Introduction to Data Analytics

T5 Bootcamp by SDAIA

SDAIA
الهيئة السعودية للبيانات والذكاء الاصطناعي
Saudi Data & AI Authority

Linear Algebra

Let's start together…

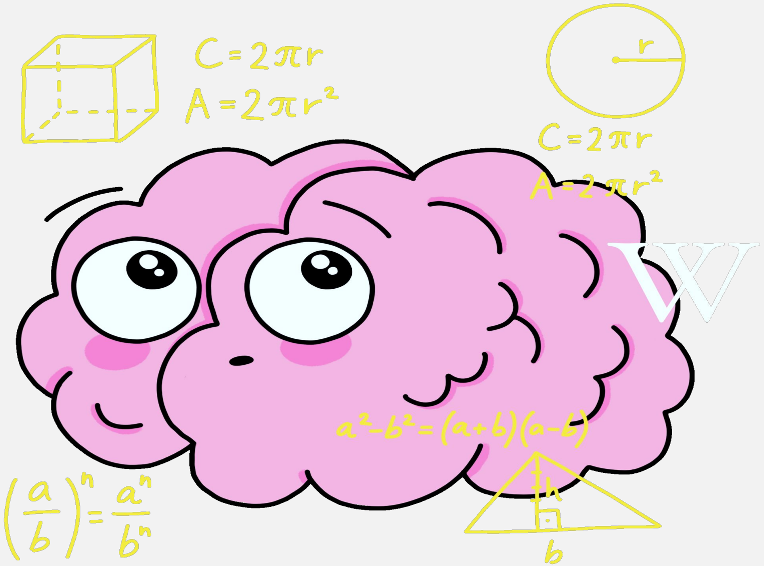
Agenda

1. Introduction to Linear Algebra
2. What is the role of Linear Algebra in Machine Learning?
3. Key areas of Linear Algebra
4. Scaler, Vector, Matrix and Tensor
5. Matrix and Vector operations
6. Linear Independence of Vectors
7. Eigenvector and Eigenvalues
8. Exercise Time

