Omar Alkendi

Chapter 1

March 02, 2020

Section 1.1

1 ----------------------------------Done

4 ----------------------------------Done

7 ----------------------------------Done

10 --------------------------------Done

Section 1.2

1 ----------------------------------Done

4 ----------------------------------Done

7 ----------------------------------Done

10 --------------------------------Done

Section 1.3

1 ----------------------------------Done

4 ----------------------------------Done

7 ----------------------------------Done

10 --------------------------------Done

13 --------------------------------Done

16 --------------------------------Done

19 --------------------------------Done

4. Given any real number, there is a real number that is greater.

a. Given any real number r, there is <u>a real number</u> s such that s is <u>greater than</u> r.

b. For any $\underline{r}$, $\underline{\exists\, s}$ such that $s > r$.

In each 1-6, fill in the blanks using a variable or variables to rewrite the given statement.

1. Is there a real number whose square is -1?

a. Is there a real number x such that $x^2 = -1$?

b. Does there exist a real number x such that $x^2 = -1$?

$x \in \mathbb{R}$

7. Rewrite the following statement less formally, without using variables. Determine as best as you can, wheather the statements are true or false.

a. There are real numbers u and v with the property that $u + v < u - v$.

Answer: There are two real numbers such that the sum of which is less than the differance; true.

b. There is a real number x such that $x^2 < x$.

Answer: There is a real the square of which is smaller than itself; true.

c. For all positive integers n, $n^2 \geq n$.

Answer: For all positive integers, the square of the integer is always greater than or equal to the integer; true.

d. For all real numbers a and b, $|a+b| \leq |a| + |b|$.

Answer: Given any real numbers, there is a real number such that the absolute value of the sum is smaller than or equal to the sum of the absolute value of each number; true.

In each of 8-13 fill in the blanks to rewrite the given statements.

10. Every nonzero real number has a reciprocal.

a. All nonzero real numbers have a reciprocal.

b. For all nonzero real numbers r, there is a reciprocal for r.

c. For all nonzero real numbers r, there is a real number s such that s is a reciprocal for r.

Which of the following sets are equal?

$A = \{a, b, c, d\}$

$B = \{B, e, a, c\}$

$C = \{d, b, a, c\}$

$D = \{a, a, d, e, c, e\}$

Answer: $A = B$ & $C = D$

4.

a. Is $2 \in \{2\}$? Answer: yes

b. How many elements are in the set $\{2, 2, 2, 2\}$? Answer: 1

c. How many element are in the set $\{2, \{2\}\}$? Answer: 2

d. Is $\{0\} \in \{\{0\}, \{1\}\}$? Answer: yes

e. Is $0 \in \{\{0\}, \{1\}\}$? Answer: No

7. Use the set-roster notation to indicate the elements in each of the following sets.

a. $S = \{n \in Z \mid n = (-1)^k$, for some integer $k\}$.

Answer: $S = \{-1, 1\}$

b. $T = \{m \in Z \mid m = 1 + (-1)^i$, for some integer $i\}$.

Answer: $T = \{0, 2\}$

10.

a. Is $((-2)^2, -2^2) = (-2^2, (-2)^2)$?

Answer: No

$(4, -4) \neq (-4, 4)$

b. Is $(5, -5) = (-5, 5)$?

Answer: No

c. Is $(8-9, \sqrt[3]{-1}) = (-1, -1)$?

Answer: yes

d. Is $(\frac{-2}{-4}, (-2)^3) = (\frac{3}{6}, -8)$?

Answer: yes

$(\frac{1}{2}, -8) = (\frac{1}{2}, -8)$

1. Let $A = \{2, 3, 4\}$ and $B = \{6, 8, 10\}$ and define a relation R from A to B as follows: For all $(x, y) \in A \times B$,

$$(x, y) \in R \text{ means } \frac{y}{x} \text{ is an integer.}$$

a. Is $4R6$? Is $4R8$? Is $(3, 8) \in R$? Is $(2, 10) \in R$?

Answer: $4R6 \to \frac{6}{4} = \frac{3}{2} \Rightarrow 4 \not R 6$

$4R8 \to \frac{8}{4} = 2 \Rightarrow 4R8$

$(3,8) \in R \to \frac{8}{3} = 2.67 \Rightarrow (3,8) \notin R$

$(2,10) \in R \to \frac{10}{2} = 5 \Rightarrow (2,10) \in R$

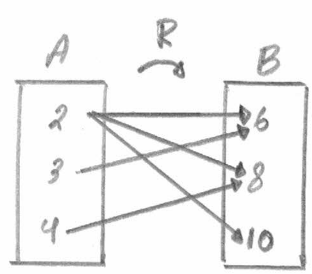
b. Write R as a set order pair.

Answer: $R = \{(2,6), (2,8), (2,10), (3,6), (4,8)\}$

c. Write the domain and co-domain of R.

Answer: Domain of R is $\{2, 3, 4\}$ and the co-domain is $\{6, 8, 10\}$

d. Draw an arrow diagram for R



4. Let $G = \{-2, 0, 2\}$ and $H = \{4, 6, 8\}$ and define a relation V from G to H as follows: for all $(x,y) \in G \times H$,

$(x, y) \in V$ means that $\frac{x-y}{4}$ is an integer.

a. Is $2V6$? Is $(-2)V(-6)$? Is $(0,6) \in V$? Is $(2,4) \in V$?

