Practice Problems in Linear Algebra

Rank, Consistency, and Gauss Elimination

Prepared for Practice

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Contents

Chapter 1

Rank of a Matrix

Problems

Problem 1.1. *Find the rank of* $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 1 & 1 \end{bmatrix}$.

Problem 1.2. *Find the rank of* $\begin{bmatrix} 2 & -1 & 3 \\ 4 & -2 & 6 \\ 1 & 0 & 1 \end{bmatrix}$.

Problem 1.3. *Find the rank of* $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

Problem 1.4. *Find the rank of* $\begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$.

Problem 1.5. *Find the rank of* $\begin{bmatrix} 3 & 6 & 9 \\ 2 & 4 & 8 \\ 1 & 2 & 3 \end{bmatrix}$.

Problem 1.6. *Find the rank of* $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 3 & 6 & 3 \end{bmatrix}$.

Problem 1.7. *Find the rank of* $\begin{bmatrix} 2 & 3 & 1 \\ 4 & 7 & 2 \\ 1 & 1 & 0 \end{bmatrix}$.

Problem 1.8. *Find the rank of* $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}$.

Problem 1.9. *Find the rank of* $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{bmatrix}$.

Problem 1.10. *Find the rank of* $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$.

Chapter 2

Consistency of Systems

Problems

Problem 2.1. *Solve consistency for system:*

$$\begin{aligned} x + y + z &= 6 \\ 2x + 3y + z &= 10 \\ x + 2y + \lambda z &= \mu \end{aligned}$$

Problem 2.2. *For what values of λ does the system* $\begin{cases} x + 2y = 3 \\ 2x + 4y = \lambda \end{cases}$ *have unique, infinite, or no solution?*

Problem 2.3. *Check consistency of:*

$$\begin{aligned} x + y + z &= 1 \\ 2x + 3y + z &= 2 \\ 3x + 4y + z &= 3 \end{aligned}$$

Problem 2.4. *Find conditions for λ for:*

$$\begin{aligned} x + y &= 2 \\ 2x + 2y &= \lambda \end{aligned}$$

Problem 2.5. *Check consistency of:*

$$\begin{aligned} x + y + z &= 3 \\ 2x + 3y + z &= 5 \\ 3x + 4y + z &= 6 \end{aligned}$$

Problem 2.6. *Find consistency for system:*

$$\begin{aligned} x + y + z &= \mu \\ x + 2y + 3z &= \lambda \\ 2x + 3y + 4z &= 5 \end{aligned}$$

Problem 2.7. *Check for solution types:*

$$\begin{aligned} x + y &= 1 \\ 2x + 2y &= \lambda \end{aligned}$$

Problem 2.8. *Find λ, μ for consistency:*

$$\begin{aligned} x + y + z &= \lambda \\ 2x + 3y + 4z &= \mu \\ 3x + 5y + 6z &= 10 \end{aligned}$$

Problem 2.9. *Check consistency:*

$$\begin{aligned} x - y + z &= 1 \\ 2x - 2y + 2z &= 2 \\ 3x - 3y + 3z &= 3 \end{aligned}$$

Problem 2.10. *Find λ for:*

$$\begin{aligned} x + 2y + 3z &= 1 \\ 2x + 4y + 6z &= 2 \\ 3x + 6y + 9z &= \lambda \end{aligned}$$

Chapter 3

Gauss Elimination Method

Problems

Problem 3.1. *Solve using Gauss elimination:*

$$\begin{aligned} x + y + z &= 6 \\ 2x + 5y + z &= 10 \\ 2x + 3y + 4z &= 13 \end{aligned}$$

Problem 3.2. *Solve using Gauss elimination:*

$$\begin{aligned} 2x + y - z &= 8 \\ -3x - y + 2z &= -11 \\ -2x + y + 2z &= -3 \end{aligned}$$

Problem 3.3. *Solve using Gauss elimination:*

$$\begin{aligned} x + y &= 2 \\ 2x - y &= 0 \end{aligned}$$

Problem 3.4. *Solve using Gauss elimination:*

$$\begin{aligned} x + y + z &= 3 \\ 2x + 3y + z &= 5 \\ 3x + 4y + 2z &= 7 \end{aligned}$$

Problem 3.5. *Solve using Gauss elimination:*

$$\begin{aligned} x - y + z &= 2 \\ 2x + y - z &= 3 \\ 3x + 2y + z &= 6 \end{aligned}$$

Problem 3.6. *Solve using Gauss elimination:*

$$\begin{aligned} 2x + 3y + z &= 1 \\ 4x + 7y + 3z &= 2 \\ 6x + 9y + 4z &= 3 \end{aligned}$$

Problem 3.7. *Solve using Gauss elimination:*

$$\begin{aligned} x + y &= 5 \\ x - y &= 1 \end{aligned}$$

Problem 3.8. *Solve using Gauss elimination:*

$$\begin{aligned} x + 2y + z &= 4 \\ 2x + 5y + z &= 9 \\ 3x + 6y + 2z &= 12 \end{aligned}$$

Problem 3.9. *Solve using Gauss elimination:*

$$\begin{aligned} 2x + y &= 5 \\ 4x + 2y &= 10 \end{aligned}$$

Problem 3.10. *Solve using Gauss elimination:*

$$\begin{aligned} x + y + z &= 1 \\ 2x + 2y + 2z &= 2 \\ 3x + 3y + 3z &= 3 \end{aligned}$$