PH 142 Spring Semester 2025: Homework 5

Due Wednesday, March 12, 2025 at 11:59 pm

Instructions: There are 5 long problems for this assignment. Please upload your solutions to Canvas when completed. 10 points will be given for attempting to solve all problems. One problem will be chosen randomly and graded in detail, out of 10 points. The sum of these scores will be the total grade, out of 20 points. Partial credit will be given. Please show all work.

1. A loop of current, I, (see Fig. 1 below) is placed inside a uniform magnetic field, $\vec{B} = -B\hat{z}$. The loop consists of two semi-circles with radii R_1 and R_2, and two straight line sections of length L.
   a. Calculate the total force on the current loop.
   b. Calculate the magnetic dipole moment of this current loop.
   c. What is the torque on the current loop due to the magnetic field?
   d. How would your answer for part (c) change if instead, $\vec{B} = B\,\hat{y} - B\hat{z}$? Here, just explain qualitatively what will happen.

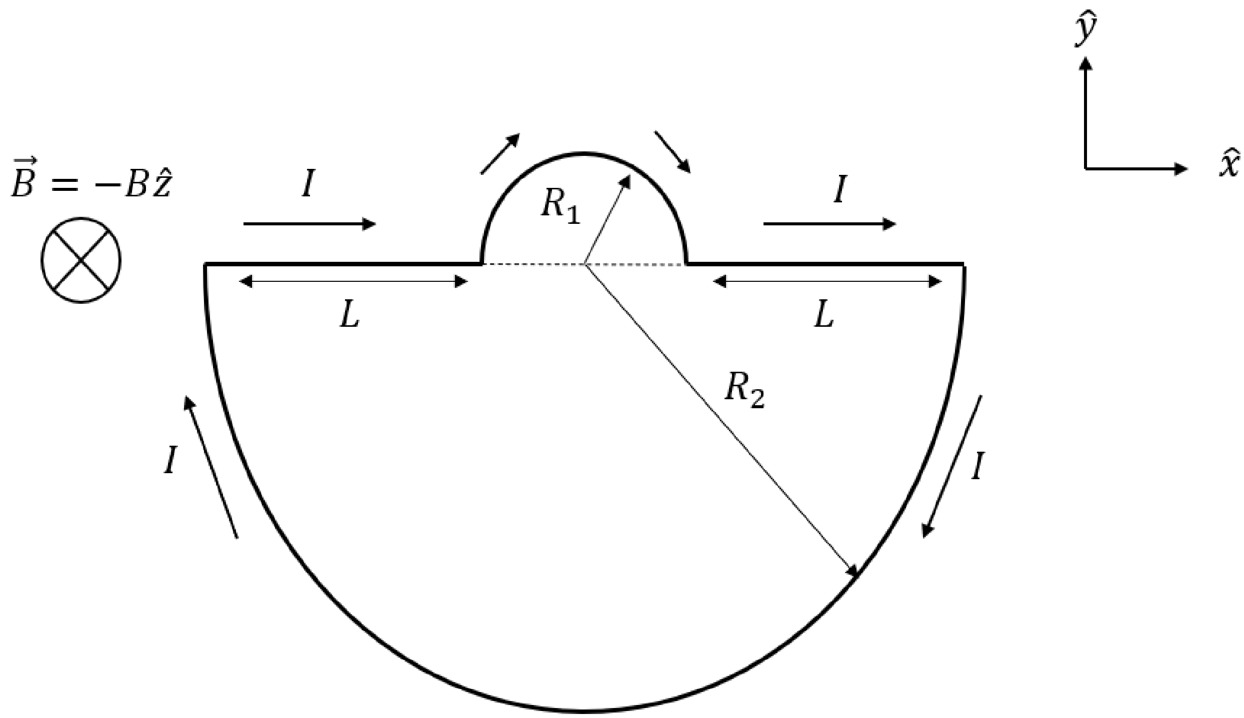

Fig. 1

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a) For any closed current loop: $\boxed{\Sigma F = 0}$

