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Chapter 2

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Section 2.1

6 --------------------------------Done

12 ------------------------------Done

18 ------------------------------Done

24 ------------------------------Done

30 ------------------------------Done

36 ------------------------------Done

42 ------------------------------Done

48 ------------------------------Done

Section 2.2

6 --------------------------------Done

12 ------------------------------Done

18 ------------------------------Done

24 ------------------------------Done

30 ------------------------------Done

36 ------------------------------Done

42 ------------------------------Done

48 ------------------------------Done

Section 2.3

6 --------------------------------Done

12 ------------------------------Done

18 ------------------------------Done

24 ------------------------------Done

30 ------------------------------Done

36 ------------------------------Done

42 ------------------------------Done

Section 2.4

6 --------------------------------Done

12 ------------------------------Done

18 ------------------------------Done

24 ------------------------------Done

30 ------------------------------Done

Section 2.5

6 --------------------------------Done

12 ------------------------------Done

18 ------------------------------Done

24 ------------------------------Done

30 ------------------------------Done

36 ------------------------------Done

42 ------------------------------Done

Write the statements in 6-9 in symbolic form using the symbols $\sim$, $\vee$, and $\wedge$ and the indicated letters to represent component statements.

6. Let s = "stocks are increasing" and i = "interest rates are steady."

a. Stocks are increasing but interest rates are steady.

Answer: $s \wedge i$

b. Neither are stocks increasing nor are interest rates are steady.

Answer: $\sim s \wedge \sim i$

Write the truth table for statement forms 12-15

12. $\sim p \vee q$

p	q	$\sim p$	$\sim p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

Determine wheather the statement forms in 16-24 are logically equivalent. In each case, construct a truth table, and include a sentence justifying your answer. Your sentence should show that you understand the meaning of logical equivalence.

18. $p \vee t$ and t

p	t	$p \vee t$
T	T	T
T	T	T
F	T	T
F	T	T

$\because$ $p \vee t$ has the same values as t

$\therefore$ $t \equiv p \vee t$

Determine whether the statements form in 16-24 are logically equivalent. In each case, construct a truth table and include a sentence justifying your answer. Your sentence should show that you understand the meaning of logical equivalence.

24. $(p \vee q) \vee (p \wedge r)$ and $(p \vee q) \wedge r$

1 p	2 q	3 r	4 $1 \vee 2$	5 $1 \wedge 3$	6 $4 \vee 5$		$4 \wedge 3$
T	T	T	T	T	T		T
T	T	F	T	F	T	≠	F
T	F	T	T	T	T		T
T	F	F	T	F	T	≠	F
F	T	T	T	F	T		T
F	T	F	T	F	T	≠	F
F	F	T	F	F	F		F
F	F	F	F	F	F		F

$\because$ $(p \vee q) \vee (p \wedge r)$ and $(p \vee q) \wedge r$ do not always have the same value

$\therefore (p \vee q) \vee (p \wedge r) \not\equiv (p \vee q) \wedge r$

Use De Morgan's laws to write negations for the statements in 25-31.

30. The dollar is at an all-time high and the stock market is at a record low.

Answer: The dollar is <u>not</u> at an all-time high or the stock market is <u>not</u> at a record low.

Assume x is a particular real number and use De Morgan's laws to write negations for the statements in 33-37.

36. $1 > x \geq -3$

Answers: $1 \leq x$ or $x < 3$

Use truth tables to establish which of the statement forms in 40-43 are tautologies and which are contradictions.

42. $((\sim p \wedge q) \wedge (q \wedge r)) \wedge \sim q$

1	2	3	4	5	6	7	8	
p	q	r	$-p$	$4 \wedge 1$	$1 \wedge 3$	$5 \wedge 6$	$\sim q$	$7 \wedge 8$
T	T	T	F	F	T	F	F	F
T	T	F	F	F	F	F	F	F
T	F	T	F	F	T	F	T	F
T	F	F	F	F	F	F	T	F
F	T	T	T	F	F	F	F	F
F	T	F	T	F	F	F	F	F
F	F	T	T	F	F	F	T	F
F	F	F	T	F	F	F	T	F

∵ The statement results in false regardless of input

∴ The statement is a contradiction.

In 48 and 49 below, a logical equivalence is derived from theorem 2.1.1. Supply a reason for each step.

