Omar Alkendi

Chapters 6,7, and 8

April 29, 2020

Section 6.1

5 --------------------------------Done

8 --------------------------------Done

9 --------------------------------Done

13 ------------------------------Done

16 ------------------------------Done

21 ------------------------------Done

24 ------------------------------Done

28 ------------------------------Done

29 ------------------------------Done

32 ------------------------------Done

Section 7.1

6 --------------------------------Done

10 ------------------------------Done

15 ------------------------------Done

18 ------------------------------Done

25 ------------------------------Done

30 ------------------------------Done

35 ------------------------------Done

40 ------------------------------Done

45 ------------------------Not-Done

51 ------------------------------Done

Section 8.1

2 --------------------------------Done

4 --------------------------------Done

5 --------------------------------Done

8 --------------------------------Done

10 ------------------------------Done

12 ------------------------------Done

16 ------------------------------Done

19 ------------------------------Done

23 ------------------------------Done

24 ------------------------------Done

Section 8.2

6 ------------------------------Done

11 ------------------------------Done

15 ------------------------------Done

20 ------------------------------Done

25 ------------------------------Done

31 ------------------------------Done

37 --------------------------Not-Done

43 ------------------------------Done

51 ------------------------------Done

Omar Alkendi

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April 29, 2020

Section 8.3

5 -----------------------------Done

10 ---------------------------Done

11 ---------------------------Done

15 ---------------------------Done

20 ---------------------------NOT-Done

26 ---------------------------NOT-Done

32 ---------------------------NOT-Done

38 ---------------------------NOT-Done

42 ---------------------------NOT-Done

5. Let $C = \{n \in \mathbb{Z} \mid n = 6r - 5 \text{ for some integer } r\}$ and
$D = \{m \in \mathbb{Z} \mid m = 3s + 1 \text{ for some integer } s\}$.
Prove or disprove each of the following statements
a. $C \subseteq D$ b. $D \subseteq C$

Part a: Proof: Suppos n is any element of $C \Rightarrow n = 6r - 5$

$$= 6r - 6 + 1$$

$$= 3(2r - 2) + 1$$

Let $s = 2r - 2 \Rightarrow 3s + 1 = 3(2r-2) + 1$

$$= 6r - 5$$

$$= n$$

$\therefore C \subseteq D$

Part b: Proof: Suppose m is any element of $D \Rightarrow m = 3s + 1$

$m(1) = 4$

$4 = 6r - 5$

$9 = 6r$

$r = \frac{3}{2} \notin \mathbb{Z}$

$\therefore D \subseteq C$

8. Write in words how to read each of the following out. Then write the shorthand notation for each set

a. $\{x \in U \mid x \in A \text{ and } x \in B\}$

The set of all x in U such that x is in A and x is in B.

$A \cap B$

b. $\{x \in U \mid x \in A \text{ or } x \in B\}$

The set of all x in U such that x is in A or x is in B

$A \cup B$

c. $\{x \in U \mid x \in A \text{ and } x \notin B\}$

The set of all x in U such that x is in A but x is not in B

$A - B$

d. $\{x \in U \mid x \notin A\}$

The set of all x in U such that x is not in A

A^c

9. Complete the following sentences without using the symbol $\cup$, $\cap$, or $-$.

a. $x \notin A \cup B$ if and only if, $x \notin A$ and $x \notin B$.

b. $x \notin A \cap B$ if and only if, $x \in A$ and $x \notin B$ or $x \notin A$ and $x \notin B$.

c. $x \notin A - B$ if and only if $x \notin A$ or $x \in B$.

16. Let $A=\{a,b,c\}$, $B=\{b,c,d\}$, and $C=\{b,c,e\}$

a. Find $A\cup(B\cap C)$, $(A\cup B)\cap C$, and $(A\cup B)\cap(A\cup C)$. Which are equal?