b) $\vec{\mu} = I\vec{A} = IA\hat{n}$

$$\boxed{\vec{\mu} = I\frac{\pi}{2}(R_1^2 + R_2^2)\,\hat{n}}$$

RHR $\rightarrow$ $\hat{n} = -\hat{z}$

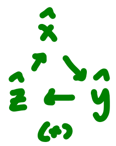
c) $\vec{\tau} = \vec{\mu} \times \vec{B}$

$$= I\frac{\pi}{2}(R_1^2 + R_2^2)(-\hat{z}) \times B(-\hat{z})$$

$\hat{z} \times \hat{z} = 0 \quad \therefore \boxed{\vec{\tau} = 0}$

d) $\vec{\tau} = \vec{\mu} \times \vec{B}$

$$= I\frac{\pi}{2}(R_1^2 + R_2^2)(-\hat{z}) \times B\hat{y} - \cancelto{0}{B\hat{z}}$$



$$-\hat{z} \times \hat{y} = -(-\hat{x}) = \hat{x}$$

$$\vec{\tau} = I\frac{\pi}{2}(R_1^2 + R_2^2)\,B\,\hat{x}$$

$\boxed{\text{The torque vector will point in the } \hat{x} \text{ direction}}$

2. (Analyzing magnet). Highly charged argon ions with charge states $^{40}Ar^{8+}$, $^{40}Ar^{9+}$, and $^{40}Ar^{10+}$ are created inside an ion source and accelerated through a potential difference, $V = 2.5$ kV inside an ultrahigh vacuum chamber. The ions enter an electric field-free region which is 5 meters long. They then enter a region of uniform magnetic field. Inside this magnetic field region, the vacuum chamber contains a 90-degree bend with radius, $R = 10$ cm. After leaving the magnetic field region, they continue to an ion capture trap 3 meters away. (See Fig. 2 below)
    a. Calculate the kinetic energy of each of the three charge states after being accelerated out of the ion source.
    b. Calculate the time-of-flight for the three charge states to leave the source and enter the magnetic field region 5 meters away.
    c. Calculate the magnetic field required to allow each charge state to pass through the 90-degree bend.
    d. Calculate the total time-of-flight for the ions to travel from the source to the final capture trap.

