Kernel: Python 3 (system-wide)

Introduction

Impulse is defined as a large force applied over a very short period, and can be used to interpret momentum, which can be further used to find properties relating to an object. Over this lab, we intend to apply the concept of impulse to predict momentum and velocity of objects in motion and determine the overall behavior of the system as a whole.

Problem 1: Happy and Sad Ball

Since the happy ball, undergoes the largest momentum change of the two balls, it's impulse, J_H, is higher than of the sad ball, J_S. This means the happy ball will have a magnitude of final momentum greater than the sad ball, since $p_f = p_i + J$. (Rightward velocity is positive and leftward velocity is negative.)

If both balls have the same drag or friction coefficients, than those forces, over a set distance, will cause the same amount of work, reducing the kinetic energy. More simply, because $W = F\Delta x$, assuming rolling friction on a flat surface is the source of the work reducing kinetic energy, $W_f = \mu_r mg\Delta x$ for *both balls*. This creates the full energy balance equations for both balls where they are rolled a distance Δx towards a wall to bounce off:

$$\frac{1}{2}m_H v_{fH}^2 = \frac{1}{2}m_H v_{iH}^2 - \mu_r m_H g\Delta x$$

$$\frac{1}{2}v_{fH}^2 = \frac{1}{2}v_{iH}^2 - \mu_r g\Delta x$$

and for the other ball,

$$\frac{1}{2}v_{fS}^2 = \frac{1}{2}v_{iS}^2 - \mu_r g\Delta x$$

rearranging, since Δx and μ_r are the same,

$$\frac{1}{2}v_{iH}^2 - \frac{1}{2}v_{fH}^2 = \frac{1}{2}v_{iS}^2 - \frac{1}{2}v_{fS}^2$$

$$v_{iH}^2 - v_{fH}^2 = v_{iS}^2 - v_{fS}^2$$

and if the initial velocities of both balls are the same, then, subtracting from both sides,

$$v_{fH}^2 = v_{fS}^2$$

$$v_{fH} = v_{fS} \equiv v_i$$

To determine which ball is the happy ball, we simply rolled both balls at the same velocity towards the wall using the above equation to rationalize that they each hit the wall at the same velocity and rebound with different impulses. We then measured the rebound distance (Δx, not to be confused with the previous variable used for solving the velocity equivalent above) before the balls stopped rolling. Using the impulse equation for both balls, where $v_f < 0$:

$$m_H v_i = m_H v_{fH} + J_H$$

$$m_H(v_i - v_{fH}) = J_H$$

and using the same simplification,

$$m_S(v_i - v_{fS}) = J_S$$

Since $J_H > J_S$, by definition:

$$m_H(v_i - v_{fH}) > m_S(v_i - v_{fS})$$

From here on it must be assumed that both balls have the same mass, otherwise it is impossible to determine which is the happy one.

$$mv_i - mv_{fH} > mv_i - mv_{fS}$$

$$-mv_{fH} > -mv_{fS}$$

$$v_{fH} < v_{fS}$$

but, since v_f is negative,

$$|v_{fH}| > |v_{fS}|$$

So the happy ball will rebound with a higher velocity than the sad one. Going back to the energy balance equation, this time solving for the rebound distance, Δx:

