

PH 142 Spring Semester 2025: Homework 2

Due Wednesday, February 5 2025, at 11:59 pm

Instructions: There are 4 long problems for this assignment. Please upload your solutions to Canvas when completed. 10 points will be given for completing all problems. One problem will be chosen randomly and graded in detail, out of 10 points. The sum of these scores will be the total grade, out of 20 points. Partial credit will be given. Please show all work.

1. An infinitely long, solid cylinder with radius  $R$  has volume charge density  $\rho(r)$  (see Fig. 1 below).
  - a. Calculate the electric field inside and outside of the cylinder for the case when the volume charge density  $\rho(r)$  is constant ( $\rho(r) = \rho$ ).
  - b. Calculate the electric field inside and outside of the cylinder for the case when the volume charge density increases linearly with the distance from the axis of the cylinder,  $\rho(r) = \beta r$ , where  $\beta$  is some arbitrary constant.

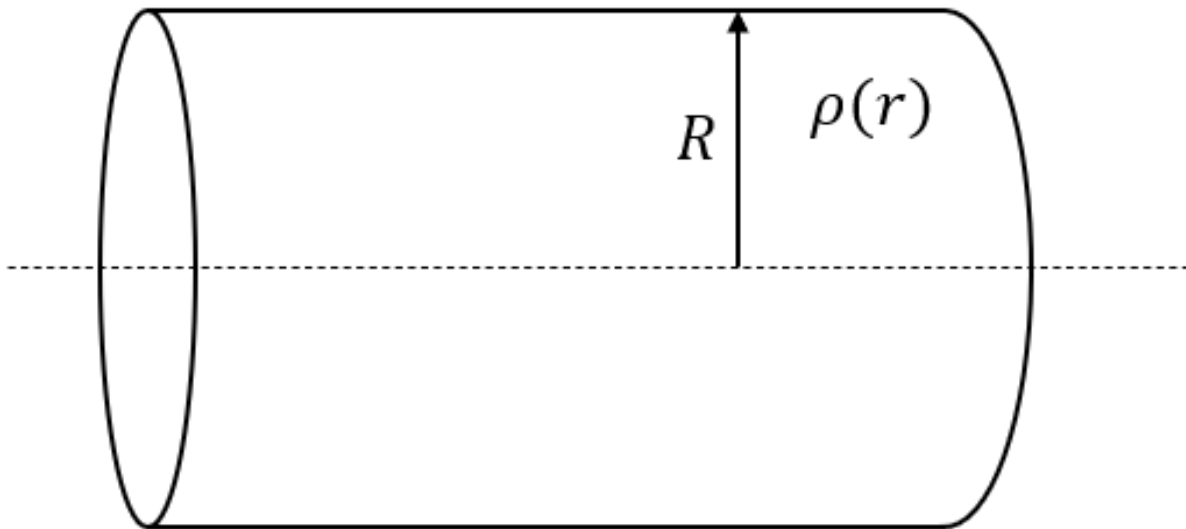


Fig. 1

2. A solid sphere of radius  $a$ , centered at the origin, carries a volume charge density of  $\rho(r) = \alpha r^2$ , where  $\alpha$  is a constant (see Fig. 2 below).
- Calculate the electric field inside and outside of the sphere. (NOTE: For a spherically symmetric system,  $dV = r^2 \sin(\theta) d\theta d\phi dr$ . Therefore, the volume integral of some function  $\rho(r)$  is given by  $\int \rho(r) r^2 \sin(\theta) d\theta d\phi dr = 4\pi \int \rho(r) r^2 dr$ )
  - Plot the resulting electric field as a function of distance  $r$  from the origin.

