

PH 142 Spring Semester 2025: Homework 4

Due Wednesday, February 26 2025, at 11:59 pm

Instructions: There are 4 long problems for this assignment. Please upload your solutions to Canvas when completed. 10 points will be given for completing all problems. One problem will be chosen randomly and graded in detail, out of 10 points. The sum of these scores will be the total grade, out of 20 points. Partial credit will be given. Please show all work.

1. A coaxial cable consists of two cylindrical conductors separated by an insulating material, or vacuum. Consider an infinitely long cable made of a metal inner cylinder with radius  $r_a$  and a metal outer cylinder with radius  $r_b$ . (see Fig. 1 below) Here, we will ignore the thickness of the conductors. The inner conductor carries positive charge per unit length,  $+\lambda$ , and the outer conductor carries negative charge per unit length,  $-\lambda$ . Calculate the potential at the following locations: (set  $V = 0$  at  $r = r_b$ )
  - a.  $r < r_a$
  - b.  $r_a < r < r_b$
  - c.  $r > r_b$
  - d. Give an expression for the potential difference between the inner and outer cylinders.
  - e. Use your results from above and the gradient of the potential to find the electric field,  $\vec{E}(r)$ , between the two conductors.
  - f. In the case where  $r_a = 5$  mm and  $r_b = 5.5$  mm, calculate the magnitude of the electric field at a distance halfway between the inner and outer conductors. Express your answer in terms of the voltage difference between the conductors,  $V_{ab}$ . How does this result compare to the answer we would get if we assumed that the two conductors make up a parallel plate capacitor separated by a distance  $d = 0.5$  mm, with voltage difference  $V_{ab}$ .

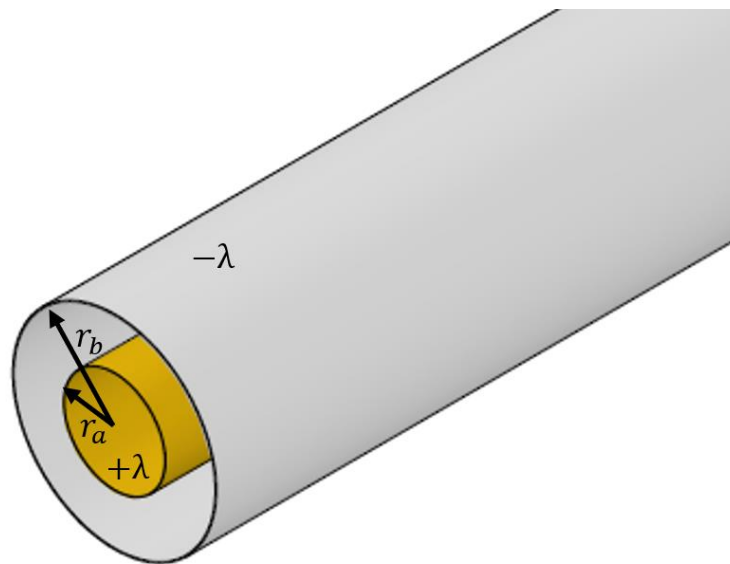


Fig. 1

2. A network of capacitors with various capacitance values is shown in Fig. 2 below.
- Calculate the equivalent capacitance of this network.
  - A potential difference,  $V_{ab} = 10 \text{ V}$ , is applied between the points  $a$  and  $b$  in the network. Calculate the potential difference ("voltage drop") across each capacitor in the network.

