$\boxed{K = \frac{1}{4\pi\epsilon_0}}$

PH 142 Spring Semester 2025: Homework 1

Due Wednesday, January 29 2025, at 11:59 pm

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Instructions: There are 4 long problems for this assignment. Please upload your solutions to Canvas when completed. 10 points will be given for completing all problems. One problem will be chosen randomly and graded in detail, out of 10 points. The sum of these scores will be the total grade, out of 20 points. Partial credit will be given. Please show all work.

1. Two point charges q_1 and q_2 are located a distance L from the origin, along the x and y axes, respectively (see Fig. 1 below).
   a. Calculate the magnitude of the electrostatic force between the two charges.
   b. Assuming now that the charges are fixed in place, when $q_1 = q_2 = +q$, calculate the electric field $\vec{E}$ (magnitude and direction) at the origin.

a) $|\vec{F}| = K\frac{|q_1 q_2|}{r^2} = \boxed{K\frac{|q_1 q_2|}{2L^2}}$

b) $\vec{E} = K\sum_i \frac{q_i}{r_i^2}\hat{r} = \boxed{-K\frac{q^+}{L^2}\hat{x} - K\frac{q^+}{L^2}\hat{y}}$

$\boxed{|\vec{E}| = \sqrt{2}\,K\frac{q^+}{L^2}}$

$\alpha = 45°$

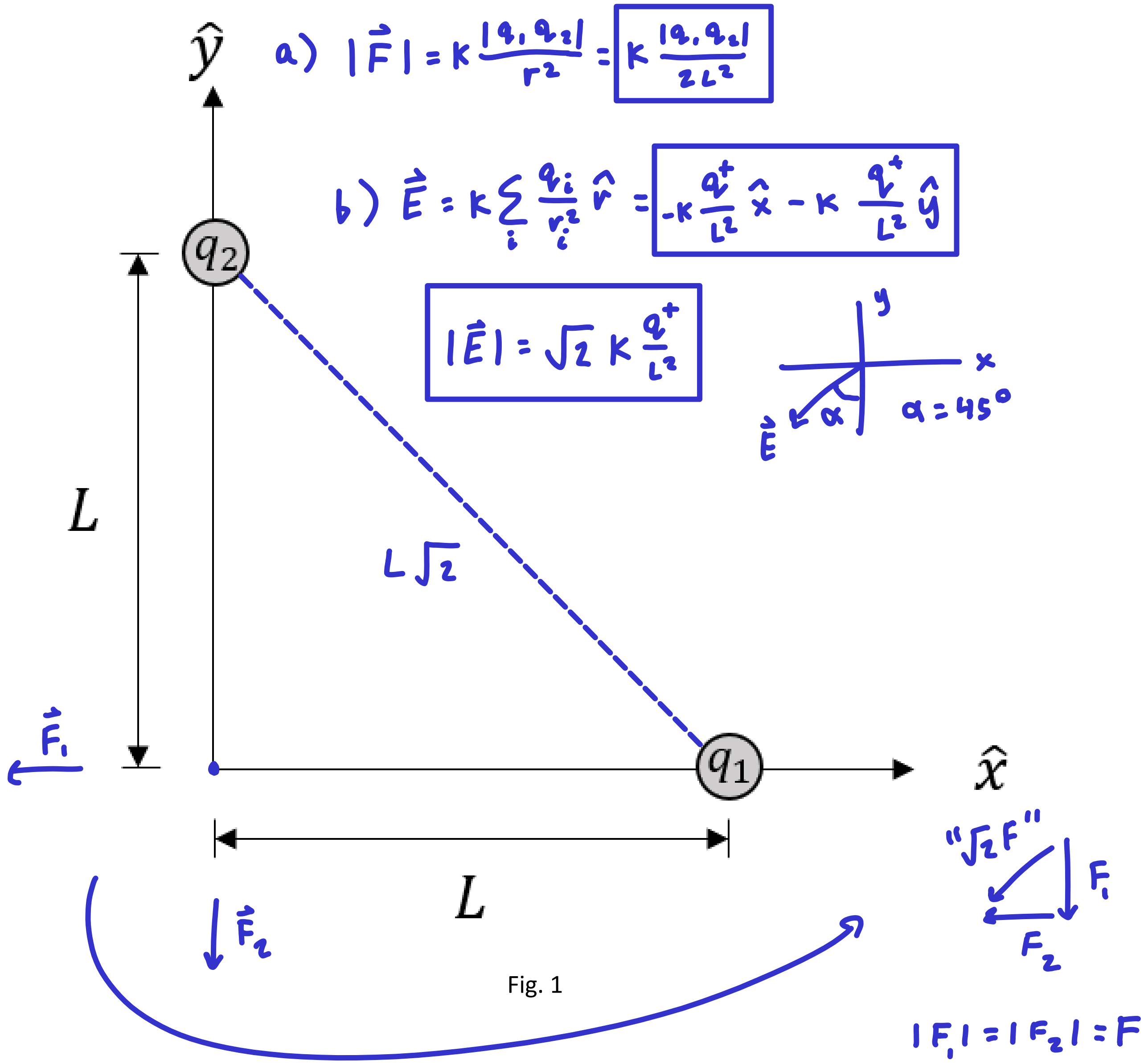
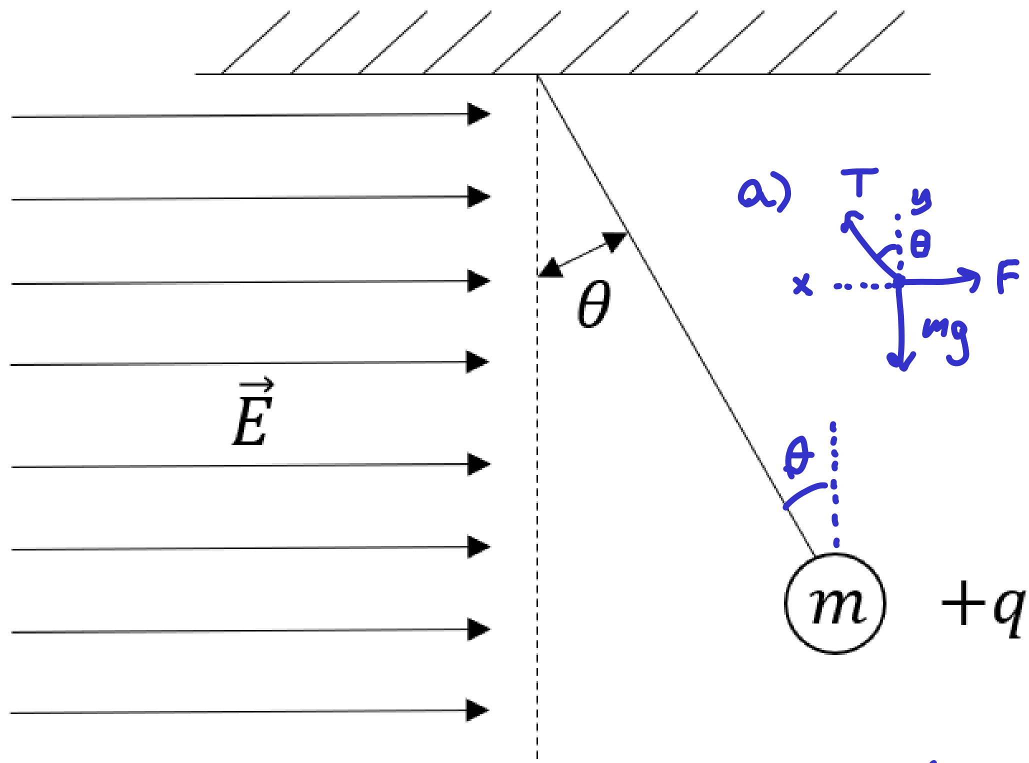


Fig. 1

$|F_1| = |F_2| = F$

2. A sphere with charge q and mass m is suspended (in stable equilibrium) with a massless string from the ceiling in a region of uniform electric field $\vec{E}$ pointing to the right ($+x$ direction). (see Fig. 2 below)
   a. Draw a free body diagram showing all forces on the sphere.
   b. Derive an expression for the angle θ (in terms of the given variables) at which this condition occurs.
   c. In the case where $m = 10$ g and $q = 10\ \mu$C, calculate the magnitude of the electric field $|\vec{E}|$ required such that $\theta = 45°$.



b) $T\cos\theta = mg$ $\quad T\sin\theta = F = qE$

$$\frac{mg}{\cos\theta} = \frac{qE}{\sin\theta} \Rightarrow mg\tan\theta = qE$$

$$\frac{qE}{mg} = \tan\theta \Rightarrow \boxed{\tan^{-1}\left(\frac{qE}{mg}\right) = \theta}$$