$A\cup(B\cap C)=\{a,b,c\}$

$(A\cup B)\cap C=\{b,c\}$

$(A\cup B)\cap(A\cup C)=\{a,b,c\}$

$A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

b. Find $A\cap(B\cup C)$, $(A\cap B)\cup C$, and $(A\cap B)\cup(A\cap C)$. Which are equal?

$A\cap(B\cup C)=\{b,c\}$

$(A\cap B)\cup C=\{b,c,e\}$

$(A\cap B)\cup(A\cap C)=\{b,c\}$

$A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

c. Find $(A-B)-C$ and $A-(B-C)$. Are these equal?

$(A-B)-C=\{a\}$

$A-(B-C)=\{a,b,c\}$

$(A-B)-C\neq A-(B-C)$

21. Let $C_i = \{i, -i\}$ for all nonnegative integers i.

a. $\bigcup_{i=0}^{4} C_i = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

b. $\bigcap_{i=0}^{4} C_i = \emptyset$

c. Are $C_0, C_1, C_2, \ldots$ mutually disjoint? Explain.

Yes, becaus no two sets of $C_0, C_1, C_2, \ldots$ have any element in common

d. $\bigcup_{i=0}^{n} C_i = \{-n, -n+1, \ldots, n-1, n\}$

e. $\bigcap_{i=0}^{n} C_i = \emptyset$

f. $\bigcup_{i=0}^{\infty} C_i = \mathbb{Z}$

g. $\bigcap_{i=0}^{\infty} C_i = \emptyset$

6.1. 24

24. Let $W_i = \{x \in \mathbb{R} \mid x > i\} = (i, \infty)$ for all nonnegative integers i

a. $\bigcup_{i=0}^{4} W_i = (0, \infty)$

b. $\bigcap_{i=0}^{4} W_i = (4, \infty)$

c. Are $W_0, W_1, W_2, \dots$ mutually disjoint? Explain

No, becaus any W_i has at least one elemens in common with any other W_i

d. $\bigcup_{i=0}^{n} W_i = (0, \infty)$

e. $\bigcap_{i=0}^{n} W_i = (n, \infty)$

f. $\bigcup_{i=0}^{\infty} W_i = (0, \infty)$

g. $\bigcap_{i=0}^{\infty} W_i = \phi$

28. Let E be the set of all even integers and O is the set of all odd integers. Is $\{E, O\}$ a partion of $\mathbb{Z}$, the set of all integers? Explain your answer.

Answer: Yes, becaus E & O don't have any elements in common, and an integer can be either odd or even but not both.

29. Let $\mathbb{R}$ be the set of all real numbers. Is $\{\mathbb{R}^+, \mathbb{R}^-, \{0\}\}$ a partion of $\mathbb{R}$? Explain your answer.

Answer: Yes, because any real number can be either a positive or a negative number or neither, and non of $\mathbb{R}^+$, $\mathbb{R}^-$, and $\{0\}$ have any elements in common.

32. a. Suppose $A = \{1\}$ and $B = \{u, v\}$. Find $\mathscr{P}(A \times B)$

$(A \times B) = \{(1, u), (1, v)\}$

$\mathscr{P}(A \times B) = \{\emptyset, \{(1, u)\}, \{(1, v)\}, \{(1, u), (1, v)\}\}.$

b. Suppose $X = \{a, b\}$ and $Y = \{x, y\}$. Find $\mathscr{P}(X \times Y)$

$(X \times Y) = \{(a, x), (a, y), (b, x), (b, y)\}$

$\mathscr{P}(X \times Y) = \{\emptyset, \{(a, x)\}, \{(a, y)\}, \{(b, x)\}, \{(b, y)\},$

