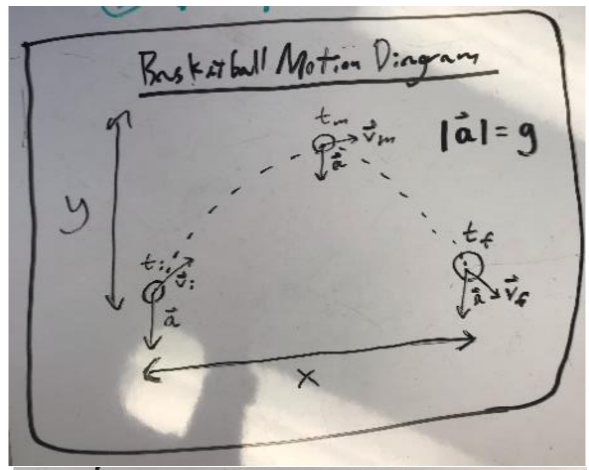
2D Motion Lab

Jonah Spector, Alec Gray, Duncan Bogdanovic

Basketball Experiment

By simply watching the video over a few times, and given the height the ball was thrown at ($y_i = 1$m), our estimations are:



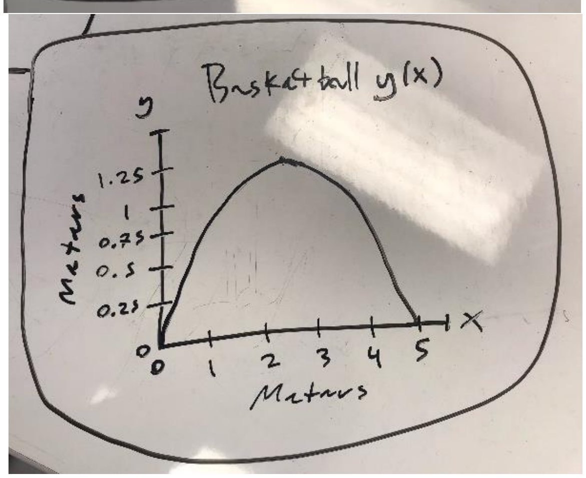
- Horizontal displacement (Δx): ~5m
- Vertical distance (Δy): ~1.25m
- Airtime (Δt): ~1s

Distance estimations were gathered by measuring based on a constant (meter stick). Time was estimated by looking at the video time bar, which was limited to 1-second precision.

From these, we sketched out our estimations of the graphs: $y(x), x(t), y(t), v_x(t), v_y(t)$

Estimation Graphs, Data and Analysis

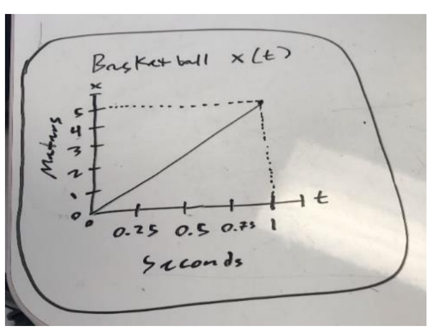
Note that all tracking graphs include data after the ball descends back to y_i. This is very evident in $y(t)$ since the origin was set at y_i in the tracking software.

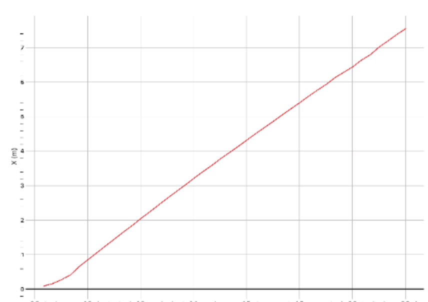


$y(x)$: Our estimation for the graph follows an arc, which comes from tracing the movement of the ball in the video. The arc here demonstrates how vertical velocity is greatly affected by gravity, whereas little horizontal velocity is lost to air resistance.

$y(x)$ (actual): We did not manage to get this graph from the iPad.