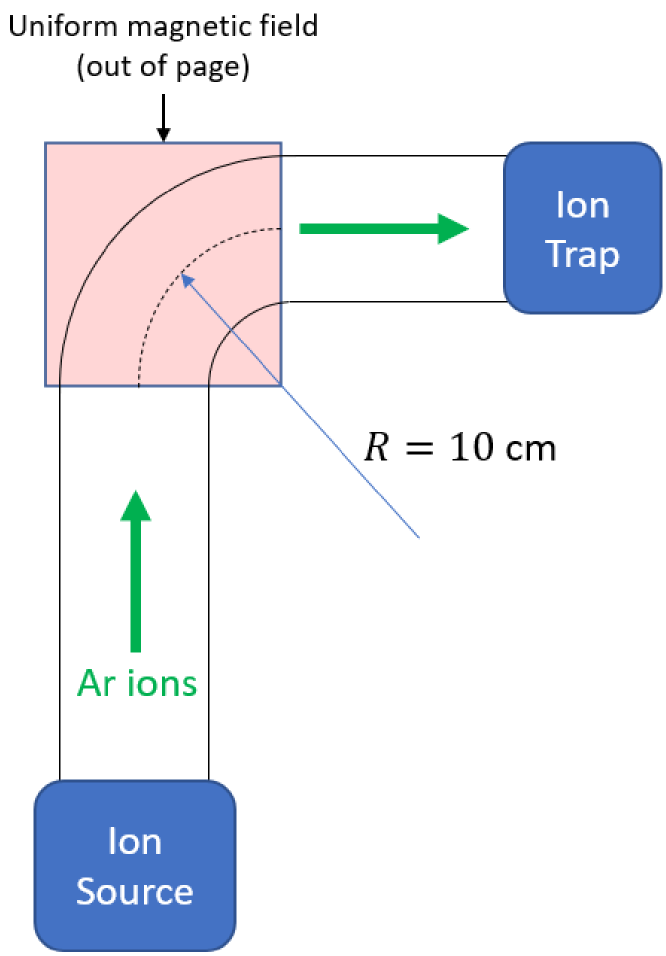


Fig. 2

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a) PE = KE

$qV = KE$

$V = 2.5\ kV$

$^{40}Ar^{+8}$ — $q = 8e$ — $KE = \boxed{20\ keV}$

$^{40}Ar^{+9}$ — $q = 9e$ — $KE = \boxed{22.5\ keV}$

$^{40}Ar^{+10}$ — $q = 10e$ — $KE = \boxed{25\ keV}$

b) $KE = \frac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2KE}{m}}$

$vt = \Delta x \rightarrow t = \frac{\Delta x}{v} = \Delta x \sqrt{\frac{m}{2KE}}$

Cool unit trick! $1\,Da = 931.4941\ MeV/c^2$

mass of 1 proton

$c = 299{,}792{,}458\ m/s$

$^{40}Ar^{+8}$ — $5m\left(\frac{40 \times 931.4941\ MeV/c^2}{2 \times 0.020\ MeV}\right)^{1/2} = 4825.697\ \frac{m}{c}\ \ (\div c \rightarrow \frac{m}{s})$

$t = \boxed{16.09\ \mu s}$

meters traveled by light in same amount of time!

. . .

$^{40}Ar^{+9}$ — $t = \boxed{15.17\ \mu s}$

$^{40}Ar^{+10}$ — $t = \boxed{14.39\ \mu s}$

c) $v = \sqrt{\frac{2qV}{m}}$ $\quad \vec{F}_B = q\vec{v} \times \vec{B}$ $\quad$ RHR + diagram + $F_T = m\frac{v^2}{r}$

$\longrightarrow m\frac{v^2}{r} = qvB \longrightarrow B = \frac{mv}{qr} = \frac{m}{qr}\sqrt{\frac{2qV}{m}}$

$= \frac{1}{qr}\sqrt{2mqV}$

variable, meaning the ions each require diff. B-fields.

$^{40}Ar^{+8}$ — $B = 160\ mT$

$^{40}Ar^{+9}$ — $B = 151\ mT$

$^{40}Ar^{+10}$ — $B = 143\ mT$

d) $\Delta x = 5m + 3m + \frac{2\pi(0.10m)}{4}$

$= 8.157\ m$

$t_{total} = \frac{t}{5\cancel{m}}(8.157\cancel{m})$ μs

$^{40}Ar^{+8}$ — $26.24\ \mu s$

$^{40}Ar^{+9}$ — $24.74\ \mu s$

$^{40}Ar^{+10}$ — $23.47\ \mu s$

3. A spatially limited magnetic field is oriented out of the page. A massless rigid wire of length L and width W, which carries a counterclockwise current, is partially immersed in the magnetic field as shown. Attached to the loop is a vertically hanging mass, m (see Fig. 3 below). Find the expression for the current necessary for this system to be in translational equilibrium.