$\{(a, x), (a, y)\}, \{(a, x), (b, x)\}, \{(a, x), (b, y)\},$

$\{(a, y), (b, x)\}, \{(a, y), (b, y)\}, \{(b, x), (b, y)\},$

$\{(a, x), (a, y), (b, x)\}, \{(a, x), (a, y), (b, y)\},$

$\{(a, x), (b, x), (b, y)\}, \{(a, y), (b, x), (b, y)\},$

$\{(a, x), (a, y), (b, x), (b, y)\}\}.$

6. Find functions defined on the set of nonnegative integers that define the sequence whose frits six terms are given.

a. $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11}$

$$f(x) = \frac{1}{2x+1} (1)^x$$

b. $0, -2, 4, -6, 8, -10$

$$f(x) = 2x(-1)^x$$

7.1.10

10. Let D be the set of all finite subsets of positive integers.
Define a function $T: \mathbb{Z}^+ \to D$ as follows: For each positive integer n,
$T(n)$ = the set of positive divisors of n.
Find the following:

a. $T(1) = \{1\}$

b. $T(15) = \{1, 3, 5, 15\}$

c. $T(17) = \{1, 17\}$

d. $T(5) = \{1, 5\}$

e. $T(18) = \{1, 2, 3, 6, 9, 18\}$

f. $T(21) = \{1, 3, 7, 21\}$

15. Let F and G be functions from the set of all real numbers to it self. Define the product functions $F \cdot G: \mathbb{R} \to \mathbb{R}$ and $G \cdot F: \mathbb{R} \to \mathbb{R}$ as follows: For all $x \in \mathbb{R}$,

$$(F \cdot G)(x) = G(x) \cdot F(x)$$
$$(G \cdot F)(x) = F(x) \cdot G(x)$$

Does $F \cdot G = G \cdot F$? Explain.

$$\because (F \cdot G)(x) = F(x) \cdot G(x)$$
$$= G(x) \cdot F(x)$$
$$= (G \cdot F)(x)$$

$\therefore F \cdot G = G \cdot F$ ∎

7.1.18

18. Find the exact values for each of the following quantities. Do not use a calculator

a. $\log_3 81 = 4$

b. $\log_2 1024 = 10$

c. $\log_3 \left(\frac{1}{27}\right) = -3$

d. $\log_2 1 = 0$

e. $\log_{10} \left(\frac{1}{10}\right) = -1$

f. $\log_3 3 = 1$

g $\log_2 2^k = k$

25. Let $A = \{2, 3, 5\}$ and $B = \{x, y\}$. Let P_1 and P_2 be the projections of $A \times B$ onto the first and the second coordinates. That is, for each pair $(a, b) \in A \times B$, $P_1(a,b) = a$ and $P_2(a, b) = b$.

a. Find $P_1(2,y)$ and $P_1(5,x)$. what is the range of P_1?

$P_1(2,y) = 2$

$P_1(5,x) =$

Range of $P_1 = A = \{2, 3, 5\}$

b. Find $P_2(2,y)$ and $P_2(5,x)$. What is the range of P_2?

$P_2(2,y) = y$

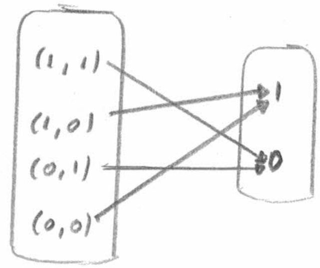
$P_2(5,x) = x$

Range of $P_2 = B = \{x, y\}$

30. Draw arrow diagrams for the Boolean functions defined by the following input/output tables

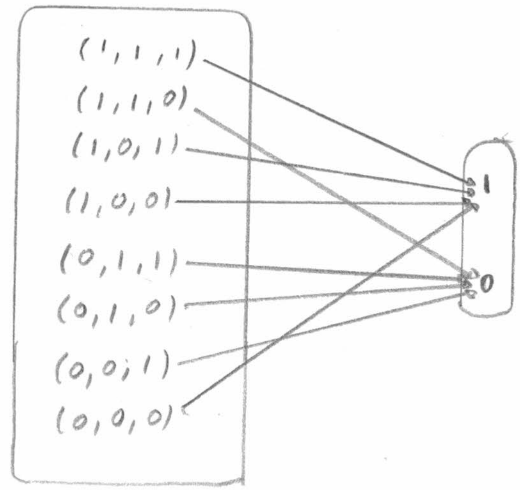
a.

