Two-Way ANOVA with Independent Measures [1]

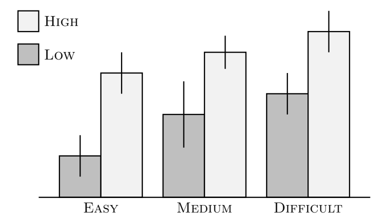
[1] equal group sizes

For a two-way experimental design, we consider three separate tests to determine 1) if Factor A has a main effect, 2) if Factor B has a main effect, and 3) if there is an interaction (AxB). Just like the other ANOVAs, these are all F-tests, with slightly different F-ratios

$$F_A = \frac{SS_A/df_A}{SS_{within}/df_{within}} \qquad F_B = \frac{SS_B/df_B}{SS_{within}/df_{within}} \qquad F_{AxB} = \frac{SS_{AxB}/df_{AxB}}{SS_{within}/df_{within}}$$

Calculating the F-Ratios

Here, for simplicity, we'll focus on the case where all of the conditions have exactly the same number of participants, which we'll call r (this replaces the sample size n). We'll use a and b to denote the number of levels for Factor A and the number of levels for Factor B, respectively. For a 2x3 design, we would have $a = 2$ and $b = 3$. With this new notation we have the following SS and df calculations



$$SS_{between} = r\sum_i (\bar{X}_i - \bar{X}_{all}) \qquad df_{between} = ab - 1$$

$$SS_{within} = \sum_i SS_i \qquad df_{within} = ab(r-1)$$

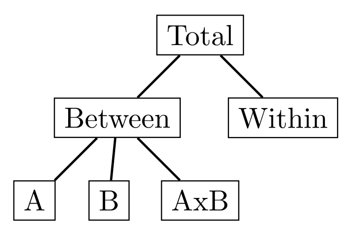
$$SS_A = rb\sum_i \left(\bar{X}_{A_i} - \bar{X}_{all}\right) \qquad df_A = a - 1$$

$$SS_B = ra\sum_i \left(\bar{X}_{B_i} - \bar{X}_{all}\right) \qquad df_B = b - 1$$

$$SS_{AxB} = SS_{total} - SS_{within} - SS_A - SS_B \qquad df_{AxB} = (a-1)(b-1)$$

Partitioning the Sum of Squares

Just like the one-way ANOVAs, a helpful tool for understanding these equations is the idea that variability can be *partitioned* into different sources. Here we have two levels of partitioning:



$$SS_{total} = SS_{between} + SS_{within} \qquad df_{total} = df_{between} + df_{within}$$

$$SS_{between} = SS_A + SS_B + SS_{AxB} \qquad df_{between} = df_A + df_B + df_{AxB}$$

Effect Sizes

The variance explained for each test is given by

$$\eta^2_A = \frac{SS_A}{SS_A + SS_{within}} \qquad \eta^2_B = \frac{SS_B}{SS_B + SS_{within}} \qquad \eta^2_{AxB} = \frac{SS_{AxB}}{SS_{AxB} + SS_{within}}$$

Example

Suppose you collect data using a 2x3 factorial design with 5 participants in each condition, and observe the data below. What are the values for the test statistics: F_A, F_B and F_{AxB}?

	Low	Medium	High
Easy	$\bar{X} = 3$	$\bar{X} = 5$	$\bar{X} = 10$
	$SS = 18$	$SS = 28$	$SS = 26$
Difficult	$\bar{X} = 1$	$\bar{X} = 3$	$\bar{X} = 2$
	$SS = 8$	$SS = 20$	$SS = 20$

Source	SS	df	MS	F
Between Groups				
Factor A				
Factor B				
Factor AxB				
Within Groups				
Total				

Solution

From the description we know that $r = 5$. Let's assume that the Easy/Difficult levels correspond to Factor A so that $a = 2$, and the Low/Medium/High levels correspond to Factor B so that $b = 3$. Once we define our factors, we can find the marginal means (e.g. $\bar{X}_{A_1}, \bar{X}_{A_2}, \cdots$) and grand mean $\bar{X}_{all}$

	Low	Medium	High	$\bar{X}_A$
Easy	$\bar{X} = 3$	$\bar{X} = 5$	$\bar{X} = 10$	6
	$SS = 18$	$SS = 28$	$SS = 26$	
Difficult	$\bar{X} = 1$	$\bar{X} = 3$	$\bar{X} = 2$	2
	$SS = 8$	$SS = 20$	$SS = 20$	
$\bar{X}_B$	2	4	6	$\bar{X}_{all} = 4$

Now we have all the ingredients to calculate

$$SS_A = rb \sum_i \left(\bar{X}_{A_i} - \bar{X}_{all}\right) = 5 \cdot 3 \left[(6-4)^2 + (2-4)^2\right] = 120 \qquad df_A = a - 1 = 2 - 1 = 1$$

$$SS_B = ra \sum_i \left(\bar{X}_{B_i} - \bar{X}_{all}\right) = 5 \cdot 2 \left[(2-4)^2 + (4-4)^2 + (6-4)^2\right] = 80 \qquad df_B = b - 1 = 3 - 1 = 2$$

Next, we need to figure out $SS_{between}$ and SS_{within}

$$SS_{between} = r \sum_i (\bar{X}_i - \bar{X}_{all}) = 5 \left[(3-4)^2 + (5-4)^2 + (10-4)^2 + (1-4)^2 + (3-4)^2 + (2-4)^2\right] = 160$$

$$SS_{within} = \sum_i SS_i = 18 + 28 + 26 + 8 + 20 + 20 = 120$$

$$df_{between} = ab - 1 = 2 \cdot 3 - 1 = 5$$

$$df_{within} = ab(r-1) = 2 \cdot 3(5-1) = 24$$

Finally, we can use the partitioning to find SS_{AxB}

$$SS_{AxB} = SS_{total} - SS_{within} - SS_A - SS_B = (160 + 120) - 120 - 120 - 80 = 60$$

$$df_{AxB} = (a-1)(b-1) = (2-1)(3-1) = 2$$

Source	SS	df	MS	F
Between Groups	260	5		
Factor A	120	1	120	F(1,24)=24
Factor B	80	2	40	F(2,24)=8
Factor AxB	60	2	30	F(2,24)=6
Within Groups	120	24	5	
Total	380	29		

Further Questions

Are these F-ratios statistically significant? What are the effect sizes?