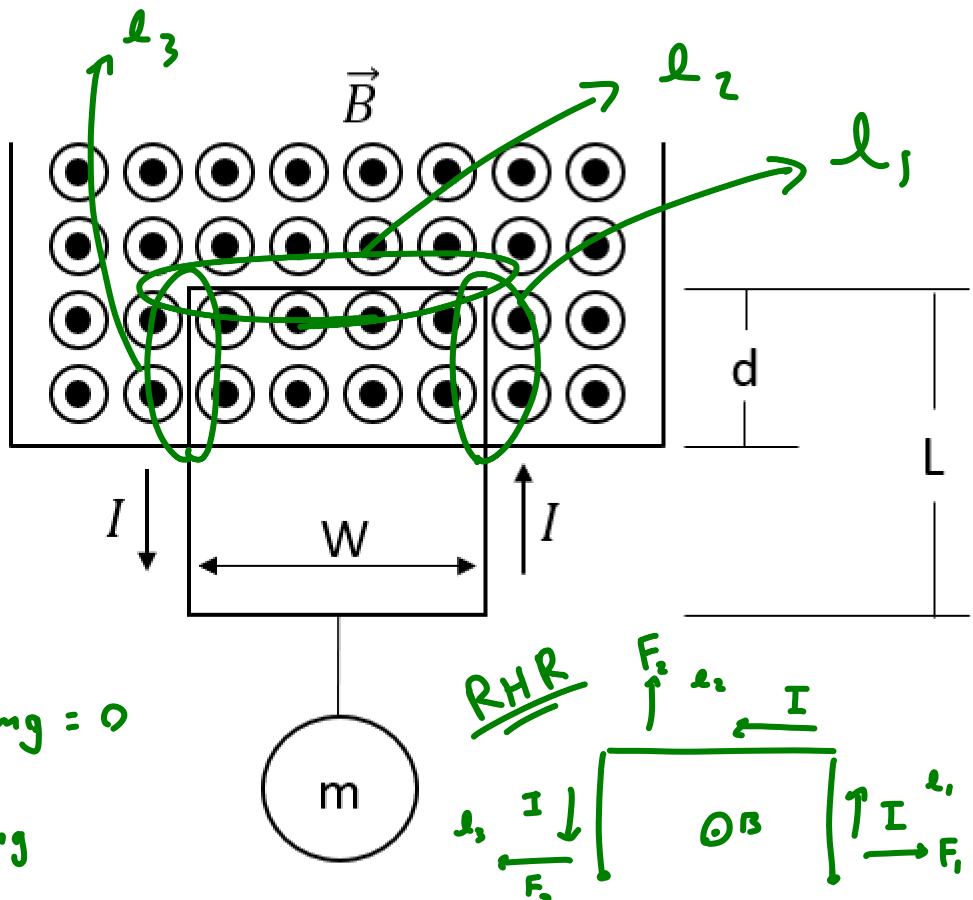

Fig. 3

$$\Sigma F_y = F_{B_y} - mg = 0$$

$$0 = F_2 - mg$$

$$0 = IWB - mg$$

$$mg = IWB$$

$$\boxed{I = \frac{mg}{WB}}$$

Since $\ell_3 = \ell_1$, $F_3 = F_1$

cancels!

$F_2 = IWB$

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4. Two long, straight wires carrying constant current, I are located along the x-axis at positions $-x$ and $+x$ oriented along the z direction (into the page, see Fig. 4 below).
    a. Calculate the total magnetic field (magnitude and direction) at P_1. → $\boxed{\vec{B}=\vec{0}}$
    b. Calculate the total magnetic field (magnitude and direction) at P_2.
    c. Calculate the total magnetic field (magnitude and direction) at P_3.