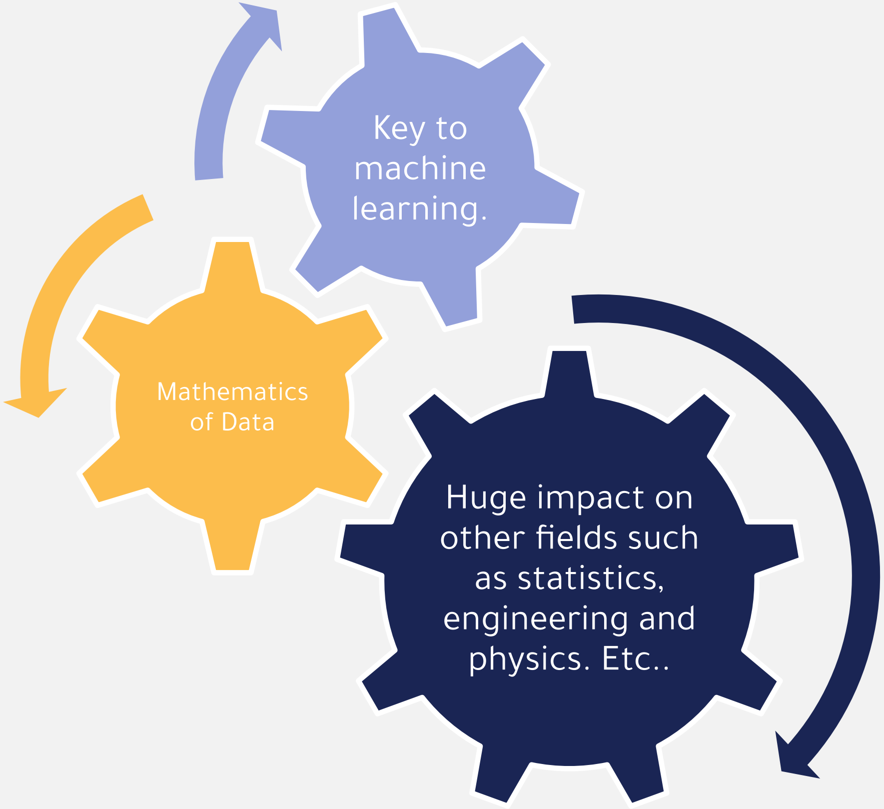
Introduction to Linear Algebra

Linear algebra is a sub-field of mathematics concerned with vectors, matrices, and operations on these data structures.

It provides a framework for representing and solving systems of linear equations and studying geometric concepts such as lines, planes, and their higher-dimensional counterparts.



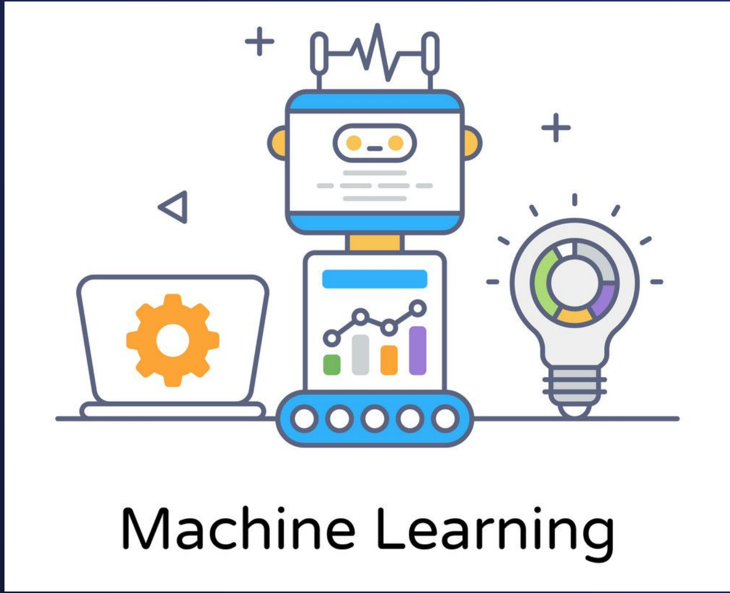
Introduction to Linear Algebra



What is the role of Linear Algebra in Machine Learning?

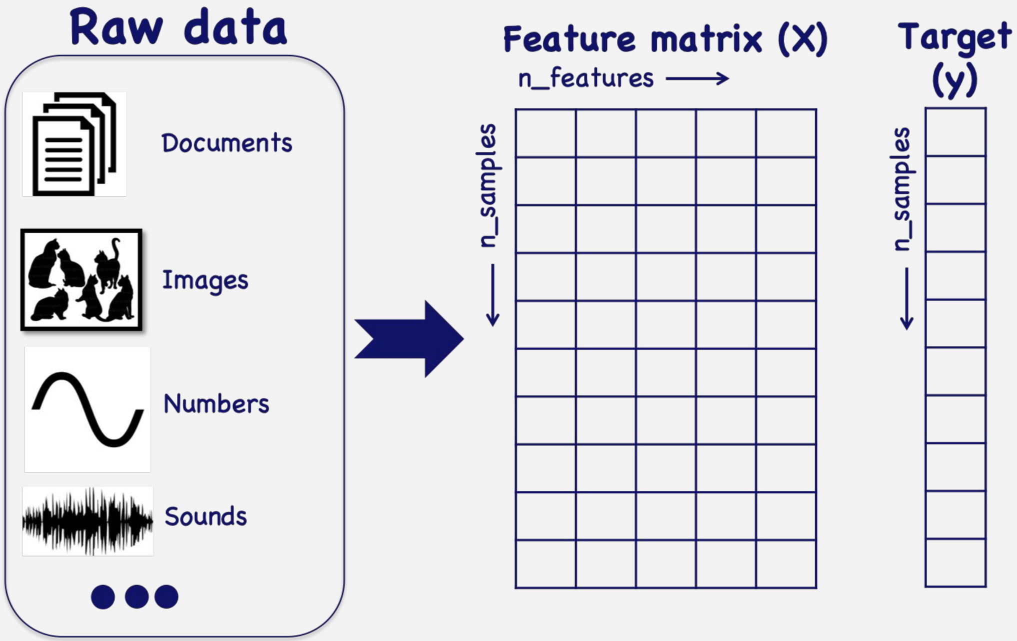
Machines or computers only understand numbers.