P	Q	R
1	1	0
1	0	1
0	1	0
0	0	1



b.

P	Q	R	S
1	1	1	1
1	1	0	0
1	0	1	1
1	0	0	1
0	1	1	0
0	1	0	0
0	0	1	0
0	0	0	1



35. Let $J_5 = \{0,1,2,3,4\}$. The $J_5 - \{0\} = \{1,2,3,4\}$.
Student A tries to define a function $R: J_5 - \{0\} \to J_5 - \{0\}$
as follows: for each $x \in J_5 - \{0\}$.

$R(x)$ is the number y such that $(xy) \bmod 5 = 1$

Student B claims that R is not well defined. Whos is right: student A or student B? Justify your answer

Answer: Student B is right, and R is not well defined, for the function does not output one unique value.

$R(3) = 2 \quad \Rightarrow \quad (3 \cdot 2) \bmod 5 = 1$

$R(3) = 7 \quad \Rightarrow \quad (3 \cdot 7) \bmod 5 = 1$

40. Let X and Y be sets, let A and B be any subset of X, and let F be a function from X to Y. Fill in the blanks in the following proof that $F(A) \cup F(B) \subseteq F(A \cup B)$.

Proof: Let y be any element in $F(A) \cup F(B)$. [We must show that y is in $F(A \cup B)$.] By definiton of union, $\underline{y \in F(A) \text{ or } y \in F(B)}$.

Case1, $y \in F(A)$: In this case, by definition of $F(A)$, $y = F(x)$ for $\underline{\text{some}}$ $x \in A$. Since $A \subseteq A \cup B$, it follows from the definition of union that $x \in A \cup B$. Hence, $y = F(x)$ for some $x \in \underline{A \cup B}$, and thus, by definition of $F(A \cup B)$, $y = \underline{F(A \cup B)}$

Case2, $y \in F(B)$: In this case, by definition of $F(B)$, $\underline{y = F(x)}$ $\underline{\text{for some}}$ $x \in B$. Since $B \subseteq A \cup B$, it follows from the definition of union that $\underline{x \in A \cup B}$. Hence, by definition of $F(A \cup B)$, $y \in F(A \cup B)$.

Therefore, regardless of whether $y \in F(A)$ or $y \in F(B)$, we have that $y \in F(A \cup B)$ [as was to be shown].

Each of 51-53 refers to the Euler phi function, denoted ϕ, which is defined as follows: For each integer $n \geq 1$, $\phi(n)$ is the number of positive integers less than or equal to to n that have no common factors with n except ± 1. For example, $\phi(10) = 4$ because there are four positive integers less than or equal to 10 that have no common factors with 10 except ± 1; namely, 1, 3, 7, 9.

51. Find the following:

a. $\phi(15) = 8$

b. $\phi(2) = 1$

c. $\phi(5) = 4$

d. $\phi(12) = 4$

e. $\phi(11) = 10$

f. $\phi(1) = 1$

2. Prove that for all integers m and n, m-n is even if, and only if, both m and n are even or both m and n are odd.

Proof | Suppose m and n are integers

Case 1, m & n are evens: $m = 2k$ & $n = 2l$ where k & l are some integers

$$m - n = 2k - 2l$$
$$= 2(k-l) \quad \text{let } k-l = a$$
$$= 2a$$

∴ By definition, m-n is even

Case 2, m & n are odd: $m = 2k+1$ & $n = 2l+1$ where k & l are some integers

$$m - n = 2k + \cancel{1} - 2l - \cancel{1}$$
$$= 2k - 2l$$
$$= 2(k-l) \quad \text{let } k-l = a$$
$$= 2a$$

