PH 142 Spring Semester 2025: Homework 6

Due Wednesday, April 2, 2025 at 11:59 pm

Instructions: There are 4 long problems for this assignment. Please upload your solutions to Canvas when completed. 10 points will be given for completing all problems. One problem will be chosen randomly and graded in detail, out of 10 points. The sum of these scores will be the total grade, out of 20 points. Partial credit will be given. Please show all work.

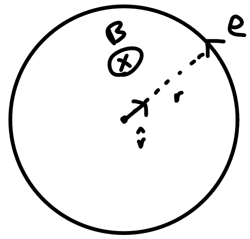
1. Hydrogen-like Uranium, U^{91+} is an ion which has one valence electron (like hydrogen) orbiting (classically) around a nucleus with charge $q = Ze$, where $Z = 92$ is the atomic number of uranium and e is the elementary charge ($e \approx 1.6 \times 10^{-19}$C). Due to the high nuclear charge, the radius of the electron orbit is approximately, $r \approx a_0/92 = 5.75 \times 10^{-13}$ m.
    a. Assuming a circular orbit, calculate the speed of the electron orbit in the ion.
    b. Calculate the electric and magnetic fields that the electron produces at the location of the nucleus. Assume that the nucleus is a point particle and that the orbit of the electron is in the x-y plane (plane of the paper).

2. A closely wound, circular coil with a diameter of 5.0 cm has 1000 turns and carries a current of 10.0 A.
    a. What is the magnitude of the magnetic field at the center of the coil?
    b. What is the magnitude of the magnetic field at a distance of 10 cm from its center?
    c. Calculate the cyclotron frequency of a single $^{20}Ne^{10+}$ ion located at the center of the coil.
    d. Assume that the magnetic field is uniform in the mid-plane of the coil. (Meaning that the value of the axial magnetic field is the same at the center of the coil and just near the wall of the coil.) If the $^{20}Ne^{10+}$ ion has a tangential kinetic energy of 1 eV, what is the cyclotron orbit radius?
    e. Under the conditions of part (d), what is the minimum current required to keep the ion from hitting the wall of the coil?

1) a) $F_E = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = F_T = m\frac{v^2}{r} \Rightarrow mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} \Rightarrow v = \sqrt{\frac{q_1 q_2}{4\pi\varepsilon_0 r m}}$

Calculator: $\boxed{v = 201{,}027{,}827 \text{ m/s}}$

b)



$\odot B \leftarrow \hat{z}$

RHR

$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{e}{r^2}\hat{r} = \boxed{4.349\times10^{15} \text{ N/C}}\ \hat{r}$

$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{qv}{r^2}(-\hat{z}) = \boxed{9.723\times10^{6} \text{ T}}$

$-\hat{z}$

2) a) $\vec{B} = \frac{n\mu_0 I a^2 \hat{r}}{2(x^2+a^2)^{3/2}} = \frac{1000(4\pi\times10^{-7})(10)(0.025)^2}{2(0.025)^3} = \boxed{0.251 \text{ T}}$

b) $\frac{1000(4\pi\times10^{-7})(10)(0.025)^2}{2(0.025^2+0.10^2)^{3/2}} = \boxed{0.0035 \text{ T}}$

c) $\omega = \frac{qB}{m}$ $\quad f = \frac{1}{2\pi}\omega$ $\quad f = \frac{qB}{2\pi m}$

calc. $\boxed{f = 1.925 \text{ MHz}}$

d) $r = \frac{mv}{qB}$ $\quad KE = \frac{1}{2}mv^2 \longrightarrow v = \sqrt{\frac{2KE}{m}}$ $\quad 1\text{eV} = 1.602\times10^{-19}\text{ J}$