48. $(p \wedge \sim q) \vee (p \wedge q) \equiv p \wedge (q \vee \sim q)$ by a

$\equiv p \wedge (\sim q \vee q)$ by b

$\equiv p \wedge t$ by c

$\equiv p$ by d

a) Distributive law of $\wedge$

b) Commulative law of $\wedge$

c) Negation law of $\vee$

d) Identity law of $\wedge t$

Construct truth table for the statement forms in 5-11

6. $(p \vee q) \vee (\sim p \wedge q) \rightarrow q$

1	2	3	4	5	6	
p	q	$\sim p$	$1 \vee 2$	$3 \wedge 2$	$4 \vee 5$	$6 \rightarrow 2$
T	T	F	T	F	T	T
T	F	F	T	F	T	F
F	T	T	T	T	T	T
F	F	T	F	F	F	T

12. Use the logical equivalence established in example 2.2.3, $p \vee q \to r \equiv (p \to r) \wedge (q \to r)$, to rewrite the following statement. (Assume x represents a fixed real number.)

If $x > 2$ or $x < -2$, then $x^2 > 4$

Answer if $x > 2$ then $x^2 > 4$, and if $x < -2$ then $x^2 > 4$

18. Write each of the following three statements in symbolic form and determine which pairs are logically equivalent. Include truth tables and a few words of explaination.

I. If it walks like a duck and it talks like a duck, then it is a duck

II. Either it does not walk like a duck or it does not talk like a duck, or it is a duck.

III. If it does not walk like a duck and it does not talk like a duck, then it is not a duck.

Solutions Let Walk like a duck = A premise
" Talk like a duck = B premise
" It is a duck = C Conclusion

$\}$ Hypothesis

Therefore, the statement written in symbolic forms

I. $(A \wedge B) \rightarrow C$

II. $(\sim A \vee \sim B) \vee C$

III. $(\sim A \wedge \sim B) \rightarrow C$

$\longrightarrow$ Continue...

I. $(A \wedge B) \rightarrow C$

II. $(\sim A \vee \sim B) \vee C$

III. $(\sim A \wedge \sim B) \rightarrow \sim C$

A	B	C	$\sim A$	$\sim B$	$\sim C$	1 $A \wedge B$	2 $\sim A \vee \sim B$	3 $\sim A \wedge \sim B$	$1 \rightarrow C$	$2 \vee C$	$3 \rightarrow \sim C$
T	T	T	F	F	F	T	F	F	T	T	T
T	T	F	F	F	T	T	F	F	F	F	T
T	F	T	F	T	F	F	T	F	T	T	T
T	F	F	F	T	T	F	T	F	T	T	T
F	T	T	T	F	F	F	T	F	T	T	T
F	T	F	T	F	F	F	T	F	T	T	T
F	F	T	T	T	T	F	T	T	T	T	T
F	F	F	T	T	F	F	T	T	T	T	F

∵ Statement I & II share the same values

∵ Statement III deos not share the same values to either I or II

∴ $I \equiv II$ & $III \not\equiv I$ & $III \not\equiv II$

Use truth tables to establish the truth of each statement in 24-27

24. A logical statement is not logically equivalent to its converse.

Proof: Let p and q be statements such that $p \to q$.

The converse is $q \to p$

p	q	$p \to q$	$q \to p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

$\because$ $p \to q$ & $q \to p$ do not share the same value

$\therefore p \to q \not\equiv q \to p$

If statement forms P and Q are logically equivalent, then $P \leftrightarrow Q$ is a tuatology. Conversly, if $P \leftrightarrow Q$ is a tuatology, then P and Q are logically equivalent. Use $\leftrightarrow$ to convent each of the logical equivalences in 29-31 to a tautology. Then use a truth table to verify each tautology.

30. $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

$p \wedge (q \vee r) \leftrightarrow (p \wedge q) \vee (p \wedge r)$

p	q	r	1 $q \vee r$	2 $p \wedge q$	3 $p \wedge r$	4 $p \wedge 1$	5 $2 \vee 3$	$4 \leftrightarrow 5$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T	T
T	F	T	T	F	T	T	T	T
T	F	F	F	F	F	F	F	T
F	T	T	T	F	F	F	F	T
F	T	F	T	F	F	F	F	T
F	F	T	T	F	F	F	F	T
F	F	F	F	F	F	F	F	T
								T

Clearly, $p \wedge (q \vee r) \leftrightarrow (p \wedge q) \vee (p \wedge r)$

is a tuatology.