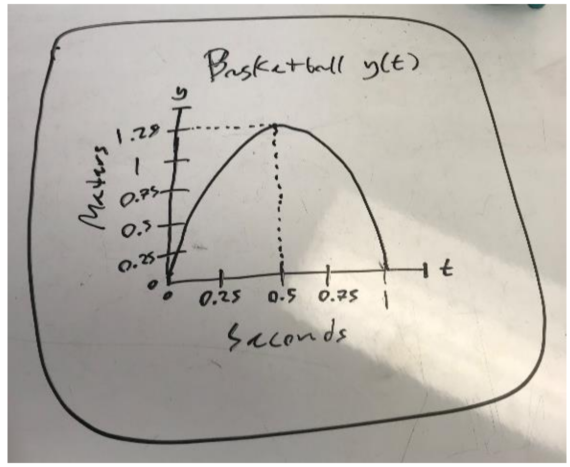
$x(t)$: Our graph follows a positive, linear equation. Given that the ball should have little total velocity lost to drag, we predicted that, given its constant speed, the ball would move a set, consistent distance over its flight.



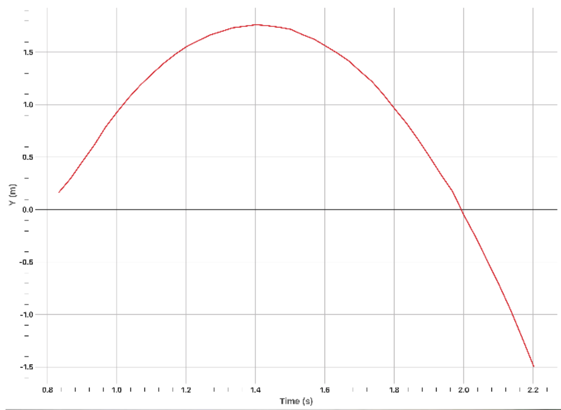


$x(t)$ (actual): Our prediction yet again aligns almost perfectly with the experimental data, despite the anomaly at the start, during the first few frames, where the ball was tracked while being thrown.

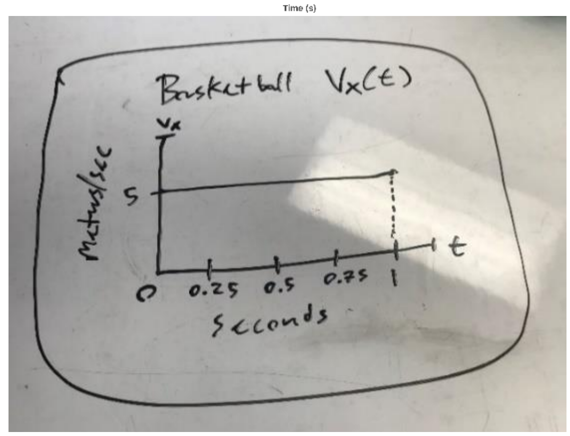
$y(t)$: Our prediction also took on the form of a parabolic arc. The impact of gravity on the velocity of the ball causes it to gradually slow down linearly, causing a parabolic position graph.



$y(t)$ (actual): Despite messing up our timescale, the overall shape of the graph is hardly impacted. However, the graph has more tracking past the desired window, extending the graph beyond the point where the ball's height returns to y_i, where the basketball was thrown from.



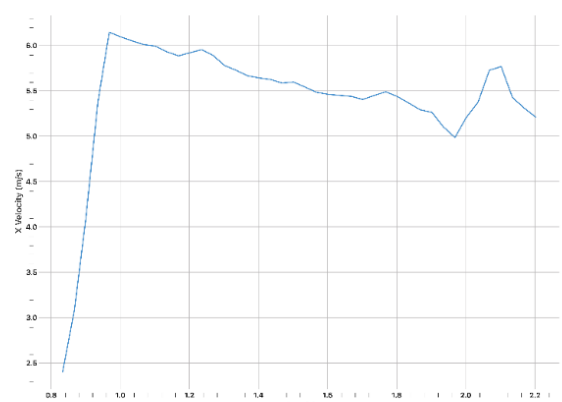
$v_x(t)$: Our estimate for horizontal velocity ends up being a constant, which is inaccurate to the real world, as this would imply that no horizontal velocity is lost to air resistance. The equation $\bar{v} = \frac{\Delta x}{\Delta t}$, with the values $\Delta x = 5$m and $\Delta t = 1$s, led us to find a constant velocity of 5m/s.