∴ By definition, m-n is even be

Case 3, either m or n is even: $m = 2k$ $n = 2l+1$ where k & l are some integers

$$m - n = 2k - 2l - 1$$
$$= 2(k-l) - 1 \quad \text{let } k-l = a$$
$$= 2a - 1$$

∴ By definition, m-n is odd

4. Define a relation P on $\mathbb{Z}$ as follows for all $m, n \in \mathbb{Z}$,

$m P n \leftrightarrow$ m and n have a common prime factor

a. Is $15 P 25$? yes $\underline{5}$

b. Is $22 P 27$? No

c. Is $0 P 5$? No

d. Is $8 P 8$? Yes

5. Let $X = \{a, b, c\}$. Recall that $\mathscr{P}(X)$ is the power set of X. Define a relation R on $\mathscr{P}(X)$ as follows:

For all $A, B \in \mathscr{P}(X)$,

$A \, R \, B \leftrightarrow$ A has the same number of elements as B

a. Is $\{a, b\} \, R \, \{b, c\}$? Yes

b. Is $\{a\} \, R \, \{a, b\}$? No

c. Is $\{c\} \, R \, \{b\}$? Yes

8. Let A be the set of all strings of a's and b's of length 4. Define a relation R on A as follows: For all $s, t \in A$,

$s\,R\,t \leftrightarrow s$ has the same first two elements as t.

a. Is abaa R abba? Yes

b. Is aabb R bbaa? No

c. Is aaaa R aaab? Yes

d. baaa R abaa? Yes

10. Let $A = \{3, 4, 5\}$ and $B = \{4, 5, 6\}$ and let R be the "less than" relation. That is, for all $(x, y) \in A \times B$,

$$x R y \leftrightarrow x < y$$

State explicitly which ordered pairs are in R and R^{-1}

$$R = \{(3,4), (3,5), (3,6), (4,5), (4,6), (5,6)\}$$

$$R^{-1} = \{(4,3), (5,3), (6,3), (5,4), (6,4), (6,5)\}$$

8.1.12

12. a. Suppose a function $F: X \to Y$ is one-to-one but not onto. Is F^{-1} a function?

Answer: No, for F^{-1} to be a function, F has to be bijective.

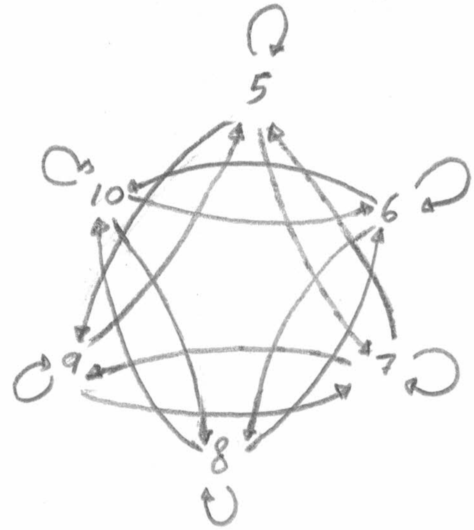
b. Suppose a function $F: X \to Y$ is onto but not one-to-one. Is F^{-1} a function?

Answer: No, for F^{-1} to be a function, F has to be bijective.

Draw the directed graphs of the relations defined in 13-18 8.1.16

16. Let $A = \{5, 6, 7, 8, 9, 10\}$ and define a relation S on A as follows: For all $x, y \in A$,

$$x \, S \, y \leftrightarrow 2 \mid (x-y)$$



8.1.19

Exercises 19-20 refer to unions and intersections of relations. Since relations are subsets of Cartesian products, their unions and intersections can be calculated as for any subsets. Given two relations R and S from A to B,

$$R \cup S = \{(x,y) \in A \times B \mid (x,y) \in R \text{ or } (x,y) \in S\}$$

$$R \cap S = \{(x,y) \in A \times B \mid (x,y) \in R \text{ and } (x,y) \in S\}.$$

