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Chapter 5

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Section 5.1

1 --------------------------------Done

8 --------------------------------Done

18 ------------------------------Done

27 ------------------------------Done

35 ------------------------------Done

44 ------------------------------Done

51 ------------------------------Done

53 ------------------------------Done

61 ------------------------------Done

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Section 5.2

3 --------------------------------Done

8 --------------------------------Done

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20 ------------------------------Done

23 ------------------------------Done

28 ------------------------------Done

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33 ------------------------------Done

37 -----------------------NOT-Done

Section 5.6

3 --------------------------------Done

7 --------------------------------Done

13 ------------------------------Done

17 ------------------------------Done

20 ------------------------------Done

24 ------------------------------Done

27 ------------------------------Done

33 --------------------------NOT-Done

39 --------------------------NOT-Done

44 --------------------------NOT-Done

Section 5.7

5 -------------------------------Done

10 ------------------------------Done

14 ------------------------------Done

19 ------------------------------Done

25 ------------------------------Done

30 ------------------------------Done

35 ------------------------------Done

39 ------------------------------Done

45 --------------------------NOT-Done

50 --------------------------NOT-Done

Write the first four terms of the sequence defined by the formulas in 1-6

1. $a_k = \frac{k}{10+k}$ for all $k \geq 1$

Answers $a_1 = \frac{1}{11}$

$a_2 = \frac{2}{12} = \frac{1}{6}$

$a_3 = \frac{3}{13}$

$a_4 = \frac{4}{14} = \frac{2}{7}$

Compute the first fifteen terms of each of the sequences in 8 and 9, and describe the general behavior of these sequences in word. (A definition of logarithm is given in Section 7.1).

8. $g_n = [\log_2 n]$ for all integers $n \geq 1$

Answer: $g_1 = \log_2 1 = 0$	$g_6 = \log_2 6$	$g_{11} = \log_2 11$
$g_2 = \log_2 2 = 1$	$g_7 = \log_2 7$	$g_{12} = \log_2 12$
$g_3 = \log_2 3$	$g_8 = \log_2 8 = 3$	$g_{13} = \log_2 13$
$g_4 = \log_2 4 = 2$	$g_9 = \log_2 9$	$g_{14} = \log_2 14$
$g_5 = \log_2 5$	$g_{10} = \log_2 10$	$g_{15} = \log_2 15$

$2^{\log_2 k} = k$

5.1.18

Let $a_0 = 2$, $a_1 = 3$, $a_2 = -2$, $a_3 = 1$, $a_4 = 0$, $a_5 = -1$, and $a_6 = -2$.
Compute each of the summations and products below.

a. $\sum_{i=0}^{6} a_i = 1$

b. $\sum_{i=0}^{0} a_i = 2$

c. $\sum_{j=1}^{3} a_{2j} = -4$

d. $\prod_{k=0}^{6} a_k = 0$

e $\prod_{k=2}^{2} a_k = -2$

Compute the summations and products in 19-28

27. $$\sum_{n=1}^{10}\left(\frac{1}{n}-\frac{1}{n+1}\right) = \frac{1}{1}-\cancel{\frac{1}{2}}+\cancel{\frac{1}{2}}-\cancel{\frac{1}{3}}+\cdots+\cancel{\frac{1}{10}}-\frac{1}{11}$$

$$= \frac{10}{11}$$

Evaluate the summation and products in 33-36 for indicated values of the variable.

35. $\left(\frac{1}{1+1}\right)\left(\frac{2}{2+1}\right)\left(\frac{3}{3+1}\right)\cdots\left(\frac{k}{k+1}\right)$; $k=3$

$$= \frac{1}{2}\cdot\frac{\cancel{2}}{\cancel{3}}\cdot\frac{\cancel{3}}{\cancel{4}_2} = \frac{1}{4}$$

Write each of 43-52 using summation or product notation.

44. $(1^3-1)-(2^3-1)+(3^3-1)-(4^3-1)+(5^3-1)$

$$= \sum_{n=1}^{5} (n^3-1)(-1)^{n+1}$$

Write each of 43-52 using summation or product notation.

51. $n + (n-1) + (n-2) + \cdots + 1$

$$= \sum_{i=0}^{n-1} (n-i)$$

Transform each of 53 and 54 by making the change of variable $i = k+1$

53. $\sum_{k=0}^{5} k(k-1)$, $i = k+1 \Rightarrow k = i-1$

Lower limit $\Rightarrow$ $i=1$

Upper limit $\Rightarrow$ $i=6$

$= \sum_{i=1}^{5} (i-1)(i-2)$

Write each of 59-61 as a single summation or product.

