

Adaptation of the GAMS-format model specs for MCMA

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This note is based on the lessons from the pilot exploration of the MCMA-Message_ix interface made in 2017 & 2018 and suggests modifications of Message_ix aimed at easing multi-criteria analysis of models specified in Message_ix. This is a substantially extended and restructured version of the original (2017) note; in particular, the proposal of the GAMS-MCMA interface was updated based on the 2018 YSSP study and the current attempts to use MCMA with the Indus model, both models developed in Message_ix. Moreover, the note includes now the MCMA architecture outline. The proposed interface focuses on Message_ix but it shall also work for models developed in the standard GAMS and continue to work with models represented by the MPS-format files.

Section 4.4 written by Volker in 2017 requires revision to account for the Message_ix modifications since 2017.

1 Introduction

The note summarizes the requirements for, and proposed implementation of the MCMA-Message_ix interface. The GAMS-format model instance specification file is further on referred to as the (core) model specs.¹ We aim at keeping the required modifications of the standard Message_ix specification at a minimum, however some modifications are necessary in order to keep MCMA applicability to models developed with other than Message_ix modeling environments.

The GAMS-format model specification integrates three elements of modeling process:

1. Core model instance specification composed of:
 - a version of specification of variables and relations (constraints);
 - a version of data defining values of the model parameters.
2. Specification of the optimization objective and optimization solver.
3. Specification of the post-optimization processing.

The note is written from the perspective of the Message_ix modifications;² therefore, the main part of the note is structured according to the above summarized elements of the GAMS specification. The main part is preceded by the outline of the approach and followed by the summary and supplementary material. Thus, the note consists of the following sections:

- Outline of the approach (Sec. 2).
- GAMS-format model specification (Sec. 3).
 - ◊ The basic assumptions (Sec. 3.1).
 - ◊ Core model requirements (Sec. 3.2).
 - ◊ Optimization of the merged model (Sec. 3.3).
 - ◊ Post-optimization processing (Sec. 3.4).
 - ◊ Technical requirements (Sec. 4).
 - ◊ Summary of the requirements for GAMS model specs (Sec. 4.6).

¹Actually, this term is used here for an model instance, i.e., a given version of the symbolic model symbolic and a selected version of data used for the model parameter.

²The corresponding MCMA modifications have already been implemented and tested.

- 39 Supplementary material is available in Appendices:
- 40 • Example of specification of outcome variables (Appendix A.1).
 - 41 • Sample of `_mc.gms` file (Appendix A.2).
 - 42 • Shared space for ENE applications of MCMA (Appendix B).
 - 43 • Functionality and architecture of the MCMA tool (Appendix C).

44 2 Outline of the approach

45 MCMA extends single-criterion model functionality by replacing optimization criterion of the core
 46 model instance (further-on called core model) by maximization of the Scalarizing Achievement Func-
 47 tion (SF), the function of the outcome variables selected to serve as criteria in a specific analysis. The
 48 SF is interactively parametrized by the users who specify preferences for criteria values. Thus, the
 49 original core model is not modified; instead, it is merged with a dynamically (for each specification of
 50 preferences) generated small MC-submodel defining the SF.

51 For effective application of MCMA to a core model instance its specification has to conform to the
 52 requirements presented in Sec. 3. These requirements are easy to follow and do not influence single-
 53 criterion (and other types of) model analysis.

54 To provide the context of the approach we now briefly summarize the Mathematical Programming
 55 view on single-criterion and multi-criteria optimization of LP models.

56 2.1 Preliminaries

57 From the mathematical programming point of view, a model represents the corresponding problem by
 58 two types of entities: variables and relations between them, all these entities but one are treated the same
 59 way, i.e., regardless of their role in the problem representation. Many problems are described by linear,
 60 often dynamic and spatial, models. A standard mathematical programming formulation of such models
 61 takes the form:

$$\underline{\mathbf{b}} \leq \mathbf{A} \cdot \mathbf{x} \leq \bar{\mathbf{b}} \quad (1)$$

62 where vector \mathbf{x} is composed of all model variables, the matrix \mathbf{A} , as well as vectors $\underline{\mathbf{b}}$ and $\bar{\mathbf{b}}$ are the
 63 model parameters. For the brevity sake we use the standard compact formulation (1) that covers also the
 64 bound-type constraints of the core model.

65 For a properly specified decision-making problem the system of relations (1) has infinitely many
 66 solutions. Thus one needs a criterion for selecting a solution that fits best the user preferences. Models
 67 of complex problems involve many variables but in model analysis one typically focuses on a small
 68 subset of all model variables. In particular, the preferences are typically represented as a function of a
 69 subset of all model variables called criteria (aka outcomes, indicators, etc). In this note we use the terms
 70 outcomes and criteria, depending on the context, as explained below.

71 To ease the discussion we shall, depending on the context, consider either all model variables as one
 72 compound³ variable \mathbf{x} or its split into components according to the roles diverse variables represent:

$$\mathbf{x} = \{\mathbf{y}, \mathbf{z}\}, \quad (2)$$

73 where vectors \mathbf{y} and \mathbf{z} are composed of variables representing outcomes and all other model variables,
 74 respectively. The criteria, denoted by \mathbf{q} , are interactively selected from outcomes \mathbf{q} by the each MCMA
 75 user; therefore, $\mathbf{q} \in \mathbf{y}$.

³Compound variable means a (multidimensional) vector of variables.

76 2.2 Single-criterion optimization

77 Mathematical Programming Problem (MPP)⁴ of a linear model (traditionally called LP or LPP) can be
78 formulated as:

$$\text{minimize } \{obj = f(\mathbf{y})\} \quad (3)$$

79 subject to:

$$\underline{\mathbf{b}} \leq \mathbf{A} \cdot \begin{bmatrix} \mathbf{y} \\ \mathbf{z} \end{bmatrix} \leq \bar{\mathbf{b}}, \quad \text{and} \quad (4)$$

$$\underline{\mathbf{y}} \leq \mathbf{y} \leq \bar{\mathbf{y}} \quad (\text{optional}), \quad (5)$$

80 where:

- 81 • the optimization criterion *obj* is defined by (3); the function $f(\mathbf{y})$ takes diverse forms, depending on
- 82 the applied representation of preferences, e.g., a selected criterion, or a utility function, or a weighted
- 83 sum of criteria, or a composite criterion defined as a function of outcome variables and penalty terms,
- 84 etc;
- 85 • reformulation (4) of the core-model (1) with taking into account (2);
- 86 • optional bounds (5) on criteria values augmenting a representation of preferences through the opti-
- 87 mization criterion (3); approaches based on application of sequences of lower/upper bounds in (5) is
- 88 called *parametric optimization* and used as a surrogate of MCA.