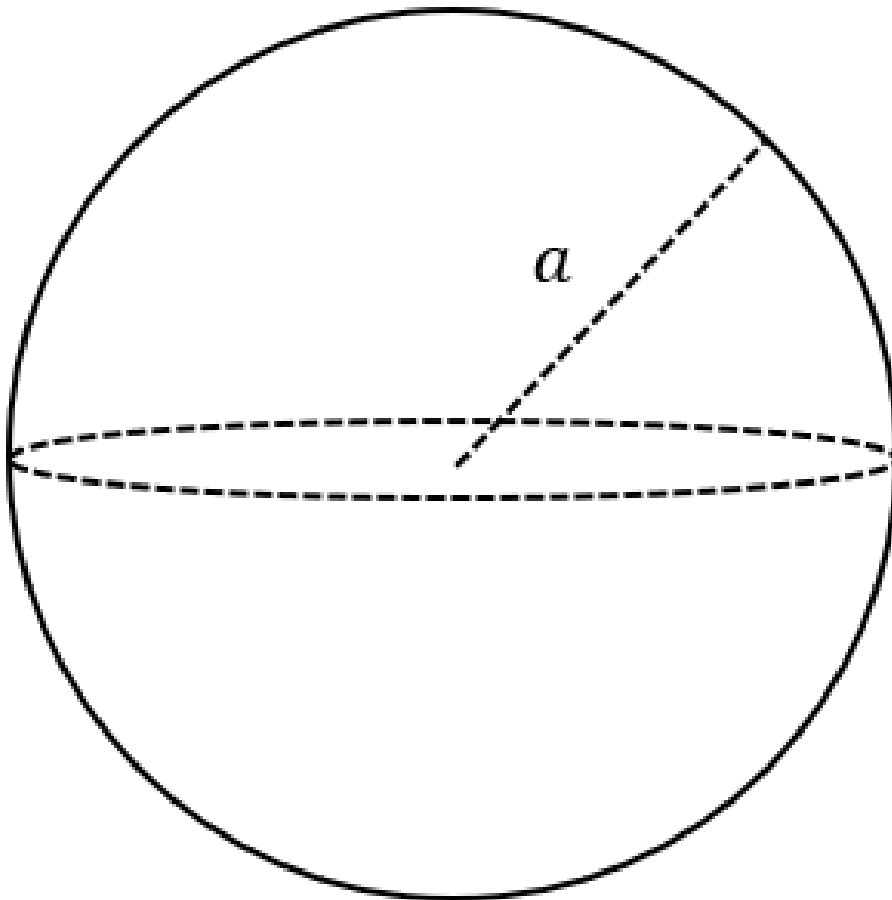


Fig. 2

3. A neutral conducting sphere has two cavities with charges  $q_a$  and  $q_b$ , and radii  $a$  and  $b$  (see Fig. 3 below).
- Calculate  $\sigma_a$ ,  $\sigma_b$ , and  $\sigma_R$  for this configuration.
  - Find the electric field outside of the conductor.
  - Find the electric field inside each cavity.
  - What is the force on  $q_a$  and  $q_b$ .
  - If we were to bring a third charge  $q_c$  near the conductor, which of your previous answers (a – d) would change, and how?

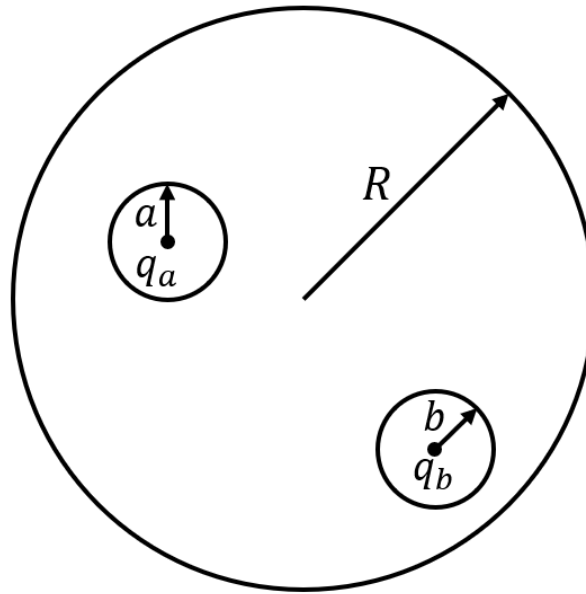


Fig. 3

4. The water molecule ( $\text{H}_2\text{O}$ ) has a dipole moment of  $6.17 \times 10^{-30} \text{ C} \cdot \text{m}$ . A single  $\text{H}_2\text{O}$  molecule in the gas phase is placed in a region of electric field with magnitude  $|\vec{E}| = 10^6 \text{ N/C}$ .
- Calculate the change in potential energy of the dipole when it changes its orientation, with respect to the electric field  $\vec{E}$ , from parallel to perpendicular.
  - The thermal translational energy (kinetic energy) of a molecule in the gas phase is given as  $KE = \frac{3}{2} k_b T$ . Calculate the temperature  $T$  at which the thermal energy is equal to the change in the potential energy of the dipole. Is this temperature large or small compared to “room temperature” (approximately 293 K).
  - Now the water molecule is oriented perpendicular to the electric field and held in place. When it is released (and allowed to rotate freely) calculate the torque induced on the dipole. Compare this answer to the peak torque specification of  $500 \text{ N} \cdot \text{m}$  found on a 2025 Maserati Quattroporte sedan.