Jonah Spector

PH 142 Spring Semester 2025: Homework 4

Due Wednesday, February 26 2025, at 11:59 pm

Instructions: There are 4 long problems for this assignment. Please upload your solutions to Canvas when completed. 10 points will be given for completing all problems. One problem will be chosen randomly and graded in detail, out of 10 points. The sum of these scores will be the total grade, out of 20 points. Partial credit will be given. Please show all work.

1. A coaxial cable consists of two cylindrical conductors separated by an insulating material, or vacuum. Consider an infinitely long cable made of a metal inner cylinder with radius r_a and a metal outer cylinder with radius r_b. (see Fig. 1 below) Here, we will ignore the thickness of the conductors. The inner conductor carries positive charge per unit length, $+\lambda$, and the outer conductor carries negative charge per unit length, $-\lambda$. Calculate the potential at the following locations: (set $V = 0$ at $r = r_b$)
    a. $r < r_a$
    b. $r_a < r < r_b$
    c. $r > r_b$
    d. Give an expression for the potential difference between the inner and outer cylinders.
    e. Use your results from above and the gradient of the potential to find the electric field, $\vec{E}(r)$, between the two conductors.
    f. In the case where $r_a = 5$ mm and $r_b = 5.5$ mm, calculate the magnitude of the electric field at a distance halfway between the inner and outer conductors. Express your answer in terms of the voltage difference between the conductors, V_{ab}. How does this result compare to the answer we would get if we assumed that the two conductors make up a parallel plate capacitor separated by a distance $d = 0.5$ mm, with voltage difference V_{ab}.

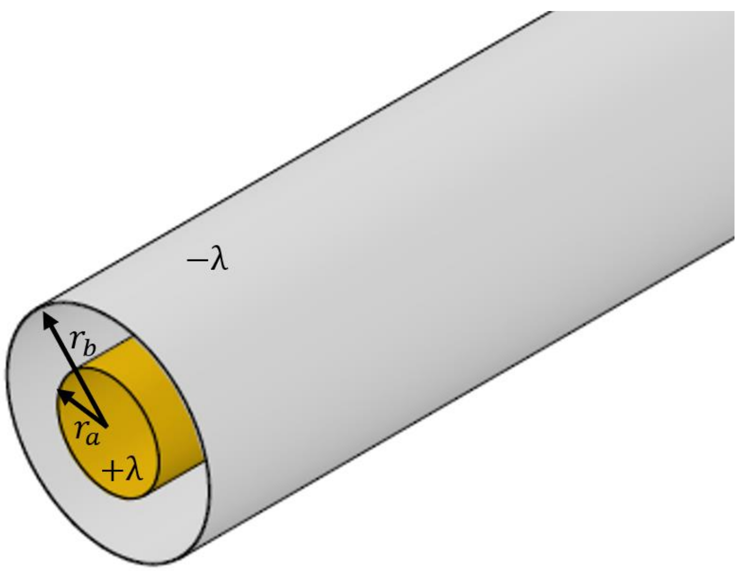


Fig. 1

e) $r < r_a$ — $Q_{enc} = 0$ — $\boxed{\vec{E}(r) = 0}$

$r_a < r < r_b$ — $Q_{enc} = L\lambda$ $\quad \dfrac{\cancel{L}\lambda}{\epsilon_0} = E(2\pi r \cancel{L})$ $\quad \boxed{\vec{E}(r) = \dfrac{\lambda \hat{r}}{2\pi r \epsilon_0}}$

$r_b < r$ $\quad Q_{enc} = \underbrace{L\lambda}_{a} - \underbrace{L\lambda}_{b} = 0$ $\quad \boxed{\vec{E}(r) = 0}$

a) $r < r_a$

$$V = -\int_{r_b}^{r} \vec{E} \cdot dr = -\int_{r_b}^{r_a} \frac{\lambda}{2\pi r \epsilon_0}\, dr - \cancel{\int_{r_a}^{r} 0\, dr}^{\,0} = -\frac{\lambda}{2\pi\epsilon_0}\left[\ln|r_a| - \ln|r_b|\right]$$

$$\boxed{V(r) = -\frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{r_a}{r_b}\right)}$$

b) $r_a < r < r_b$

$$V = -\int_{r_b}^{r} \vec{E} \cdot dr = -\int_{r_b}^{r} \frac{\lambda}{2\pi r \epsilon_0}\, dr \qquad \boxed{V(r) = -\frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{r}{r_b}\right)}$$

c) $r_b < r$

$$V = -\int_{r_b}^{r} \vec{E} \cdot dr = -\int_{r_b}^{r} \frac{\lambda}{2\pi r \epsilon_0}\, dr \qquad \boxed{V(r) = -\frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{r}{r_b}\right)}$$

d) $\boxed{V = -\dfrac{\lambda}{2\pi\epsilon_0} \ln\left(\dfrac{r_a}{r_b}\right)}$

f) $\boxed{\vec{E} = \dfrac{\lambda}{2\pi\epsilon_0} \dfrac{1}{(0.25\,\text{mm})}}$ normal

vs.