Fig. 2

c) $E = mg\tan\theta / q$

$$E = (0.010\text{ kg})(9.8\text{ m/s}^2)\tan(45°)/(10^{-5}\text{ C})$$

$$= \boxed{9.8 \times 10^3\text{ N/C}}$$

3. A line of charge with uniform linear charge density $\lambda = Q/L$ is located along the y axis (see Fig. 3 below).
   a. Calculate the electric field, $\vec{E}$, at the location P. (Assume that the charge Q is positive.)
   b. Find an expression for the electric field in the limit that $x \gg L$ (i.e., very far away from the line charge).

a)

$$Q = \lambda L \longrightarrow dq = \lambda\, dy \qquad dE = k\frac{dq}{r^2}\hat{r} = k\frac{\lambda\, dy}{r^2}\hat{r}$$

$$dE_y = \frac{kQ}{L}\frac{dy}{(x^2+y^2)}\sin(\alpha) = \frac{kQ}{L}\frac{y\, dy}{(x^2+y^2)^{3/2}} \qquad = \frac{kQ}{L}\frac{dy}{r^2}\hat{r} = \frac{kQ}{L}\frac{dy\,\hat{r}}{(x^2+y^2)}$$

$$E_y = \frac{kQ}{L}\int_0^L \frac{y\, dy}{(x^2+y^2)^{3/2}} = \frac{kQ}{L}\left[\frac{-1}{\sqrt{x^2+y^2}}\right]_0^L = \boxed{\frac{kQ}{L}\left[\frac{1}{x} - \frac{1}{\sqrt{x^2+L^2}}\right]} \quad E_y$$

$$dE_x = \frac{kQ}{L}\frac{dy}{(x^2+y^2)}\cos(\alpha) = \frac{kQ}{L}\frac{x\, dy}{(x^2+y^2)^{3/2}} \qquad E_x = \frac{kQ}{L}\int_0^L \frac{x\, dy}{(x^2+y^2)^{3/2}}$$

Subs

$y = x\tan(\theta)$

$dy = x\sec^2(\theta)\, d\theta$

$$\Rightarrow \frac{kQ}{L}\int_0^L \frac{x^2\sec^2(\theta)\, d\theta}{(x^2+x^2\tan^2(\theta))^{3/2}} \Rightarrow \frac{kQ}{L}\int_0^L \frac{x^2\sec^2(\theta)\, d\theta}{(x^2(1+\tan^2(\theta)))^{3/2}}$$

$$\frac{kQ}{L}\int_0^L \frac{x^2\sec^2\theta\, d\theta}{(x^2\sec^2\theta)^{3/2}} \Rightarrow \frac{kQ}{L}\int_0^L \frac{x^2\sec^2\theta}{x^3\sec^3\theta}\, d\theta$$

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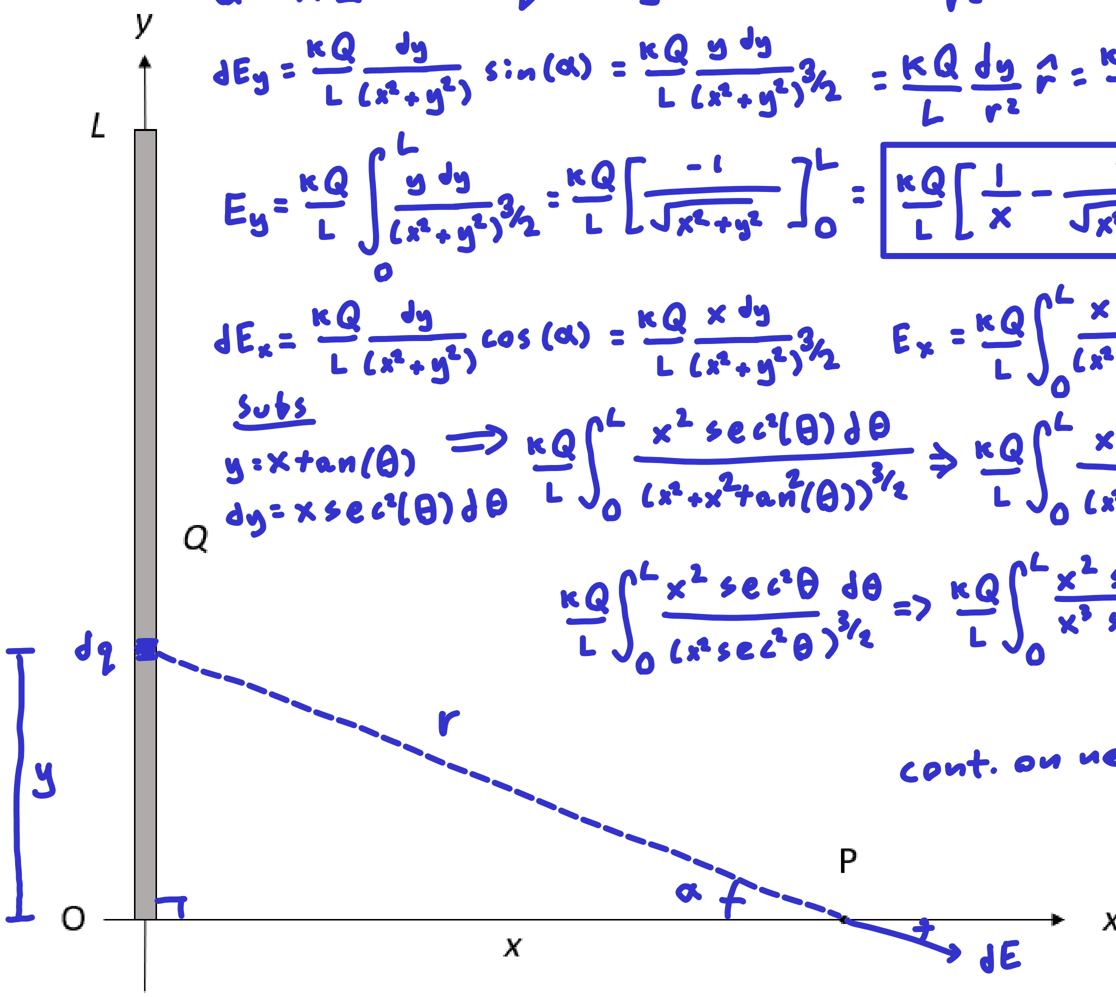