- Unlike traditional programming, where instructions are predefined, machine learning operates by extracting **patterns** from data.



What is the role of Linear Algebra in Machine Learning?

Numerical representation of data facilitates machine understanding and problem-solving.



What is the role of Linear Algebra in Machine Learning?

REASONS FOR LEARNING LINEAR ALGEBRA BEFORE MACHINE LEARNING

To gain a better understanding of how the different machine learning algorithms really work under the hood.

Better Machine Learning Algorithms

Enhanced graphic processing

Improving Statistics

What is the role of Linear Algebra in Machine Learning?

REASONS FOR LEARNING LINEAR ALGEBRA BEFORE MACHINE LEARNING

Mastery of relevant linear algebra concepts is essential for becoming proficient in Data Science and Machine Learning field.

What exactly that I need to learn to be a successful Data Scientist?

Key areas of Linear Algebra

A. NOTATIONS

Notation in linear algebra enables you to read algorithm descriptions in papers, books, and websites to understand the algorithm's working.

$$\vec{x}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Key areas of Linear Algebra

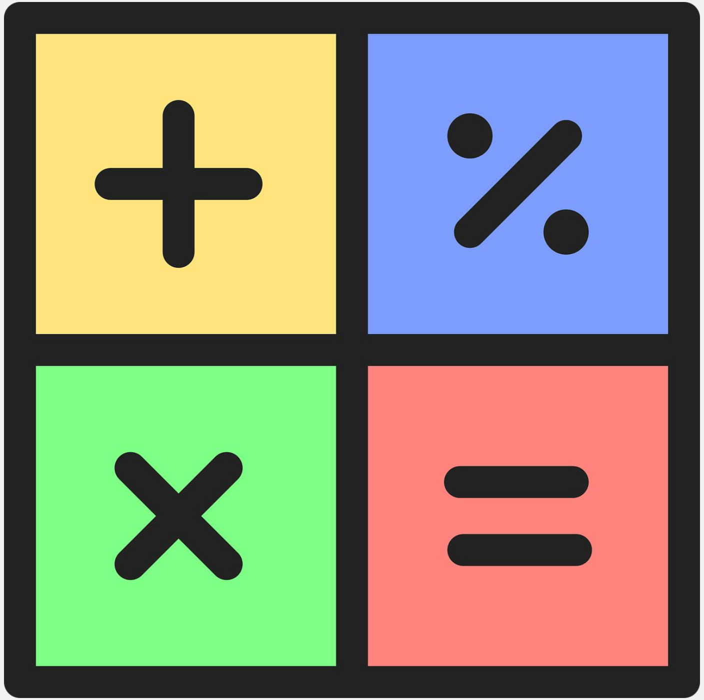
B. OPERATIONS → العمليات

Vectors and matrices can make concepts clearer, and it can also help in the description, coding, and even thinking capability.

In linear algebra, it is required to learn the basic operations such as **addition, multiplication,** etc.