$$\frac{1}{2}v_f^2 = \frac{1}{2}v_i^2 - \mu_r g \Delta x$$

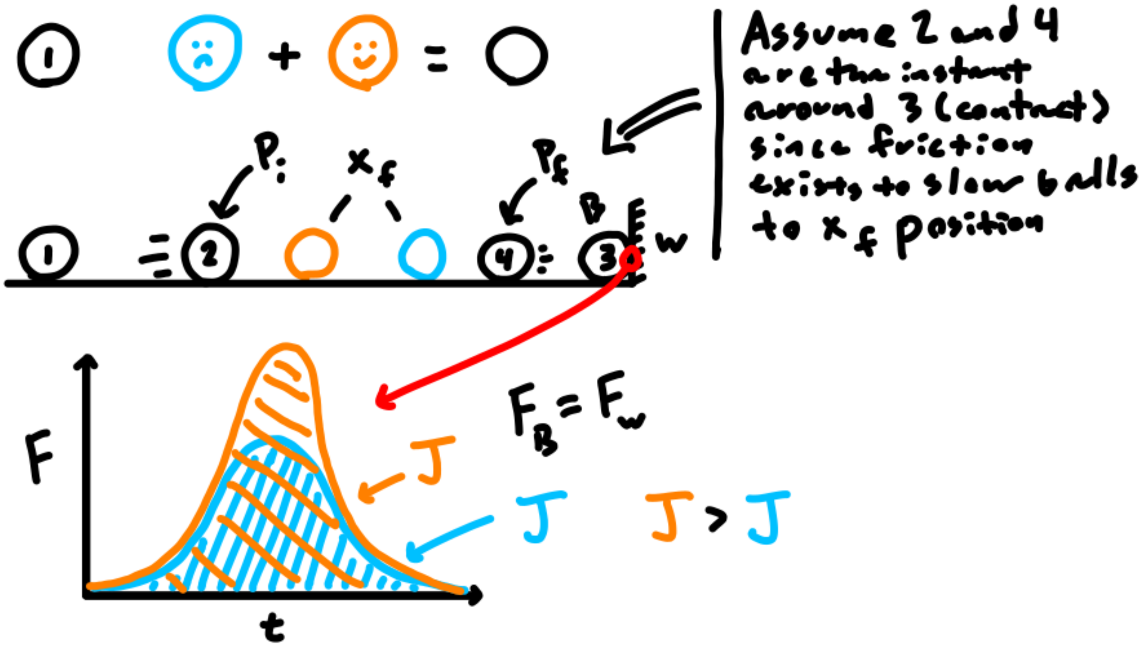
Since the balls roll to a stop, the final velocity is 0, and the initial velocity is the "final velocity" of the rebound already found in the inequality above.

$$0 = \frac{1}{2}v_f^2 - \mu_r g \Delta x$$

$$\frac{1}{2}v_f^2 = \mu_r g \Delta x$$

$$\frac{1}{2\mu_r g}v_f^2 = \Delta x$$

Since $v_f \propto \sqrt{\Delta x}$ and $|v_{fH}| > |v_{fS}|$, it can be reasoned that **the happy ball will have a bigger rebound distance.** Using this method, we determined which ball was the happy ball.



Since the impulse of the happy ball is greater, and momentum is conserved, the change in momentum that is the impulse is also applied to the block it hits when rolling down the ramp. If the happy ball is released at a set distance from the end of the ramp at a set angle, where it just barely knocks the block down, since the sad ball will have a smaller impulse, it will not be able to overcome the threshold of change in momentum required to knock the block over.

Testing this, with the blocks we determined were the happy and sad ones, the theory holds up.

Problem 2: Pushing Chairs Apart

The system is both people and both chairs all together, and it begins at a momentum of zero. Since momentum is conserved, any positive momentum in one object implies negative momentum in the other. The impulse generated by the push acts equally on both people and their chairs. Person A is combined with chair A and likewise for person B. Person A is the one who pushes against the other with the force plate.

