_pop_america)
    fig1_ax2.set_title("Change in Population vs Time")
    fig1 ax2.set xlabel("Year")
    fig1_ax2.set_ylabel("Change (mil/year)")
    fig1.tight_layout()
```

Sample Graph (American Population)



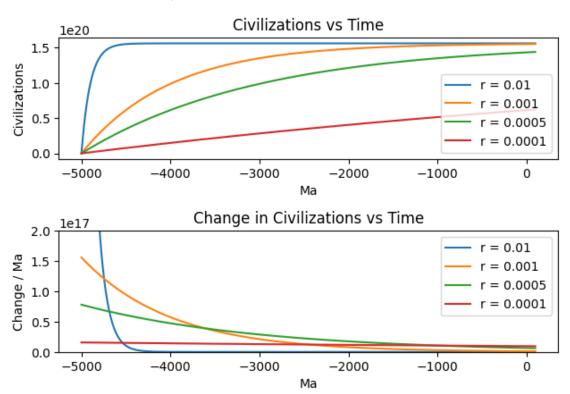
The next task is to find r for civilizations in the Universe, as this is the model that works best for the Universe. This is also the last unknown needed before a graph can be made, since $P_0 = 10^{10}$ and t = 0 is -5000 Ma (which will go to the present time and a bit past it). There is really no way to estimate what this may be, and all that really matters is that the graph looks nice. So multiple r values will be graphed to see what looks best.

```
[8]: # Possible rates at which all remaining planets become civilized / Ma
r1 = 0.01
r2 = 0.0001
r3 = 0.0005
r4 = 0.0001

time1, pop1, d_pop1 = \
    make_civ_data(r1, init_pop, k_civ, init_time, end_time)
time2, pop2, d_pop2 = \
    make_civ_data(r2, init_pop, k_civ, init_time, end_time)
time3, pop3, d_pop3 = \
    make_civ_data(r3, init_pop, k_civ, init_time, end_time)
time4, pop4, d_pop4 = \
    make_civ_data(r4, init_pop, k_civ, init_time, end_time)
```

```
fig2, (fig2_ax1, fig2_ax2) = plt.subplots(2)
fig2.suptitle("Population of the Universe (No War)")
fig2_ax1.plot(time1, pop1, label=str(f"r = {r1}"))
fig2_ax1.plot(time2, pop2, label=str(f"r = {r2}"))
fig2_ax1.plot(time3, pop3, label=str(f"r = {r3}"))
fig2_ax1.plot(time4, pop4, label=str(f"r = {r4}"))
fig2_ax1.set_title("Civilizations vs Time")
fig2_ax1.set_xlabel("Ma")
fig2 ax1.set ylabel("Civilizations")
fig2_ax1.legend()
fig2_ax2.plot(time1, d_pop1, label=str(f"r = {r1}"))
fig2_ax2.plot(time2, d_pop2, label=str(f"r = {r2}"))
fig2_ax2.plot(time3, d_pop3, label=str(f"r = {r3}"))
fig2_ax2.plot(time4, d_pop4, label=str(f"r = {r4}"))
fig2_ax2.set_title("Change in Civilizations vs Time")
fig2_ax2.set_xlabel("Ma")
fig2_ax2.set_ylabel("Change / Ma")
fig2_ax2.set_ylim([0,0.2 * 10**18])
fig2_ax2.legend()
fig2.tight_layout()
```

Population of the Universe (No War)



The best of these seems to be r=0.001, so from here on, the rate at which planets become civilizations per million years is 0.1%. Now, to calculate Picker times, which is the entire point of this model, death rates must be found. It can be assumed that each civilization, once at the carrying capacity, has a natural, equal birth and death rate. Since the number of civilizations can be calculated at anytime, the total death rate can too. Finding the total population of the universe from the number of civilizations and multiplying that by the death rate gives the number of deaths in any period of time.

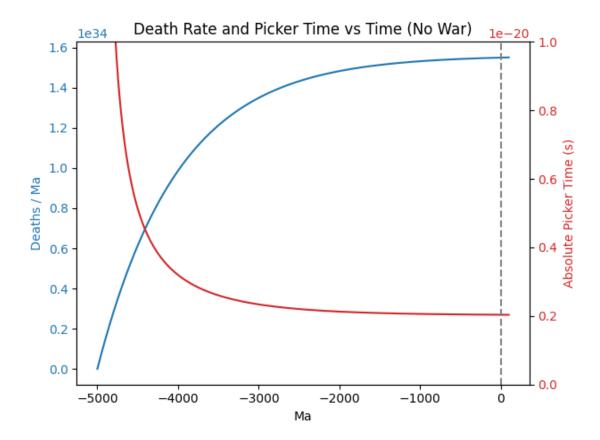
Continuing to ignore the Transcendentalists' War, where P_{civ} is the number of civilizations in the Universe at any given time, the death rate can be calculated as,

$$\frac{\mathrm{d}D}{\mathrm{d}t} = dK_{world}P_{civ}(t)$$

The death rate in America, as used earlier, is r = 0.0084 per year, or 0.84%. Rounding, 1% can be used as the natural death rate of all civilizations in the Universe at carrying capacity, ignoring how it changes with the population and development of a civilization.