عموماً العمليات الحسابية:

جمع، طرح، ضرب



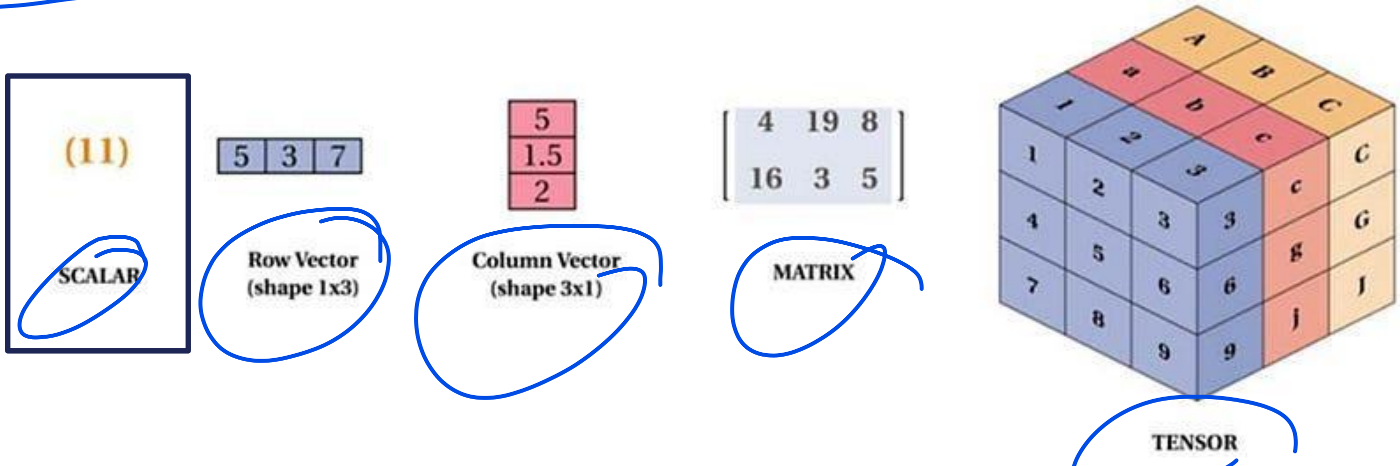
Key areas of Linear Algebra

C. MATRIX FACTORIZATION → تمييز العمليات

- Matrix factorization is commonly utilized in solving systems of linear equations, image and signal denoising, and reducing the dimensionality of data for analysis and visualization purposes.