19. Let $A = \{2,4\}$ and $B = \{6,8,10\}$ and define relations R and S as follows: For all $(x,y) \in A \times B$,

$$x R y \leftrightarrow x \mid y \quad \text{and}$$
$$x S y \leftrightarrow y - 4 = x$$

State explicitly which ordered pairs are in $A \times B$, R, S, $R \cup S$, $R \cap S$.

$$A \times B = \{(2,6), (2,8), (2,10), (4,6), (4,8), (4,10)\}$$

$$R = \{(2,6), (2,8), (2,10), (4,8)\}$$

$$S = \{(2,6), (4,8)\}$$

$$R \cup S = \{(2,6), (2,8), (2,10), (4,8)\}$$

$$R \cap S = \{(2,6), (4,8)\}$$

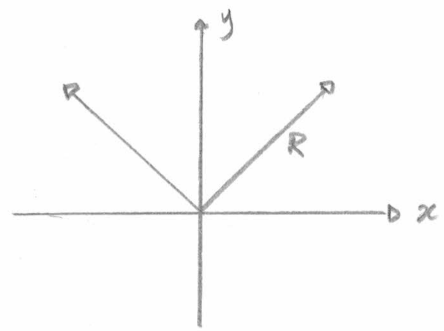
8.1.23

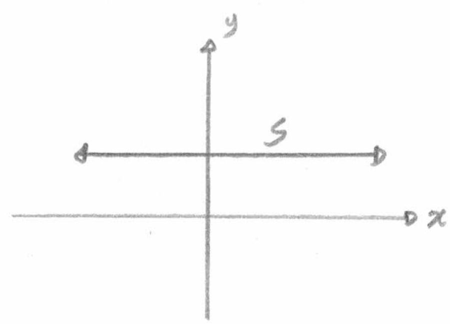
23. Define relations R and S on $\mathbb{R}$ as follows:

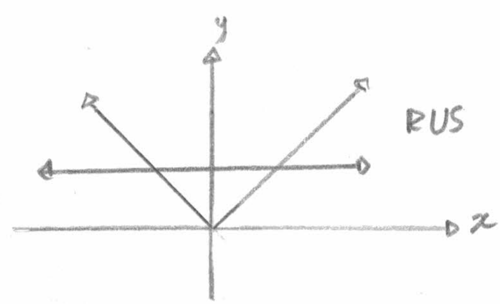
$R = \{(x,y) \in \mathbb{R} \times \mathbb{R} \mid y = |x|\}$ and

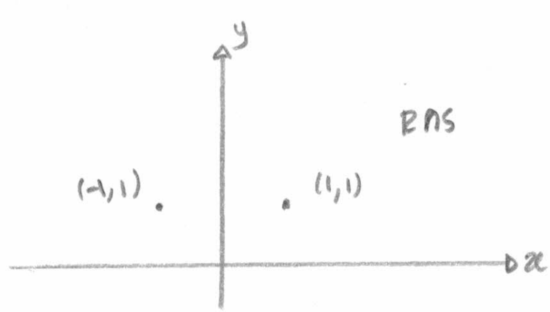
$S = \{(x,y) \in \mathbb{R} \times \mathbb{R} \mid y = 1\}$

Graph R, S, $R \cup S$, $R \cap S$ in the cartesian plane.









24. In Example 8.1.7, the result of the query SELECT Patient_ID#, Name FROM S WHERE Primary_Diagnosis = X is the projection onto the first two coordinates of the intersection of the set $A_1 \times A_2 \times A_3 \times \{X\}$ with the database

a. Find the result of the query SELECT Patient_ID#, Name FROM S WHERE Primary_Diagnosis = pneumonia.

Results - 574329, Tak Kurosawa
- 011985, John Schmidt

b. Find the result of the query SELECT Patient_ID#, Name FROM S WHERE Primary_Diagnosis = appendicitis.