89 2.3 Multi-Criteria submodel

90 MCMA exploits the concept of maximization of a Scalarizing Achievement Function (SF) that repre-
91 sents the user preferences; the SF is interactively parametrized by the user during the analysis. While
92 the SF interpretation is intuitive and easy, its specification in terms of mathematical programming is not
93 straightforward; therefore the SF is specified through an auxiliary LP model, further on called the MC
94 sub-model, which is generated for each preference specification. The MC sub-model is merged with
95 the core model (4) and optimized with the same solver as used for the single-criterion optimization of
96 the core model. Therefore, MCMA does not involve any modification of the core model (4), except
97 of skipping (5) (because including bounds on criteria values would cut-off a part of the set of feasible
98 solutions of the core model).

99 The MC-submodel defines small sets of own variables and relations, as well as uses the core-model
100 variables representing criteria $\mathbf{q} \in \mathbf{y}$. We stress that the merged (MC-submodel and core model) model
101 has the same set of feasible solutions as the core model (4).

102 The MC-submodel takes the form:

$$\text{maximize } \{sf = SF(\mathbf{q})\} \quad (6)$$

subject to:

$$\underline{\mathbf{d}} \leq \mathbf{D} \cdot \begin{bmatrix} sf \\ \mathbf{v} \\ \mathbf{q} \in \mathbf{y} \end{bmatrix} \leq \bar{\mathbf{d}}, \quad (7)$$

103 where:

- 104 • variable *sf* is equal to the value of $SF(\mathbf{q})$; discussion of the $SF(\cdot)$ interpretation and specification is
- 105 beyond the scope of this note;⁵
- 106 • the auxiliary variables \mathbf{v} are generated for defining the *sf* and for internal scaling of the criteria values;
- 107 • variables \mathbf{q} represent criteria and are shared with the core model (4);⁶

⁴Aka optimization problem.

⁵Detailed discussion of the SF is available e.g., in [3], an overview in [8], methodological background in [12].

⁶Note that the other (not selected to be criteria) outcome variables (i.e., $\mathbf{y} \setminus \mathbf{q}$) are not included into the MC-submodel.

108 • \mathbf{D} , $\underline{\mathbf{d}}$, $\bar{\mathbf{d}}$ are parameters of the MC-submodel.

109 Note that relations (6) and (7) define an optimization submodel that needs to be merged with a
 110 core model defining all logical and physical relations that variables \mathbf{q} (representing criteria) have to
 111 conform to.

112 2.4 Merged MC-submodel and a core model

113 The MC-submodel and the core-model are merged into an instance of optimization task (shown in Fig-
 114 ure 2 on page 13 as *Optimization Instance*). The structure of the merged model’s constraints is illustrated
 115 by the merged model Jacobian shown in Figure 1. The optimization criterion of the merged model is
 116 defined by (6).

	Variables		
merged models’ variables	sf	\mathbf{v}	\mathbf{y} \mathbf{z}
MC-submodel variables	sf	\mathbf{v}	$\mathbf{q} \in \mathbf{y}$
core-submodel variables			\mathbf{y} \mathbf{z}
MC-submodel parameters		\mathbf{D}	$\mathbf{0}$
core-model parameters	$\mathbf{0}$	\mathbf{A}	

Figure 1: Structure of the Jacobian of the merged MC-submodel and the core model.

117 The numbers of rows and columns of the MC-submodel are small, usually between 15 and 50,
 118 depending on the criteria number and the needs of adaptive criteria scaling. Therefore, the computational
 119 requirements of the MCMA are practically the same as of single-criterion optimization.

120 To complete the outline of the merged MC and core models let us comment on sharing the \mathbf{q} between
 121 the MC-submodel and the core model. This is easily achieved by using the same names of \mathbf{q} variables
 122 in both models, i.e., MC-submodel and the core-model instance. To assure this consistency, the MCMA
 123 user selects the criteria \mathbf{q} from the list of outcome variables \mathbf{y} specified in the core model. In order to
 124 ease the selection the names of \mathbf{y} should be provided in the master.cfg configuration file (see Sec. 4.5).

125 3 GAMS-format model specification

126 Specification of the GAMS-format model prepared for MCMA should contain the following elements,
 127 each discussed in the corresponding section:

- 128 • Specification of the outcome variables \mathbf{y} (Sec. 3.2);
- 129 • Declaration of the merged model and its optimization (Sec. 3.3);
- 130 • Post-optimization processing (Sec. 3.4).

131 Moreover, the GAMS-format specs needs to conform to the technical requirements summarized in
 132 Sec. 4.

133 3.1 The basic assumptions

134 Although the proposed interface focuses on enabling MCMA of models developed in `Message_ix`,
 135 it also aims at such core model specification that as well supports diverse types of model analysis, in
 136 particular enables use of the specification without any modifications for:

- 137 • single-criterion optimization,
- 138 • parametric single-criterion optimization in form of (5), and

139 • multi-criteria analysis with the MCMA.

140 The selection of each of these model analysis types shall be done by specification of the correspond-
141 ing option in the GAMS execution command but should not require any modifications of the model
142 specification. Such an approach shall greatly ease the model analysis process (composed of single- and
143 multi-criteria optimizations) and well as support consistency of both types of analysis. Moreover, the
144 given specification shall be self-contained, i.e., the optimization can be run on any computer running
145 GAMS without accessing the model development environment.

146 MCMA executes GAMS⁷ through the command:

```
147 gams master.gms -ll=2 -lo=2 --mcma=_mc.gms
```

148 Therefore, it is easy to implement a *conditional compilation* of parts of the GAMS specs; this can be
149 done by enclosing the corresponding parts between the following pairs of the GAMS commands:

150 • to include only for MCMA: \$If not set mcma \$GOTO label_a and \$LABEL label_a

151 • to exclude only for MCMA: \$If set mcma \$GOTO label_b and \$LABEL label_b

152 Note that such a conditional compilation not only enables using one GAMS-specs for both single-
153 and multi-criteria analysis but it also can be used (together with diverse options replacing the command
154 option `--mcma`) for defining diverse single-criterion optimization tasks, e.g., for diverse objective func-
155 tions or parametric optimizations.

156 3.2 Specification of the outcome variables

157 As pointed out in Sec. 2.1, outcome variables are typically specified⁸ in models developed for facili-
158 tating model-based decision analysis and support. There are at least three popular model specification
159 approaches in which some outcome variables are not explicitly defined but it is easy to define them
160 through simple modifications of the specification:

161 • Single-criterion optimization objective (goal function) is often defined in LP as a neutral row. Such a
162 row can be duplicated as a right-hand side of equation defining the corresponding outcome variable.