Scaler, Vector, Matrix and Tensor

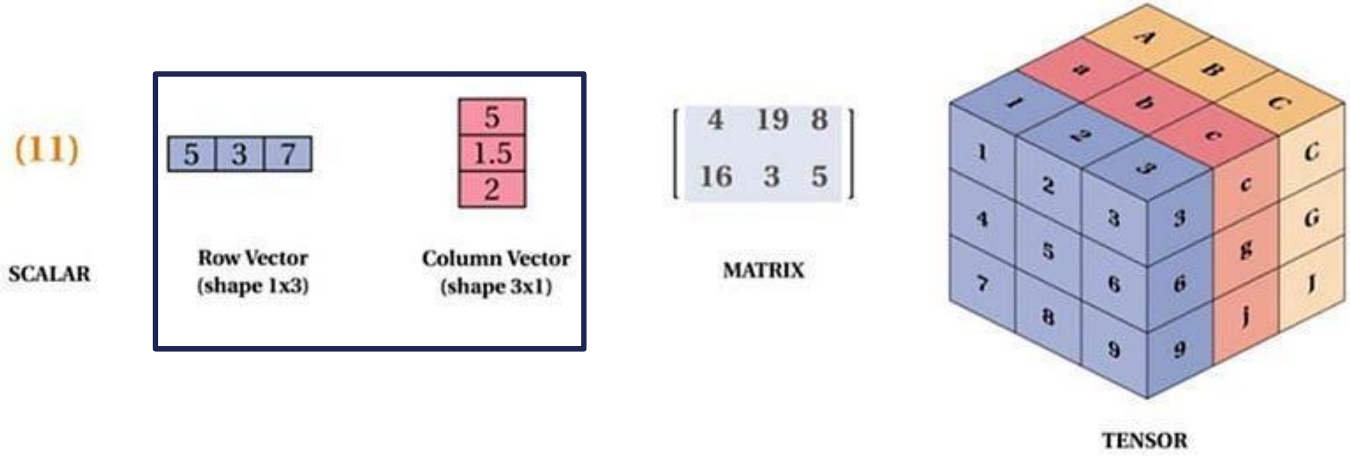
SCALAR



- A single point or a single data is called a scalar. It cannot be divided further.
- In the figure above 11 is a scalar, which is a single data. Scalars are zero-dimensional.

Scaler, Vector, Matrix and Tensor

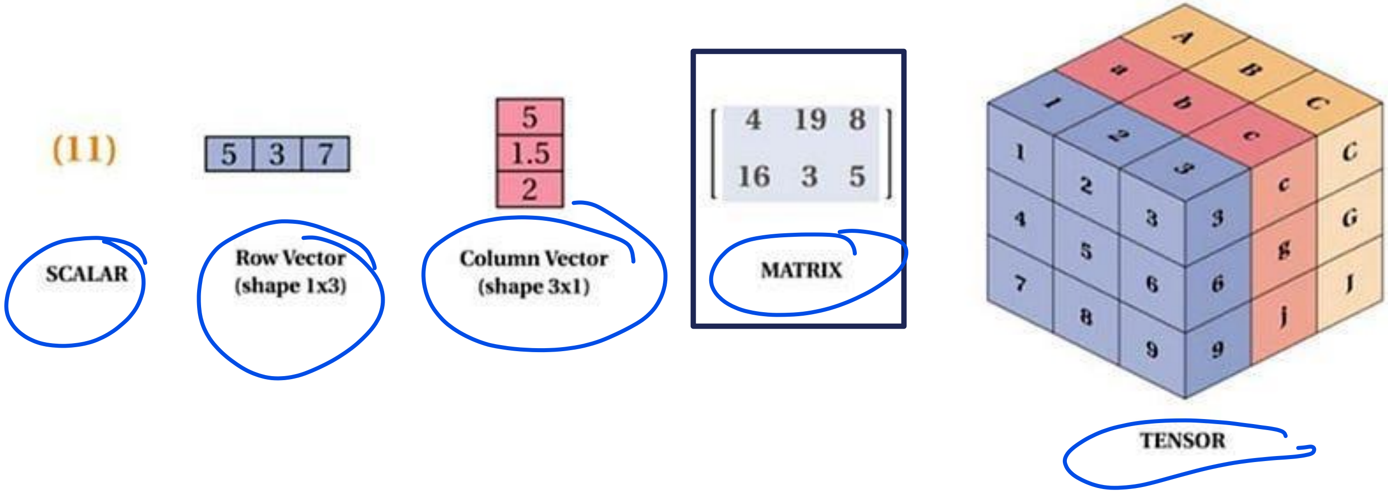
VECTOR



- If we have multiple scalars, then it is called a vector.
- Vectors are of 2 types. Row vector which is horizontally filled and Column Vector which is vertically filled. Vectors are one-dimensional.