36. Taking the long view on your education, you go to the Prestige Corporation and ask what you should do in college to be hired when you graduate. The personnel director replies that you will be hired only if you major in mathematics or computer science, get a B averag or better, and take accounting. You do, in fact, become a math major, get a B^+ average, and take accounting. You return to prestige corporation, make a formal application, and are turned down. Did the personnel director lie to you?

Solutions Let p = You will be hired

q = You are a math major w/ averag B, and took accounting

p only if means $\sim q \to \sim p$

or equivalently $p \to q$

	p	q	$p \to q$
	T	T	T
	T	F	F
★	F	T	T
	F	F	T

So, it is true that a person might not get hired even if the requirements are met.

∴ No, the personnel director did not lie. ∎

Use the contrapositive to write the statements in 42 and 43 in if-then form in two ways.

42. Being divisible by 3 is a necessary condition for this number to be divisible by 9.

a) If a number is not divisible by 3, it is not divisible by 9.

b) If a number is divisible by 9, it is divisible by 3.

In 47-50 (a) use logical equivalences $p \to q \equiv \sim p \vee q$ and $p \leftrightarrow q \equiv (\sim p \vee q) \wedge (\sim q \vee p)$ to rewrite the given statement forms without using the symbol $\to$ or $\leftrightarrow$, and (b) use the logical equivalence $p \vee q \equiv (\sim p \wedge \sim q)$ to rewrite each statement form using only $\wedge$ and $\sim$.

48. $p \vee \sim q \to r \vee q$

a) $\sim (p \vee \sim q) \vee (r \vee q)$

b) $(p \vee \sim q) \wedge \sim (r \vee q)$

$\sim (\sim p \wedge q) \wedge (\sim r \wedge \sim q)$

Using truth tables to determine whether the argument forms in 6-11 are valid. Indicate which columns represents the premises and which represent the conclusion, and include a sentence explaining how the truth table supports your answer. You explaination should show that you understand what it means for a form of argument to be valid or invalid

6. $p \to q$

$q \to p$

$\therefore p \vee q$

p	q	$p \to q$	$q \to p$	$p \vee q$
		Premises	Premises	Conclusion
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	F

$\because$ Both premises are true, yet the conclusion is false

$\therefore$ The argument is invalid

12. Use truth tables to show that the following forms of argument are invalid.

a. $p \to q$

q

$\therefore p$

(converse error)

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

$\because$ Both false premises results in a true conclusion

$\therefore$ The argument form is invalid ■

b. $p \to q$

$\sim p$

$\therefore \sim q$

(inverse error)

p	q	$\sim p$	$\sim q$	$p \to q$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

$\because$ Both premises are true, yet their conclusion is false

$\therefore$ The argument form is invalid

Use truth tables to show that the argument form referred to in 13-21 are valid. Indicate whic columns represent the premises and which represent the conclusion, and include a sentence explaining how the truth table supports your answer. Your explaination should show that you understand what it means for a form of argument to be valid.

18. Example 2.3.5 (a)

$p \vee q$

$\sim q$

$\therefore p$

Premises

Conclusion

p	q	$\sim q$	$p \vee q$
T	T	F	T
T	F	T	T
F	T	F	T
F	F	T	F

$\because$ When both premises are true, the conclusion is true.

$\therefore$ The argument form is valid $\blacksquare$

Some of the arguments in 24-32 are valid, whereas others exhibit the converse or the inverse error. Use symbols to write the logical form of each argument. If the argument is valid, identify the rule of inference that guarantees its validity. Otherwise, state whether the converse or the inverse error is made.

24. If Jules solved the problem, then Jules obtained the answer 2.

Jules obtained the answer 2.

$\therefore$ Jules solved the problem.

Lets p = solved the problem.
& q = Obtained the answer 2.

Logical form:

$$p \to q$$
$$q$$
$$\therefore p$$

Answer: A <u>converse error</u> is made.