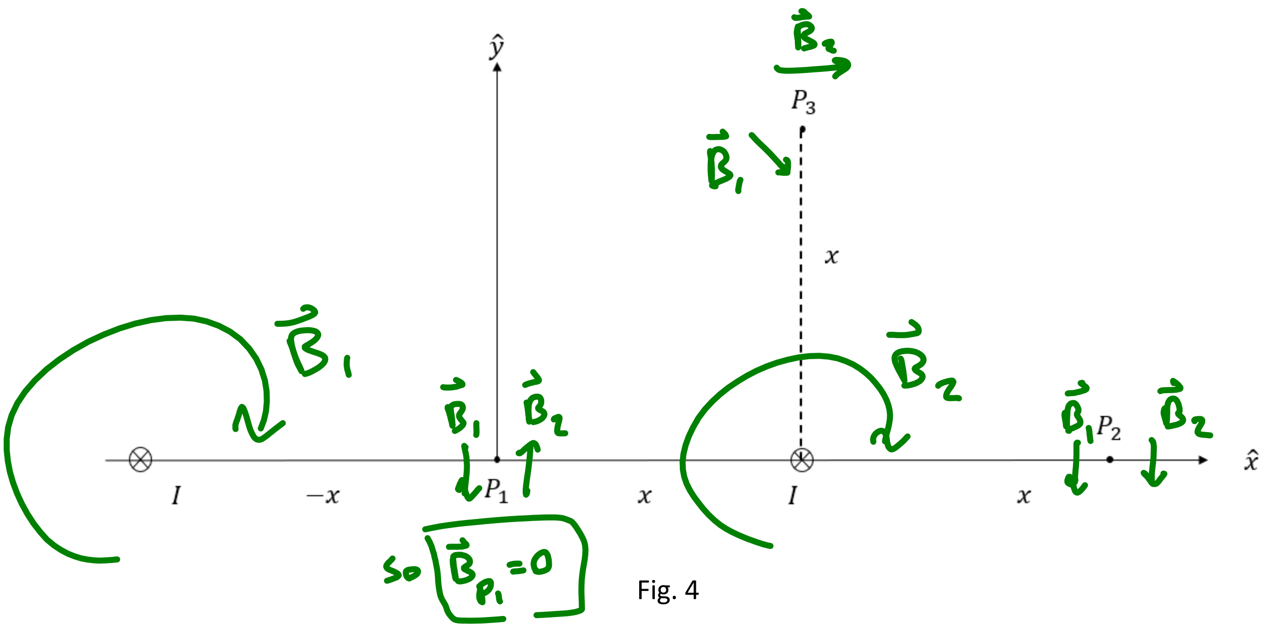

Fig. 4

5. Two loops of current with radius R are separated a distance R along the z-axis. Both loops carry constant current I (see Fig. 5 below). Develop an expression for the total magnetic field and field gradient (slope) along the z-axis for the two configurations shown in the figure below ($\vec{B}(z)$). Take $z = 0$ to be the midpoint between the two coils. NOTE: (a) indicates where the currents are in the same direction and (b) is where the currents are in the opposite direction.