163 • Single-criterion optimization objective is sometimes defined as a relation composed of diverse ele-
164 ments (such as components of costs or penalty terms or weighted sum of sub-criteria, etc). In such
165 situations a number of outcome variables can be defined by assignments to the corresponding element
166 of such a composite objective.

167 • Another approach replaces bounds (5) on outcomes by defining lower/upper bounds on constraints rep-
168 resenting the corresponding outcomes; e.g., total emissions or use of resources. Also such constraints
169 can be replaced by explicit definitions of outcome variables.

170 3.2.1 Naming convention for outcome variables

171 In order to avoid conflicts in names of variables and constraints of the merged MMP, and to enable the
172 outcome variables⁹ of core model to represent criteria in MCMA, the following two naming **require-**
173 **ments** need to be met in the GAMS model specification:

174 1. The first three characters of all names of variables and constraints must differ from the
175 `mC_` string.

176 2. Names of outcome variables have to conform to the following requirements:

177 • the variable name length is maximum 8 characters;

178 • the variable is not indexed.

179 3.2.2 Example of `mcma_variables.gms`

180 As an example, below the three blocks of code that need to be included in `mcma_variables.gms` are pro-
181 vided for a MESSAGEix model with three criteria, i.e. cumulative discounted system costs, cumulative

⁷For the MC-submodel integrated with the core model.

⁸Although rarely called *outcome* variables.

⁹A typically small subset of all variables to be presented for interactive selection of criteria.

182 GHG emissions and cumulative water use. In addition, the file mcma_variables.gms may contain any
183 additional (and syntactically correct) definitions, e.g., an additional MODEL statement for testing single
184 criteria model versions with the newly defined variables as optimization criterion.

185 Declaration of equations in which the criteria variables are defined:

186 Equations

187 COST_CUMULATIVE summation of cumulative total discounted system costs

188 EMISSION_CUMULATIVE summation of cumulative total GHG emissions

189 WATER_CUMULATIVE summation of cumulative total water consumption

190 ;

191 Definition of (non-indexed) variables with a maximum of eight characters:

192 Variables

193 CUMCOST

194 CUMGHG

195 CUMWATER

196 ;

197 Definition of equations (example only for one equation fully spelled out, the other two replaced by ...):

198 COST_CUMULATIVE..

199 CUMCOST =E=

200 sum((node,year), discountfactor(year) * COST_NODAL(node, year))

201 ;

202 EMISSION_CUMULATIVE.. . . .

203 WATER_CUMULATIVE.. . . .

204 3.2.3 Guidelines for specification of outcome variables

205 In order to enable an appropriate MCMA of the core model the model specification should conform to
206 the following **requirements**:

- 207 1. The core model should not define bounds on outcome variables that represent preferences
208 (e.g., acceptable range of values for each outcome), see (5) and the associated discussion
209 in Sec. 2.2; such bounds should only be defined if needed for representation of logical or
210 physical constraints on values of outcomes.¹⁰
- 211 2. For each potential¹¹ criterion a corresponding outcome variable needs to be defined.
212 Specification of all suitable outcome variables not only enables specification of diverse
213 MCMA instances but also can provide additional characteristics of model solutions (out-
214 come variables, even if not used as criteria, typically represent diverse informative met-
215 rics or indices).
- 216 3. Each outcome variable should either have a precisely defined measurement unit (e.g.,
217 monetary or physical) or be an established indicator. In other words, values of such
218 variables should have for the model analysts a clear interpretation in terms of the corre-
219 sponding unit. This feature is important for preference specification during the MCMA.
220 Therefore, we suggest to refrain from defining outcome variables representing e.g., a
221 composite objective or aggregated metrics unless their values have clear meaning.

¹⁰Recall that the GF of traditional optimization is typically defined by a neutral row, i.e., there is no constraint on optimization objective.

¹¹During the model specification many potential criteria can be considered. Choice of a criteria set is interactively done for each MCMA instance (see Sec. C.1). Criteria selection for subsequent MCMA instances often depends on analysis of earlier defined instances. Therefore, it is rational to include into the core model specification definitions of possibly all outcome variables (note that defining another MCMA instance is by far easier than preparation of a new model instance and then starting a new MCA analysis).

- 222 4. One should define in the model specification as many outcome variables as helpful for
 223 evaluating different aspects of the model solution. Such definitions are very easy dur-
 224 ing the model development and do not increase computational requirements. A surplus
 225 (compared to a typically small number of outcomes initially considered for criteria) of
 226 outcome variables increase flexibility and efficiency of model instance analysis; more-
 227 over, outcome variables are also helpful in model verification.
 228 5. Finally, it is recommended to include into the process of model verification runs of selfish
 229 optimization, i.e., to optimize each outcome variables separately.

230 3.3 Declaration of the merged model and its optimization

231 MCMA solver generates for each specification of the user preference the MC-submodel in the GAMS-
 232 format and stores in a file specified with the `--mcma` option of the run command:

```
233 gams master.gms -ll=2 -lo=2 --mcma=_mc.gms
```

234 Example of the `_mc.gms` file is provided in Appendix A.2. Therefore, merging the core model with
 235 the MC-submodel can be easily implemented by inserting just before the `SOLVE` statement (used for
 236 single-criterion optimization of the core model) the following statement:

```
237 $If set mcma $INCLUDE %mcma%
```

238 The included file contains also the declarations of the name of the merged MC-submodel and of the
 239 optimization task. In order to suppress (within MCMA optimization runs) single-criterion optimization
 240 of the core model its `SOLVE` statement(s) should be conditionally excluded from the compilation, e.g.,
 241 in the way suggested in Sec. 3.1.

242 The core model specification should conform to the following **requirements**:

- 243 1. All specified entities (sets, parameters, variables, equations) of the model will be used in
 244 MCMA. This is equivalent to the statement (included in the `_mc.gms` file, see it sample
 245 in Sec. A.2):

```
246 MODEL model_name / all / ;
```


 247 Therefore, parts of the specification not needed for the MCMA of the core model should
 248 be excluded from the compilation, e.g., as suggested in Sec. 3.1.
- 249 2. Specification of the model instance has to be self-contained. In particular, there is no
 250 possibility of passing to MCMA arguments of the GAMS-execution command-line (e.g.,
 251 for selecting a data set). To replace the command-line option, e.g., for the GDX data file
 252 one can include in `MESSAGE-MCA_master.gms` the statement `$SETGLOBALdata`.
- 253 3. For the data-path (full directory name) separators the only / (slash) character should be
 254 used. Although also the \ (backslash) character works on a PC-version of GAMS it does
 255 not work on unix machines. Note that the slash separator works with GAMS correctly on
 256 all platforms.

257 3.4 Post-optimization processing

258 The post-processing specified with the core model is executed after successful optimization and the
 259 generated files are stored in the `wdir` or in its optional `output` subdirectory.