```
d_pop = d_pop2
d = 0.01 \# per year
d = d*1_000_000 \# per Ma
death_rate = k_world*d*pop_civ
# Deaths / Ma to deaths / seconds
death_rate_per_sec = death_rate/1_000_000/365/24/60/60
absolute_picker_time = 1/death_rate_per_sec
fig3, fig3_ax1 = plt.subplots()
fig3_ax1.plot(time, death_rate, color='tab:blue')
fig3_ax1.set_title("Death Rate and Picker Time vs Time (No War)")
fig3_ax1.set_xlabel("Ma")
fig3_ax1.set_ylabel("Deaths / Ma", color='tab:blue')
fig3_ax1.tick_params(axis='y', labelcolor='tab:blue')
fig3_ax2 = fig3_ax1.twinx()
fig3_ax2.plot(time, absolute_picker_time, color='tab:red')
fig3 ax2.set ylabel("Absolute Picker Time (s)", color='tab:red')
fig3_ax2.tick_params(axis='y', labelcolor='tab:red')
fig3_ax2.set_ylim([0,10**-20])
fig3_ax2.vlines(x=0, ymin=0, ymax=10**50, linestyles='dashed', color='gray')
fig3.tight_layout()
print(f"The absolute Picker time at 0 Ma: A = {absolute_picker_time[-1]}_\( \)
 ⇔seconds")
print(f"A = {absolute_picker_time[-1]*(10**18)} attoseconds")
```

The absolute Picker time at 0 Ma: A = 2.0339195330319226e-21 seconds A = 0.0020339195330319228 attoseconds



It takes light 0.247 attoseconds to travel the average bond length of molecular hydrogen. So, around 120 Pickers control the universe in the time it takes a photon to cross a hydrogen atom.

1.4 Transcendentalists' War

Let the goal of the Transcendentalists be to maintain a stable absolute Picker time below the value found above. The War lasts between -130 Ma and 15 Ma, with its plateau between -100 and -15 Ma. There are two methods to increase the death rate while maintaining a constant population:

1) increase the birth and death rate of individual civilizations en masse and 2) increase the birth and death rate of civilizations themselves in the Universe.

The Transcendentalist civilizations are already assumed to have completed the first method on their own civilizations. They are, however, such a small minority of the civilizations in the Universe (although advanced) that they may be ignored, since the majority are Cascade Theorists. The primary method Transcendentalists use is civilization seeding and destruction. Basically, evolved beings capable of creating civilizations are plucked from one planet and sent to another (capable of hosting civilization) to seed a new civilization. Then, after a few thousand years, where to planet's population reaches K_{world} , the planet is obliterated. If new planets are being seeded and destroyed at a significant rate every second, it will have a significant impact on the total death rate of the Universe.

While, as mentioned earlier, interstellar colonization like this takes too much energy and ought to be ignored, it is still easier than imposing method 1 on civilizations. However, it also does not

require terraforming and the Transcendentalists do have access to significant energy.

Thus, the original equation for the growth of civilizations can be adjusted to include b and d (not to be confused with the b and d of individual civilizations),

$$\frac{\mathrm{d}P_{civ}}{\mathrm{d}t} = (b-d)(K_{civ}-P_{civ})$$

The natural r as found above is 0.1% per Ma. Since d was assumed to be 0, it must follow that the natural b is 0.1% per Ma. The Trancendentalists increase the birth rate of civilizations with seeding and the death rate with destuction. This destruction is assumed to be instantenous. This creates a new equation, where the subscript T marks Trancendentalists',

$$\frac{\mathrm{d}P_{civ}}{\mathrm{d}t} = (r + b_T - d_T)(K_{civ} - P_{civ})$$

$$b_T - d_T = 0$$

The values of b_T and d_T can therefore be anything as long as they are the same, for the Trancendentalists destroy ever planet they seed. This also results in a new death rate equation, which must now take into account the previously absent d for civilizations, each with an assumed poulation of K_{world} ,

$$\boxed{\frac{\mathrm{d}D}{\mathrm{d}t} = dK_{world}P(t) + d_TK_{world}P(t)}$$

The next step is to find a function for $d_T(t)$, since the War only occurs in a set range of time, with ramp up and slow down periods. Assuming the function should begin at t=-130 Ma and end at t=15 Ma, with a plateau between t=-100 Ma and t=-15 Ma, it can be written as a logistic function,

$$d_T(t) = \frac{L}{1 + e^{-k(t - t_0)}}$$

This specific function is really only used to make the graphs look nice and like r before, the constants are found by looking for a nice graph. In this case, the nice looking value of d_T was found through trial and error before making the actual plot.

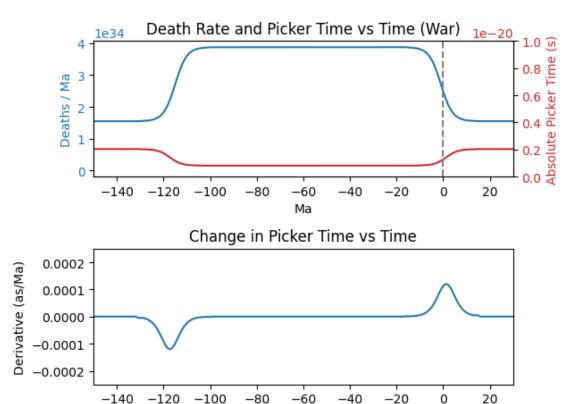
```
[10]: def make_trans_data(supremum, change_rate, carry, pop, time):
    d_trans = np.zeros(np.size(time))
    x = np.linspace(0, supremum, 30)