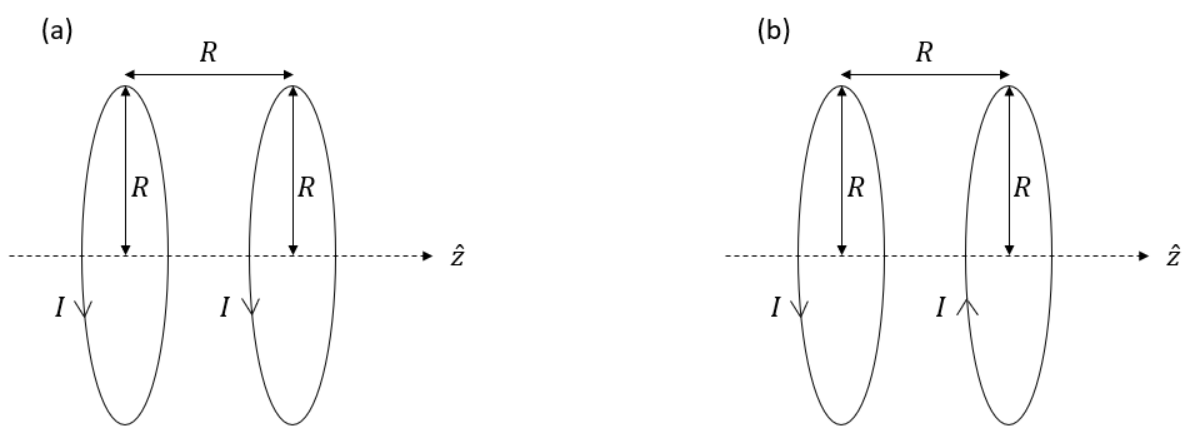

Fig. 5

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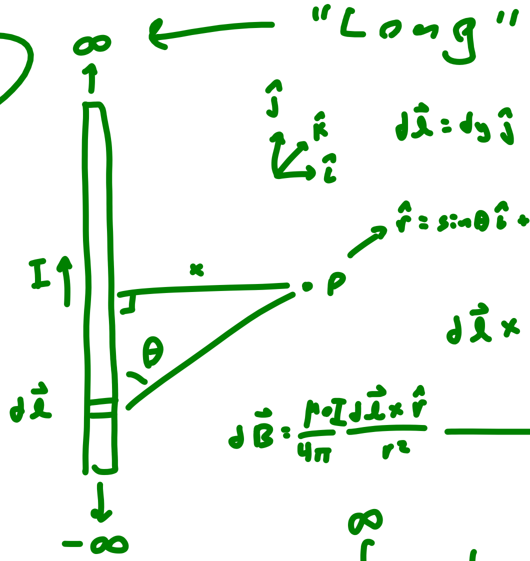


$d\vec{l} = dy\,\hat{j}$

$\sin\theta = \frac{x}{r}$

$\hat{r} = \sin\theta\,\hat{i} + \cos\theta\,\hat{j}$

$$d\vec{l} \times \hat{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & dy & 0 \\ \sin\theta & \cos\theta & 0 \end{vmatrix} = 0 + 0 + (-dy\sin\theta)\hat{k}$$

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l}\times\hat{r}}{r^2} \longrightarrow d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{(-dy\sin\theta)\hat{k}}{r^2} = -\frac{\mu_0 I}{4\pi}\frac{x\,dy}{(\sqrt{x^2+y^2})^3}\hat{k}$$

$$\vec{B} = -\frac{\mu_0 I}{4\pi}\int_{-\infty}^{\infty}\frac{x\,dy}{(\sqrt{x^2+y^2})^3}\hat{k} = -\frac{\mu_0 I}{4\pi}\left[\frac{y}{x^2\sqrt{x^2+y^2}}\right]_{-\infty}^{\infty} x\hat{k} \Rightarrow B = \frac{\mu_0 I}{2\pi x}$$

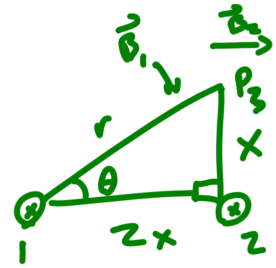
4a

$$\boxed{\vec{B}_{P_1} = \vec{0}}$$

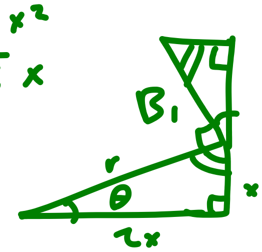
4b

$$\boxed{\vec{B}_{P_2} = -\hat{y}\left(\frac{\mu_0 I}{2\pi x} + \frac{\mu_0 I}{2\pi\cdot 3x}\right)}$$