Fig. 3

$$\cos(\alpha) = \frac{x}{r}$$

$$\sin(\alpha) = \frac{y}{r}$$

$$\frac{KQ}{L}\int_0^L \frac{x^2\sec^2\theta}{x^3\sec^3\theta}\,d\theta \Rightarrow \frac{KQ}{L}\int_0^L \frac{\cos\theta}{x}\,d\theta \Rightarrow \frac{KQ}{L}\frac{1}{x}\left[\sin\theta\right]_0^L$$

sub

$$\Rightarrow \frac{KQ}{L}\frac{1}{x}\left[\frac{y}{\sqrt{x^2+y^2}}\right]_0^L \Rightarrow \frac{KQ}{\cancel{L}}\frac{1}{x}\frac{\cancel{L}}{\sqrt{x^2+L^2}} \Rightarrow \boxed{E_x = \frac{KQ}{x\sqrt{x^2+L^2}}}$$

b) $x >> L$: $E_x = \frac{KQ}{x\sqrt{x^2+0}} = \frac{KQ}{x^2}$ $\boxed{E_x \propto \frac{1}{x^2}}$

$$E_y = \frac{KQ}{L}\left[\frac{1}{x} - \frac{1}{\sqrt{x^2+0^2}}\right] = \frac{KQ}{L}\left[\frac{1}{x} - \frac{1}{x}\right] = 0 \quad \boxed{E_y = 0}$$

4. Four point charges q_1, q_2, q_3, and q_4 are arranged in a square, with length L, centered at the origin. (see Fig. 4 below)
   a. For the case where $q_1 = q_2 = q_3 = q_4 = +q$, calculate the electric field at location P_1 along the x axis.
   b. For the case where $q_1 = q_4 = +q$, and $q_2 = q_3 = -q$, calculate the electric field at location P_1 along the x axis.
   c. For the case where $q_1 = q_3 = q_4 = +q$, and $q_2 = -q$, calculate the electric field at location P_1 along the x axis.
   d. For the case where $q_1 = q_2 = q_3 = q_4 = +q$, calculate the electric field at the origin.

defs.