Scaler, Vector, Matrix and Tensor

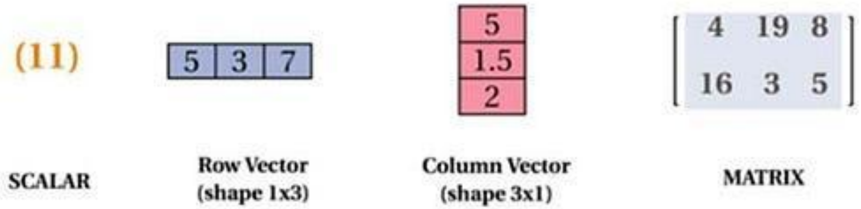
MATRIX

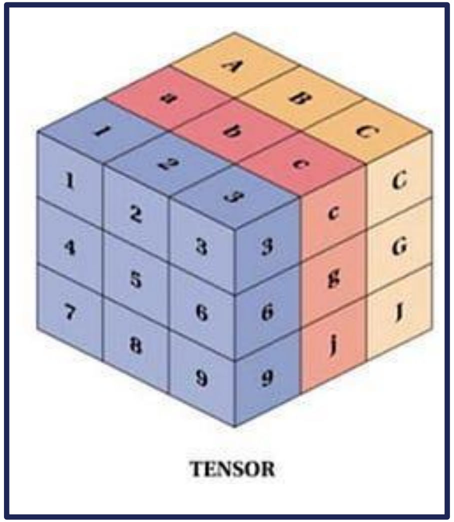


- A matrix is a group of vectors. It has a vector, filled horizontally and a vector filled vertically.
- Matrices(plural of matrix) are two-dimensional. A table is a common example of matrix.

Scaler, Vector, Matrix and Tensor

TENSOR





- A tensor is a group of matrices which is three-dimensional.
- A common example is Rubik's cube which is 3D.

Matrix and Vector operations

WHAT ARE MATRIX OPERATIONS?

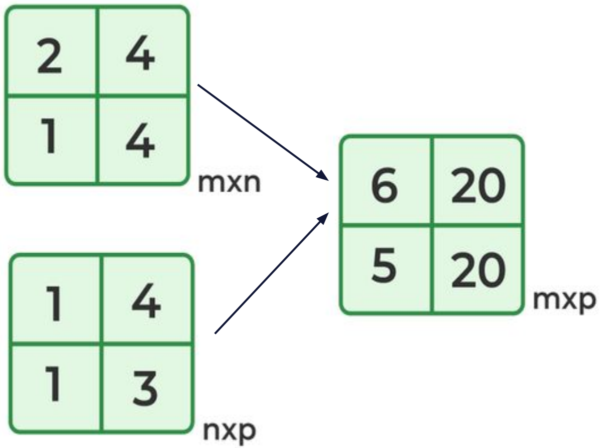
Matrix operations are the operations that are used to combine various matrices to form a single matrix.

The following are some of the important matrix operations.

- Addition - matrix operations
- Subtraction - matrix operations
- Multiplication - matrix operations
- Transpose operation of a matrix أعكس صفوف أعمدة
- Inverse operation of a matrix

م تحويلة

ما بعرف



Matrix and Vector operations

MATRIX-SCALAR OPERATIONS

ضرب العناصر

If you multiply a scalar to a matrix, you do so with every element of the matrix.

The image below illustrates this perfectly for multiplication:

$$\underset{\substack{\uparrow \\ \text{scalar}}}{3} \quad \cdot \quad \underset{\substack{\uparrow \\ \text{matrix}}}{\begin{pmatrix} 1 & 2 \\ -5 & -1 \end{pmatrix}}$$

Matrix and Vector operations

MATRIX-VECTOR MULTIPLICATION

Multiplying a matrix by a vector can be thought of as multiplying each row of the matrix by the column of the vector.

The output will be a vector that has the same number of rows as the matrix.

$$\underbrace{\begin{bmatrix} 1 & 0 & 3 \\ 3 & 1 & 1 \end{bmatrix}}_{\text{Matrix}} \times \underbrace{\begin{bmatrix} 7 \\ 1 \\ 2 \end{bmatrix}}_{\text{Vector}} = \begin{bmatrix} 13 \\ 24 \end{bmatrix}$$

Matrix and Vector operations

MATRIX-MATRIX ADDITION AND SUBTRACTION

The matrices must have the same dimensions and the result is a matrix that has also the same dimensions.

