In each of 3-15 a sequence is defined recursively. Use iteration to guess an explicit formula for the sequence. Use formulas from 5.2 to simplify your answer

5. $c_k = 3c_{k-1} + 1$, for all integers $k \geq 2$
$c_1 = 1$

	-1			
$c_2 = 3(1) + 1 = 4$	$3 = 3 \times 1$		$+1 = 4$	$3 + 1$
		$+3 = 3$		
$c_3 = 3(4) + 1 = 13$	$12 = 3 \times 4$		$+1 = 13$	$3^2 + 3^1 + 1$
		$+9 = 3^2$		
$c_4 = 3(13) + 1 = 40$	$39 = 3 \times 13$		$+1 = 4$	$3^3 + 3^2 + 3 + 1$
		$+27 = 3^3$		
$c_5 = 3(40) + 1 = 121$	$120 = 3 \times 40$		$+1 = 121$	$3^4 + 3^3 + 3^2 + 3 + 1$

Sum of a geometric sequence

$$\sum_{i=1}^{n} 3^{i-1} = \frac{3^{n+1-1} - 1}{3 - 1} = \frac{3^n - 1}{2}$$

$$c_n = \frac{3^n - 1}{2}$$

In each of 3-15 a sequence is defined recursively. Use iteration to guess an explicit formula for the sequence. Use the formulas from Section 5.2 to simplify your answers whenever possible.

10. $h_k = 2^k - h_{k-1}$, for all integers $k \geq 1$
$h_0 = 1$

$h_1 = 2 - 1 = 1$

$h_2 = 2^2 - (2-1) = 3$ $\quad +2$

$h_3 = 2^3 - (2^2 - (2-1)) = 5$ $\quad +2$

$h_4 = 2^4 - (2^3 - (2^2 - (2-1))) = 11$ $\quad +5$

$\vdots$

$h_n = 2^n - 2^{n-1} + 2^{n-2} + \cdots + (-1)^n 1$

$= (-1)^n (1 - 2 + \cdots - 2^{n-1} + 2^n)$

$= \sum_{n=1}^{k} (-1)^n (-2)^n$

$= (-1)^n \dfrac{(-2)^{n+1} - 1}{-2-1} = (-1)^n \dfrac{[(-2)^{n+1} - 1]}{-3}$ ✓

$$h_n = \frac{2^{n+1} - (-1)^{n+1}}{3} \quad \text{for all } n \geq 0$$

In each of 3-15 a sequence is defined recursively. Use iteration to guess an explicit formula for the the sequence. Use formulas from Section 5.2 to simplify your answers whenever possible.

14. $x_k = 3x_{k-1} + k$, for all integers $k \geq 2$
$x_1 = 1$

n	0
3	1
2	2
1	3
0	n

$$x_2 = 3(1) + 2 = 3+2 = 3^1(1) + 3^0 \cdot 2$$

$$x_3 = 3(3+2) + 3 = 3\cdot(3+2) + 3 = 3^2(1) + 3^1(2) + 3^0(3)$$

$$x_4 = 3(3(3+2)+3)+4 = 3(3(3+2)+3)+4 = 3^3(1) + 3^2(2) + 3^1(3) + 3^0(4)$$

$$x_n = 3^n \cdot 0 + 3^{n-1}(1) + 3^{n-2}(2) + \dots + 3^2(n-2) + 3^1(n-1) + 3^0(n)$$

$$= (3^{n-1} + 3^{n-2} + \dots) + (3^{n-2} + 3^{n-3} \dots) + (3^{n-3} + 3^{n-4} + \dots) + \dots + (3+1) + (1)$$

Double Sum $\displaystyle\sum_{l=m}^{m=n} \sum_{l=0}^{m} 3^{l-1}$

$$= \sum_{l=m}^{m=n} \frac{3^{l-1+1} - 1}{3-2}$$

$$= \sum_{l=m}^{m=n} \frac{3^{l} - 1}{2} = \frac{1}{2} 3 \sum_{l=m}^{m=n} (3^{l-1}) - \frac{1}{2} \sum_{l=n}^{m=n} l$$

$$= \frac{3}{2}\left(\frac{\cdot 3^{n+1-1} - 1}{3-2}\right) - \frac{1}{2} n$$

$$= \frac{3^{n+1} - 3 - 2n}{4} \text{ for all } n \geq 1$$

5.7.19

19. A worker is promised a bonus if he can increase his productivity by 2 units a day every day for a period of 30 days. If on day he produced 170 units, how many units must he produce on day 30 to qualify for a bonus?