$r_2 = r_3 = \sqrt{a^2 + \frac{1}{4}L^2}$ $\quad \cos\alpha = \frac{b}{r_1}$

$r_1 = r_4 = \sqrt{b^2 + \frac{1}{4}L^2}$ $\quad \cos\beta = \frac{a}{r_2}$

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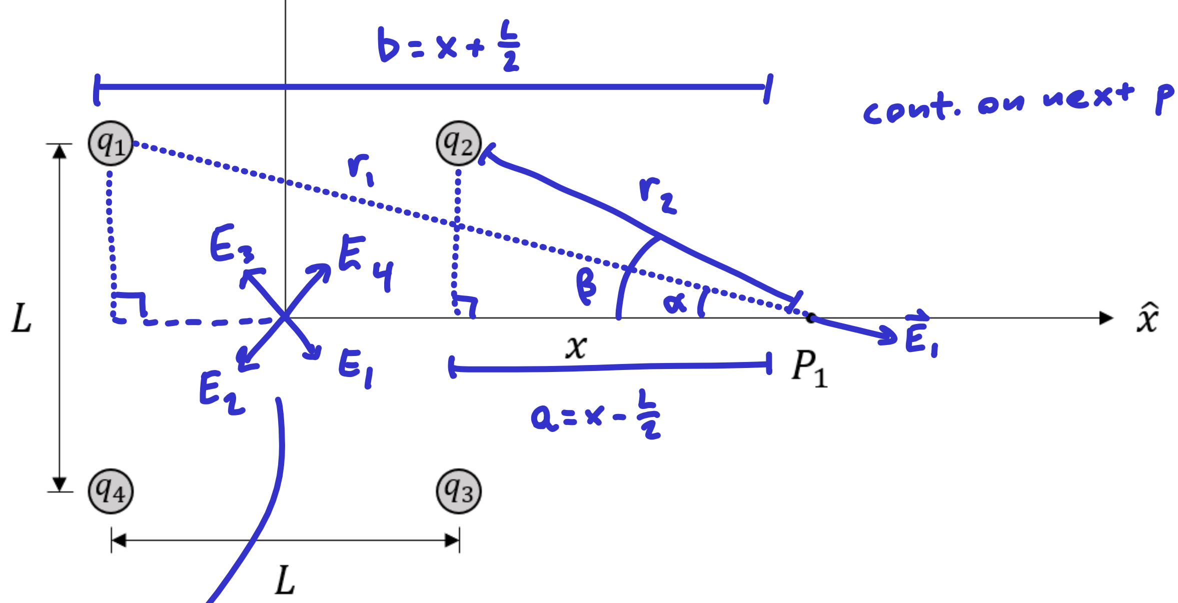

Fig. 4

d) $\boxed{\vec{E} = \vec{0}}$

defs

$r_2 = r_3 = \sqrt{a^2 + \frac{1}{4}L^2}$ $\cos\alpha = \frac{b}{r_1}$ 1 •   2 •

$r_1 = r_4 = \sqrt{b^2 + \frac{1}{4}L^2}$ $\cos\beta = \frac{a}{r_2}$ 4 •   3 •

a) $\vec{E} = K\left(\frac{2q^+}{r_1^2}\hat{r}_1 + \frac{2q^+}{r_2^2}\hat{r}_2\right) = K\left(\frac{2q^+}{r_1^2}\cos\alpha + \frac{2q^+}{r_2^2}\cos\beta\right)\hat{x}$

$E_y = 0 \leftarrow$ symmetry $= \boxed{2Kq^+\left(\frac{b}{r_1^3} + \frac{a}{r_2^3}\right)\hat{x}}$

b) Same as (a) but with diff charges

$$\boxed{\vec{E} = 2K\left(\frac{q^+ b}{r_1^3} - \frac{q^- a}{r_2^3}\right)\hat{x}}$$

subtraction since negative charge attracts the positive test charge

c) $q_2 = -q$

$\hat{y}$ is X by symmetry

$$\vec{E} = \underbrace{\frac{2q^+K}{r_1^2}\cos\alpha\,\hat{x}}_{\frac{2q^+Kb}{r_1^3}\hat{x}} + K\frac{q^+}{r_2^2}\hat{r} + K\frac{q^-}{r_2^2}\hat{r}$$

$K\frac{q^+}{r_2^2}\cos\beta\,\hat{x}$

$= K\frac{q^+ a}{r_2^3}\hat{x}$

$K\frac{q^+}{r_2^2}\sin\beta\,\hat{y}$ $\sin\beta = \frac{L/2}{r_2}$

$= \frac{KL}{2}\frac{q^+}{r_2^3}\hat{y}$

$$\boxed{\vec{E} = \frac{2q^+Kb}{r_1^3}\hat{x} + K\frac{q^+ a}{r_2^3}\hat{x} + \frac{KL}{2}\frac{q^+}{r_2^3}\hat{y} - K\frac{q^- a}{r_2^3}\hat{x} - \frac{KL}{2}\frac{q^-}{r_2^3}\hat{y}}$$