260 MC-submodel defines its postprocessing also in the `_mc.gms` file; therefore, several MCMA spe-
 261 cific files are stored in each `wdir`. Names of all files generated by or for MCMA start with the under-
 262 score (`_`) character, thus are easily recognized.

263 4 Technical requirements for the model specs

264 The GAMS-format core model specification can be provided in two ways: either as a single file or as
 265 a zip-archive. The corresponding technical (i.e., other than those concerning core model specification,

266 which are discussed above) requirements for each of these ways are presented below. We precede this
267 presentation by summarizing the requirements common for both ways.

268 **4.1 General requirements for GAMS-format specs**

- 269 1. In order to avoid conflicts with names of files generated by MCMA, the core-model files
270 should not start with the underscore (.) character.
- 271 2. Names of the provided files should:
 - 272 • be composed of Latin letters, optionally include digits, and/or the underscore (.) char-
273 acter and/or a single dot (.);
 - 274 • NOT include blanks (spaces, tabs);
 - 275 • NOT include non-alphanumeric characters, except of the two listed above.

276 **4.2 Requirements for preparing a single-file model instance**

277 A single-file specification can be used for small models. In addition to the requirements specified above
278 only two obvious **requirements** should be met:

- 279 1. The model specs should be defined in one file (i.e., no `$INCLUDE` statements can be
280 used for integrating model instance entities).
- 281 2. The file name with the model instance specs should have extension `gms` (e.g., `nexus4.gms`).
282 The root name should be no longer than 12 characters, shall have a letter as first character,
283 and must not contains spaces.

284 For single-file model specifications MCMA derives names of outcome variables through parsing
285 the specification and selecting names of non-indexed variables that conforms to the naming convention
286 described in Sec. 3.2.1.

287 **4.3 Requirements for preparing a zipped-archive**

288 Models having complex specification and/or large data sets are often specified in GAMS through several
289 files, optionally organized in directory/folder structure. In order to rationally support diverse structures
290 of such files we have defined simple requirements for the corresponding zip-archive. In addition to the
291 requirements presented above, the following **requirements** should be met:

- 292 1. The files can be organized into a directory (folder) structure.
- 293 2. Names of all files (including directories) shall conform to the file-name requirements
294 described in Sec. 4.2.
- 295 3. All files shall be zipped into one file. Zip-archives nested into the uploaded archive will
296 not be processed by MCMA. The zip-archive should have the file name extension `zip`
297 (e.g., an uploaded archive named as `nexus7.zip`). The root of the archive name should be
298 no longer than 12 characters, and conform to file-naming convention (see Sec. 4.1).
- 299 4. Two files located in the root directory of the uploaded archive have the prescribed names
300 and the required content:
 - 301 • `master.gms` containing the master (main) part of the model specification. This file
302 will be become the argument of the GAMS solver call. `Message_ix` typically names
303 the master file either `MESSAGE_master.gms` or `MESSAGE-MCA_master.gms`.
304 Therefore, renaming such a file to `master.gms` fulfils this requirement.
 - 305 • `master.cfg` with configuration data for the MCMA analyses (see Sec. 4.5 for de-
306 tails).
- 307 5. Optional directory named `output` is considered as a place-holder for full solution of
308 optimization run of each MCMA iteration. Such directory, if provided in zip-archive will
309 be ignored. In any case, for each iteration an `output` directory will be created in the
310 corresponding working directory, and made available for optional¹² location of the full

¹²If defined in the provided problem specification.

311 solution (specified in the post-processing part of the GAMS specs) of the corresponding
312 optimization run.

313 6. The uploaded zip-archive should contain only files needed for the core model specifica-
314 tion. Consider that all files included in the uploaded archive are used in every (typically
315 several hundreds of) MCMA iteration. Therefore, including into the archive redundant
316 files (especially results of analysis, logs, tests, or diverse versions of specification, etc)
317 causes substantial overhead.

318 4.4 Comments on using the MESSAGE_{ix}-based models

319 *This Section was written by Volker in 2017. We need to check if:*

- 320 • *it is consistent with the current Message_{ix} version;*
- 321 • *we indeed need different master.gms for single-criterion and MCMA; maybe conditional compilation*
322 *proposed above would be a better solution?*

323 When applying the interactive MCMA tool with MESSAGE_{ix}-based models the requirements for
324 models consisting of multiple files that are uploaded as a zip archive hold (see Section 4.3). In order to
325 prepare the MESSAGE_{ix} code for use with MCMA tool, basically three modifications to the standard
326 code need to be made:

- 327 • MESSAGE-MCA_{master.gms} (renamed to master.gms, see below) should be used as the
328 entry point for the model code (instead of MESSAGE_{master.gms}),
- 329 • in MESSAGE-MCA_{master.gms} the GDX data file for the MCMA variable needs to be
330 specified via the command \$SETGLOBALdata, and
- 331 • a file called mcma_variables.gms in which the outcome variables¹³ are defined needs to
332 be created.

333 When calling MESSAGE_{ix} via MESSAGE-MCA_{master.gms}, the file MESSAGE-MCA_{run.gms} is
334 used to execute the various blocks of GAMS code that are part of MESSAGE_{ix}. In the file MESSAGE-
335 MCA_{run.gms}, after the definition of the core model, the file mcma_variables.gms is included. As men-
336 tioned above, when creating the zip-archive for upload to the MCMA, the file MESSAGE-MCA_{master.gms}
337 needs to be renamed to master.gms. The mcma_variables.gms includes customized GAMS code that in
338 a set of equations defines outcome variables, i.e., the variables out of which criteria will be selected
339 interactively in MCMA. As described in Section 3.2.1 these outcome variables should conform to some
340 requirements, particularly their length is limited to eight characters and they should not be indexed.

341 Further, the file mcma_variables.gms should be self-contained in the sense that the file includes the
342 definition of variable names, as well as the declaration and definition of the equations which are used in
343 the assignment of the variables.

344 4.4.1 Files to be included in the zip archive

345 In principle the content of the model folder of the message_{ix} git repository can be included in the
346 zip archive. While considering which files include in the zip-archive note that MCMA consists of many
347 iterations. For each iteration a working directory is created and a corresponding optimization is run
348 in this directory. For efficiency, the zip-archive is not copied there; instead symbolic links are created
349 from the working directory to each file/directory of the zip-archive. Therefore all files needed for the
350 model instance definition need to be in the archive. However, it should be noted that possibly not all files
351 and subfolders in the message_{ix}-git folder are needed. For instance, at present the MACRO sub-folder
352 under model is generally not needed and can be removed from the zip-archive if not needed for analysis
353 of results of each iteration.

