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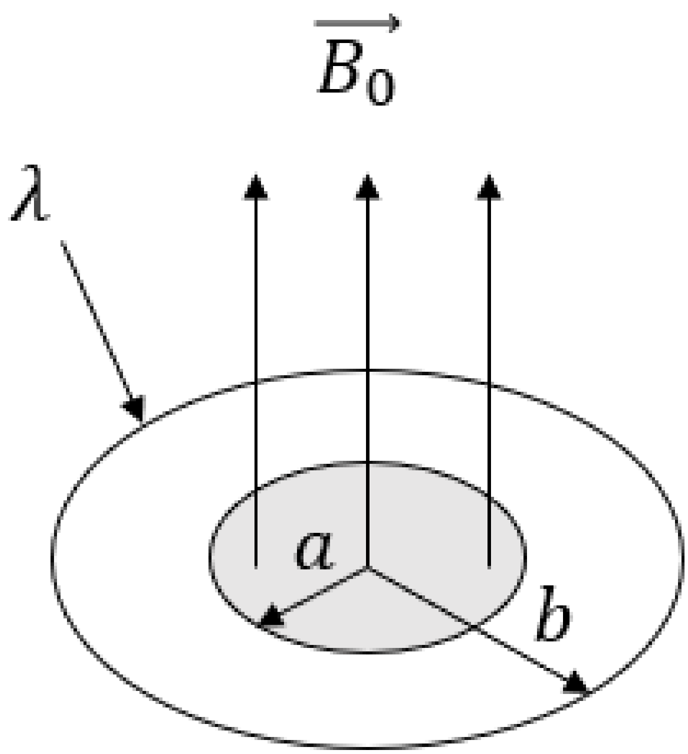
PH 142 Spring Semester 2025: Homework 8

Due Friday, April 18, 2025 at 11:59 pm

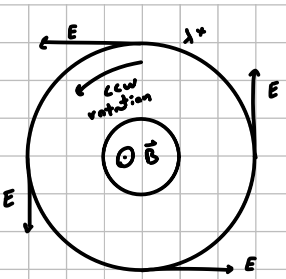
Instructions: There are 4 long problems for this assignment. Please upload your solutions to Canvas when completed. 10 points will be given for attempting all problems. One problem will be chosen randomly and graded in detail, out of 10 points. The sum of these scores will be the total grade, out of 20 points. Partial credit will be given. Please show all work.

1. A line charge λ is glued onto the rim of a wheel of radius b and mass m, which is suspended horizontally so that it is free to rotate. (see figure below) In the central region, out to radius a, there is a uniform magnetic field $\overrightarrow{B_0}$, pointing up. The magnetic field is suddenly switched off. Now we will analyze what happens in the system after the magnetic field is turned off.
   a. Will the disc rotate or not? If the answer is yes, in which direction will it rotate? (Indicate the direction on the diagram or your own sketch.)
   b. What is the magnitude and direction of the electric field that is generated when the magnetic field is turned off.
   c. If the disc will rotate, calculate the torque and angular momentum of the disc.

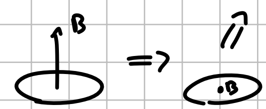


1) a) $\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$ E is const at r=b Assuming λ^+, $\vec{F}=q\vec{E}$

$E(2\pi b) = -\frac{d}{dt}(B\pi a^2)$ $\nabla\times E = -\frac{\partial B}{\partial t}$ $\frac{dB}{dt} = \downarrow$ $-\frac{dB}{dt} = \uparrow$



$E(2\pi b) = -\pi a^2 \frac{dB}{dt}$



$\nabla\times E = \uparrow$

$\therefore E = \circlearrowleft$

(RHR)

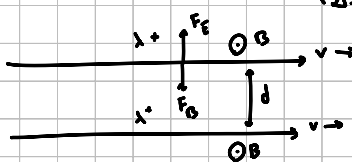
b) $\boxed{E = \frac{-a^2}{2b}\frac{dB}{dt}}$ ← $\neq 0$, since $B_0 \to 0$

Direction is CCW.

c) $\tau = Fb$ $F = qE \longrightarrow \tau = bqE$ $q = \lambda 2\pi b \longrightarrow \tau = 2\pi\lambda b^2\left(\frac{-a^2}{2b}\frac{dB}{dt}\right) \Rightarrow \boxed{\tau = -a^2 b\lambda\pi\left(\frac{dB}{dt}\right)}$