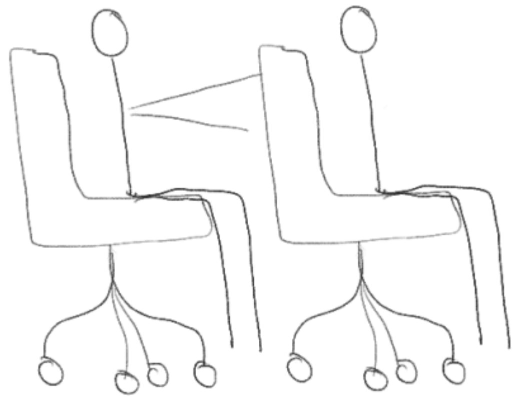
$0 = p_{iA} = p_{iB}$

$J = J_A = J_B$

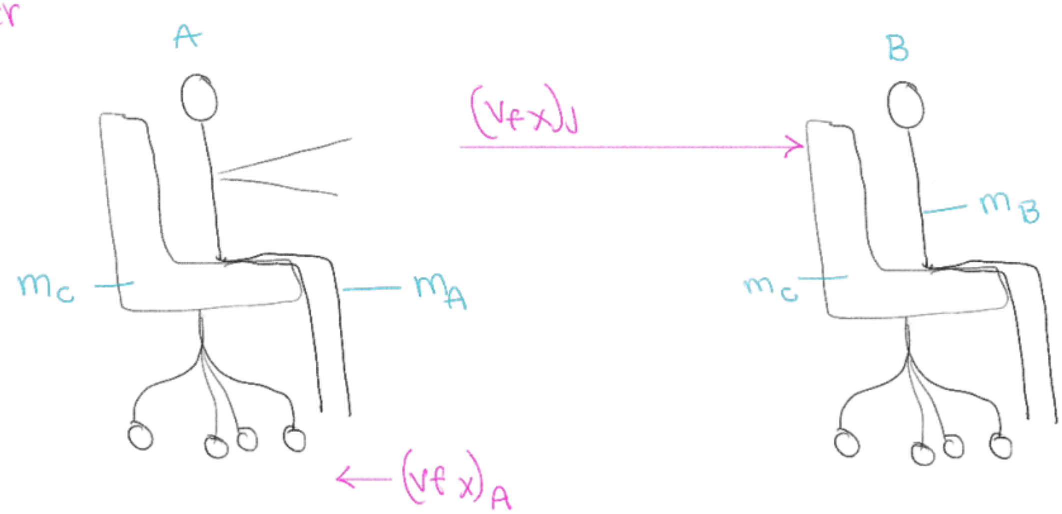