61. $\left(\prod_{k=1}^{n} \frac{k}{k+1}\right) \cdot \left(\prod_{k=1}^{n} \frac{k+1}{k+2}\right)$

$$= \prod_{k=1}^{n} \frac{k}{k+2}$$

5.1.75

Compute each of 62-76. Assume the value of the variables are restricted so that the expressions are defined.

75. $\binom{n}{n-1} = \frac{n!}{(n-1)!(\cancel{n}-\cancel{n}+1)!} = \frac{n\cancel{(n-1)!}}{\cancel{(n-1)!}(1)!} = \underline{\underline{n}}$

5.1.88

Use the algorithm you developed for exercise 87 to conver the integers in 88-90 to hexadecimal notation.

88. 287

$$
\begin{array}{r}
017 \\
16\,\overline{)\,287} \\
\underline{16} \\
127 \\
\underline{112} \\
015
\end{array}
\qquad
\begin{array}{r}
01 \\
16\,\overline{)\,17} \\
\underline{16} \\
1
\end{array}
$$

$\xrightarrow{LE}$

$F11_{16}$

$\xleftarrow{BE}$

$11F_{16}$

3. For each positive integer n, let $P(n)$ be the formula

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

a. Write $P(1)$. Is $P(1)$ true? $P(1) = 1 \Rightarrow$ True

b. Write $P(k)$. $P(k) = \frac{k(k+1)(2k+1)}{6}$

c. Writ $P(k+1)$ $P(k+1) = \frac{(k+1)(k+2)(2k+3)}{6}$

d. In a proof by mathematical induction that the formula holds for all integers $n \geq 1$, what must be shown in the inductive step?

Show that $P(k+1)$ is true.

Prove each statement in 6-9 using mathematical induction. Do not derive them from theorem 5.2.2 or theorem 5.2.3.

8. For all integers $n \geq 0$, $1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1} - 1$

Proof | Let $P(n) = 2^{n+1} - 1$

$P(0) = 2^{1+0} - 1 = 1$ True

$P(k) = 2^{k+1} - 1$

$P(k+1) = 2^{k+2} - 1$

$$2^{k+2} - 1 = 1 + 2 + 2^2 + \cdots + 2^k + 2^{k+1}$$

$$= 2^{k+1} - 1 + 2^{k+1}$$

$$= 2^{k+1}(1+1) - 1$$

$$= 2^{k+1}(2) - 1$$

$$2^{k+2} - 1 = 2^{k+2} - 1$$

$\because$ $P(k+1)$ is true & $P(0)$ is true

$\therefore$ The statement is true by mathematical induction. ∎

5.2.14

Prove each statement in 10-17 by mathematical induction.

14. $\sum_{i=1}^{n+1} i \cdot 2^i = n\,2^{n+2} + 2$ for all integers $n \geq 0$

Proof | Let $P(n) = n\,2^{n+2} + 2$

$P(0) = 0 + 2 = 2$ True

$P(k) = k\,2^{k+2} + 2$

$P(k+1) = (k+1)\,2^{k+3} + 2$

Inductive Step *

$$(k+1)2^{k+3} + 2 = \sum_{i=1}^{k+2} i\,2^i$$

$$= \sum_{i=1}^{k+1} i\,2^i + (k+2)\,2^{k+2}$$

$$= k\,2^{k+2} + 2 + (k+2)2^{k+2}$$

$$= k2^{k+2}(1+1) + 2 + 2^{k+3}$$

$$= k2^{k+3} + 2^{k+3} + 2$$

$$(k+1)2^{k+3} + 2 = (k+1)2^{k+3} + 2$$

$\because$ $P(0)$ is true and $P(k+1)$ is true

$\therefore$ The statement is true by mathematical induction. ■

5. 2. 15

Prove each statement in 10-17 by mathematical induction.

15. $\sum_{i=1}^{n} i(i)! = (n+1)! - 1$, for all integers $n \geq 1$

Proof | Let $P(n) = (n+1)! - 1$

$P(1) = 2 - 1 = 1$ True

$P(k) = (k+1)! - 1$

$P(k+1) = (k+2)! - 1$

$$(k+2)! - 1 = \sum_{i=1}^{k+1} i(i)!$$

$$= \sum_{i=1}^{k} i(i)! + (k+1)(k+1)!$$

$$= (k+1)! - 1 + (k+1)(k+1)!$$

$$= (k+1)!(1 + k + 1) - 1$$

$$= (k+1)!(k+2) - 1$$

$$(k+2)! - 1 = (k+2)! - 1$$

$\because$ $P(1)$ is true & $P(k+1)$ is true

$\therefore$ The statement is true by mathematical induction ∎

5.2.20

Use the formula for the sum of the first n Integers and/or the formula for the sum of a geometric sequence to evaluate the sums in 20-29 or to write them in closed form.

