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Quiz

04/22/20

1) Sum $\{100, 104, 108, \ldots, 1000\}$

$= 4\{25, 26, 27, \ldots, 250\}$

$= 4\left(\frac{250(251)}{2}\right) - \frac{24(25)}{2}$

$= 500(251) - 12(25)$

$= 125500 - (300) = 125200$

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2) Sum 2^i where $i \in \mathbb{Z}$, $4 \leq i \leq 10$

$= \sum_{i=4}^{10} 2^i = \sum_{i=0}^{10} 2^i - \sum_{i=0}^{4} 2^i = \frac{2^{11}-1}{2-1} - \frac{2^5-1}{2-1}$

$= 2^{11} - 1 - (2^5 - 1)$

$= 2^{11} - 2^5$

$= 2048 - 32$

$= 2016$

3) company A $\rightarrow A_0 = \$ 10^6$ increasing $\frac{1}{6} \times 10^6$ / year

Company B $\rightarrow B_0 = \$ 10^6$ increasing 12% year

When will $A_k = B_k$ in the future

$$A_k = A_0 + k \cdot \frac{1}{6} \cdot 10^6$$

$$B_k = B_0 (0.12)^k$$

$$\Sigma A_k = A_0 + \frac{A_0}{6} \cdot \frac{k(k+1)}{2} = A_0 \left(1 + \frac{k(k+1)}{12}\right)$$

$$\Sigma B_k = B_0 \left(\frac{(0.12)^{k+1} - 1}{0.12 - 1} \right)$$

Let $\Sigma A_k = \Sigma B_k$: $\cancel{B_0} \left(\frac{(0.12)^{k+1} - 1}{0.12 - 1} \right) = \cancel{A_0} \left(1 + \frac{k(k+1)}{12}\right)$

$$A_0 = B_0$$

$$\frac{(0.12)^{k+1} - 1}{\underbrace{0.12 - 1}_{-0.88}} = 1 + \frac{k(k+1)}{12}$$

$$(0.12)^{k+1} - 1 = -0.88 - \frac{0.88 k(k+1)}{12}$$

$$0.12 \cdot 0.12^k = -0.88 - \frac{0.88 k(k+1)}{12}$$

$\frac{12}{100}$

$$0.12^k = -\frac{88}{12} - \frac{88 k(k+1)}{144}$$

$$0.12^k = \frac{-88 \cdot 12 - 88 k(k+1)}{144}$$

$$0.12^k + \frac{88 k(k+1)}{144} = \frac{-88 \times 12}{144}$$ Solve by graphing

4) Prove $\binom{N}{J} < \frac{N!}{J!}$ $\quad 1 < J < N \quad J, N \in \mathbb{Z}^+$

Proof/ $\binom{N}{J} < \frac{N!}{J!}$

$$\frac{N!}{J!(N-J)!} < \frac{N!}{J!}$$

$$\frac{(N-J)!\cdots N}{J!\cancel{(N-J)!}}$$

$$\frac{(N-J+1)\cdots(N-1)N}{J!} < \frac{N!}{J!} \qquad \blacksquare$$

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