$p_f = p_i + J$

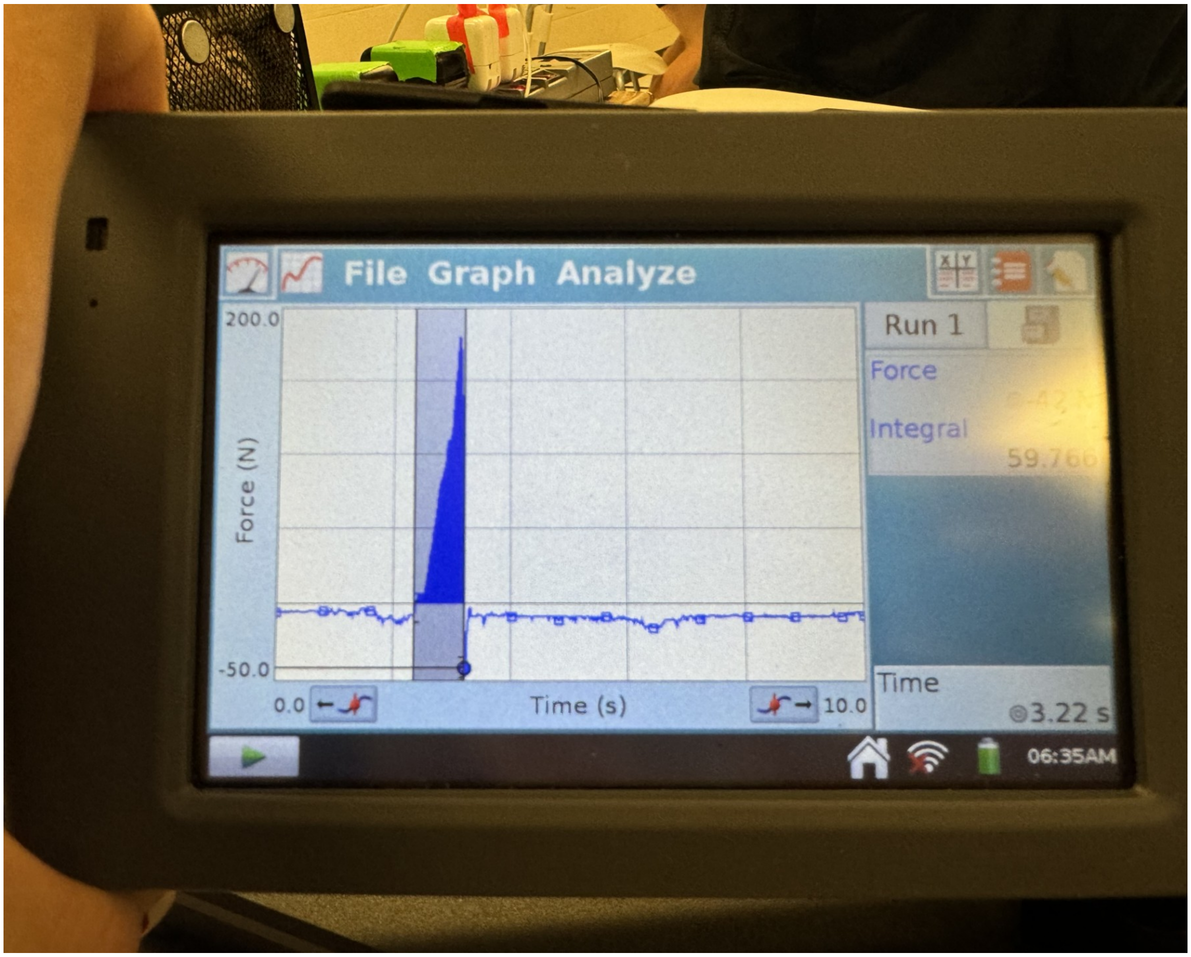
$p_A = J \quad p_B = J$

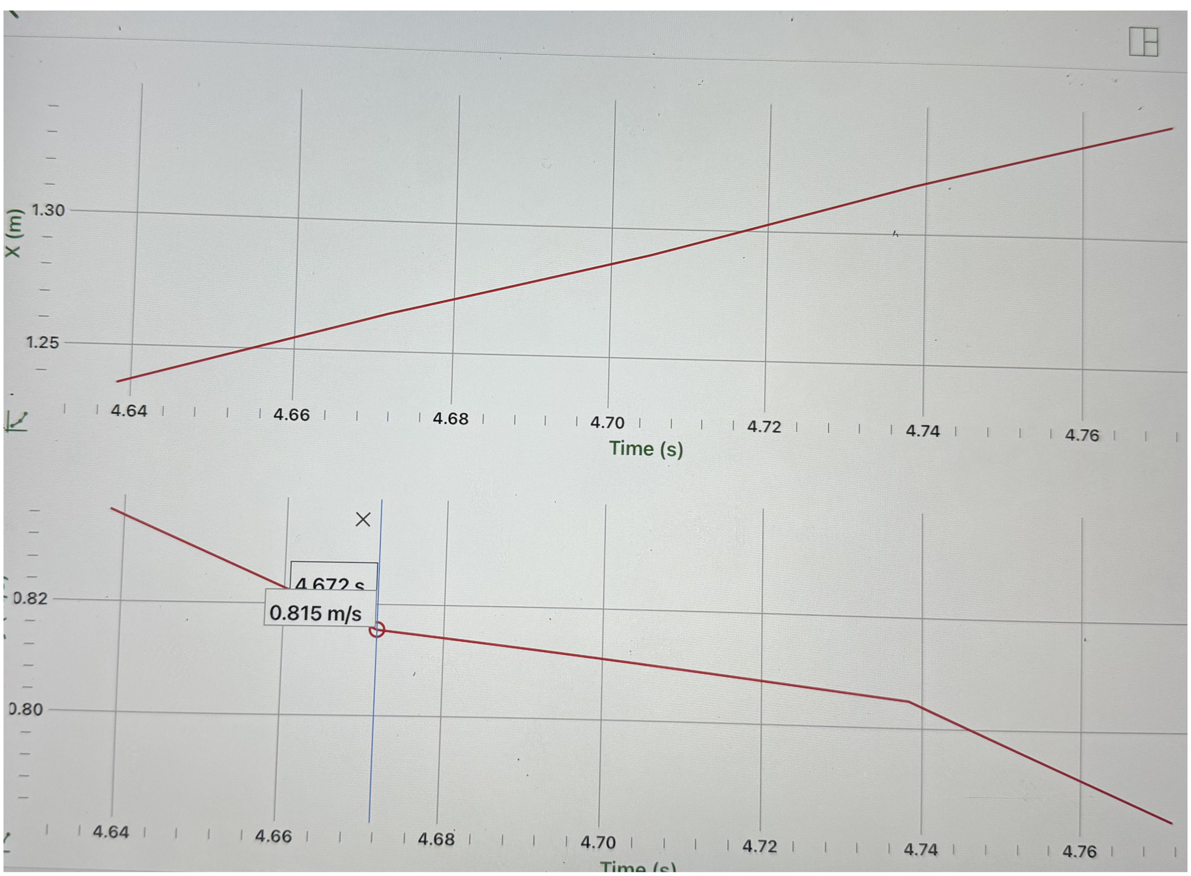
Before:



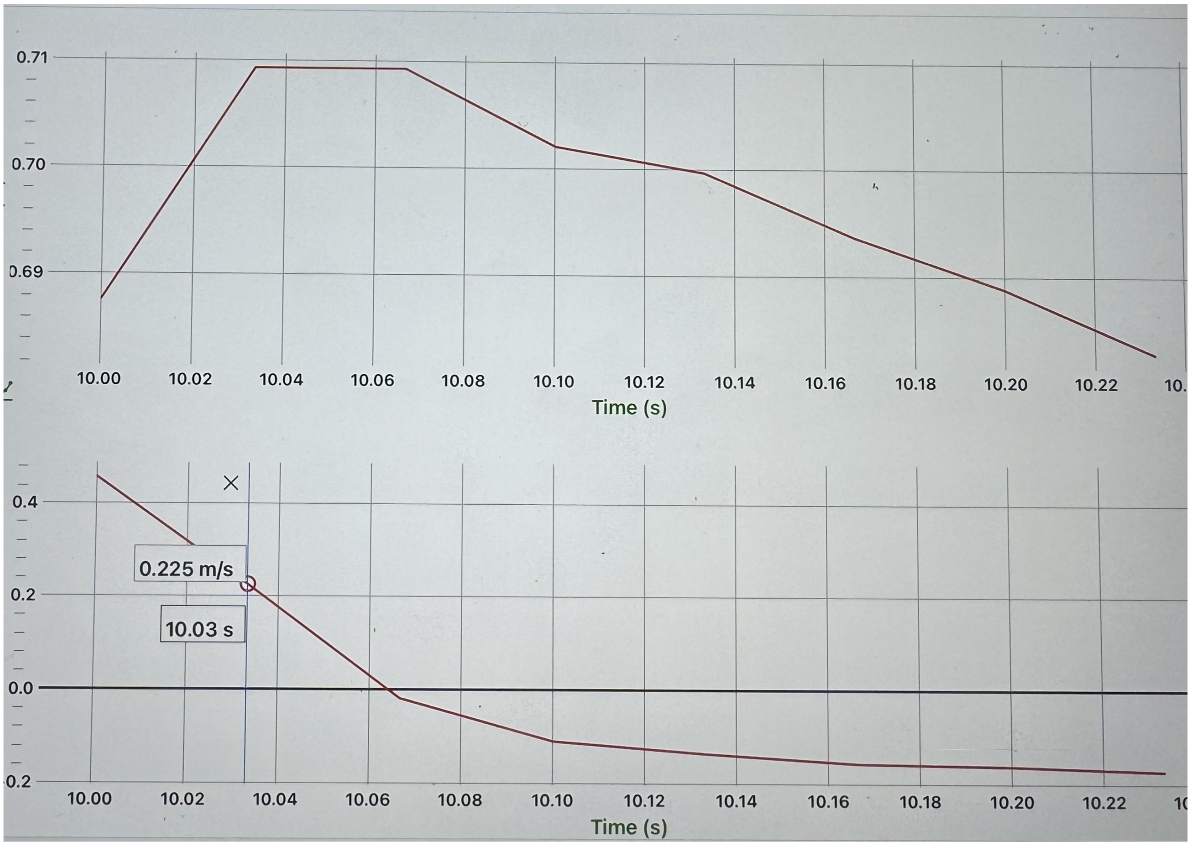
After




File Graph Analyze
200.0
Force (N)
-50.0
0.0
Time (s)
10.0
Run 1
Force
Integral
59.766
Time
3.22 s
06:35AM


X (m)
1.30
1.25
4.64
4.66
4.68
4.70
4.72
4.74
4.76
Time (s)
4.672 s
0.815 m/s
0.82
0.80
4.64
4.66
4.68
4.70
4.72
4.74
4.76

A Velocity



B Velocity

$$0 = p_{iA} = p_{iB}$$

$$J = J_A = J_B$$

$$p_f = p_i + J$$

$$p_A = J \qquad p_B = J$$

In [5]:

```
impulse              = 60 # Ns, measured from force plate integration of contact force over time

mass_a               = (168*4.55)/9.81# kg, person + chair
mass_b               = (145*4.55)/9.81 # kg, person + chair
mass_force_plate     = 4.535 # kg

est_final_vel_a      = -impulse/(mass_force_plate + mass_a) # m/s
est_final_vel_b      = impulse/mass_b # m/s
```

We measured using the iPads the final velocity after the impulse force on both A and B.