20. $4+8+12+16+\cdots+200$

$= 4(1+2+3+4+\cdots 50)$

$= 4 \cdot \frac{50(51)}{2}$

$= 2 \cdot 50 \cdot 51$

$= 100 \cdot 51$

$= 5100$

5.2.23

Use the formulas for the sum of the first n integers and/or the formula for the sum of a geometric sequence to evaluate the sums in 20-29 or to write them in closed form.

23. $7+8+9+10+\dots+600$

$$= \frac{600(601)}{2} - 6 - 5 - 4 - 3 - 2 - 1$$

$$= 300(601) - 21$$

$$= 180279$$

Use the formulas for the sum of the first n integers and for formula for the sum of a geometric sequence to evaluat the sums in 20-29 or to write them in closed form.

28. $1 + \frac{1}{2} + \frac{1}{2^2} + \cdots + \frac{1}{2^n}$, where n is a positive integer

$$= \frac{2^{-(n+1)} - 1}{2^{-1} - 1}$$

$$= -2 \cdot 2^{-n-1} + 2$$

$$= -2^{-n} + 2$$

$$= \frac{-1}{2^n} + 2$$

$$= 2 - \frac{1}{2^n}$$

30. Find a formula in n, a, m, and d for the sum $(a+md)+(a+(m+1)d)+(a+(m+2)d)+\cdots+(a+(m+n)d)$, where m and n are integers, $n \geq 0$, and a and d are real numbers. Justify your answer.

$$(a+(m+n)d) = a+md+nd$$

$$\sum_{n=0}^{n}(a+(mn)d) = \sum_{n=0}^{n}(a+md+nd)$$

$$= (a+md)(n+1) + \frac{n(n+1)d}{2}$$

$$= (n+1)\left(a+md+\frac{nd}{2}\right)$$

Justification

Let $P(n) = (n+1)\left(a+md+\frac{nd}{2}\right)$

$P(0) = (a+md)$ True

$P(k) = (k+1)\left(a+md+\frac{kd}{2}\right)$

$P(k+1) = (k+2)\left(a+md+(k+1)\frac{d}{2}\right)$

Inductive step* $(k+2)\left(a+md+(k+1)\frac{d}{2}\right) = (k+1)\left(a+md+\frac{kd}{2}\right)+(a+(m+k+1)d)$

$$= (k+1)\left(a+md+\frac{kd}{2}\right)+(a+md+(k+1)d)$$

$$= (k+1)\left(a+md+\frac{kd}{2}+d\right)+a+md$$

$$= (k+1)\left(a+md+\frac{1}{2}d(k+2)\right)+a+md$$

$(k+2)\left(a+md+(k+1)\frac{d}{2}\right) = (k+2)\left(a+md+\frac{1}{2}d(k+1)\right)$

True by mathematical induction. ■

5.2.33

Find the mistakes in the proof fragments in 33-35

33. Theorem: for any integer $n \geq 1$,

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

"Proof (by mathematical induction): Certainly the theorem is true for $n=1$ because $1^2 = 1$ and $\frac{1(1+1)(2\times1+1)}{6} = 1$. So the basis step is true

For the inductive step, suppose that for some integer $k \geq 1$,

$k^2 = \frac{k(k+1)(2k+1)}{6}$. We must show that

$(k+1)^2 = \frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$ ";

Mistakes: 1) The inductive step.

