

1) Is $0.555... = 0.\bar{5} \in \mathbb{Q}$ rational ?

Proof

$$\text{Let } 0.\bar{5} = x$$

$$\begin{array}{r} 5.\bar{5} = 10x \\ \hline 5 = 9x \end{array}$$

$\therefore 0.\bar{5} = x = \frac{5}{9}$, which is a rational number by the definition of rational. \blacksquare

2) Prove $r, s \in \mathbb{Q} \rightarrow (2r + 3s) \in \mathbb{Q}$

Proof

Let $r = \frac{a}{b}$ and $s = \frac{c}{d}$, where a, b, c , and d are integers.

by definition, r and s are rational numbers.

$$2r = \frac{2a}{b} \quad \text{and} \quad 3s = \frac{3c}{d}$$

$$\text{Let } 2a = p \quad \text{and} \quad 3c = q$$

p and q are integers because they are the product of integers.

$$2r = \frac{p}{b} \quad \text{and} \quad \frac{q}{d}$$

$\therefore 2r$ and $3s$ are the ratios of two integers

$\therefore 2r$ and $3s$ are rational numbers

$\therefore 2r + 3s$ is the sum of two rational numbers

$\therefore 2r + 3s$ is rational \blacksquare

3) Prove $r, s \in \mathbb{Z}^+$, $r \% 2 = 1$, $s \% 2 = 0 \rightarrow r \cdot s \% 2 = 0$

Proof

Let r and s be some positive integers where r is odd and s is even

$$r = 2k + 1$$

$$s = 2m$$

(Rewrite r and s as the product/sum of integers)

$$r \cdot s = (2k + 1)(2m)$$

$$= 4km + 2m$$

(Distribution Law)

$$= 2(2km + m)$$

(Factoring 2)

$$= 2(\text{Some integer})$$

(Rewrite)

\therefore The product of even and odd integers is an even integer. \square

4) Steps?

Mistakes

1. Start by stating the word proof.
2. Include all theorem (self contained)
3. State conclusion at the end
4. Explain Every step in English.

1. Arguing from example
2. Circle reasoning
3. Confusing what is known w/ what to be shown
4. Jumping to conclusion