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Quiz

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1) Sum $\{100, 104, 108, \ldots, 1000\}$

$= 4\{25, 26, 27, \ldots, 250\}$

$= 4\left(\frac{250(251)}{2}\right) - \frac{24(25)}{2}$

$= 500(251) - 12(25)$

$= 125500 - (300) = 125200$

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2) Sum 2^i where $i \in \mathbb{Z}$, $4 \leq i \leq 10$

$= \sum_{i=4}^{10} 2^i = \sum_{i=0}^{10} 2^i - \sum_{i=0}^{4} 2^i = \frac{2^{11}-1}{2-1} - \frac{2^5-1}{2-1}$

$= 2^{11} - 1 - (2^5 - 1)$

$= 2^{11} - 2^5$

$= 2048 - 32$

$= 2016$

3) Company A → $A_0 = \$10^6$ increasing $\frac{1}{6} \times 10^6$ / year

Company B → $B_0 = \$10^6$ increasing 12% year

When will $A_k = B_k$ in the future

$$A_k = A_0 + K \cdot \frac{1}{6} \cdot 10^6$$

$$B_k = B_0 (0.12)^K$$

$$\Sigma A_K = A_0 + \frac{A_0}{6} \cdot \frac{K(K+1)}{2} = A_0 \left(1 + \frac{K(K+1)}{12}\right)$$

$$\Sigma B_K = B_0 \left(\frac{(0.12)^{K+1} - 1}{0.12 - 1}\right)$$

Let $\Sigma A_k = \Sigma B_k$: $B_0 \left(\frac{(0.12)^{K+1} - 1}{0.12 - 1}\right) = A_0 \left(1 + \frac{K(K+1)}{12}\right)$

$$A_0 = B_0$$

$$\frac{(0.12)^{K+1} - 1}{0.12 - 1} = 1 + \frac{K(K+1)}{12}$$

$0.12 - 1 = -0.88$

$$(0.12)^{K+1} - 1 = -0.88 - \frac{0.88K(K+1)}{12}$$

$$0.12 \cdot 0.12^K = -0.88 - \frac{0.88K(K+1)}{12}$$

$\frac{12}{100}$

$$0.12^K = -\frac{88}{12} - \frac{88K(K+1)}{144}$$

$$0.12^K = \frac{-88 \cdot 12 - 88K(K+1)}{144}$$

$$0.12^K + \frac{88K(K+1)}{144} = \frac{-88 \times 12}{144}$$

Solve by graphing

4) Prove $\binom{N}{J} < \frac{N!}{J!}$ $\quad 1 < J < N \quad J, N \in \mathbb{Z}^+$

Proof | $\binom{N}{J} < \frac{N!}{J!}$

$$\frac{N!}{J!(N-J)!} < \frac{N!}{J!}$$

$$\frac{(N-J)! \cdots N}{J!(N-J)!}$$

$$\frac{n \cdot (N-1)N}{J!} < \frac{N!}{J!}$$