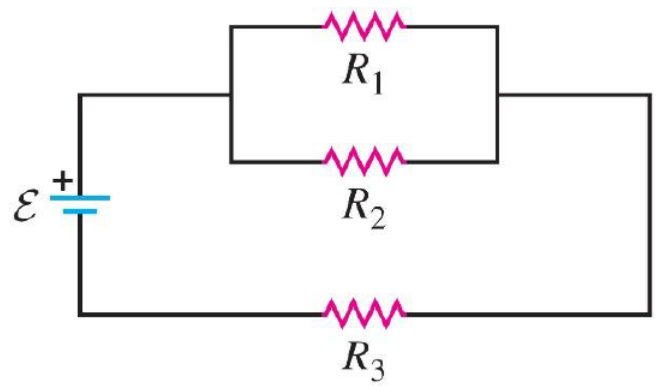
$r = \frac{m}{qB}\sqrt{\frac{2KE}{m}} \longrightarrow r = 2.56\times10^{-4}\text{ m}$

$r = \boxed{0.256 \text{ mm}}$

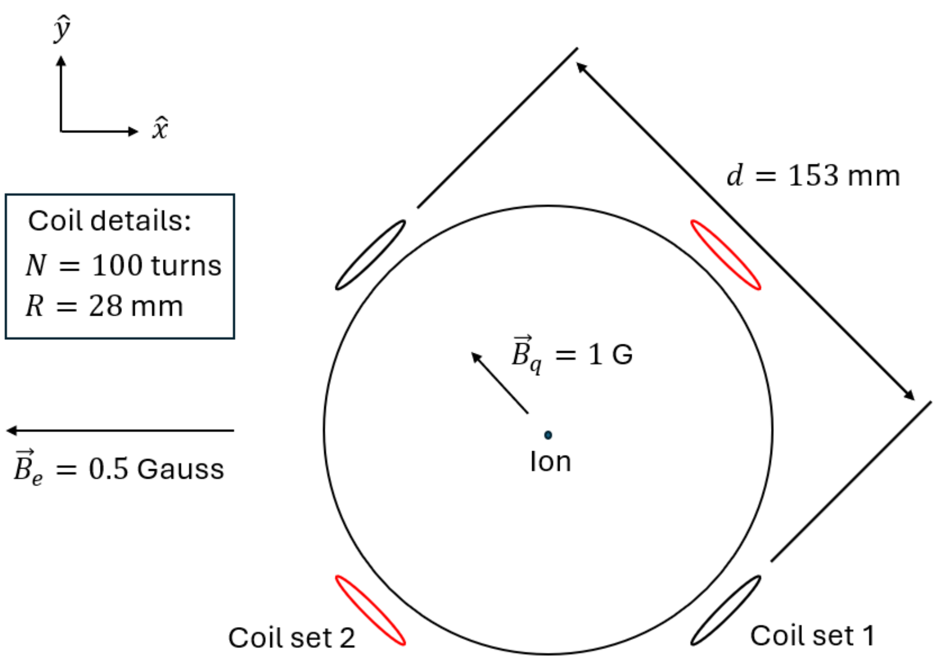
e) $r = \frac{m}{qB}\sqrt{\frac{2KE}{m}}$ $\quad B = \frac{n\mu_0 I}{2\cancel{a^3}}\frac{\cancel{a^2}}{1}\frac{1}{a}$

$r = \frac{m}{q}\frac{2a}{n\mu_0 I}\sqrt{\frac{2KE}{m}} \Rightarrow I = \frac{2m}{q n\mu_0}\sqrt{\frac{2KE}{m}} \Rightarrow I = \boxed{100 \text{ mA}}$

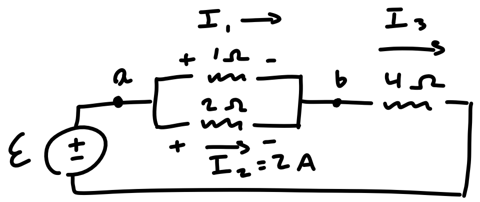
3. In the figure below, $R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$, and $R_3 = 4\ \Omega$. If the current I_2 through the resistor R_2 is 2 A, what is the current I_3 through the resistor R_3?