354 In order to assure correct and efficient runs the following guidelines should be observed:

¹³See Sec. 3.2.1 for requirements that names of such variables should conform to, and Sec. 3.2.3 for description of outcome variables.

- 355 1. The GDX data file needs to be included in the `data` sub-folder of `model` for the model
 356 to successfully run.
- 357 2. Log files from possible runs of `master.gms` (typically named `master.l??`) should
 358 not be included in the zip-archive.¹⁴
- 359 3. `MESSAGEix` by default stores its results in GDX format in the `output` sub-folder. In-
 360 cluding this sub-folder in zip-archive is optional. However, if it is included then it will be
 361 removed by the MCMA. In any case, an empty `output` sub-folder is created in working
 362 directory created for each MCMA iteration, and the results of the corresponding opti-
 363 mization can be stored there.
- 364 4. Other than log-files redundant (i.e., not needed for specification of the model instance)
 365 files can be included in zip-archive (and will be linked to each working-directory). How-
 366 ever, including many such files is likely undesired.

367 **4.5 Content of the `master.cfg` file**

368 Two types of data (namely, the list of outcome variables names and the approximate dimension of the
 369 single-criterion optimization problem) should be provided. This is done by a small, free-format text¹⁵
 370 file named `master.cfg` and is composed of the corresponding two parts separated by an empty line:

- 371 1. List of names of outcome variables, i.e., model variables that shall be available for interactive selection
 372 of criteria (for each MCMA instance). Each name should be defined in a separate line. Optional, but
 373 recommended, content (after a space separating it from the name) is considered as comment, i.e., is
 374 not be processed MCMA but it is useful for a quick reference of each variable meaning, especially if
 375 the names are not self-explanatory.
- 376 2. Approximate dimensions¹⁶ of the single-criterion optimization model, specified by two pairs (each in
 377 separate line) of an integer number and a keyword denoting either `rows` or `cols`, respectively. The
 378 numbers correspond to thousands of rows/columns. E.g., the following two lines define the dimension
 379 of a model composed of less than 1000 rows, and $1000 \leq \text{cols} < 2000$:

380 0 rows
 381 1 cols

382 Lines with the first character `#` followed by a space are considered as comments that offer optional
 383 ad-hoc documentation.

384 Annotated `master.cfg` sample is included in the pilot example outlined in Sec. B.2 and in Ap-
 385 pendix A.3.

386 *We should discuss whether such a file can be easily generated by `Message_ix` or should be prepared*
 387 *"manually".*

388 **4.6 Summary of the requirements for GAMS model specs**

389 *Maybe would be useful here?*

390 **A Examples of files**

391 **A.1 Example of specification of outcome variables**

392 *to be added here.*

¹⁴In the next revision of MCMA the `master.l??` files will also (like the `output` now) be removed.

¹⁵I.e., spaces serve as word separators, initial and trailing spaces are ignored.

¹⁶This information is used by the MCMA task manager for queuing the optimization tasks.

393 A.2 Sample of `_mc.gms` file

394 The content of `_mc.gms` file copied below was generated by MCMA solver for the simplest case, i.e.,
395 selfish optimization of one (CUMCOST) criterion. In such case the MC-submodel is composed of only
396 one variable and one constraint. Typical MC-submodels are composed of several dozens of variables
397 and constraints. Note that declaration of the CUMCOST variable is commented because all outcome
398 variables should be declared in the core model.

399 The `_mc.gms` files are composed of three parts:

- 400 • specification of the MC-submodel
- 401 • specification of the optimization task (which includes the core model)
- 402 • specification of post-optimization processing of the MC-submodel (post-optimization processing of
403 the core model, if desired, is specified in the core model). Note that in this part also value of the
404 other criterion (CUMINV) is reported although this criterion is not included in the SF for the selfish
405 optimization of the CUMCOST criterion.

```
406 **** MC-submodel specs *****
407 *   Model variables (defined in model.gms) are commented.
408
409 Variables
410 *       CUMCOST
411 *   Auxiliary MC variable mC_gf_c (defined by the modified GF) to be max.:
412     mC_gf_c ;
413
414 Equations
415     mC_gf_r ;
416
417 *   Modified GF: now defines aux. var. mC_gf_c (to be max.)
418 mC_gf_r .. +1.00000e+00 * CUMCOST   =e= -1.00000e+00 * mC_gf_c ;
419
420 **** Optimization task specs *****
421
422 Model LP_MC_PART /all/ ;
423 Solve LP_MC_PART using lp maximizing mC_gf_c ;
424
425 * End of the MC-part of LP *****
426
427 **** MC-submodel post-processing *****
428
429 * Put MC-part solution
430 FILE mc_out / "/p/ime/smt_work/mcma_tst/files/00545/1244/2468/_mc.sol" / ;
431 mc_out.nd = 5;
432 mc_out.nr = 2;
433 mc_out.nw = 12;
434 mc_out.tw = 9;
435 put mc_out;
436
437 put 'mC_gf_c'; put mC_gf_c.l / ;
438 put 'CUMCOST'; put CUMCOST.l / ;
439 put 'CUMINV'; put CUMINV.l / ;
440
441 putclose mc_out;
```

442 A.3 Example of the `master.cfg` file

443 *To be added here.*

444 **B Shared space for ENE Program applications of MCMA**

445 Working space for the ENE program applications of MCMA has been created at the `/p/ene` file
446 system. To ease testing and bug fixing it is recommended (but not necessary) to copy there model
447 instances prepared for MCMA. We will soon provide there a link to `wdir` directories which shall ease
448 discussions on the problems and access to full solutions of the underlying optimization tasks.

449 **B.1 Directory location and structure**

450 The directory `/p/ene/mca` has been created on the unix file system as the ENE collaboration space
451 for the MCMA applications. This directory is mounted on PCs as `P:\ene.mca`.¹⁷

452 The directory has currently the following subdirectories with the corresponding suggested use:

- 453 • *examples* - for generic examples of using MCMA and MCAA.
- 454 • *models* - for model instances; each model should be placed in a separate sub-directory. Please use
455 unix-style dir/file naming convention (short names starting with a letter and composed of only letters
456 and digits (if really desired than an underscore and a dot can be used). Currently¹⁸ there are two
457 directories with models.
- 458 • *tests* - for exploring/storing diverse tests. Currently tests of Cplex numerical problems are stored there.

459 **B.2 Pilot example**

460 The pilot example for exploring the MCMA interface to model instances generated from the MESSAGE-
461 IX was prepared by Volker on Apr. 5th 2017 as the zip-archive. Based on this example the test case was
462 prepared by:

- 463 • renaming file `MESSAGE-MCA_master.gms` to `master.gms`
- 464 • adding the configuration file `master.cfg`).
- 465 • creating zip-archive `volker_apr.zip` containing only the needed files.

