PH 142 Spring Semester 2025: Homework 7

Due Wednesday, April 9, 2025 at 11:59 pm

Instructions: There are 4 long problems for this assignment. Please upload your solutions to Canvas when completed. 10 points will be given for attempting all problems. One problem will be chosen randomly and graded in detail, out of 10 points. The sum of these scores will be the total grade, out of 20 points. Partial credit will be given. Please show all work.

1. An ideal battery supplies an emf of 50 V and is connected to two resistors (R_1 and R_2) and two capacitors (C_1 and C_2). (see Fig. 1 below) $R_2 = 3.0\ \Omega$, $C_1 = 2.0\ \mu$F, and $C_2 = 5.0\ \mu$F. After the capacitors have been fully charged, the charge on $C_1 = 10.0\ \mu$C.
    a. Calculate the final charge on capacitor C_2.
    b. Calculate the resistance R_1 needed to achieve the values given above.

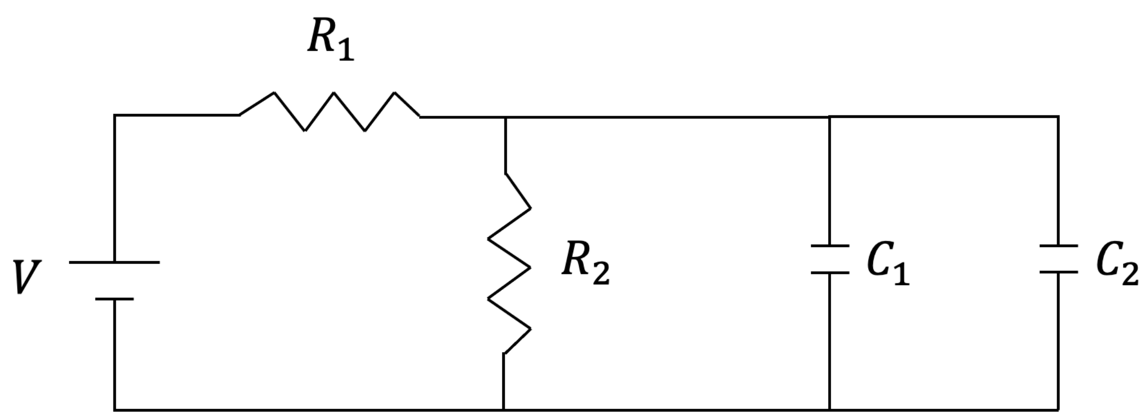
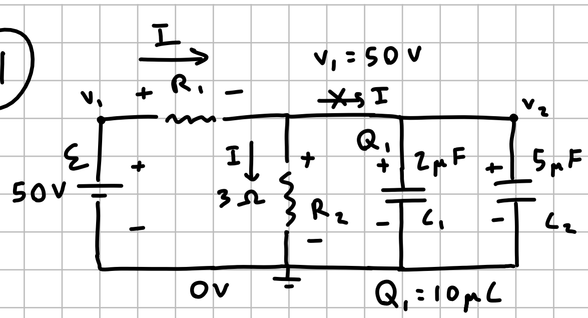


Fig. 1

2. A capacitor with capacitance C is connected in series to a resistor with resistance R and a battery with emf V. At $t = 0$, the circuit is complete.
    a. How much energy is stored in the capacitor when it is fully charged?
    b. The power output of the battery is $P = iV$, where $i(t) = I_0 \exp(-t/RC)$. The instantaneous energy supplied by the battery in an infinitesimal time dt is given as Pdt. Calculate the total energy supplied by the battery from $t = 0$ to when the system reaches a steady state.
    c. The rate of consumption of electrical energy in the resistor is $P_R = i^2 R$. In an infinitesimal time interval dt, the amount of electrical energy consumed by the resistor is $P_R dt$. Calculate the total energy consumed by the resistor from $t = 0$ to when the system reaches a steady state.
    d. What fraction of the total energy supplied by the battery is stored in the capacitor?
    e. What fraction of the total energy supplied by the battery is consumed in the resistor?

1



a) $Q_1 = C_1 V_2$

$10\mu C = 2\mu F\, V_2 \longrightarrow V = 5V$

$Q_2 = C_2 V_2$

$= 5\mu F\,(5V) = \boxed{25\mu C}$

b) $V = IR$ $\quad V_2 = IR_2$ $\quad \varepsilon - V_2 = IR_1$ $\quad \boxed{R_1 = 27\,\Omega}$