36. Given the following information about a computer program, find the mistake in the program.

a. There is an undeclared variable (UV) or there is a syntax error (SE) in the first five lines.

b. If there is a syntax error (SE) in the first five lines, then there is a missing semicolon (MS) or a variable name is misspelled (VN).

c. There is not a missing semicolon.

d. There is not a misspelled variable name.

$UV \vee SE$

$SE \rightarrow (MS \vee VN)$

$\sim MS$

$\sim VN$

$\therefore \sim SE$

$\therefore UV$

Answer: There is an undeclared variable ∎

In 41-44 a set of premises and a conclusion are given. Use the valid argument forms listed in Table 2.3.1 to deduce the conclusion from the premises, giving a reason for each step as in example 2.3.8. Assume all variables are statement variables.

42.
a. $p \vee q$
b. $q \to r$
c. $p \wedge s \to t$
d. $\sim r$
e. $\sim q \to u \wedge s$
f. $\therefore t$

(1) $q \to r$ premise b.
$\sim r$ premise d.
$\therefore \sim q$ by Modus Tollens

(2) $p \vee q$ premise a.
$\sim q$ by conclusion 1
$\therefore p$ by Ellemination

(3) $\sim q \to u \wedge s$ premise e.
$\sim q$ by conclusion 1
$\therefore u \wedge s$ by Modus Ponens

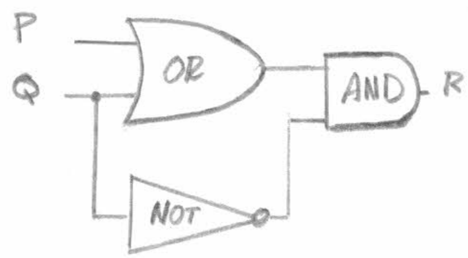
(4) $u \wedge s$ by conclusion 3
$\therefore s$ by Specialization

(5) p by Conclusion 2
s by Conclusion 3
$\therefore p \wedge s$ by Conjunction

(6) $p \wedge s \to t$ premise c.
$p \wedge s$ by Conclusion (5)
$\therefore t$ by Modus Ponens

In 5-8, write an input/output table for the circuit in the referenced exercise.

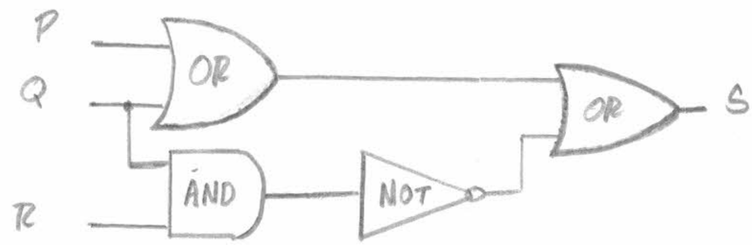
6. Exercise 2.



P	Q	$\sim Q$	$P \vee Q$	R $(P \vee Q) \wedge \sim Q$
1	1	0	1	0
1	0	1	1	1
0	1	0	1	0
0	0	1	0	1

In 9-12, find the Boolean expression that correspond to the circuit in the refrenced exercise.

12. Exercise 4



Answer: $(P \vee Q) \vee \sim(Q \wedge R)$

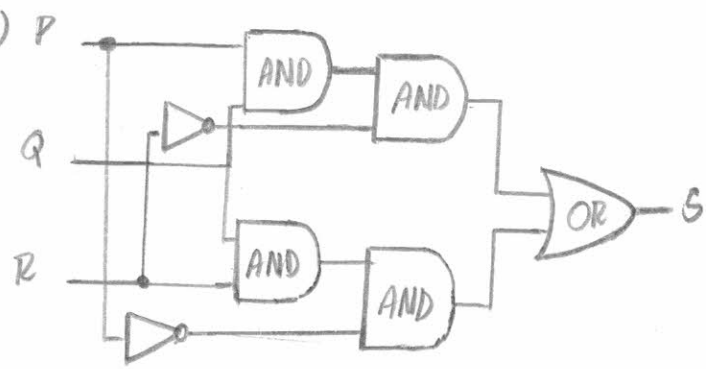
For each of the tables in 18-21, construct (a) a Boolean expression having the given table as its truth table and (b) a circuit having the given table as its input/output table.

18.