4. An ion is located in a trap that is mounted at the center of a vacuum chamber (see figure below of cross-sectional view). The earth generates a magnetic field of approximately $\vec{B}_e = 0.5$ Gauss in the $-\hat{x}$ direction. Two sets of magnetic field coils (Black = coil set 1, Red = coil set 2) are arranged perpendicular to each other and centered on the location of the ion. Given the coil specifications ($N = 100$ turns and $R = 28$ mm), what current is needed in each set of coils to generate a net magnetic field of 1 G along the desired quantization axis direction, $\vec{B}_q$, (along the coil set 1 axis direction)?



3)



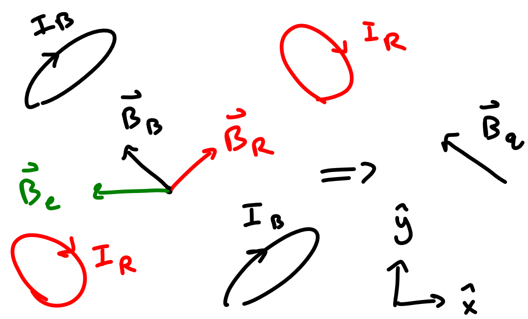
$I_3 = I_2 + I_1$ $\qquad V = IR$

$V_{ab} = I_2 R_2 = 2A(2\Omega) = 4V$

$I_1 = V_{ab}/R_1 = 4V/1\Omega = 4A$

$\Big\}\; I_3 = 4A + 2A = \boxed{6A}$

4)



$\vec{B}_e = \begin{pmatrix} 1/2 \\ 0 \end{pmatrix} G \qquad \vec{B}_B = \begin{pmatrix} -\sqrt{2}/2 \\ \sqrt{2}/2 \end{pmatrix} B_B (2) \qquad \vec{B}_R = \begin{pmatrix} \sqrt{2}/2 \\ \sqrt{2}/2 \end{pmatrix} B_R (2)$

2 coils

$\vec{B}_q = \begin{pmatrix} -\sqrt{2}/2 \\ \sqrt{2}/2 \end{pmatrix} G$

$$\begin{pmatrix} -\sqrt{2}/2 \\ \sqrt{2}/2 \end{pmatrix} = \begin{pmatrix} 1/2 \\ 0 \end{pmatrix} + \overbrace{\begin{pmatrix} -\sqrt{2} \\ \sqrt{2} \end{pmatrix} B_B + \begin{pmatrix} \sqrt{2} \\ \sqrt{2} \end{pmatrix} B_R}^{Ax}$$

$$\begin{pmatrix} -\sqrt{2}/2 \\ \sqrt{2}/2 \end{pmatrix} - \begin{pmatrix} 1/2 \\ 0 \end{pmatrix} = \begin{pmatrix} -\sqrt{2} & \sqrt{2} \\ \sqrt{2} & \sqrt{2} \end{pmatrix} \begin{pmatrix} B_B \\ B_R \end{pmatrix}$$

$$\begin{pmatrix} B_B \\ B_R \end{pmatrix} = \begin{pmatrix} 0.323 \\ 0.176 \end{pmatrix} G \Longleftarrow \begin{pmatrix} B_B \\ B_R \end{pmatrix} = \begin{pmatrix} -\sqrt{2} & \sqrt{2} \\ \sqrt{2} & \sqrt{2} \end{pmatrix}^{-1} \begin{pmatrix} 1/2(1-\sqrt{2}) \\ \sqrt{2}/2 \end{pmatrix}$$

For axial $\vec{B}$ of coil: (w/ n loops) $\qquad 10\,000\,G = 1\,T$

$$\vec{B} = \frac{n \mu_0 I a^2 \,\hat{r}}{2(a^2 + x^2)^{3/2}} \qquad 0.323 \cdot 10^{-4}\,T = \frac{(100)(4\pi \times 10^{-7}) I_B (0.026)^2}{2(0.026^2 + 0.0765^2)^{3/2}}$$

$$I = \frac{2B(a^2 + x^2)^{3/2}}{n \mu_0 a^2} \qquad \boxed{I_B = 354\,mA} \qquad \boxed{I_R = 193\,mA}$$