Results - 466581, Mary Lazars
- 778400, Jamal Baskers

8.2. 6

In 1-8, a number of relations are defined on the set $A = \{0,1,2,3\}$
For each relation:

a. Draw the directed graph.

b. Determine whether the relation is reflexive

c. " " " " " symmetric

d. " " " " " transitive

Give a counterexample in each case in which the relation does not satisfy one of the properties

6. $R_6 = \{(0,1), (0,2)\}$

a.

b. Not reflexive $(0,0) \notin R_6$

c. Not symmetric $(0,1) \in R_6$ but $(1,0) \notin R_6$

d. Not reflexive $(0,1) \in R_6$ & $(0,2) \in R_6$ but $(1,2) \notin R_6$

8.2.11

In 9-33, determine whether the given relation is reflexive, symmetric, transitive, or none of these. Justify your answer.

11. D is the relation on $\mathbb{R}$ as follows: for all $x, y \in \mathbb{R}$,
$x\,D\,y \Leftrightarrow xy \geq 0$

D is reflexive: $x\,D\,x \Leftarrow x \cdot x \geq 0$
$$x^2 \geq 0$$

D is symmetric: $x\,D\,y \to y\,D\,x$

$$xy \geq 0 \to yx \geq 0$$

D is not transitive: $x\,D\,y$ & $y\,D\,z \to x\,D\,y$

counterexample: let $x=1$, $y=0$, $z=-1$

$x\,D\,y \quad (1)(0) \geq 0$

$y\,D\,z \quad (0)(-1) \geq 0$

$x\,D\,z \quad (1)(-1) \ngeq 0$

8.2.15

In 9-33, determine whether the relation is reflexive, symmetric, transitive, or non of these. Justify your answer.

15. D is the "divides" relation on $\mathbb{Z}^+$: For all positive integers m and n, $mDn \Leftrightarrow m|n$

D is reflexive: $mDm = m|m$

D is not symmetric: $mDn \rightarrow nDm$

counterexamples $m=3$ & $n=6$

$3|6$ but $6 \nmid 3$

D is transitive: mDn & $nDp \rightarrow mDn$

$m|n$ means $n=mk$ where k is some integer

$n|p$ means $p=nl$ where l is some integer

$m|p = m|m(kl)$

8. 2. 20

In 9-33, determine whether the given relation is reflexive, symmetric, transitive, or non of theses. Justify your answers.

20. Let $X = \{a, b, c\}$ and $\mathcal{P}(X)$ be the power set of X (the set of all subsets of X) A relation E is defined on $\mathcal{P}(X)$ as follows: For all $A, B \in \mathcal{P}(X)$, $A E B \Leftrightarrow$ the number of elements in A equals the number of elements in B.

- E is reflexive: $A E A$ — A has the same number of elements as A which is true for any subset.

E is symmetric: $A E B \rightarrow B E A$ by the definition of E

E is transitive: $A E B$ & $B E C \rightarrow A E C$ by the definition of E

8.2.25

In 19-33 determine whether the given relation is reflexive, symmetric, transitive, or non of these. Justify your answer.

25. Let A be the set of all strings of a's and b's of length 4. Define a relation R on A as follows: for all $s, t \in A$, $sRt \Leftrightarrow$ s has the same first characters as t.

R is reflexive: xRx x has the same first two characters as x

R is symmetric: $sRt \to tRs$ by the definition of R

R is transitive: $sRt \cap tRn \to sRn$ by the definition of R

In 19-33, determine whether the given function is reflexive, symmetric, transitive, or non of these. Justify your answer.

31. Let A be the set of people living in the world today. A relation R is defined on A as follows: For all $p, q \in A$,

$$pRq \Leftrightarrow p \text{ lives within 100 miles of } q$$

R is reflexive: pRp a person lives within 100 miles of himself.

