Lab 9: Electron Beam

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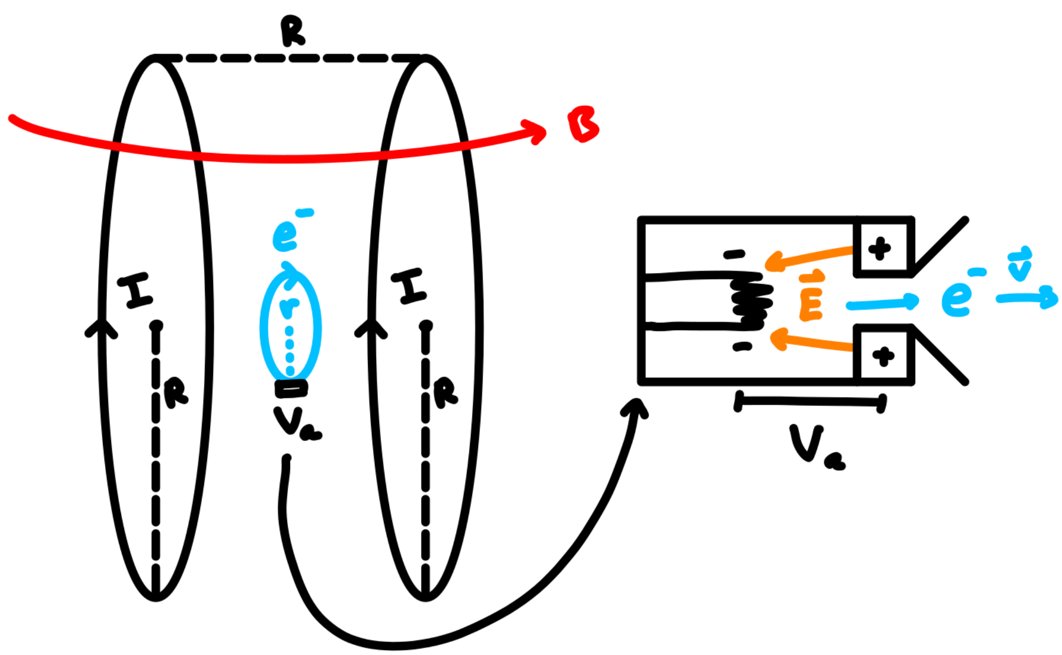
1 Introduction

Magnetic fields can feel very foreign, almost as if they operate in an entirely different plane of existence, occupying the same space, but not visible. However, their impacts on the world can be observed through either the movement of magnetic materials, or the deflection of particles. Today We exploit this impact to visualize the impact that magnetic fields have on electron beams, helping to demonstrate how magnetic fields interact, an essential principle to understand for almost any system utilizing electrical current and/or electromagnets.

2 Derivation

The apparatus has two Helmholtz coils separated by a distance equal to their radius R with an electron gun in the center to give the electrons an accelerating voltage of V_a. After the initial kick, the Lorentz force causes the electrons to orbit with a set radius r since the velocity vector $\vec{v}$ is perpendicular to the magnetic field $\vec{B}$. Using the right-hand rule (and inverting the direction of the force due to the negative charge of the electron), the direction of the orbit can be found, as shown in the drawing.



First, to find the velocity of the electrons leaving the accelerating voltage, we use two equations:

$$U = qV_a \qquad K = \frac{1}{2}mv^2$$

Since energy is conserved:

$$qV_a = \frac{1}{2}mv^2 \implies v = \sqrt{\frac{2qV_a}{m}}$$

The magnitude of the force on the electron in a perpendicular magnetic field is:

$$|F_B| = qvB$$

The direction of the force is $\hat{r}$, or towards the center of the orbit.

Applying this to the circular motion and centripetal acceleration:

$$F_{\text{net}} = ma_c = m\frac{v^2}{r} = F_B = qvB \implies m\frac{v^2}{r} = qvB$$

Solving for velocity:

$$v = \frac{rqB}{m}$$

Isolating q/m in terms of known variables:

$$\frac{rqB}{m} = \sqrt{\frac{2qV_a}{m}} \implies \frac{(rqB)^2}{m^2} = \frac{2qV_a}{m}$$

$$\frac{q(rB)^2}{m} = 2V_a \implies \boxed{\frac{q}{m} = \frac{2V_a}{(rB)^2}}$$

Substituting the Helmholtz coil equation, which has been derived many times by us and is given on the lab document:

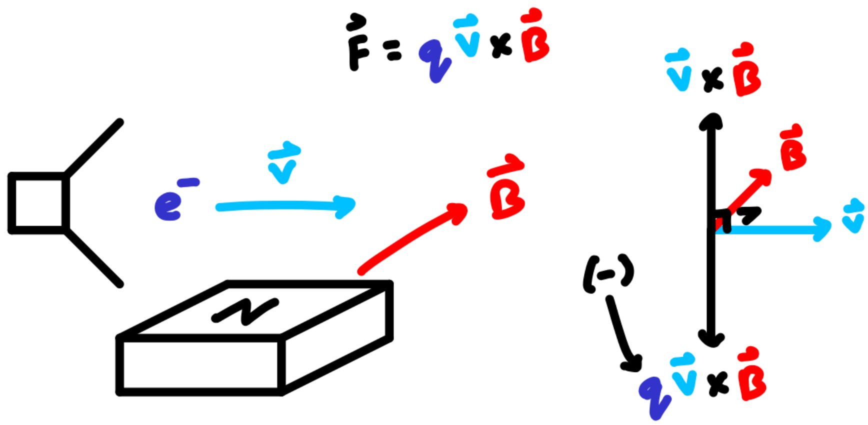
$$\frac{q}{m} = \frac{2V_a}{r^2}\left(\frac{5}{4}\right)^3\left(\frac{R}{n\mu_0 I}\right)^2 \implies \frac{q}{m} = \frac{125}{32}V_a\left(\frac{R}{n\mu_0 Ir}\right)^2$$

3 Problem 1: Magnetic Deflection

Materials: Helmholtz apparatus, power supply, bar magnet

Method: Power up the apparatus to 100 V, generating an electron beam. Move the Bar magnet close to the vacuum tube. Note the electron beam's behavior.

When the North pole of the magnet is held up to the electron beam, our prediction and reasoning is as follows (given that field lines leave North poles and return to South poles):



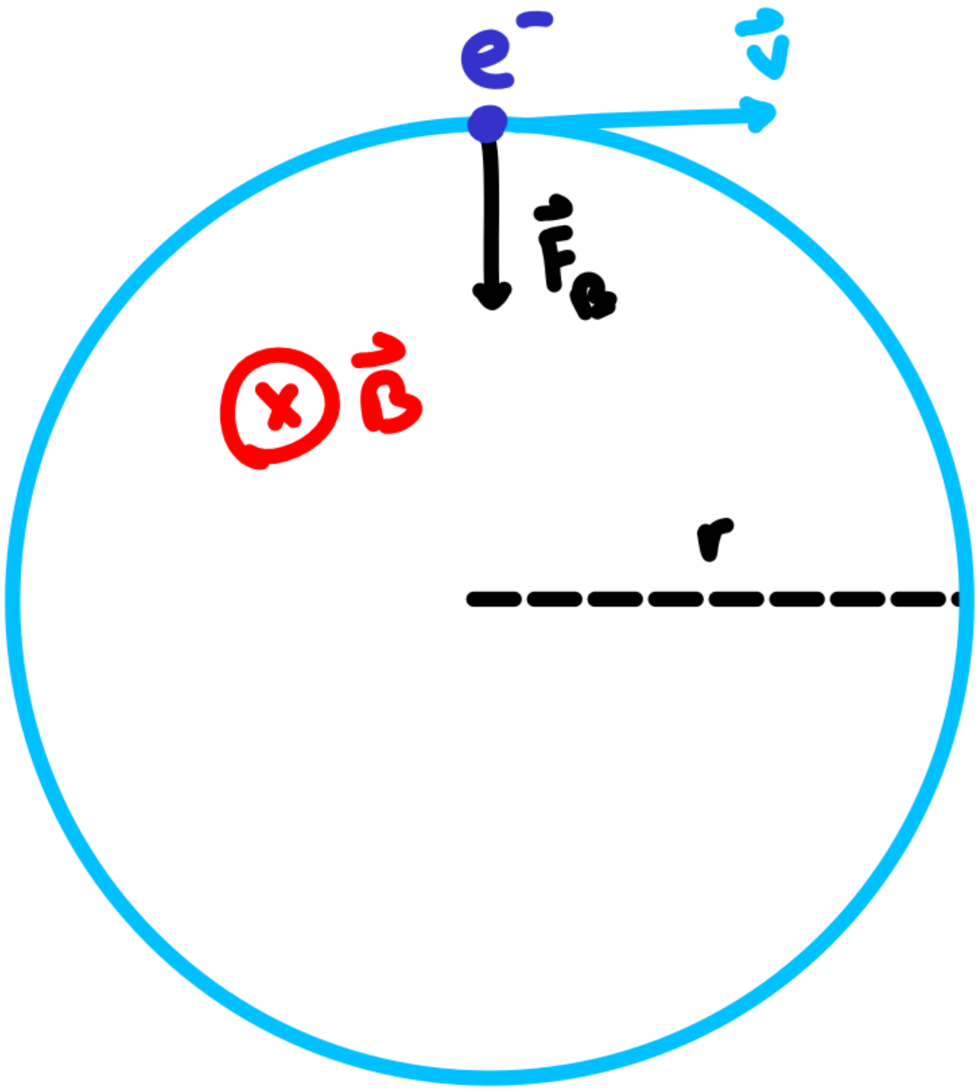
The Lorentz force will cause the beam to deflect downwards. Our prediction matches reality, which gives evidence that the electrons are negatively charged, since if they were positive they would deflect upwards. There is no deflection to or from the magnet, implying that the magnet has no electric charge and therefore $\vec{E}$ field, which would interact with the electrons according to $\vec{F} = q\vec{E}$.