$\boxed{\vec{E} = \epsilon_0 \dfrac{A}{d} = \epsilon_0 \dfrac{A}{(0.3\,\text{mm})}}$ parallel plate

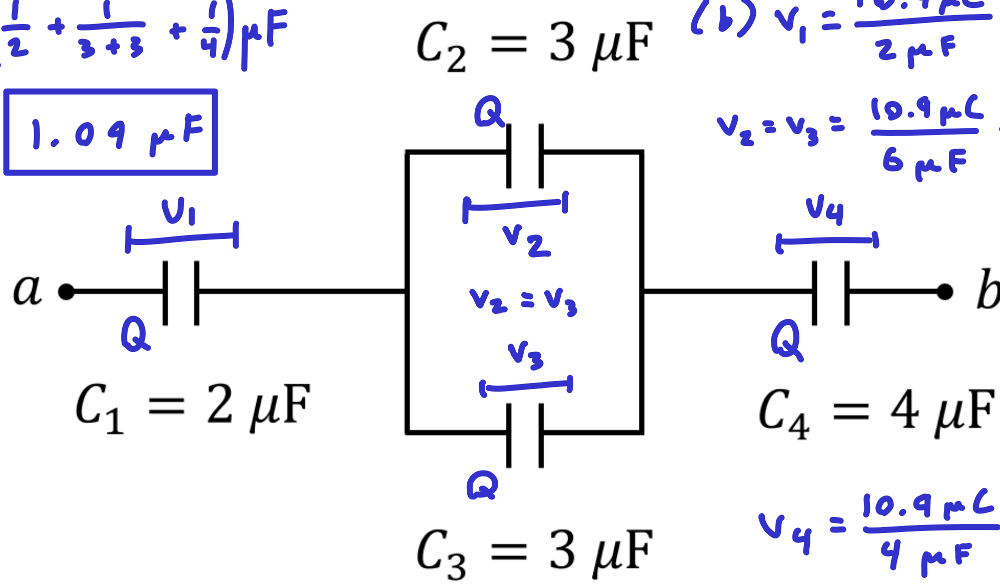
2. A network of capacitors with various capacitance values is shown in Fig. 2 below.
    a. Calculate the equivalent capacitance of this network.
    b. A potential difference, $V_{ab} = 10$ V, is applied between the points a and b in the network. Calculate the potential difference ("voltage drop") across each capacitor in the network.

(a) $C_{eq} = \left(\frac{1}{2} + \frac{1}{3+3} + \frac{1}{4}\right)^{-1} \mu F$

$= \boxed{1.09\ \mu F}$

$C_{eq} = \frac{Q}{V_{ab}}$

$\Rightarrow Q = 10.9\ \mu C$



(b) $V_1 = \frac{10.9\ \mu C}{2\ \mu F} = \boxed{5.45\ V}$

$V_2 = V_3 = \frac{10.9\ \mu C}{6\ \mu F} = \boxed{1.82\ V}$

$V_4 = \frac{10.9\ \mu C}{4\ \mu F} = \boxed{2.725\ V}$

Fig. 2

3. A parallel plate capacitor has a separation distance of $d = 2.5$ mm and stores 10 J of energy.
    a. If the separation distance is decreased to 1 mm, what is the energy stored if the capacitor is disconnected from the voltage source used to charge it?
    b. If the separation distance is decreased to 1 mm, what is the energy stored if the capacitor stays connected to the voltage source used to charge it?

(a) $u = \frac{1}{2}\epsilon_0 E^2$ $\quad U = uAd = \frac{1}{2}\epsilon_0 E^2 Ad = \frac{1}{2}\epsilon_0 \frac{V^2}{d^2} Ad = \frac{1}{2}\underbrace{\epsilon_0 \frac{A}{d}}_{C} V^2 \to V = 0$

$\therefore \boxed{U = 0}$

$Ed = V \to E = \frac{V}{d} \to E^2 = \frac{V^2}{d^2}$

(b) $10\ J = \overbrace{\frac{1}{2}\epsilon_0 \frac{A}{2.5\ mm} V^2}^{\text{"m"}} \Rightarrow 10\ J = m \frac{1}{2.5\times10^{-3}\ m} \Rightarrow m = 10\ J\,(2.5\times10^{-3}\ m)$