You add or subtract each value of the first matrix with its corresponding value in the second matrix.

$$\begin{bmatrix} 8 & 5 \\ 2 & 3 \end{bmatrix} \overset{+}{+} \begin{bmatrix} 9 & 5 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 17 & 10 \\ 3 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 5 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 9 & 5 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix}$$

لازم تكون
ليهم - الحجم
نفس ال size

Matrix and Vector operations

م الحجم لازم

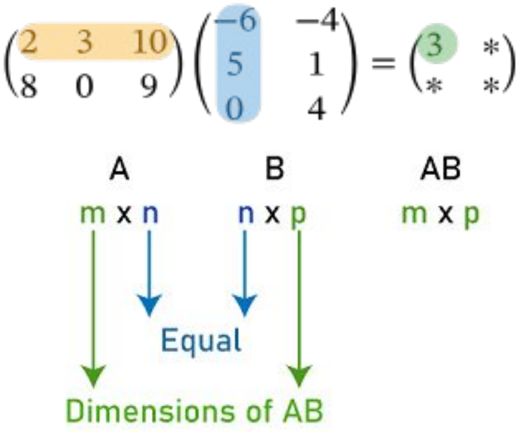
نفس

عدد

MATRIX-MATRIX MULTIPLICATION

You can only multiply matrices together if the number of the first matrix's columns matches the number of the second matrix's rows.

The result will be a matrix with the same number of rows as the first matrix and the same number of columns as the second matrix.



و تختلف

في النواتج

Matrix and Vector operations

TRANSPOSE OPERATION OF A MATRIX

The transpose of matrix A is an operator that flips a matrix over its diagonal. In other words, it switches the row and column indices of a matrix. This operation produces another matrix of order $n \times m$ denoted by A^T

$$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}_{2 \times 3} \qquad A^T = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}_{3 \times 2}$$

Matrix and Vector operations

IDENTITY MATRIX

The number 1 is an identity because everything you multiply with 1 is equal to itself. Every matrix that is multiplied by an identity matrix is equal to itself.

$1 \times 1 \quad [1]$

$2 \times 2 \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$3 \times 3 \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

etc.

Matrix and Vector operations

INVERSE OPERATION OF A MATRIX

عملية التحويل

For any matrix A its inverse is found only when A is a square matrix and its determinant is equal to 1, such that:

$A \times A^{-1} = A^{-1} \times A = I$, where I is the identity matrix. $= I$

الحدود الأول نقلبهم
الثاني نغير الإشارة

For a 2x2 matrix the inverse is:

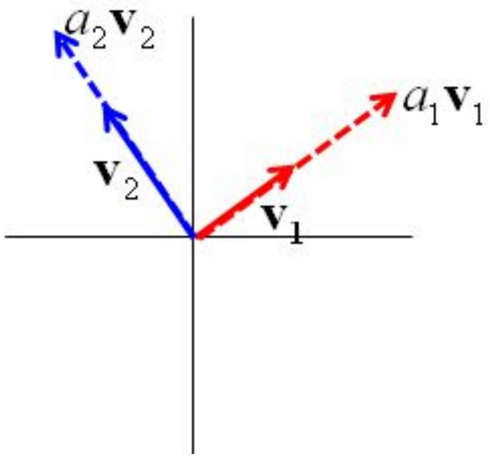
مهم

$$\underbrace{\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1}}_{A^{-1}} = \underbrace{\frac{1}{ad-bc}}_{\text{Determinant}} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Linear Independence of Vectors

A set of vectors is said to be linearly independent if no vector in the set can be expressed as a linear combination of the other vectors in the same set.

الفرق بين



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Linear Independence of Vectors

Let S be a linear space. Some vectors $x_1, x_2, x_3, \ldots \in S$ are said to be linearly independent if and only if they are not linearly dependent.

It follows from this definition that, in the case of linear independence,

Implies

$$\alpha_1 x_1 + \alpha_2 x_2 + \ldots + \alpha_n x_n = 0$$

$$\alpha_1 = \ldots = \alpha_n = 0$$

In other words, when the vectors are linearly independent, their only linear combination that gives the zero vector as a result has all coefficients equal to zero.