$L = \tau\Delta t = -a^2 b\lambda\pi\left(\frac{\Delta B}{\Delta t}\right)\Delta t = -a^2 b\lambda\pi(0 - B_0) \Rightarrow \boxed{L = a^2 b\lambda\pi B_0}$

2)



$F_E = \frac{\lambda^2 L^2}{4\pi\epsilon_0}\frac{1}{d^2}$ $B = \frac{\mu_0}{4\pi}\frac{\lambda L v}{d^2}$ $\cancel{\frac{\lambda^2 L^2}{4\pi}}\frac{1}{\epsilon_0}\cancel{\frac{1}{d^2}} = \frac{\mu_0}{\cancel{4\pi}}\frac{\cancel{\lambda^2 L^2} v^2}{\cancel{d^2}}$

$F_B = \lambda L v B \Longrightarrow F_B = \frac{\mu_0}{4\pi}\frac{\lambda^2 L^2 v^2}{d^2}$

$\frac{1}{\epsilon_0} = \mu_0 v^2 \Rightarrow \boxed{v = \frac{1}{\sqrt{\mu_0\epsilon_0}} = c}$

3)

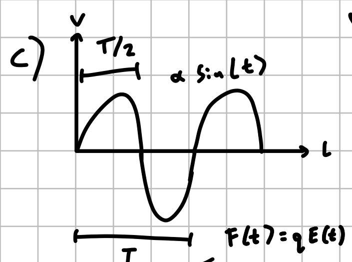
a) $\omega_0 = 1/\sqrt{LC} = 1/\sqrt{2\times10^{-6}H\cdot 2\times10^{-12}F} = 500\times10^6 \longrightarrow f_0 = \frac{\omega_0}{2\pi} = \boxed{\frac{250}{\pi} MHz}$

b) $Ed = V \rightarrow E = \frac{V}{d} = \frac{100V}{10^{-3}m} = 100{,}000\frac{N}{C} = \boxed{100\ kN/C}$

$V(t) = V_0\sin(\omega_0 t - \delta)$ $E(t) = \frac{V(t)}{d} = \frac{V_0}{d}\sin(\omega_0 t)$

$= E_0\sin(\omega_0 t)$

c)



$F(t) = qE(t)$

$a(t) = \frac{1}{m}F(t) = \frac{q}{m}E(t) = \frac{qE_0}{m}\sin(\omega_0 t)$

$v(t) = \frac{q}{m}\frac{V_0}{d}\int_0^t \sin(\omega_0\tau)\,d\tau$

$v(t) = \frac{q}{m}\frac{V_0}{d}\left(-\frac{1}{\omega_0}\cos(\omega_0 t) + \frac{1}{\omega_0}\right)$

$x(t) = \frac{q}{m}\frac{V_0}{d}\int_0^t\left(-\frac{1}{\omega_0}\cos(\omega_0\tau) + \frac{1}{\omega_0}\right)d\tau$

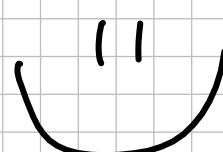
$= \frac{q}{m}\frac{V_0}{d}\left[-\frac{1}{\omega_0^2}\sin(\omega_0\tau) + \frac{\tau}{\omega_0}\right]_0^t$

$x(t) = \frac{q}{m}\frac{V_0}{d}\frac{1}{\omega_0}\left(t - \frac{1}{\omega_0}\sin(\omega_0 t)\right)$ $\omega_0 = \frac{2\pi}{T} \rightarrow T = \frac{2\pi}{\omega_0}$

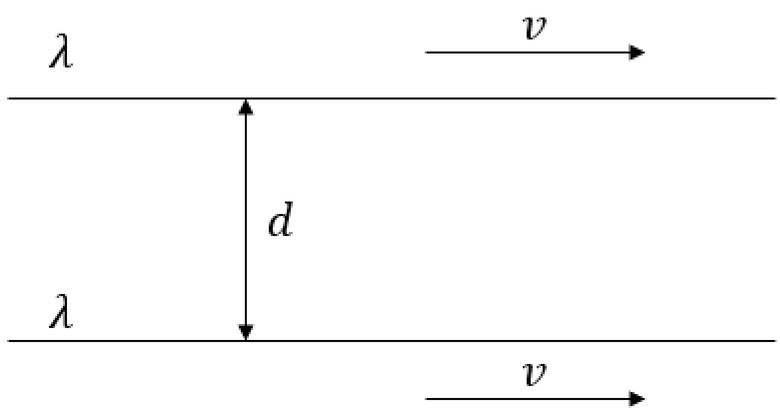
$\frac{1}{2}T = \frac{\pi}{\omega_0}$

$x\left(\frac{\pi}{\omega_0}\right) = \frac{q}{m}\frac{V_0}{d}\frac{1}{\omega_0}\left(\frac{\pi}{\omega_0} - \frac{1}{\omega_0}\underbrace{\sin(\pi)}_{=0}\right)$

