One-Way ANOVA with Repeated Measures

Suppose we wish to determine whether a set of sample means $(\bar{X}_1, \bar{X}_2, \cdots)$ show a statistically significant difference when the samples are dependent. Similar to the one-way ANOVA with independent measures, for a one-way ANOVA with *repeated measures* from k groups we want to quantify variability between groups and variability within groups. However, with repeated measures, we can now attribute some of the within groups variability to individual differences! Rather than using "within groups" variability in the denominator we will use "error" variability:

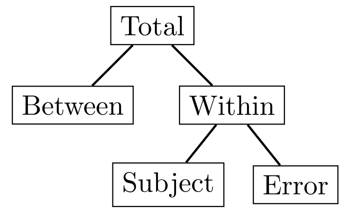

Figure 1: Partitioning the Sum of Squares for the One-Way ANOVA with Repeated Measures

$$F = \frac{SS_{\text{between groups}}/df_{\text{between groups}}}{SS_{\text{error}}/df_{\text{error}}}$$

$$SS_{\text{between groups}} = \sum_{\text{groups}} n_i(\bar{X}_i - \bar{X}_{\text{All}})^2 \qquad df_{\text{between groups}} = k - 1$$

$$SS_{\text{within groups}} = \sum_{\text{groups}} \sum_{\text{scores}} (X - \bar{X}_i)^2 \qquad df_{\text{within groups}} = N - k$$

$$SS_{\text{between subjects}} = k \sum_{\text{subjects}} (\bar{X}_s - \bar{X}_{\text{All}})^2 \qquad df_{\text{between subjects}} = n - 1$$

$$SS_{\text{error}} = SS_{\text{within groups}} - SS_{\text{between subjects}} \qquad df_{\text{error}} = (n-1)(k-1)$$

The notation is as before, but now we also have "subject means" $\bar{X}_s$. Just like the independent measures ANOVA, we can take advantage of the fact that variability can be "partitioned" to make the calculations easier. Here we have two levels of partitioning: first, the total variability, which is exactly as it was partitioned before ...

$$SS_{\text{total}} = \sum (X - \bar{X}_{\text{All}})^2$$
$$SS_{\text{total}} = SS_{\text{between groups}} + SS_{\text{within groups}}$$
$$df_{\text{total}} = df_{\text{between groups}} + df_{\text{within groups}} = N - 1$$

Then as the second level, we further partition the "within groups" variability...

$$SS_{\text{within groups}} = SS_{\text{between subjects}} + SS_{\text{error}}$$
$$df_{\text{within groups}} = df_{\text{between subjects}} + df_{\text{error}}$$

Finally, the effect size, Variance Accounted For, with the repeated measures ANOVA also has the individual differences removed

$$\eta^2 = \frac{SS_{\text{between groups}}}{SS_{\text{between groups}} + SS_{\text{error}}}$$

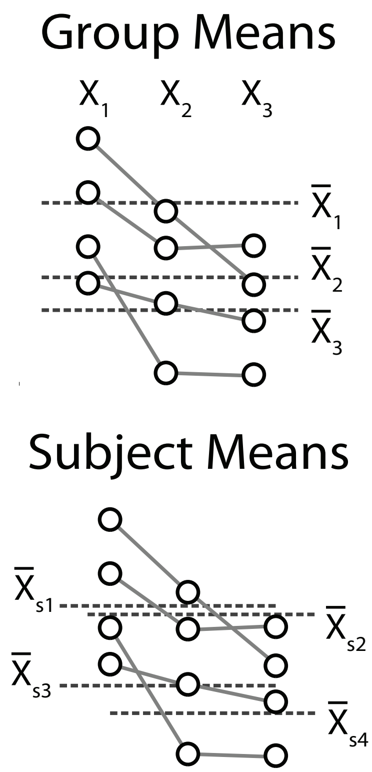

Figure 2: With repeated measures anova, we control for individual differences by using the mean for each subject.

Example

Suppose you collect data using a repeated-measures design with 5 participants in three different conditions, and observe the data below. What is the value for the test statistic F?

	Condition 1	Condition 2	Condition 3	$\bar{X}_s$
Subject 1	5	2	2	
Subject 2	4	2	3	
Subject 3	7	3	5	
Subject 4	4	2	3	
Subject 5	0	1	2	
$\bar{X} =$				
$SS =$				

Source	SS	df	MS	F
Between Groups				
Within Groups				
Between Subjects				
Error				
Total				

Solution

	Condition 1	Condition 2	Condition 3	$\bar{X}_s$
Subject 1	5	2	2	3
Subject 2	4	2	3	3
Subject 3	7	3	5	5
Subject 4	4	2	3	3
Subject 5	0	1	2	1
$\bar{X} =$	4	2	3	
$SS =$	26	2	6	

Let's start by figurout out $SS_{\text{between groups}}$. From the table, we see we have $n = 5$ subjects so $N = kn = 3 \cdot 5 = 15$. After calculating the group means, the one thing we are missing is the "grand mean" $\bar{X}_{\text{All}} = (5+4+7+\cdots)/15 = 3$ Now we can fill in

$$SS_{\text{between groups}} = \sum_{\text{groups}} n_i(\bar{X}_i - \bar{X}_{\text{All}})^2 = 5(4-3)^2 + 5(2-3)^2 + 5(3-3)^2 = 10$$
$$df_{\text{between groups}} = k - 1 = 3 - 1 = 2$$

Now let's figure out $SS_{\text{within groups}}$

$$SS_{\text{within groups}} = \sum_{\text{groups}} \sum_{\text{scores}} (X - \bar{X}_i)^2 = \sum_{\text{groups}} SS_i = 26 + 2 + 6 = 34$$
$$df_{\text{within groups}} = N - k = 15 - 3 = 12$$

Now let's figure out $SS_{\text{between subjects}}$

$$SS_{\text{between subjects}} = k \sum_{\text{subjects}} (\bar{X}_s - \bar{X}_{\text{All}})^2 = 3[(3-3)^2 + (3-3)^2 + (5-3)^2 + (3-3)^2 + (1-3)^2] = 3[8] = 24$$
$$df_{\text{between groups}} = n - 1 = 5 - 1 = 4$$

Finally, to figure out SS_{error}, we'll use the formulas for partitioning the sum of squares ...

$$SS_{\text{error}} = SS_{\text{within groups}} - SS_{\text{between subjects}} = 34 - 24 = 10$$
$$df_{\text{error}} = (n-1)(k-1) = (5-1)(3-1) = (4)(2) = 8$$

Source	SS	df	MS	F
Between Groups	10	2	5	F=4
Within Groups	34	12		
Between Subjects	24	4		
Error	10	8	1.25	
Total	44	14		