It should be $1^2 + 2^2 + \cdots + k^2 = \frac{k(k+1)(2k+1)}{6}$

2) What must be show

Should be $1^2 + 2^2 + \cdots + k^2 + (k+1)^2 = \frac{(k+1)((k+1)+1)(2(k+1)+1)}{6}$

Find the first four terms of each of the recursively defined sequences in 1-8

3. $C_k = k(C_{k-1})^2$, for all integers $k \geq 1$ & $C_0 = 1$

$C_0 = 1$

$C_1 = 1(1)^2 = 1$

$C_2 = 2(1)^2 = 2$

$C_3 = 3(2)^2 = 12$

5.6.7

Find the first four term of each of recursively defined sequences in 1-8

7. $u_k = k u_{k-1} - u_{k-2}$, for all integers $k \geq 3$, $u_1 = 1$, $u_2 = 1$

$u_1 = 1$

$u_2 = 1$

$u_3 = 3(1) - 1 = 2$

$u_4 = 4(2) - 1 = 7$

13. Let $t_0, t_1, t_2, \ldots$ be defined by the formula $t_n = 2 + n$ for all integers $n \geq 0$. Show that the sequence satisfies the recurrence relation

$$t_k = 2t_{k-1} - t_{k-2}$$

$t_k = 2 + k$

$t_{k-1} = 1 + k$

$t_{k-2} = k$

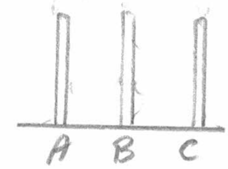
$2 + k = 2(k+1) - k$

$2 + k = 2 + k$

5.6.17

Tower of Hanoi with adjacency Requirement: Suppose that in addition to the requirement that they never move a larger disk on top of a smaller one, the priests who move the disks of the Tower of Hanoi are also allowed only to move disk one by one from one pole to an adjacent pole. Assume poles A and C are at the two ends of the row and pole B is in the middle. Let

$$a_n = \left[\begin{array}{l}\text{the minimum number of moves} \\ \text{needed to transfer a tower of } n \\ \text{disks from pole A to pole C}\end{array}\right]$$



a. Find a_1, a_2, and a_3.

b. Find a_4

c. Find a recurrence relation for $a_1, a_2, a_3, \ldots$

Part a: $a_1 = 2$ $a_2 = 8$

$a_3 = 26$

Part b: $a_4 = 80$

Part c: $a_n = 3a_{n-1} + 2$ for all integers $n \geq 0$

$a_2 = 2 + 1 + 2 + 1 + 2$
$= 8$

$a_3 = 8 + 1 + 8 + 1 + 8$
$= 26$

$a_4 = 26 + 1 + 26 + 1 + 26$
$= 60 + 18 + 2 = 80$

5.6.20

20. Tower of Hanoi Poles in a circle: Suppose that instead of being lined up in a row, the three poles for the original Tower of Hanoi are placed in a circle. The monks move disks one by one from one pole to another, but they may only move disk one over in a clockwise direction and the may never move a larger disk over a smaller one. Let C_n be the minimum number of moves needed to transfer a pile of n disk from one pole to the next adjacent pole in the clockwise direction

a. Justify the inequality $C_k \le 4C_{k-1} + 1$ for all integers $k \ge 2$

b. The expression $4C_{k-1} + 1$ is not the minimum number of move need to transfer a pile of k disks from on pole to anothe. Explain why $C_3 \neq 4C_2 + 1$

From A→B	From A→C
$B_1 = 1$	$C_1 = 2$
$B_2 = 5$	$C_2 = 7$
$B_3 = 15$	$C_3 = 21$

Part a) $S_3 \le 21$ & $S_2 = 5$ & $S_1 = 1$

$S_3 \le 4(5) + 1 \Rightarrow S_3 \le 21$

$S_2 \le 4(1) + 1 \Rightarrow S_2 \le 5$

$C_k \le 4C_{k-1} + 1$ for all integers $k \ge 2$

Part b) Le $C_k = 4C_{k-1} + 1$ & $C_1 = 1$, $C_2 = 5$

Case A→B: $15 = 4(5) + 1$

$15 \neq 21$

$\therefore C_k \neq 4C_{k-1} + 1$

5.6.24

In 24 - 34 F_0 , F_1 , F_2 ,... is the Fibonacci sequence

24. Use the recurrence relation and values F_0 , F_1 , F_2 , ... given in example 5.6.6 to compute F_{13} and F_{14}

$F_0 = 0$

$F_1 = 1$

$\vdots$

$F_{11} = 144$

$F_{12} = 233$

$F_{13} = 377$

$F_{14} = 610$

In 24-34, $F_0, F_1, F_2, \ldots$ is the Fibonacci sequence

27. Prove that $F_k^2 - F_{k-1}^2 = F_k F_{k+1} - F_{k+1} F_{k-1}$, for all integers $k \geq 1$

<u>Proof</u> $F_k^2 - F_{k-1}^2 = (F_k + F_{k-1})(F_k - F_{k-1})$

$= (F_{k+1})(F_k - F_{k-1})$

$= F_{k+1} F_k - F_{k+1} F_{k-1}$ ∎

5.7.5

In each of 3-15 a sequence is defined recursively. Use iteration to guess an explicit formula for the sequence. Use formulas from 5.2 to simplify your answer