$= U\,(1.5\times10^{-3}\ m)$ $\quad -1\ mm$

$U = \boxed{16.66\ J}$

4. (Electrostatic deflector). Highly charged argon ions with charge states $^{40}Ar^{8+}$, $^{40}Ar^{9+}$, and $^{40}Ar^{10+}$ are created inside an ion source and accelerated through a potential difference, $U = 2.5$ kV inside an ultrahigh vacuum chamber. The ions enter an electric field-free region which is 5 meters long and travel through an electrostatic bender towards an ion trap (recall problem from last week). One way to prevent the ions from entering the trap is to insert an electrostatic deflector (see Fig. 3 below). The deflector consists of two parallel plates of length $L = 10$ cm, separated by a distance $d = 2$ cm. The plates of the deflector can be biased to generate a potential difference between them which can be used to alter the trajectory of the ions as they travel through this region.
    a. Calculate the minimum potential difference between the plates, V, required to divert the $^{40}Ar^{9+}$ ions from the center of the beamline to one of the plates.
    b. Is this potential difference large enough to divert charge states $^{40}Ar^{+}$ through $^{40}Ar^{16+}$? If so, explain why. If not, calculate the minimum potential difference required to divert these charge states. NOTE: All charge states are initially accelerated through the same 2.5 kV potential.
    c. Suppose that we are interested in only sending $^{40}Ar^{9+}$ ions through the deflector to the ion trap. Assuming that the total pathlength from the ion source to the entrance of the deflector is 7.5 meters, and that all charge states leave the ion source at the same time, is there a way to adjust the potential V to allow only the $^{40}Ar^{9+}$ ions through the deflector? (Consider what happens when $V = 0$ V)
    d. Is there a particular time-of-flight required to accomplish the task in part (d)? If so, what timing resolution is required to ensure that only $^{40}Ar^{9+}$ ions make it to the trap an no other neighboring charge states can enter.

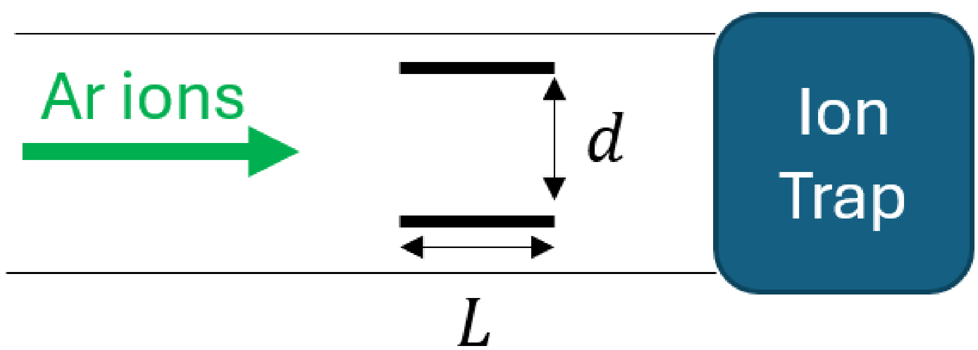
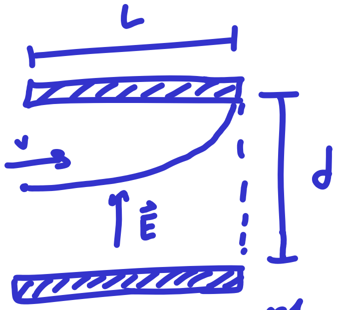


Fig. 3

a)



assume uni. electric field

$t = \frac{L}{v}$ — max time to deflect

$\frac{d}{2} = \frac{1}{2} a t^2$ — min accel. needed

$$\Rightarrow a = \frac{v^2 d}{L^2} \Rightarrow F_E = m\frac{v^2 d}{L^2} = qE$$

$$m\frac{v^2 d}{L^2} = qE \qquad Ed = V \longrightarrow E = \frac{V}{d} \quad \Rightarrow \quad m\frac{v^2 d}{L^2} = q\frac{V}{d} \Rightarrow \boxed{V = \frac{m v^2 d^2}{L^2 q}}$$

nums. from HW #3

$$V = \frac{(40 \times 1.66 \times 10^{-27}\,\text{kg})(329{,}498.524\,\text{m/s})^2 (0.02\,\text{m})^2}{(0.10\,\text{m})^2 (9 \times 1.602 \times 10^{-19})}$$

$$= \boxed{200\,\text{V}}$$

b) Plugging the other ions' values in $\boxed{\text{doesn't change the voltage}}$

This is because, even though the F on the more charged ions is greater, the init. velocity is too.

c) The different ions will spread out over the 7.5 meters and so the plates can be turned off for a few micro-seconds to let a specific type through while being on the rest of the time to deflect unwanted ions.

est. from HW 3, Q3, (b)

d)

HW3 →

Ar^{8+} $310{,}654.187\,\frac{m}{s} \cdot t = 7.5\,m \longrightarrow \sim 24\,\mu s$

Ar^{9+} $329{,}498.524\,\frac{m}{s} \cdot t = 7.5\,m \longrightarrow \sim 22\,\mu s$

Ar^{10+} $347{,}321.940\,\frac{m}{s} \cdot t = 7.5\,m \longrightarrow \sim 21\,\mu s$

$\boxed{\sim 1\,\mu s}$

Possible!