466 The created zip-archive was stored in `/p/ene/mca/examples/gams17` and was used for test-
467 ing the MCMA-GAMS interface described in this note. Note that the zip-archive contains the annotated
468 configuration file `master.cfg`), which can easily be adapted to other model instances.

469 **C Functionality and architecture of the MCMA tool**

470 Providing the required MCMA functionality for supporting multiple-criteria model analysis demands in-
471 tegration of many components developed for various needs and by diverse developers. The implemented
472 infrastructure is therefore complex and has hierarchical modular structure. However, the users typically
473 prefer to neither explore software architecture nor be involved in software configuration and mainte-
474 nance. In order to meet these typical preferences, MCMA computational infrastructure is transparent
475 for the users, who access the needed functionality through an easy User Interface (UI) provided through
476 commonly used web browsers. Some readers however, might be curious about the MCMA computa-
477 tional infrastructure; therefore we start the MCMA tool description with the overview of main MCMA
478 components, and follow (in Section C.3 with the description of MCMA functions directly controlled by
479 the users.

480 **C.1 Stages of the MCMA**

481 The MCMA structures the analysis into three stages (each accessible through the corresponding button
482 of the top MCMA menu, cf Section C.3):

¹⁷The PC mount has been so far arranged for only few ENE colleagues. If you want to have this directory mounted on your PC account, then please create the MIS ticket and make Pat Wagner its co-owner.

¹⁸MM: update this (it was current in 2017).

483 1. Problem. In this stage a core model instance is processed into the form suitable for MCMA. The
 484 model instance defines a set of feasible solutions as well as outcome variables out of which subsets are
 485 interactively selected to serve as criteria. At this stage the user uploads a file with the model instance
 486 specification. The uploaded file is processed in the background; if no processing errors occur then the
 487 problem (i.e., the model instance) becomes available for MCA.
 488 2. Instance. For a given problem several MCA instances can be defined. An instance is defined by
 489 interactively selected (from the list of the model outcome variables) criteria; For each criterion the user
 490 interactively defines its type (minimize or maximize) and optionally the criterion name (by default the
 491 selected variable name is used as the criterion name). After an instance is defined, the corresponding
 492 utopia and approximation of nadir values are automatically computed. Next, the initial analysis is
 493 automatically generated. For large models this stage can take several hours (or even days). After
 494 completion of this stage, the user can start interactive analyses.
 495 3. Analysis. Several analyses can be generated for each instance (the first one is generated automati-
 496 cally; the user can easily generate more analyses through a simple form). Each analysis is composed
 497 of iterations. Each iteration is defined by the user preferences specified interactively in terms of As-
 498 piration/Reservation (A/R) values. Several iterations (for each analysis) are generated automatically
 499 in order to provide an initial view on the criteria trade-offs. For each iteration (after the correspond-
 500 ing preferences are defined) the underlying parameterized optimization task (OT) is generated. The
 501 OT consists of the problem core model merged with the LP submodel corresponding to the multiple-
 502 criteria optimization defined for the A/R values specified by the user. Each OT is run in a separate
 503 `wdir` (working directory) where the optimization process logs and full solution are stored. The crite-
 504 ria values are available through the MCMA; values of all model variables are available in the `wdir`,
 505 if the GAMS-format model specification includes generation of solution (and possibly other desired
 506 information).

507 C.2 Architecture of MCMA computational infrastructure

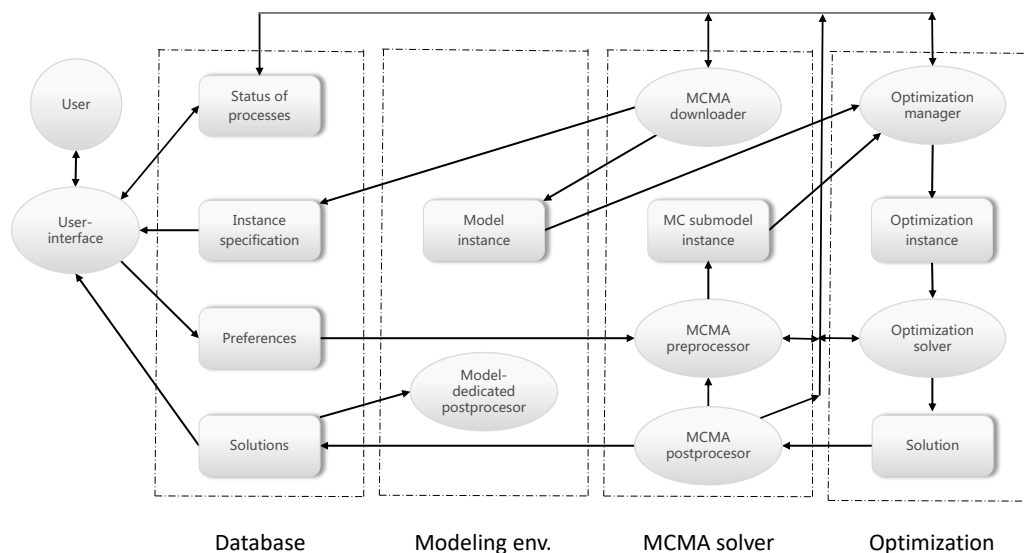


Figure 2: Modular infrastructure for multiple-criteria model analysis.

508 The main hindrance in wide applications of the MCMA methodology is the amount of work and
 509 expertise needed for such applications; the architecture described here illustrates well this issue. Tradi-
 510 tional methods of model analysis are relatively easy to use with standard optimization tools, especially
 511 if integrated with the modeling environment used for the model development. The environment actually
 512 implemented and described in this paper removes this hindrance by making the multiple-criteria model

513 analysis even easier to apply than to use traditional optimization-based environments for iterative model
514 analysis that requires advanced modeling skills.

515 To enable use of MCMA with models developed in diverse modeling environments, the MCMA
516 architecture shown in Figure 2 consists of four modular and interlinked components that are linked
517 (in a way transparent for the MCMA users) with modeling environments used for development and
518 maintenance of the model instances uploaded to MCMA for analysis:

- 519 • The user-interface.
- 520 • Database handling all data needed for all MCMA processes.
- 521 • MCMA solver.
- 522 • Optimization module.

523 The fifth top-component shown in Figure 2, labeled *Modeling env.*, does not belong to the MCMA
524 infrastructure; however, we show it because the user needs a modeling tool for the model development.

525 The short characteristics of the top-level components are as follows:

526 **UI:** The user communicates with the MCMA exclusively through the User Interface (UI) application,
527 implemented in Java, installed at a Tomcat servlet container, thus providing users with the MCMA
528 interface through Web-browsers. The UI is presented in Section C.3.