When the magnet is placed slightly higher or lower, the electrons also deflect slightly to and from the magnet, respectively. This is not because they are attracted or repelled by the magnet, but is caused due to the fact that the field lines become angled.

4 Problem 2: Round-Abouts

Materials: Helmholtz apparatus, power supply

Method: Adjust the power supply current to 1 A and the voltage to 100 V. Note the electron beam's behavior at different currents.



The higher the current, the smaller the radius of the electron beam loop (our prediction and result). This makes sense just be looking at the cyclotron radius equation and the magnetic field's proportionality (from the Helmholtz equation):

$$r = \frac{mv}{qB} \qquad B \propto I$$

Likewise, according to the equation above, a higher accelerating voltage, and therefore velcoity, increases the radius (our prediction and result).

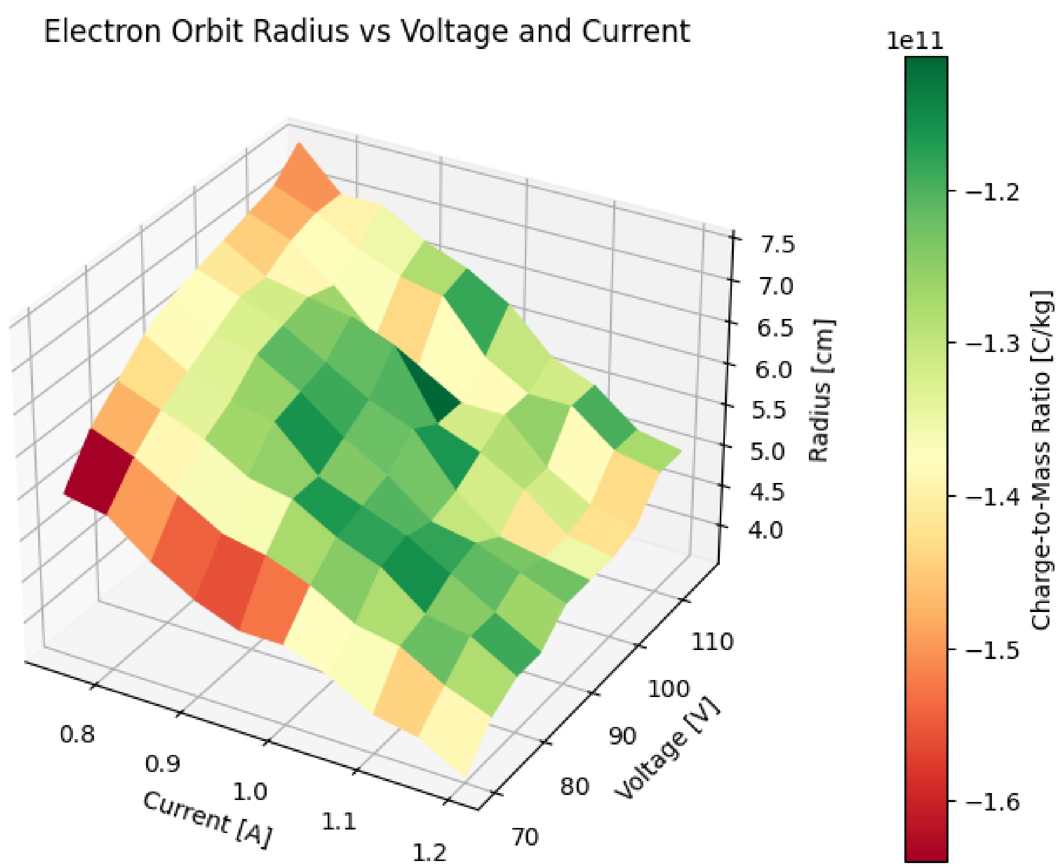
5 Problem 3

Materials: Helmholtz apparatus, power supply

Method: Measure the radius of the electron beam orbit at voltages between 70 V and 115 V with jumps of 5 V, where at each voltage trying out currents between 0.75 A and 1.2 A, with jumps of 0.05 A. Record this data and perform analysis.