5. $c_k = 3c_{k-1} + 1$, for all integers $k \geq 2$

$c_1 = 1$

	-1			
$c_2 = 3(1) + 1 = 4$	$3 = 3 \times 1$		$+1 = 4$	$3 + 1$
		$+3 = 3$		
$c_3 = 3(4) + 1 = 13$	$12 = 3 \times 4$		$+1 = 13$	$3^2 + 3^1 + 1$
		$+9 = 3^2$		
$c_4 = 3(13) + 1 = 40$	$39 = 3 \times 13$		$+1 = 4$	$3^3 + 3^2 + 3 + 1$
		$+27 = 3^3$		
$c_5 = 3(40) + 1 = 121$	$120 = 3 \times 40$		$+1 = 121$	$3^4 + 3^3 + 3^2 + 3 + 1$

Sum of a geometric sequence

$$\sum_{i=1}^{n} 3^{i-1} = \frac{3^{n+1-1} - 1}{3 - 1} = \underline{\frac{3^n - 1}{2}}$$

$$c_n = \frac{3^n - 1}{2}$$

In each of 3-15 a sequence is defined recursively. Use iteration to guess an explicit formula for the sequence. Use the formulas from Section 5.2 to simplify your answers whenever possible.

10. $h_k = 2^k - h_{k-1}$, for all integers $k \geq 1$

$h_0 = 1$

$$h_1 = 2 - 1 = 1$$

$$h_2 = 2^2 - \overset{1}{(2-1)} = 3$$

$$h_3 = 2^3 - \overset{3}{(2^2 - (2-1))} = 5$$

$$h_4 = 2^4 - \overset{5}{(2^3 - (2^2 - (2-1)))} = 11$$

$\vdots$

$$\underbrace{h_1, h_2}_{+2}, \quad \underbrace{h_2, h_3}_{+2}, \quad \underbrace{h_3, h_4}_{+5}$$

$$h_n = 2^n - 2^{n-1} + 2^{n-2} + \cdots + (-1)^n 1$$

$$= (-1)^n (1 - 2 + \cdots - 2^{n-1} + 2^n)$$

$$= \sum_{n=1}^{k} (-1)^n (-2)^n$$

$$= (-1)^n \frac{(-2)^{n+1} - 1}{-2-1} = (-1)^n \frac{[(-2)^{n+1} - 1]}{-3}$$

$$h_n = \frac{2^{n+1} - (-1)^{n+1}}{3} \quad \text{for all } n \geq 0$$

5.7.14

In each of 3-15 a sequence is defined recursively. Use iteration to guess an explicit formula for the the sequence. Use formulas from Section 5.2 to simplify your answers whenever possible.

14. $x_k = 3x_{k-1} + k$, for all integers $k \geq 2$
$x_1 = 1$

n	0
3	1
2	2
1	3
0	n

$$x_2 = 3(1) + 2 = 3+2 = 3^1(1) + 3^0 \cdot 2$$

$$x_3 = 3(3+2) + 3 = 3\cdot(3+2)+3 = 3^2(1) + 3^1(2) + 3^0(3)$$

$$x_4 = 3(3(3+2)+3)+4 = 3(3(3+2)+3)+4 = 3^3(1) + 3^2(2) + 3^1(3) + 3^0(4)$$

$$x_n = 3^n \cdot 0 + 3^{n-1}(1) + 3^{n-2}(2) + \dots + 3^2(n-2) + 3^1(n-1) + 3^0(n)$$

$$= (3^{n-1} + 3^{n-2} + \dots) + (3^{n-2} + 3^{n-3} \dots) + (3^{n-3} + 3^{n-4} + \dots) + \dots + (3+1) + (1)$$

Double Sum $\displaystyle\sum_{l=m}^{m=n} \sum_{l=0}^{m} 3^{l-1}$

$$= \sum_{l=m}^{m=n} \frac{3^{l-1+1} - 1}{3-2}$$

$$= \sum_{l=m}^{m=n} \frac{3^{l} - 1}{2} = \frac{1}{2} 3 \sum_{l=m}^{m=n} (3^{l-1}) - \frac{1}{2} \sum_{l=n}^{m=n} l$$

$$= \frac{3}{2}\left(\frac{\cdot 3^{n+1-1} - 1}{3-2}\right) - \frac{1}{2} n$$

$$= \frac{3^{n+1} - 3 - 2n}{4} \quad \text{for all } n \geq 1$$

5. 7. 19

19. A worker is promised a bonus if he can increase his productivity by 2 units a day every day for a period of 30 days. If on day he produced 170 units, how many units must he produce on day 30 to qualify for a bonus?