529 **DB:** Dedicated data-base, manages all persistent data of MCMA. The DB is implemented as an instance
530 of an RDBMS (relational database management system). The schema of the MCMA DB is far too
531 complex to be even outlined in this paper. We only list below examples of data to illustrate the data
532 scope:

- 533 • Users and user groups with privileges of members.
- 534 • Configuration of the MCMA components; e.g., of solvers to specify functionality options available
535 for diverse users and applications.
- 536 • Status of all processes run by the MCMA components. This data provides a back-bone for orga-
537 nizing the MCMA workflows.
- 538 • Specifications of uploaded model instances.
- 539 • Parameters of the preferences defined by the user.
- 540 • The MC-submodel solutions of each analysis iteration.

541 We also point out that handling MCMA component configuration data through a RDBMS greatly
542 improves robustness and maintenance of such rather complex systems.

543 **Modeling environment:** the two-way linkage between the model development environment and MCMA
544 is composed of:

- 545 • Model instance conforming to the requirements summarized in Section 4 is interactively uploaded
546 to MCMA.
- 547 • Solution of each MCMA iteration is provided to the user for optional, model-dedicated postpro-
548 cessing of analysis results.

549 Three formats of model instances are supported:

- 550 • the standard MPS-format;¹⁹
- 551 • the GAMS-format²⁰ provided as a single *.gms file or as a structured collection of GAMS
- 552 • a structured collection of GAMS-based model specification and data files generated from the
553 `Message_ix` platform [5].

554 Solutions are provided in formats corresponding to the model instance, i.e., either a standard MPS-
555 format output file or the format defined in GAMS specification for the output.

556 **MCMA-solver:** Dedicated solver, written in C++, transparent for the MCMA users. It: (1) processes
557 the uploaded model instance, (2) generates the MC-submodel instances for user-defined preferences,
558 (3) prepares data and working space for optimization solver, (4) queues the optimization tasks, and
559 (5) postprocesses optimization results for making them available to the user through the UI.

560 **Optimization:** The dedicated task manager handles optimization jobs generated and queued by MCMA-

¹⁹MPS (from: Mathematical Programming System) widely used file format for specification of linear- and mixed-integer-programming problems.

²⁰General Algebraic Modeling System (GAMS), see e.g., [2].

561 solver, i.e., allocates each of them to one of optimization solvers distributed over the workstation net-
 562 work. MCMA uses the same solvers as the single criterion optimization of the corresponding core
 563 models. Before the selected solver is executed, a dedicated application merges the MC-submodel
 564 and core-model instances.

565 The workflows between elements of the MCMA infrastructure are actually hidden from the MCMA
 566 users, who control the flows only through the UI described in Section C.3. Therefore, we only briefly
 567 summarize the basic workflows and actions triggered by specification of preferences for each iteration:

- 568 1. Preference parameters are stored in the DB and the iteration status is updated in the DB; then the
 569 MCMA-solver is called, and the user may either wait for the solution, or switch to another iteration.
- 570 2. The MCMA-solver reads the iteration data from the DB, generates the MC-submodel, stores it on
 571 the server file-system, and updates the iteration status in the DB, which queues the corresponding
 572 optimization task.
- 573 3. The optimization manager allocates the task execution. The manager is actually a daemon-type ap-
 574 plication, i.e., it runs in the background, frequently checks the queued tasks, and allocates them on
 575 available servers and solvers whenever resources allow.
- 576 4. MCMA solvers are of three types:
 - 577 • Preprocessor: it merges the MC-LP with the model instance representation and generates input files
 578 for the selected optimizer, executes the suitable optimizer, waits until optimization finishes, and then
 579 calls the postprocessor.
 - 580 • Optimizers: solvers of the optimization problems.
 - 581 • Postprocessor: extracts from the optimization results solution of the MC-LP part, stores it in the
 582 DB, and calls back the MCMA-solver. The full solution remains available for the user.
- 583 5. MCMA-solver processes the solution, in particular prepares data for generation of the chart presenter
 584 to the user.

585 Each MCMA application updates, at the beginning and at the end of execution, the task status in the
 586 DB. Thus, each application can check the status and provide the user with the corresponding information,
 587 e.g., about execution stage for yet unfinished jobs, or charts and values for finished iterations.

588 C.3 User interface

589 Following the Structured Modeling (SM) paradigm and the corresponding model instance analysis cycle,
 590 the MCMA processes are structured to help the users in effective and efficient analysis through a simple,
 591 error-tolerant interface. The User Interface (UI) summarized here provides users with flexible control of
 592 the all analysis elements, and thus enables organization of the process according to diverse and changing
 593 users' needs. The interface is available through Web-browsers, thus allows for the anytime-anywhere
 594 access; in particular, the analysis can be paused anytime; the defined tasks are anyway processed in
 595 background by servers, results stored and made available whenever the user decides to continue analysis.

596 C.3.1 User control of the top-level functionality

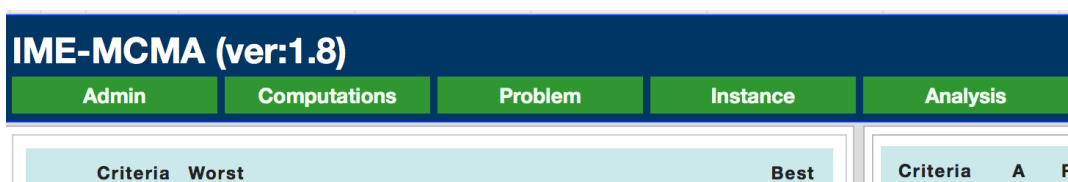


Figure 3: User-interface to control the MCMA workflows.

597 The main five top-level functions of the UI are accessed through the green buttons shown in Figure 3;
 598 each of them opens access to the underlying structure of dialogues. Here, we only outline the provided
 599 functionality:

600 **Admin:** Each MCMA user has private space for handling model instances and results of analysis. How-
601 ever, a group of users may share their models and results. Therefore, MCMA supports administration
602 of user groups, and privileges of group members.

603 **Computations:** MCMA provides information on the status of optimization tasks that have been gener-
604 ated but not yet finished, see Section C.2 for the summary of the corresponding components.

605 **Problem:** The corresponding set of dialogues supports uploading of model instances provided in one
606 of the formats discussed above. The uploaded model instance is considered as the MCA problem.

607 **Instance:** For each problem the user may define several analysis instances. Each MCA instance is
608 specified by the interactively defined criteria. The definition of each criterion is composed of the
609 following selected:

- 610 • Corresponding outcome variables. The list of such variables is extracted from the uploaded model
611 instance; filters for names of variables support selection for large models.
- 612 • Criterion type (either minimized or maximized).
- 613 • Criterion name. This is optional because the criterion name is initialized to be the same as the name
614 of the corresponding outcome variable.