R is symmetric: $pRq \to qRp$ if person lives within 100 miles of me, then I live within 100 miles of them.

R is not transitive: $pRq \cap qRs \to p\cap s$

Counterexample: $p = 0$, $q = 70$, $s = 140$

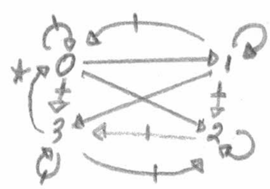
$pRq \cap qRs$ but $p \not R s$

8.2.51

In 51 - 53 R, S, and T are relations defined on $A = \{0, 1, 2, 3\}$.

51. Let $R = \{(0,1)\ (0,2), (1,1), (1,3), (2,2), (3,0)\}$.
Find R', the transitive closure of R.

$R' = \{(0,0), (0,1), (0,2), (0,3), (1,0), (1,1), (1,2)$
$(1,3), (2,2), (3,0), (3,1), (3,2), (3,3)\}$



In each of 3-14, the relation R is an equivalence relation on the set A. Find a distinct equivalence class of R.

5. $A = \{1, 2, 3, 4, \ldots, 20\}$. R is defined on A as follows:

$$\text{For all } x, y \in A, \quad x R y \Leftrightarrow 4 \mid (x-y)$$

$\{1, 5, 9, 13, 17\}$
$\{2, 6, 10, 14, 18\}$
$\{3, 7, 11, 15, 19\}$
$\{4, 8, 12, 16, 20\}$

In each of 3-14, the relation R is an equivalence relation on the set A.
Find a distinct equivalence class of R.

10. $A = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$. R is defined on A as follows:
for all $m, n \in \mathbb{Z}$

$$mRn \Leftrightarrow 3 \mid m^2 - n^2$$

$[5] = \{-5, -4, -2, -1, 1, 2, 4, 5\}$

$[0] = \{-3, 0, 3\}$

$[-5]$ or $[5]$	$25 - 25 = 0 \Rightarrow 3 \mid 0$
$[-4]$ or $[4]$	$25 - 16 = 9 \Rightarrow 3 \mid 9$
$[-3]$ or $[3]$	$25 - 9 = 16 \Rightarrow 3 \nmid 16$
$[-2]$ or $[2]$	$25 - 4 = 21 \Rightarrow 3 \mid 21$
$[-1]$ or $[1]$	$25 - 1 = 24 \Rightarrow 3 \mid 24$
$[0]$	$25 - 0 = 25 \Rightarrow 3 \nmid 25$

In each of 3-14, the relation R is an equivalence relation on the set A. Find a distinct equivalence class of R.

11. $A = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$. R is defined on A as follows For all $(m, n) \in A$

$$m \, R \, n \Leftrightarrow 4 \mid (m^2 - n^2).$$

$[0] = \{-4, -2, 0, 2, 4\}$

$[1] = \{-3, -1, 1, 3\}$

$[-4]$ or $[4]$	$16 - 16 = 0 \Rightarrow 4 \mid 0$
$[-3]$ or $[3]$	$16 - 9 = 7 \Rightarrow 4 \nmid 7$
$[-2]$ or $[2]$	$16 - 4 = 12 \Rightarrow 4 \mid 12$
$[-1]$ or $[1]$	$16 - 1 = 15 \Rightarrow 4 \nmid 15$
$[0]$	$16 - 0 = 16 \Rightarrow 4 \mid 16$

15. Determine which of the following congruence relations are true and which are false.

a. $17 \equiv 2 \bmod 5 \Rightarrow 17-2=15$ & $5 \mid 15$

True

b. $4 \equiv -5 \bmod 7 \Rightarrow 4+5=9$ & $7 \nmid 9$

False

c. $-2 \equiv -8 \pmod{3} \Rightarrow -2+8=6$ & $3 \mid 6$

True

d. $-6 \equiv 22 \pmod{2} \Rightarrow -6-22=-28$ & $2 \mid -28$

True