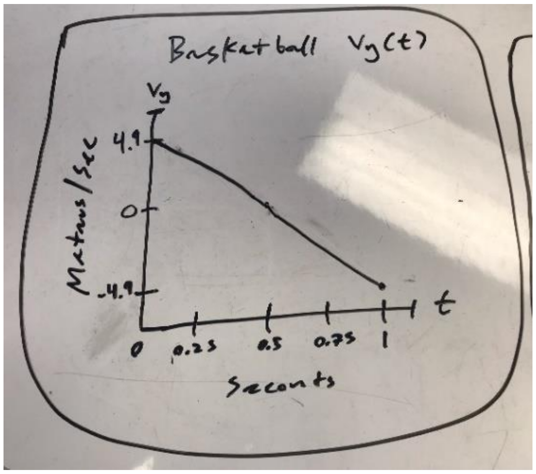
$v_x(t)$ (actual): Our graph, as mentioned above, fails to account for air resistance, but it is likely that the roughness of the tracked data is due to a combination of air resistance and wind, although the video may have been shaky too. Note that this footage did not include a bounce.

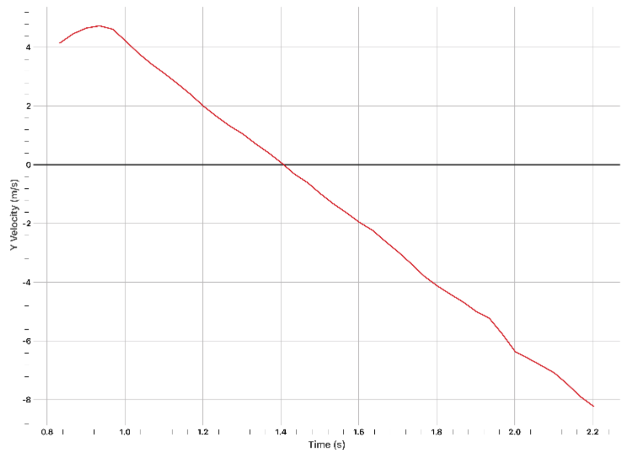
$v_y(t)$: A negative linear slope indicates that the ball's velocity would decrease over time due to the acceleration of gravity. We set 0 as a center point since, at the peak of its arc, its velocity becomes negative. To estimate the starting velocity, given our estimated values and the known velocity at the top of the arc, we used the equation $v_f^2 = v_i^2 + 2a\Delta y$.

$$0 = v_i^2 + 2g\Delta y$$
$$v_i^2 = -2g\Delta y$$
$$v_i = \sqrt{-2g\Delta y}$$
$$\boxed{4.949\ m/s = \sqrt{-2(-9.8m/s^2)(1.25m)}}$$



$v_y(t)$ (actual): Our prediction was highly accurate for this model, barring a few moments such as the acceleration from the throw and what looks like some slight tracking problems towards the end of the graph.