المعاملات لازم تكون كلها
أصفار

Linear Independence of Vectors

Example Let x_1 and x_2 be 2x1 column vectors defined as follows:

$$x_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad x_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Consider a linear combination of these two vectors with coefficients a_1 and a_2

Linear Independence of Vectors

إذا كانت صفر مستقلة

إذا لا غير مستقلة

Example Let x_1 and x_2 be 2x1 column vectors defined as follows:

Consider a linear combination of these two vectors with coefficients a_1 and a_2

$$\alpha_1 x_1 + \alpha_2 x_2$$

$$= \alpha_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \alpha_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \alpha_1 \cdot 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \alpha_2 \cdot 1 \end{bmatrix}$$

$$= \begin{bmatrix} \alpha_1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \alpha_2 \end{bmatrix}$$

$$= \begin{bmatrix} \alpha_1 + 0 \\ 0 + \alpha_2 \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}$$

Then, we have that

$$\alpha_1 x_1 + \alpha_2 x_2 = 0$$

if and only if

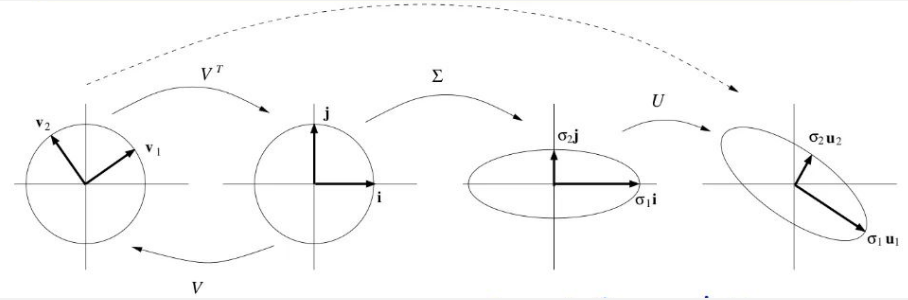
$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

مستقلة

Eigenvalue and Eigenvector

Eigenvalues are associated with eigenvectors in Linear algebra.

The word 'Eigen' is of German Origin which means 'characteristic'. The ese are the characteristic value that indicates the factor by which eigenvectors are stretched in their direction.



Eigenvalue and Eigenvector

Eigenvalues and Eigenvectors and their use in Machine Learning and AI

- In machine learning, eigenvalues and eigenvectors are used to represent, perform operations on data, and to train machine learning models.

- They are also used to develop algorithms for tasks such as image recognition, natural language processing, and robotics.

Eigenvalue and Eigenvector

= رمز

What is Eigenvectors Formula?

The eigenvector of any matrix is calculated using the formula,

$$Av = \lambda v$$

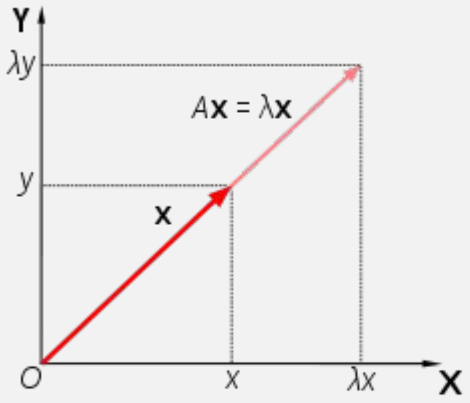
القانون

where,

رمز = λ is the eigenvalue

v is the eigenvector

= مصفوفة



Eigenvalue and Eigenvector

Examples of EigenValue and EigenVector Applications:

Principal Component Analysis (PCA) is a widely used to reduce the number of features while retaining as much information as possible.

الهدف هنا نقلل من التركيبات

Singular Value Decomposition (SVD) is a technique for image compression commonly used in applications like image storage and transmission.

في التخزين وتحويل الصور

Thank you

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