Answer: $2V6? \rightarrow \frac{2-6}{4} = -1 \Rightarrow 2V6$

$(-2)V(-6)? \rightarrow (-6) \notin H \Rightarrow (-2) \not V (-6)$

$(0,6) \in V? \rightarrow \frac{0-6}{4} = \frac{-3}{2} \Rightarrow (0,6) \notin V$

$(2,4) \in V? \rightarrow \frac{2-4}{4} = \frac{-1}{2} \Rightarrow (2,4) \notin V$

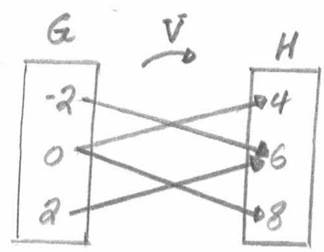
b. Write V as a set of ordered pairs.

Answer: $V = \{(-2, 6), (0,4), (0,8), (2,6)\}$

c. Write down the domain and codomain of V.

Answer: The domain of V is $\{-2, 0, 2\}$ and the co-domain is $\{4, 6, 8\}$

d. Draw an arrow diagram for V.



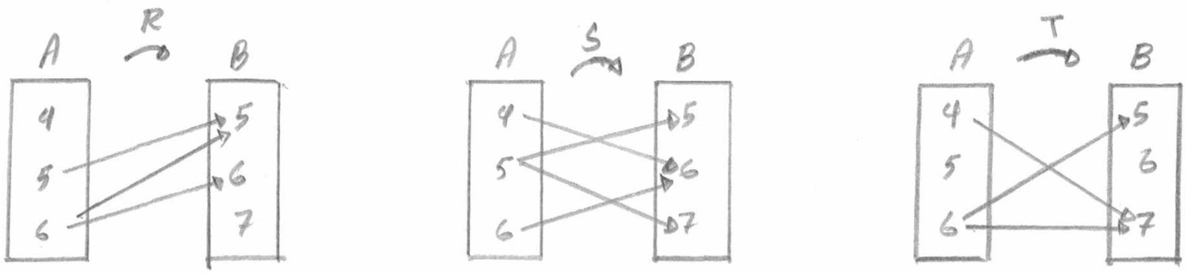
7. Let $A = \{4, 5, 6\}$ and $B = \{5, 6, 7\}$ and define relations R, S, and T from A to B as follows:

For all $(x, y) \in A \times B$, $\quad (x, y) \in R$ means that $x \geq y$

$(x, y) \in S$ means that $\frac{x-y}{2}$ is an integer

$T = \{(4, 7), (6, 5), (6, 7)\}$.

a. Draw an arrow diagram for R, S, and T



b. Indicate whether any of relations R, S, and T are functions.

Answer: - R is not a function because $(4, y) \notin R$ & $(6,6) \in R$ & $(6,5) \in R$ but $5 \neq 6$

- S is not a function because $(5,5) \in R$ & $(5,7) \in R$ but $5 \neq 7$

- T is not a function because $(5, y) \notin T$ & $(6,5) \in T$ & $(6,7) \in T$ but $5 \neq 7$

10. Find four relation from $\{a, b\}$ to $\{x, y\}$ that are not function from $\{a, b\}$ to $\{x, y\}$

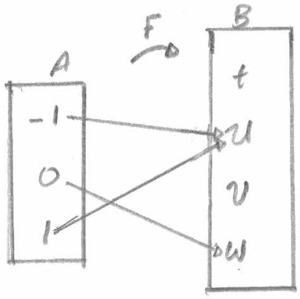
Answer: $A = \{(a, x)\}$

$B = \{(a, x), (a, y)\}$

$C = \{(b, x)\}$

$D = \{(b, x), (b, y)\}$

13. Let $A=\{-1,0,1\}$ and $B=\{t,u,v,w\}$. Define a function $F: A \to B$ by the following arrow diagram.



a. Write the domain and co-domain of F

Answer: Domain of F is $\{-1,0,1\}$ and it's co-domain is $\{t,u,v,w\}$.

b. Find $F(-1)$, $F(0)$, and $F(1)$.

Answer: $F(-1)=u$

$F(0)=w$

$F(1)=u$

16. Let f be the squaring function defined in example 1.3.6. Find $f(-1)$, $f(0)$, and $f(\frac{1}{2})$

Answer: $f(-1) = (-1)^2 = 1$

$f(0) = (0)^2 = 0$

$f(\frac{1}{2}) = (\frac{1}{2})^2 = \frac{1}{4}$

19. Define functions f and g from R to R by the following formulas:

for all $x \in R$

$$f(x) = 2x \quad \text{and} \quad g(x) = \frac{2x^3 + 2x}{x^2 + 1}$$

Does $f = g$? Explain.

Solution: $g(x) = \dfrac{2x^3 + 2x}{x^2 + 1} = \dfrac{2x\cancel{(x^2+1)}}{\cancel{(x^2+1)}} = 2x = f(x)$

$\because f(x) = g(x) \ \forall x$ in R

$\therefore f = g$ ∎