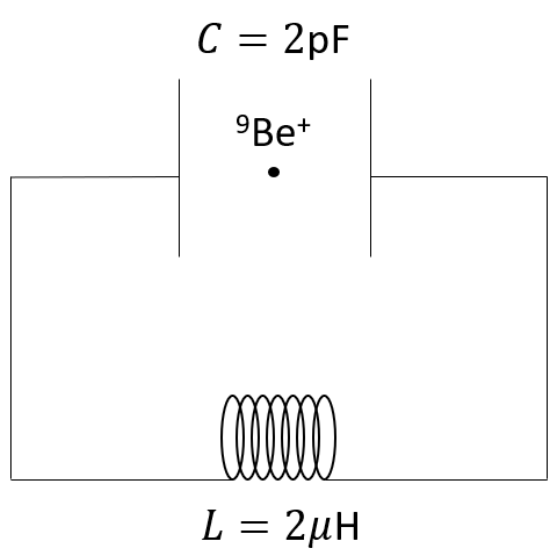
$= \frac{q}{m}\frac{V_0}{d}\frac{\pi}{\omega_0^2} = \frac{1.602\times10^{-19}C}{9\times1.66\times10^{-27}kg}\frac{100V}{10^{-3}m}\frac{\pi}{(500\times10^6)^2} \Rightarrow \boxed{\Delta x = 13.47\mu m}$



2. Suppose you have two infinite straight-line charges λ, a distance d apart, moving in the same direction at constant speed. (see figure below) What speed would be required in order for the magnetic attraction to balance out the electrostatic force of repulsion? Is this value reasonable?



3. A radiofrequency ion trap can roughly be modeled as a parallel plate capacitor connected to an inductor. An ion can be placed in between the plates of the capacitor and it will experience a force due to the electric field between the plates. As this electric field changes direction, the ion will be pushed towards one plate and then another and it will oscillate around the midplane of the capacitor. (see figure below)
   a. Calculate the resonant frequency of this *L* – *C* circuit.
   b. Assume that the distance between the capacitor plates is 1 mm. If the trap is initially charged to 100 V, calculate the magnitude of the electric field inside the trap.
   c. If the ion starts from rest at the center of the trap, how far will it be displaced during one half period of the voltage oscillation? (HINT: Think about the average electric field over one half cycle of the oscillation. This average field leads to an average force, etc.)



4. A solenoidal coil with 35 turns of wire is wound tightly around another coil with 500 turns. The inner solenoid has radius R_1 and length L_1. The initial current in the inner solenoid is I_1 and the current is increasing with rate dI_1/dt.
   a. Derive an expression for the average magnetic flux through each turn of the inner solenoid.
   b. Calculate the mutual inductance of the two solenoids.
   c. If $R_1 = 2.5$ cm, $L_1 = 1.0$ m, $I_1 = 0.1$ A, and $dI_1/dt = 1600.0$ A/s, calculate the emf in the outer solenoid due to the changing current in the inner solenoid.

a) $B = \mu_0 n_1 I_1 = \mu_0 \frac{N_1}{L_1} I_1$ $\quad \Phi_1 = B_1 A_1 = \mu_0 \frac{N_1}{L_1} I_1 (\pi R_1^2)$ $\quad \boxed{\Phi_1 = \mu_0 \pi R_1^2 \frac{N_1}{L_1} I_1}$

b) $M = \frac{N_2 \Phi_1}{I_1} \Rightarrow \boxed{M = \frac{\mu_0 \pi R_1^2}{L_1} N_1 N_2}$ $\quad$ c) $\mathcal{E}_2 = -M \frac{dI_1}{dt} = -\frac{\mu_0 \pi R_1^2}{L_1} N_1 N_2 \frac{dI_1}{dt}$

$\mathcal{E}_2 = \boxed{69.1\,\text{mV}}$