$a_0 = 170$

$a_n = a_0 + 2n$

$a_{30} = 170 + 2(30) = 260$

25. A certain computer algorithm executes twice as many operations when it is run with an input of size k as when it is run with an input of size k-1 (where k is an integer greater than 1). When the algorithm is run with an input of size 1, it executes seven operations. How many operations does it executes when it is run with an input of size 25?

$a_n = a_0 r^{n-1}$ & $a_0 = 7$ & $r = 2$

$a_n = 7 \cdot 2^{n-1}$

$a_{25} = 7 \cdot 2^{24} = 117440512$

In 28-42 use mathematical induction to verify the correctness of the formula you obtained in the refrenced exercise.

30. Exercise 5. $\quad C_k = 3C_{k-1} + 1$

$$C_n = \frac{3^n - 1}{2} \quad \text{where } n \geq 1$$

Proof | Let $C(n) = C_n = \frac{3^n - 1}{2}$

$C(1) = 1 \qquad$ True

Assume $C(k) = \frac{3^k - 1}{2}$ is true

Inductive Step

$$C(k+1) = \frac{3^{k+1} - 1}{2} = 3C_{(k+1)-1} + 1$$

$$= 3C_k + 1$$

$$= 3 \cdot \frac{3^k - 1}{2} + 1$$

$$= \frac{3^{k+1} - 3 + 2}{2}$$

$$\frac{3^{k+1} - 1}{2} = \frac{3^{k+1} - 1}{2}$$

$\because$ $C(0)$ & $C(k+1)$ are true

$\therefore$ $C_n = \frac{3^n - 1}{2}$ is correct by mathematical induction

In 38 - 42 use mathematical induction to verify the correctness of the formula you obtained in the referenced exercise.

35. Exercise 10. $h_k = 2^k - h_{k-1}$ $\quad \forall k \in \mathbb{Z}$ & $k \geq 1$

Formula: $h_n = \dfrac{2^{n+1} - (-1)^{n+1}}{3}$ $\quad \forall n \in \mathbb{Z}$ & $n \geq 0$

Proof | $h_0 = \dfrac{3}{3} = 1 \Rightarrow h_0$ is true

$$h_{k+1} = \frac{2^{k+2} - (-1)^{k+2}}{3} = 2^{k+1} - \frac{2^{k+1} - (-1)^{k+1}}{3}$$

$$= \frac{3 \cdot 2^{k+1}}{3} - \frac{2^{k+1} + (-1)(-1)^{k+1}}{3}$$

$$= \frac{2 \cdot 2^{k+1} + (-1)^{k+2}}{3}$$

$$\frac{2^{k+2} - (-1)^{k+2}}{3} = \frac{2^{k+2} - (-1)^{k+2}}{3}$$

$\because$ h_0 & h_{k+1} are true

$\therefore$ $h_n = \dfrac{2^{n+1} - (-1)^{n+1}}{3}$ is correct by mathematical induction

In 28-42 use mathematical induction to verify the correctness of the formula you obtained in the referenced exercise. 5.7.39

39. Exercise 14. $x_k = 3x_{k-1} + k$ for all integers $k \geq 2$

$$x_n = \frac{3^{n+1} - 3 - 2n}{4} \quad \text{for all integers } n \geq 1$$

Proof | $x_1 = \frac{3^2 - 3 - 2}{4} = \frac{4}{4} = 1$ true

Assume x_k is true

$$x_{k+1} = \frac{3^{k+2} - 3 - 2(k+1)}{4} = 3\left(\frac{3^{k+1} - 3 - 2k}{4}\right) + (k+1)$$

$$= \frac{3^{k+2} - 3 - 6(k+1)}{4} + \frac{4(k+1)}{4}$$

$$\frac{3^{k+2} - 3 - 2(k+1)}{4} = \frac{3^{k+2} - 3 - 2(k+1)}{4}$$

$\because$ x_1 & x_{k+1} are true

$\therefore$ $x_n = \frac{3^{n+1} - 3 - 2n}{4}$ for all $n \geq 1$ is correct

by mathematical induction. ∎