4c



$r = \sqrt{x^2+4x^2} = \sqrt{5x^2} = \sqrt{5}\,x$



$|\vec{B}_1| = \frac{\mu_0 I}{2\pi}\frac{1}{\sqrt{5}\,x}$

$B_{1x} = B_1\frac{2x}{r}$

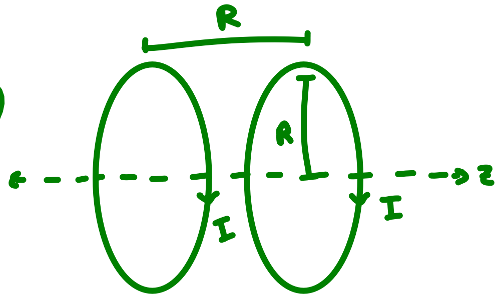
$B_{1y} = B_1\frac{x}{r}$

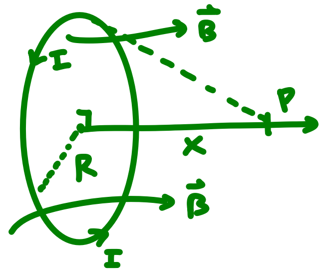
$x = r\cos\theta$

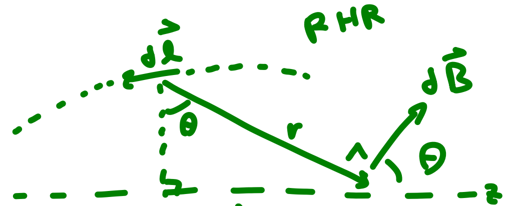
$x = B_1\frac{2x}{r}$

$$\boxed{\vec{B}_{P3} = \frac{\mu_0 I}{2\pi x}\hat{x} + \frac{\mu_0 I}{5\pi x}\hat{x} - \frac{\mu_0 I}{10\pi x}\hat{y}}$$

5

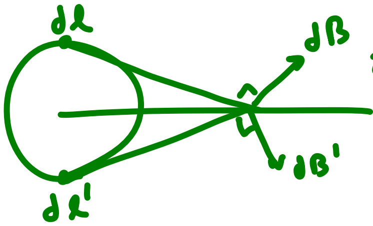






$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l}\times\hat{r}}{r^2}$$

Since $d l \perp \hat{r} \longrightarrow dB = \frac{\mu_0 I}{4\pi}\frac{dl}{R^2+x^2}$



y-comp cancel.

z-comp.

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{dl}{R^2+z^2}\cos\theta\,\hat{z}$$

renamed var

$$\cos\theta = \frac{R}{\sqrt{R^2+z^2}}$$

$$= \frac{\mu_0 I}{4\pi}\int\frac{R\,dl}{(R^2+z^2)^{3/2}}\hat{z}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi}\frac{R}{(R^2+z^2)^{3/2}}\underbrace{\int dl}_{2\pi R}\hat{z}$$

$$\vec{B} = \frac{\mu_0 I}{2}\frac{R^2}{(R^2+z^2)^{3/2}}\hat{z}$$

a

$$\boxed{\vec{B}(z) = \frac{\mu_0 I}{2}R^2\left(\frac{\hat{z}}{(R^2+(z\pm\frac{1}{2}R)^2)^{3/2}}\right)}$$

Both loops $\pm\frac{1}{2}R$ from $z=0$

$$\boxed{\frac{\partial\vec{B}}{\partial z} = \frac{\mu_0 I}{2}R^2\left(-3(R^2+2(z\pm\tfrac{1}{2}R))^{-5/2}\right)\hat{z}}$$

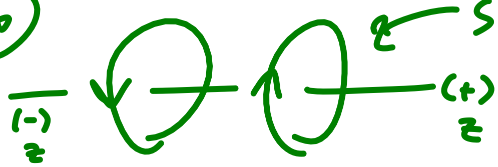
Scratch work

$$\frac{d}{dx}\left[(a+(x+b)^2)^{-3/2}\right]$$

$$\cancel{2}\cdot -\frac{3}{\cancel{2}}(a+2(x+b))^{-5/2}$$

$$-3(a+2(x+b))^{-5/2}$$

b



Swap $+\frac{1}{2}R$ term sign

$$\boxed{\vec{B}(z) = \frac{\mu_0 I}{2}R^2\left(\frac{1}{(R^2+(z-\frac{1}{2}R)^2)^{3/2}} - \frac{1}{(R^2+(z+\frac{1}{2}R)^2)^{3/2}}\right)\hat{z}}$$

$$\boxed{\frac{\partial\vec{B}}{\partial z} = \frac{\mu_0 I}{2}R^2\left(-3(R^2+2(z-\tfrac{1}{2}R))^{-5/2} + 3(R^2+2(z+\tfrac{1}{2}R))^{-5/2}\right)\hat{z}}$$