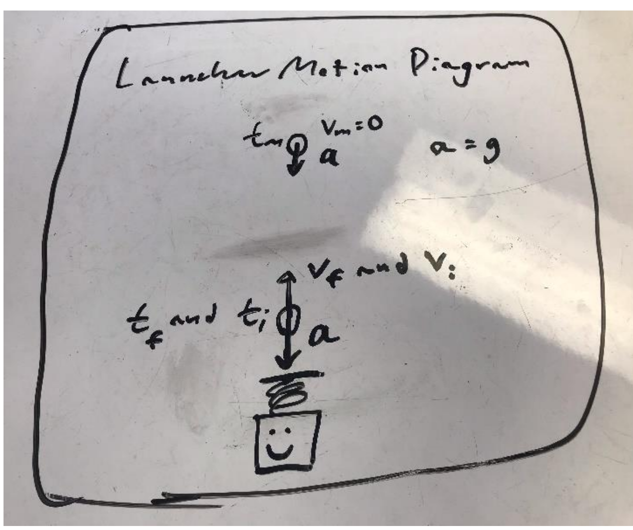


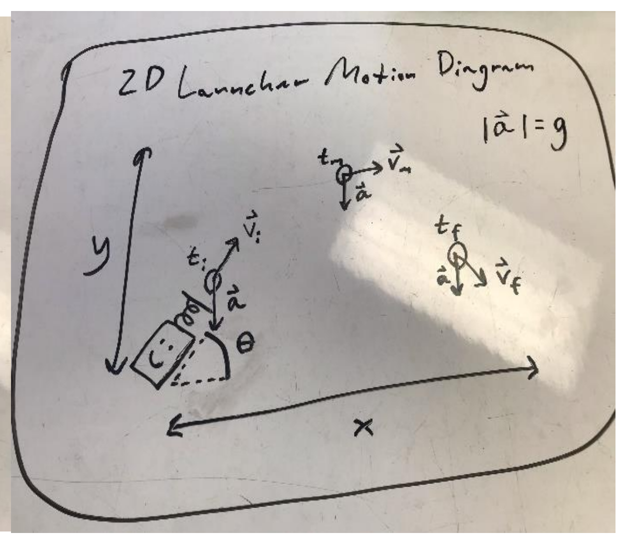
Conclusion

The basic forms (linear, parabolic) and whether certain values were negative or positive of our estimations and the actual data all matched perfectly, ignoring the slight impact of air resistance on $v_x(t)$. Our main issue was estimating the flight time at 1 second instead of the more reasonable 2 seconds as is seen in the actual tracked graphs. However, all the estimated values, including time, are within the same order of magnitude as their respective actual values. And, in some cases, they are only less than a few units off.

Launcher Experiment

Our first effort to solve the problem at hand was to draw a few motion diagrams to determine what our values of acceleration, position, and velocity should be labeled and where they might be over time.





We then tested the launcher vertically and measured the max height the ball reached with a set of meter sticks next to it. By subtracting the height of the launcher, 28cm, from the height the ball reached, ~1.5m, we found that the ball traveled 1.22m (y_m) vertically from the end of the launcher (were the acceleration of the launcher ends and free fall begins). Using this, the initial velocity of the ball can be calculated:

$$v_f^2 = v_i^2 + 2a\Delta y$$
$$v_i^2 = 2a\Delta y - v_f^2$$
$$v_i = \sqrt{2a\Delta y - v_f^2}$$

Since the velocity of the ball at the top of the vertical test is 0, as it is the max height before the velocity becomes negative, the equation becomes:

$$v_i = \sqrt{2g\Delta y}$$

Plugging values in, we get the initial velocity of a ball launched by the launcher on the second setting:

$$v_i = \sqrt{2(9.8m/s^2)(1.22m)}$$
$$\boxed{v_i = 4.889m/s}$$

The x and y components of the velocity vector when the launcher is rotated, then, are given by $v_x = v_i \cos\theta$ and $v_y = v_i \sin\theta$. (The x/y axis is fixed relative to gravity, not the launcher.) Since θ is variable, the equation for Δx, the distance from the launcher to the cup, should rely only on the known v_i and given θ. The distance between the launcher and cup, ignoring air resistance and other factors, is given by:

$$\Delta x = v_{xi}\Delta t$$
$$\Delta x = v_i \cos\theta\, \Delta t$$

So, Δt must be solved for in known terms using the equation $v_f = v_i + a\Delta t$ and the vertical velocity. Since the peak of the trajectory occurs halfway through its flight (if it lands at the same initial height, which in this case is the end of the launcher's tube), and the vertical velocity there is 0:

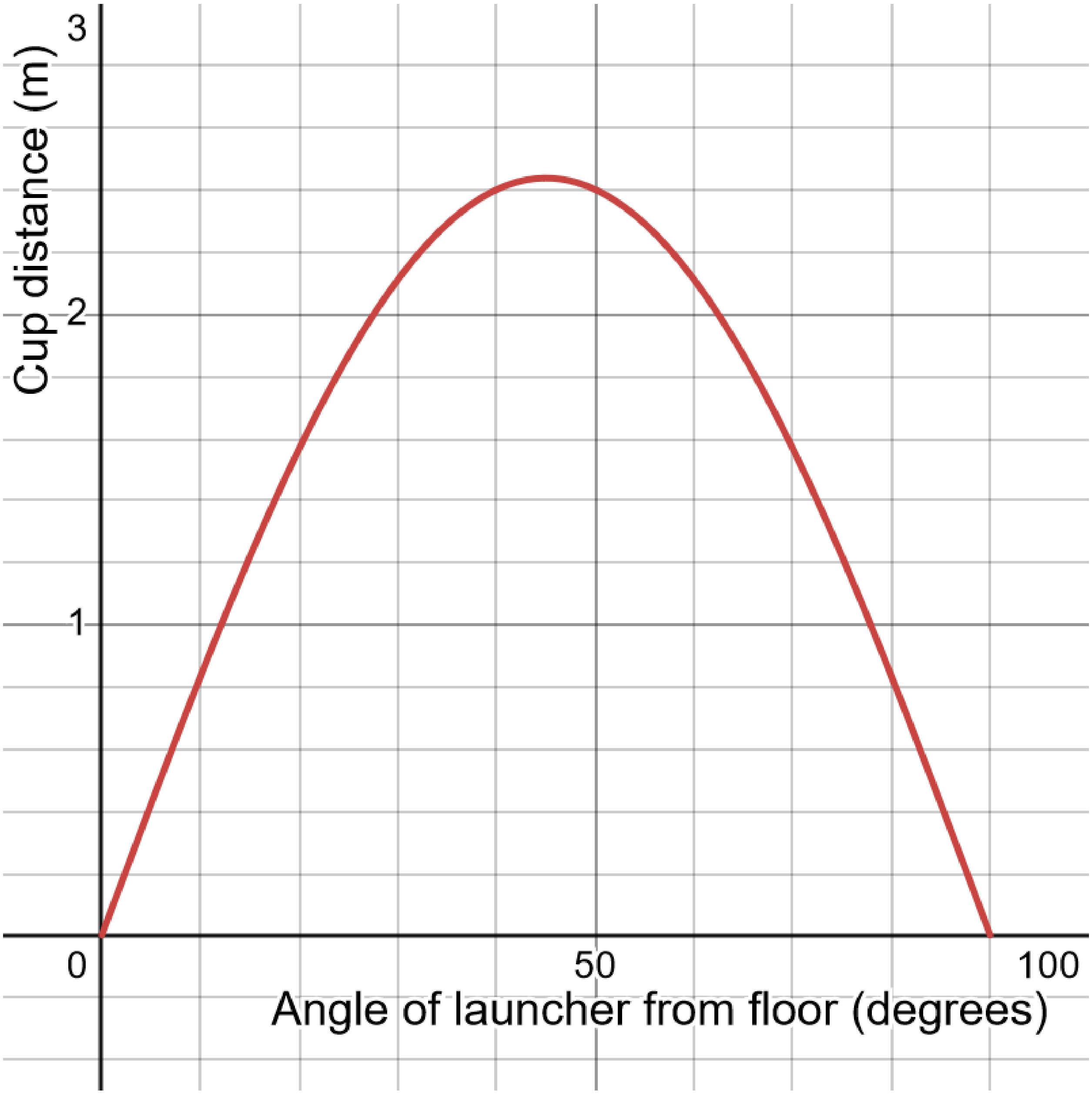
$$0 = v_i \sin\theta - g\frac{\Delta t}{2}$$

$$\frac{\Delta t}{2} = \frac{v_i \sin\theta}{g}$$

Combining the equations, we can get Δx only in terms of the known v_i and given θ:

$$\boxed{\Delta x = \frac{2v_i^2 \sin\theta \cos\theta}{g}}$$

We did not have time to test if this equation is accurate for placing the cup, but we can plot the estimated Δx against all reasonable θ's (in degrees):



Conclusion

We sadly did not get the chance to test our equation, but, theoretically, it would give us the correct distance to put the cup from the launcher to catch the ball. Our measurement of the test launch's max height may also have been off, since the rules were held by a person and the max height itself was measured without any tools beyond human sight.