In [7]:

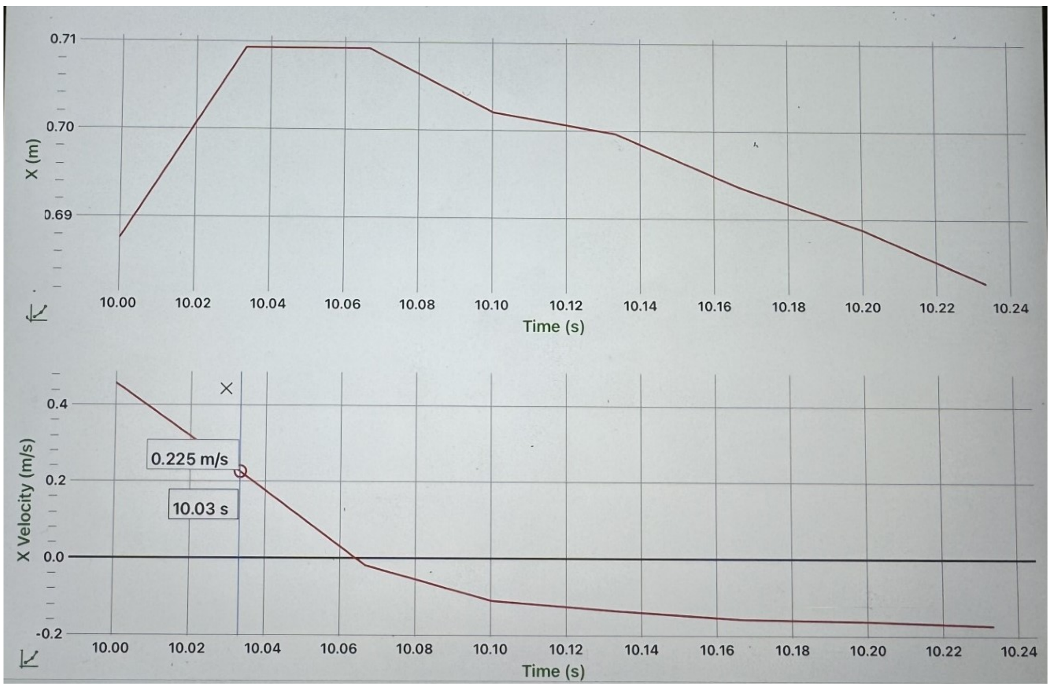
```
actual_final_vel_a = -0.5 # m/s
actual_final_vel_b = 0.815 # m/s

print(f"Error A: {abs(actual_final_vel_a - est_final_vel_a)} m/s")
print(f"Error B: {abs(actual_final_vel_b - est_final_vel_b)} m/s")
```

Out[7]:
```
Error A: 0.22766531994186845
Error B: 0.07715611974232672
```

The errors in this experiment may be due to the scale of our measurements. The size of the floor tiles was our chosen reference scale (12in X12in), but the numbers were skewed due to the point of reference in the video (the square tiles got smaller the further away they were from the iPad).

Problem 3: Heading a Soccer Ball



To find the average force of a headbutt, we need to find the time it takes to impart a certain momentum (impulse) on the ball. Using the impulse equation, the force can be calculated like:

$$p_f = p_i + J$$

$$p_f = p_i + F\Delta t$$

$$F = \frac{p_f - p_i}{\Delta t}$$

$$F = \frac{\Delta p}{\Delta t}$$

Force is by definition the rate of change of momentum as this equation shows. From the video of the head butt, we gather the contact time, Δt, and the net change in momentum, Δp. The change in momentum is calculated by finding the difference in signed velocity and multiplying that by the mass of the ball. Using the iPads and a scale, we calculated the following estimation for the average force of the headbutt:

In [6]:
```
ball_mass     = 0.416 # kg
contact_time  = 0.03 # s
init_ball_vel = -0.475 # m/s
end_ball_vel  = 0.225 # m/s

print(f"Average force: {(ball_mass*(end_ball_vel - init_ball_vel))/
contact_time} N")
```

Out[6]:
```
Average force: 9.706666666666665 N
```

This average force aligns with expectations, as the ball wasn't thrown hard enough to see much impact. The ball's low mass and the fact that it doesn't need much force to overcome its inertia, are the main factors helping to demonstrate that the force calculated is reasonable.

Conclusion

Throughout this lab we applied the principles of conservation of energy, momentum, and impulse. For instance, in problem 1, using both conservation of energy and momentum, it was possible to differentiate the happy ball from the sad ball. For problem 2, our errors were all within reason (less than 1 m/s) and helping to demonstrate the accuracy and reliability of the principles of impulse and conservation of energy. Lastly, the final calculation for the headbutt force aligned with expectations, where we applied impulse and the definition of force as being the rate of change of momentum. All of our numbers represent the physical changes in momentum and forces applied between objects accurately.