$a_0 = 170$

$a_n = a_0 + 2n$

$a_{30} = 170 + 2(30) = 260$

5.7.25

25. A certain computer algorithm execues twice as many operations when it is rum with an input of size k as when it is run with an input of size k-1 (Where k is an integer greater than 1). When the algorithm is run with an input of size 1, it executes seven operations. How many operations does it executes when it is run with an input of size 25?

$a_n = a_0 r^{n-1}$ $\quad$ & $\quad$ $a_0 = 7$ $\quad$ & $\quad$ $r = 2$

$a_n = 7 \cdot 2^{n-1}$

$a_{25} = 7 \cdot 2^{24} = 117440512$

In 28-42 use mathematical induction to verify the correctness of the formula you obtained in the refrenced exercise.

30. Exercise 5. $C_k = 3C_{k-1} + 1$

$$C_n = \frac{3^n - 1}{2} \quad \text{where } n \geq 1$$

Proof | Let $C(n) = C_n = \dfrac{3^n - 1}{2}$

$C(1) = 1$ True

Assume $C(k) = \dfrac{3^k - 1}{2}$ is true

Inductive Step $C(k+1) = \dfrac{3^{k+1} - 1}{2} = 3C_{(k+1)-1} + 1$

$$= 3C_k + 1$$

$$= 3 \cdot \frac{3^k - 1}{2} + 1$$

$$= \frac{3^{k+1} - 3 + 2}{2}$$

$$\frac{3^{k+1} - 1}{2} = \frac{3^{k+1} - 1}{2}$$

$\because$ $C(0)$ & $C(k+1)$ are true

$\therefore$ $C_n = \dfrac{3^n - 1}{2}$ is correct by mathematical induction

In 38 - 42 use mathematical induction to verify the correctness of the formula you obtained in the referenced exercise.

35. Exercise 10. $h_k = 2^k - h_{k-1} \quad \forall k \in \mathbb{Z} \ \& \ k \geq 1$

Formula: $h_n = \dfrac{2^{n+1} - (-1)^{n+1}}{3} \quad \forall n \in \mathbb{Z} \ \& \ n \geq 0$

Proof | $h_0 = \dfrac{3}{3} = 1 \Rightarrow h_0$ is true

$$h_{k+1} = \frac{2^{k+2} - (-1)^{k+2}}{3} = 2^{k+1} - \frac{2^{k+1} - (-1)^{k+1}}{3}$$

$$= \frac{3 \cdot 2^{k+1}}{3} - \frac{2^{k+1} + (-1)(-1)^{k+1}}{3}$$

$$= \frac{2 \cdot 2^{k+1} - (-1)^{k+2}}{3}$$

$$\frac{2^{k+2} - (-1)^{k+2}}{3} = \frac{2^{k+2} - (-1)^{k+2}}{3}$$

$\because$ h_0 & h_{k+1} are true

$\therefore$ $h_n = \dfrac{2^{n+1} - (-1)^{n+1}}{3}$ is correct by mathematical induction

In 28-42 use mathematical induction to verify the correctness of the formula you obtained in the referenced exercise. 5.7.39

39. Exercise 14. $x_k = 3x_{k-1} + k$ for all integers $k \geq 2$

$$x_n = \frac{3^{n+1} - 3 - 2n}{4} \quad \text{for all integers } n \geq 1$$

Proof | $x_1 = \frac{3^2 - 3 - 2}{4} = \frac{4}{4} = 1$ true

Assume x_k is true

$$x_{k+1} = \frac{3^{k+2} - 3 - 2(k+1)}{4} = 3\left(\frac{3^{k+1} - 3 - 2k}{4}\right) + (k+1)$$

$$= \frac{3^{k+2} - 3 - 6(k+1)}{4} + \frac{4(k+1)}{4}$$

$$\frac{3^{k+2} - 3 - 2(k+1)}{4} = \frac{3^{k+2} - 3 - 2(k+1)}{4}$$

$\because$ x_1 & x_{k+1} are true

$\therefore$ $x_n = \frac{3^{n+1} - 3 - 2n}{4}$ for all $n \geq 1$ is correct

by mathematical induction. ∎