P	Q	R	S
1	1	1	0
1	1	0	1
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

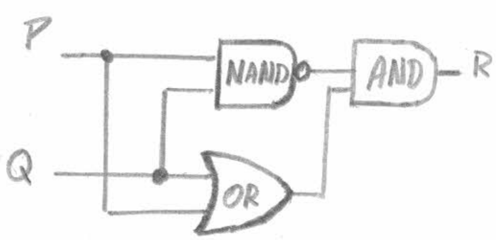
a) $(P \wedge Q \wedge \sim R) \vee (\sim P \wedge Q \wedge R)$

b)



24. The lights in the classroom are controlled by two switches: one at the back and one at the front of the room. Moving either switch to the opposite position turns the lights off if they are on or on if they are off. Assume the lights are installed so that when both switches are in the down position, the lights are off. Design a circuit to control the switches.

P	Q	R	1 $\sim(P \wedge Q)$	2 $P \vee Q$	$1 \wedge 2$
1	1	0	0	1	0
1	0	1	1	1	1
0	1	1	1	1	1
0	0	0	1	0	0



For the circuits corresponding to the Boolean expressions in each of 30 and 31 there is an equivalent circuit with at most two logic gates. Find such a circuit.

30. $(P \wedge Q) \vee (\sim P \wedge Q) \vee (\sim P \wedge \sim Q)$

$(P \wedge Q) \vee \sim P(Q \vee \sim Q)$

$(P \wedge Q) \vee \sim P(t)$

$(P \wedge Q) \vee \sim P$

$(\sim P \vee P) \wedge (\sim P \vee Q)$

$t \wedge (\sim P \vee Q)$

$\sim P \vee Q$

Represent the decimal integers in 1-6 in binary notation.

6. 1424 $= 1024 + 256 + 128 + 16$

$$= 2^{10} + 2^{8} + 2^{7} + 2^{4}$$

$$= \underline{\underline{0101 \quad 1001 \quad 0000_2}}$$

Represent the integers 7-12 in decimal notation.

12. $101\ 1011_2 = 2^6 + 2^4 + 2^3 + 2^1 + 2^0$

$$= \underbrace{64 + 16}_{80} + \overbrace{8 + 2}^{10} + 1$$

$$= \underline{\underline{91_{10}}}$$

Perform the arithmetic in 13-20 using binary notation.

18.

$$\begin{array}{r} \overset{1\,1\;\;1}{\overset{1\,1\;\;1}{\cancel{1}\ 1010_2}} \\ -\ \ 1101_2 \\ \hline 1101_2 \end{array}$$

Answer: $\underline{\underline{1101_2}}$

Check:

1's: 0010_2

2's: 0011_2

$$\begin{array}{r} 1\ 1010_2 \\ 0011_2 \\ \hline \underline{\underline{1101_2}} \end{array} +$$

Find the 8-bit two's complements for the integers in 23-26

24. $67_{10} = 2^6 + 2^1 + 2^0$

$= 0100\ 0011_2$

1's : $1011\ 1100_2$

2's : $\underline{\underline{1011\ \ 1101_2}}$

Find the decimal representations for the integer with the 8-bit representations given in 27-30.

30. $1011\ 1010_2 = BA_{16} = 11 \times 16^1 + 10 \times 16^0$

$= 176_{10} + 10_{10}$

$= \underline{\underline{186_{10}}}$

Use 8-bit representations to compute the sums in 31-36

36. $123_{10} + (-94_{10})$

$123_{10} = 7B_{16} = 0111\ 1011_2$

$94_{10} = 5E_{16} = 0101\ 1110_2$

1's: $1010\ 0001_2$

$-94_{10} = 1010\ 0010_2$

2's: $1010\ 0010_2$

$$
\begin{array}{rr}
123_{10} + (-94_{10}) = & 0111\ 1011_2 \\
+ & 1010\ 0010_2 \\
\hline
& 1|0001\ 1101_2
\end{array}
$$

Answer: $\underline{\underline{0001\ 1101_2}}$

Convert the integers in 41-43 from hexadecimal to binary notation.

42. $B53DF8_{16}$

$$\begin{array}{cccccc} B & 5 & 3 & D & F & 8 \\ 1011 & 0101 & 0011 & 1101 & 1111 & 1000 \end{array}$$

$= \underline{\underline{1011\ 0101\ 0011\ 1101\ 1111\ 1000_2}}$