# The segement where d_trans changes - a period of 30 Ma
    d_trans_grow_seg = np.zeros(30)

# Apply logistic curve
for i in range(0, 30):
    d_trans_grow_seg[i] = supremum / (1 + exp(-change_rate*(i-15)))
```

```
# Copy reversed segement
   d_trans_shrink_seg = np.flip(d_trans_grow_seg)
   # Splices everything together in one array
   d_trans[4870:4900] = d_trans_grow_seg
   d_{trans}[4900:4985] = supremum
   d_trans[4985:5015] = d_trans_shrink_seg
    # Find the actual number of deaths / unit time
   death_rate_trans = d_trans*carry*pop
   return death rate trans, d trans
max_d_trans = 15_000 # per Ma
k_trans = 0.4 # Sigmoid steepness
death_rate_trans, d_trans = make_trans_data(max_d_trans, k_trans, k_world,__
 →pop_civ, time)
# Using the previous death rate and just adding the new deaths
total_death_rate = death_rate + death_rate_trans
total_death_rate_per_sec = total_death_rate/1_000_000/365/24/60/60
absolute_picker_time = 1/total_death_rate_per_sec
d_absolute_picker_time = np.gradient(absolute_picker_time)
# Useful data
print(f"Civ. population at 0 Ma: {pop_civ[5000]}")
print(f"d_T at 0 Ma: {d_trans[5000]}")
print(f"Civ. population at -50 Ma: {pop civ[4950]}")
print(f"d_T at -50 Ma: {d_trans[4950]}")
worlds_dest_per_year_0ma = pop_civ[5000] * (d_trans[5000]/1_000_000)
worlds_dest_per_year_neg50ma = pop_civ[4950] * (d_trans[4950]/1_000_000)
print(f"Worlds destroyed per year (0 Ma): {worlds_dest_per_year_0ma}")
print(f"Worlds destroyed per year (-50 Ma): {worlds_dest_per_year_neg50ma}")
print(f"The absolute Picker time at 0 Ma: A = {absolute_picker_time[5000]}_\_
 ⇔seconds")
print(f"A = {absolute_picker_time[5000]*(10**18)} attoseconds")
print(f"Change in absolute Picker time at 0 Ma: A' =_
 →{d_absolute_picker_time[5000]} seconds/Ma")
print(f"A' = {d_absolute_picker_time[5000]*(10**18)} attoseconds/Ma")
```