[1]:
```python
from math import *

import matplotlib
import matplotlib.pyplot as plt
import numpy as np

# Constants
R = 0.165  # m
n = 130  # m
mu_0 = pi * 4e-7  # N/A^2


# Calculats q/m
def charge_mass_ratio(V, I, r):
    a = (125 / 32) * V
    b = (R / (n * mu_0 * I * r)) ** 2

    return a * b


# Voltage and current ranges
V = np.array([70, 75, 80, 85, 90, 95, 100, 105, 110, 115])  # V
I = np.array([0.75, 0.80, 0.85, 0.90, 0.95, 1, 1.05, 1.10, 1.15, 1.20])  # A

# Recorded data (100 points!)
r = np.array(
    [
        [0.055, 0.054, 0.050, 0.047, 0.045, 0.045, 0.043, 0.040, 0.039, 0.036],
        [0.060, 0.058, 0.055, 0.052, 0.051, 0.049, 0.046, 0.045, 0.042, 0.040],
        [0.063, 0.061, 0.059, 0.056, 0.055, 0.052, 0.050, 0.046, 0.045, 0.042],
        [0.066, 0.063, 0.061, 0.060, 0.055, 0.053, 0.051, 0.048, 0.045, 0.042],
        [0.068, 0.065, 0.064, 0.061, 0.057, 0.054, 0.050, 0.049, 0.047, 0.045],
        [0.069, 0.067, 0.065, 0.062, 0.059, 0.057, 0.051, 0.047, 0.046, 0.045],
        [0.070, 0.067, 0.066, 0.065, 0.063, 0.055, 0.053, 0.050, 0.046, 0.045],
        [0.071, 0.069, 0.065, 0.060, 0.058, 0.055, 0.055, 0.050, 0.047, 0.046],
        [0.072, 0.070, 0.067, 0.065, 0.064, 0.058, 0.055, 0.055, 0.051, 0.050],
        [0.075, 0.070, 0.070, 0.067, 0.065, 0.062, 0.057, 0.055, 0.051, 0.050],
    ]
)  # m

I, V = np.meshgrid(I, V)

# (-) since electron is negatively charged
```

```python
q_m = -charge_mass_ratio(V, I, r)

# Plotting
fig, ax = plt.subplots(subplot_kw={"projection": "3d"}, figsize=(20, 6))
scamap = plt.cm.ScalarMappable(cmap="RdYlGn")
fcolors = scamap.to_rgba(q_m)

ax.plot_surface(I, V, r * 100, facecolors=fcolors, cmap="RdYlGn")
ax.set_title("Electron Orbit Radius vs Voltage and Current")
ax.set_xlabel("Current [A]")
ax.set_ylabel("Voltage [V]")
ax.set_zlabel("Radius [cm]")

cbar = fig.colorbar(scamap, ax=plt.gca())
cbar.ax.set_ylabel("Charge-to-Mass Ratio [C/kg]")

plt.show()

# Results
print("=== Charge-to-Mass Ratios [C/kg] ===")
print(f"Data Average:              {np.average(q_m):.2e}")
print(f"                       min {np.min(q_m):.2e}")
print(f"                       max {np.max(q_m):.2e}")
print(f"                       std  {np.std(q_m):.2e}")
print(f"CODATA (Real):             {-1.75882000838e11:.2e}")
```




```
=== Charge-to-Mass Ratios [C/kg] ===
Data Average:              -1.32e+11
                       min -1.64e+11
                       max -1.11e+11
                       std  1.04e+10
CODATA (Real):             -1.76e+11
```

5.1 Analysis

Our average prediction is -1.32e+11 C/kg, whereas the actual value is -1.76e+11. Even with 100 measurements being averaged together, somehow our prediction is off by 0.44e+11 C/kg. At the very least, the two values are within the same order of magnitude. The minimum value for the charge-to-mass ratio of the electron, -1.64e+11, is much closer though, only being off by 0.12e+11. However, this means every measurement predicted a charge-to-mass ratio *below* the absolute magnitude of the real value.

This could have been due to unaccounted for impedance or other electrical issues with the power supply and Helmholtz coils, or issues measuring the radius of the electron beam orbit. Another interesting point is that the minimum value was the *first* value measured. This could imply issues with repeated measurements *over time*, like what was done. The electron gun may have accumulated error over time due to overuse at many voltages.

6 Conclusion

Electromagnets are used in a wide variety of applications, from medical instruments to electronics components. Understanding the charge-to-mass ratios of the fundamental particles is essential for understanding the properties of matter and their applications. Despite getting a charge-to-mass ratio of -1.32e+11 C/kg for the electron (which didn't match reality very closely), the principles used to calculate it apply to many other areas of physics and engineering.