$5V = I \cdot 3\Omega$ $\quad 50V - 5V = 5/3\ A \cdot R_1$

$I = 5/3\ A$

2

a) $\boxed{U = \frac{1}{2}CV^2}$ $\quad I = 0 \rightarrow V_R = IR = 0$

b) $P = IV = I_0 e^{-t/RC} V \longrightarrow P\,dt = I_0 e^{-t/RC} V\,dt$

$$\int P\,dt = E = I_0 V \int_0^\infty e^{-t/RC}\,dt = I_0 V \left[-RCe^{-t/RC}\right]_0^\infty$$

$$= I_0 V\left[0 - (-RC)\right]$$

$$= \boxed{I_0 V R C}$$

c) $P_R = I^2 R = R\left(I_0 e^{-t/RC}\right)^2 \longrightarrow P\,dt = R I_0^2\, e^{-2t/RC}\,dt$

$$\int P\,dt = E = R I_0^2 \int e^{-2t/RC}\,dt = R I_0^2 \left[-\frac{RC}{2} e^{-2t/RC}\right]_0^\infty$$

$$= \boxed{R^2 I_0^2 C/2}$$

d) $\frac{E_C}{E_\varepsilon} = \frac{1/2\, CV^2}{CV^2} = \boxed{1/2}$ $\quad$ e) $\frac{E_R}{E_\varepsilon} = \frac{1/2\, CV^2}{CV^2} = \boxed{1/2}$

3. Consider the circuit shown in Fig. 2 below. Calculate the three currents I_1, I_2, and I_3.

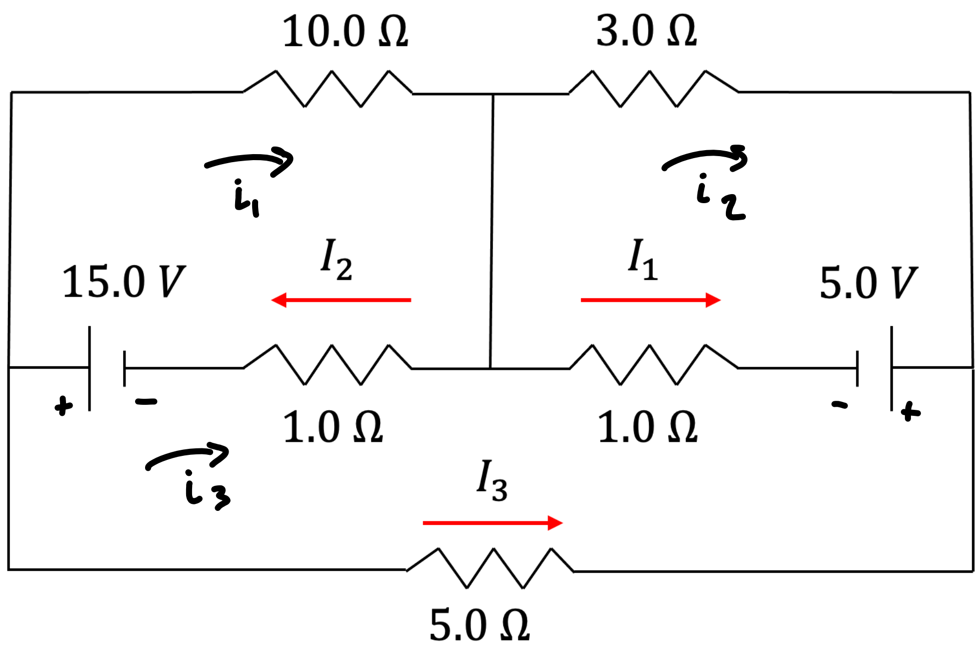


Fig. 2

4. A low-pass filter consists of a capacitor with capacitance $C = 10\ \mu\text{F}$ and a resistor with resistance $R = 10\ \text{k}\Omega$. Calculate the corner frequency, f_{3dB}, for this filter. If we wanted to keep the resistance fixed and reduce the corner frequency by a factor of 3, what capacitance would we need to use instead.

(3)

$i_1: \quad 10i_1 + i_1 - i_3 - 15 = 0$

$i_2: \quad 3i_2 + 5 + i_2 - i_3 = 0$

$i_3: \quad 15 + i_3 - i_1 + i_3 - i_2 - 5 + 5i_3 = 0$

$I_2 = i_1 - i_3$

$I_1 = i_3 - i_2$

$I_3 = -i_3$

$$\begin{pmatrix} 15 \\ -5 \\ -10 \end{pmatrix} = \begin{pmatrix} 11 & 0 & -1 \\ 0 & 4 & -1 \\ -1 & -1 & 7 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} \longrightarrow \begin{matrix} i_1 = 1.229 \text{ A} \\ i_2 = -1.621 \text{ A} \\ i_3 = -1.485 \text{ A} \end{matrix} \quad \boxed{\begin{matrix} I_1 = 0.136 \text{ A} \\ I_2 = 2.714 \text{ A} \\ I_3 = 1.485 \text{ A} \end{matrix}}$$

(4)

$$f = \frac{1}{2\pi RC} = \frac{1}{2\pi(10\times10^3\Omega)(10\times10^{-6}\text{F})} = \frac{1}{0.2\pi} \approx \boxed{1.592 \text{ Hz}}$$

$$f' = \frac{f}{3} = 0.53 \text{ Hz} = 1/(2\pi(10\times10^3\Omega)\, c')$$

$$c' = 3.0\times10^{-5} \text{ F} = \boxed{30\,\mu\text{F}}$$