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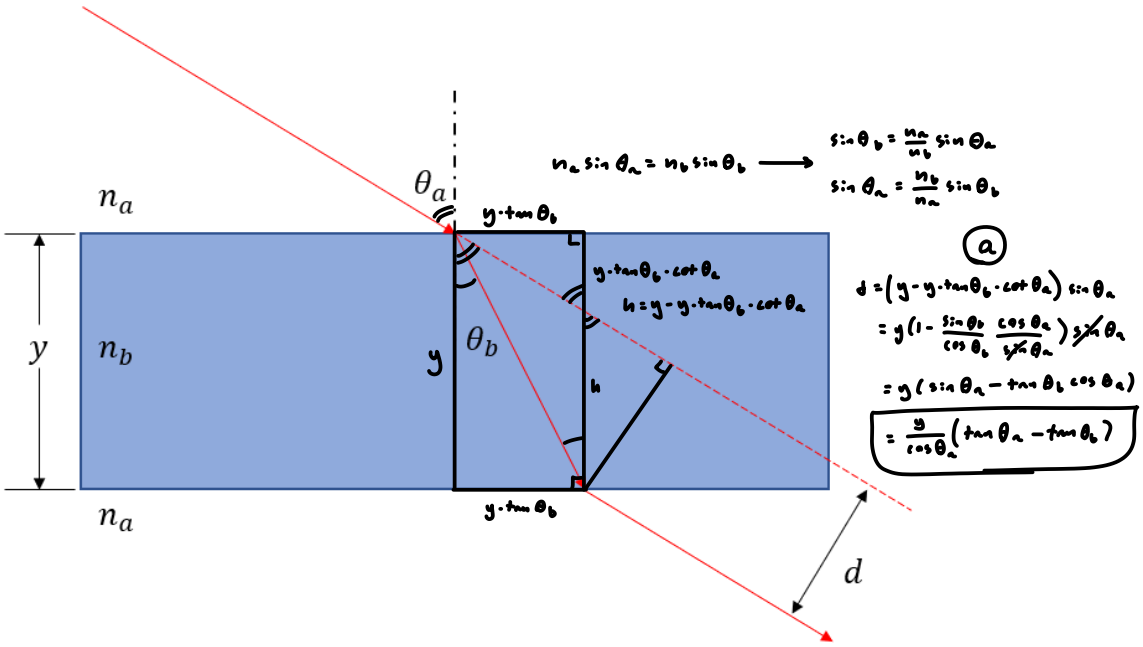
PH 142 Spring Semester 2025: Homework 10

Due Wednesday, May 7, 2025 at 11:59 pm

Instructions: There are 2 long problems for this assignment. Please upload your solutions to Canvas when completed. 10 points will be given for attempting all problems. One problem will be chosen randomly and graded in detail, out of 10 points. The sum of these scores will be the total grade, out of 20 points. Partial credit will be given. Please show all work.

1. A light ray initially traveling in air enters a piece of glass with index of refraction, $n$ and thickness $y$ at an angle $\theta_a$. The ray is diffracted with at an angle $\theta_b$ and propagates in the glass. When the light ray exits the glass, it travels parallel to the incoming ray and is displaced by a distance $d$. (see figure below)




   a. Derive an expression for the displacement of the light ray in terms of the given variables.
   b. For a light ray with a wavelength of $\lambda_0 = 633$ nm (in vacuum), calculate the wavelength of the light inside the glass with $n = 1.52$.
   c. For a glass thickness of $y = 2.0$ cm and incident angle $\theta_a = 55°$, calculate the time it takes for light to travel through the glass and the displacement, $d$ when the light ray exits the glass.
   d. At what incident angle, $\theta_a$, will the displacement be equal to zero.

(1) (b) $\frac{n_2}{n_1} = \frac{\lambda_1}{\lambda_2} \Rightarrow \frac{1.52}{1} = \frac{633\times10^{-9}\,m}{\lambda_2} \Rightarrow \boxed{\lambda = 416\,nm}$

(c) $\theta_a = 55°$ $\quad y = 2\,cm$ $\quad \sin\theta_b = \frac{n_a}{n_b}\sin\theta_a \Rightarrow \theta_b = \sin^{-1}\left(\frac{n_a}{n_b}\sin\theta_a\right)$ $\quad \theta_b = 32.609°$

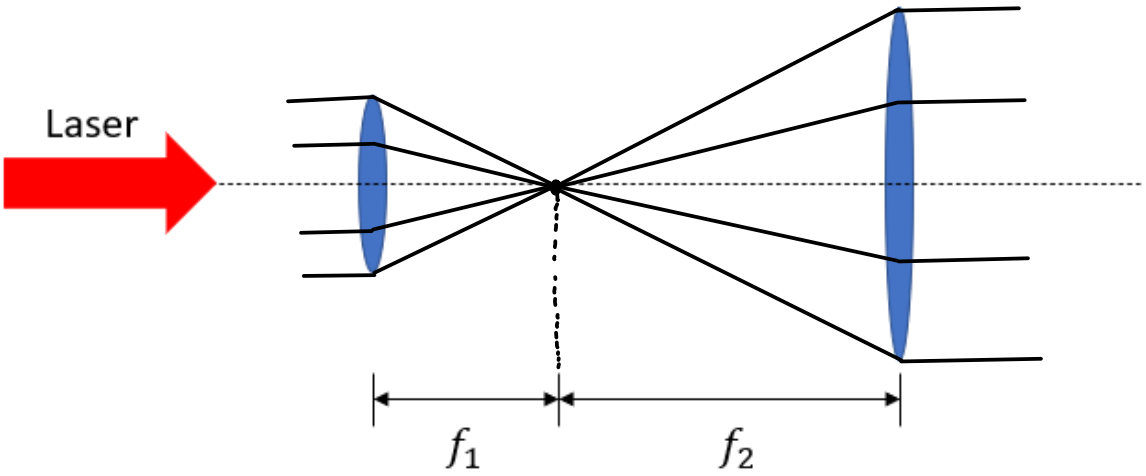
$d = \frac{0.02}{\cos 55°}(\tan 55° - \tan 32.609°) = \boxed{2.75\,cm}$ $\quad \frac{n_2}{n_1} = \frac{v_1}{v_2} \longrightarrow v_2 = \frac{n_1}{n_2}v_1 = \frac{1}{1.52}c = 197\,231\,880.263\,m/s$

$x = y/\cos\theta_b = 0.02/\cos 32.609° = 2.37\,cm$ $\quad t = d/v = \boxed{0.12\,ns}$

(d) $\theta_a = \theta_b \longrightarrow \tan\theta_a - \tan\theta_b = 0 \longrightarrow d = 0$

$\sin\theta_a = 1.52\sin\theta_a \longrightarrow \sin\theta_a - 1.52\sin\theta_a = 0 \longrightarrow \sin\theta_a(1-1.52) = 0 \longrightarrow \sin\theta_a = 0$ $\quad \boxed{\theta_a = 0°}$

2. A beam expander consists of two lenses with focal lengths $f_1$ and $f_2$ (see figure below).




a. Derive an expression for the magnification of this system. (a) $M = \frac{f_2}{f_1}$

b. For an incoming laser beam with a diameter of 1 mm, $f_1 = 25$ mm, $f_2 = 75$ mm, calculate the outgoing diameter of the beam (after the lens on the right).

$M = \frac{75\,mm}{25\,mm} = 3$ $\quad d = 3 \cdot 1\,mm = \boxed{3\,mm}$