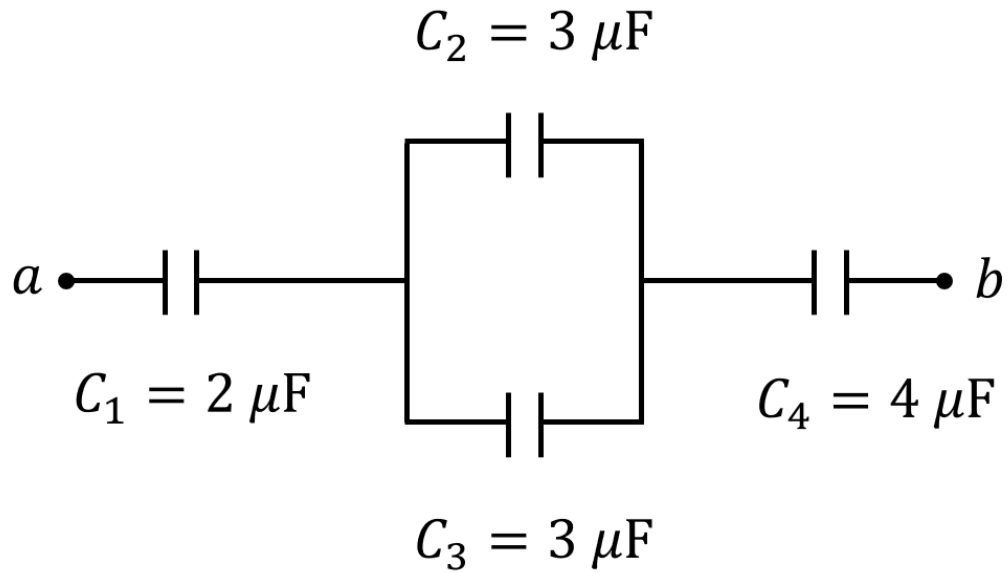


Fig. 2

3. A parallel plate capacitor has a separation distance of  $d = 2.5 \text{ mm}$  and stores  $10 \text{ J}$  of energy.
- If the separation distance is decreased to  $1 \text{ mm}$ , what is the energy stored if the capacitor is disconnected from the voltage source used to charge it?
  - If the separation distance is decreased to  $1 \text{ mm}$ , what is the energy stored if the capacitor stays connected to the voltage source used to charge it?

4. (Electrostatic deflector). Highly charged argon ions with charge states  $^{40}\text{Ar}^{8+}$ ,  $^{40}\text{Ar}^{9+}$ , and  $^{40}\text{Ar}^{10+}$  are created inside an ion source and accelerated through a potential difference,  $U = 2.5$  kV inside an ultrahigh vacuum chamber. The ions enter an electric field-free region which is 5 meters long and travel through an electrostatic deflector towards an ion trap (recall problem from last week). One way to prevent the ions from entering the trap is to insert an electrostatic deflector (see Fig. 3 below). The deflector consists of two parallel plates of length  $L = 10$  cm, separated by a distance  $d = 2$  cm. The plates of the deflector can be biased to generate a potential difference between them which can be used to alter the trajectory of the ions as they travel through this region.
- Calculate the minimum potential difference between the plates,  $V$ , required to divert the  $^{40}\text{Ar}^{9+}$  ions from the center of the beamline to one of the plates.
  - Is this potential difference large enough to divert charge states  $^{40}\text{Ar}^{+}$  through  $^{40}\text{Ar}^{16+}$ ? If so, explain why. If not, calculate the minimum potential difference required to divert these charge states. NOTE: All charge states are initially accelerated through the same 2.5 kV potential.
  - Suppose that we are interested in only sending  $^{40}\text{Ar}^{9+}$  ions through the deflector to the ion trap. Assuming that the total pathlength from the ion source to the entrance of the deflector is 7.5 meters, and that all charge states leave the ion source at the same time, is there a way to adjust the potential  $V$  to allow only the  $^{40}\text{Ar}^{9+}$  ions through the deflector? (Consider what happens when  $V = 0$  V)
  - Is there a particular time-of-flight required to accomplish the task in part (d)? If so, what timing resolution is required to ensure that only  $^{40}\text{Ar}^{9+}$  ions make it to the trap and no other neighboring charge states can enter.

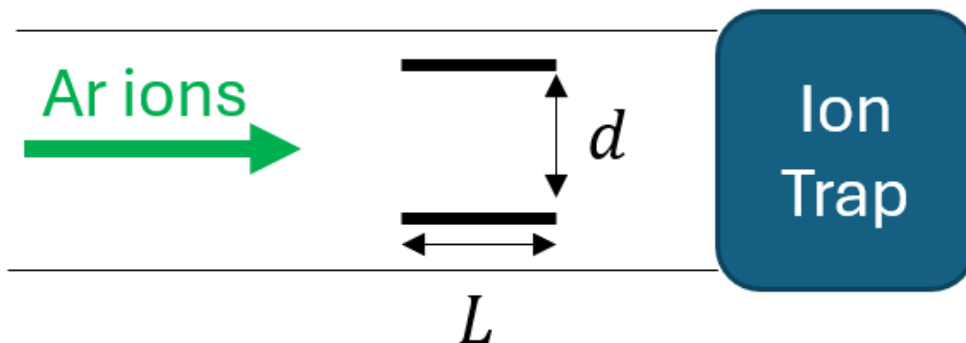


Fig. 3