```
print(f"A' = \{d_absolute_picker_time[5000]*(10**18)/1_000_000\} attoseconds/
  ⇔year")
fig4, (fig4_ax1, fig4_ax3) = plt.subplots(2)
fig4 ax1.plot(time, total death rate, color='tab:blue')
fig4 ax1.set title("Death Rate and Picker Time vs Time (War)")
fig4_ax1.set_xlabel("Ma")
fig4_ax1.set_xlim([-150, 30])
fig4_ax1.set_ylabel("Deaths / Ma", color='tab:blue')
fig4_ax1.tick_params(axis='y', labelcolor='tab:blue')
fig4_ax2 = fig4_ax1.twinx()
fig4_ax2.plot(time, absolute_picker_time, color='tab:red')
fig4_ax2.set_ylabel("Absolute Picker Time (s)", color='tab:red')
fig4_ax2.tick_params(axis='y', labelcolor='tab:red')
fig4_ax2.set_ylim([0,10**-20])
fig4_ax2.vlines(x=0, ymin=0, ymax=10**50, linestyles='dashed', color='gray')
fig4 ax3.plot(time, d absolute picker time*(10**18))
fig4_ax3.set_title("Change in Picker Time vs Time")
fig4_ax3.set_ylabel("Derivative (as/Ma)")
fig4_ax3.set_ylim([-0.00025,0.00025])
fig4_ax3.set_xlabel("Ma")
fig4_ax3.set_xlim([-150, 30])
fig4.tight_layout()
Civ. population at 0 Ma: 1.549515065343281e+20
d_T at 0 Ma: 6019.68509831322
Civ. population at -50 Ma: 1.548977215497985e+20
d T at -50 Ma: 15000.0
Worlds destroyed per year (0 Ma): 9.327592748458784e+17
Worlds destroyed per year (-50 Ma): 2.3234658232469775e+18
The absolute Picker time at 0 Ma: A = 1.2704478102769586e-21 seconds
A = 0.0012704478102769587 attoseconds
Change in absolute Picker time at 0 Ma: A' = 1.1309597313076625e-22 seconds/Ma
A' = 0.00011309597313076625 \text{ attoseconds/Ma}
A' = 1.1309597313076625e-10 attoseconds/year
```



Finally, some values for the Transcendentalists' War can be used:

Time	Civilizations Destroyed per Year	Percent of Total
-50 Ma	2.324e+18	1.5%
0 Ma	9.328e + 17	0.6%

Ma

Or, in terms of galaxies, using the Milky Way as reference. At 0 Ma, 59,794,871,795 galaxies were destroyed and seeded every year in the War. At 50 Ma, 148,974,358,974 galaxies were destroyed and seeded every year. These numbers are beyond comprehension.

The new absolute Picker time for when the story takes place, is 0.00127 attoseconds, close to half of the previously calculated value. The rate of change is 1.131e-10 attoseconds per year. Before this can be understood in terms of relative Picker time, this means nothing really. So, γ must be found.

1.5 Numerical Calculations for Gamma

Since $\mathcal{R} = \gamma \mathcal{A}$, the time conversion factor can be calculated based on a desired relative Picker time.

I may later justify this γ using some values for n and T. But for now, γ is simply given by the Pickers through their Oracles.

```
Relative Picker time at -50 Ma: 641.0099039167923 years
Relative Picker time at 0 Ma: 1000 years
Relative Picker time at 20 Ma: 1601.7537885716981 years
Change in relative Picker time per year: 2807.3523208122374 seconds per year
Gamma: 2.4822743401891596e+31
```

This shows how absurd the doctrine of (Super-)Choosers is. Maybe even the absurdity of the Transcendentalists, although they at least managed to shave around 1200 years from the Picker time. A full human lifetime during the decline of the Transcendentalists' War would add practically nothing to the minimum relative Picker time.

1.6 Numerical Calculations for Changers

Here, a Changer killed 10 billion people in an instant. The Python speaks for itself.

Relative Picker time 0 Ma: 1000.0 years

Relative Picker time 0 Ma (where a Changer kills 10 billion people in one second):

999.999 years

Difference: 0.395 seconds