615 **Analysis:** The user may define for each MCMA instance several analyses. Each analysis is composed
616 of iterations outlined above and discussed detail below. Defining several analyses is especially useful
617 for extensive MCMA in which each analysis is composed of many iterations; moreover, separate
618 analyses can have different focus and/or be done by different users.

619 **Contact:** Reporting problems and questions is supported by the corresponding white button (at the top
620 right corner of the blue control panel shown in Figure 5). The user comments are stored in the
621 DB together with automatically assembled information about the situation in which the contact is
622 used. Thus, the developers can easily recreate the situation for exploring and handling the reported
623 problem.

624 The interactive specification of preferences is the main MCMA activity. Therefore we comment
625 below on this part in more detail than on the other MCMA elements.

626 C.3.2 Preparatory computations

627 Before providing the user with access to the interactive analysis, MCMA performs several background
628 tasks in order to provide the user with initial information on the Pareto set. These tasks are automatically
629 generated and run after the user defines a new instance or a new analysis (the initial analysis is also
630 generated automatically). We briefly summarize these background computations.

631 For each new instance the utopia and nadir values are computed. This requires $4 \star N$ automatically
632 generated optimization tasks, where N is the criteria number. First N tasks compute the utopia values,
633 for each criterion by the selfish optimization of the corresponding outcome variable. Next $3 \star N$ tasks
634 sequentially improve approximation of the nadir values. After completing these tasks, the initial analysis
635 is automatically created.

636 For each new analysis $N + 1$ iterations are automatically generated to provide the user with initial set
637 of solutions. This set consists of N iterations of selfish optimizations (i.e., for each only one criterion is
638 active), and one iteration with the so-called compromise preferences, i.e., \mathbf{q}^a and \mathbf{q}^r set for each criterion
639 at equal (in terms of the fractions of utopia and nadir range) values. Thus the user starts specification of
640 his/her preferences with knowledge about the extreme (selfish-criterion) and compromise preferences.

641 C.3.3 Interactive specification of preferences

642 After the automatically generated iterations are completed, the user takes full control of further iter-
643 ations. The main interaction screen is composed of the two panels shown in Figures 4 and 5, which
644 illustrate the left and right parts of the screen, respectively: (1) the left-side chart shows the distribu-
645 tion of criteria values, and (2) the right-side control panel supports specification of preferences for next
646 iteration, as well the selection of the displayed iteration.

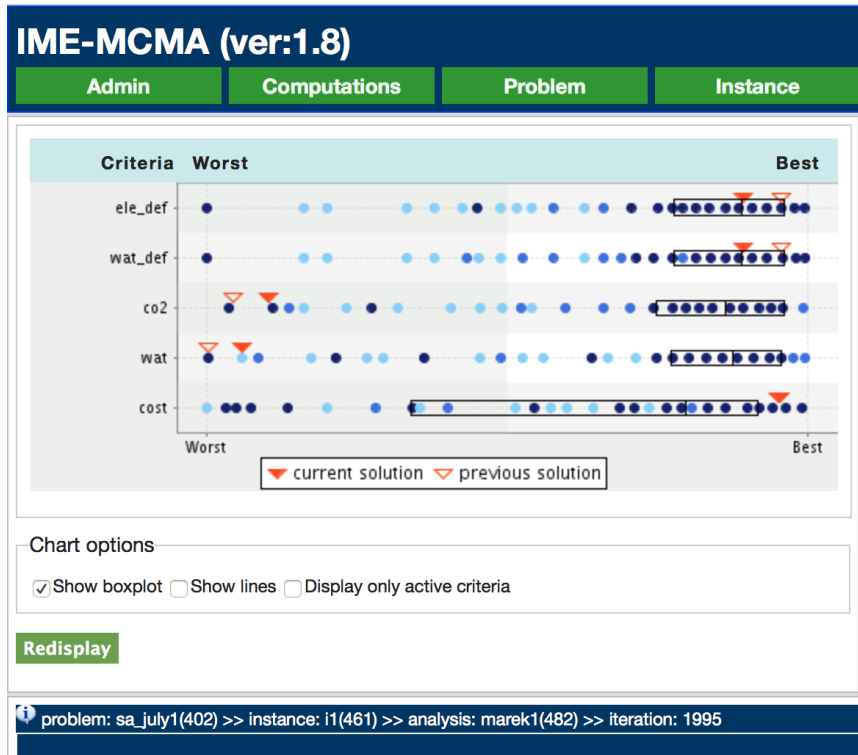


Figure 4: Distribution of the criteria values.

647 The user can select, using the choice list shown at the bottom of Figure 5, any iteration as the basis for
 648 further analysis. For the selected iteration the user, typically, first explores the Pareto-solutions obtained
 649 in the previous iterations of the current analysis, and then specifies preferences for the next iteration.

650 The criteria values of the previously obtained (within the same analysis) solutions are presented
 651 in the chart composed of normalized parallel coordinates shown in Figure 4. Each dot corresponds to
 652 a solution; dots in darker colors represent aggregates of several solutions having similar criterion values.
 653 The details of the underlying solution(s) is displayed (as a hint, not shown in the Figure) on demand,
 654 when the user points to a selected solution. The boxes cover the two middle quartiles of solutions; the
 655 best and worst 25% of solutions are displayed on the right- and left-sides of the corresponding box,
 656 respectively.

657 The red triangles shown in the chart point to the criteria values of the current (solid) and previous
 658 (empty) solutions, respectively. Thus the user easily sees which (and by how much) some criteria were
 659 improved or compromised (compared to the previous solution) by the recently specified preferences.

660 The bottom blue panel provides information about the model and analysis instances, as well as the
 661 current iteration of the analysis. The little light-blue icon shown at the left corner (marked by the letter *i*)
 662 enables access to the full information on the corresponding optimization run, and the solution provided
 663 by the solver.

664 Preferences for a new desired trade-off between criteria values are then expressed through aspiration
 665 and reservation values for each criterion, respectively. While defining new preferences one should con-
 666 sider that the basis solution is Pareto-efficient, i.e., an improvement of one criterion (or more criteria)
 667 is possible only, if at least one other criterion will worsen. In other words, the user shall decide which
 668 criteria he/she wants to improve and which to compromised to make the desired improvement possible.
 669 An improvement of a criterion performance can be triggered by setting a more ambitious (closer to the
 670 corresponding utopia value) reservation value for this criterion, optionally augmented by also higher
 671 aspiration. Also optionally, one can select a criterion (or criteria) to compromise; this can be done by
 672 relaxing (i.e., worsening) the corresponding reservation value(s). In such a way the user preferences are
 673 defined for each iteration.

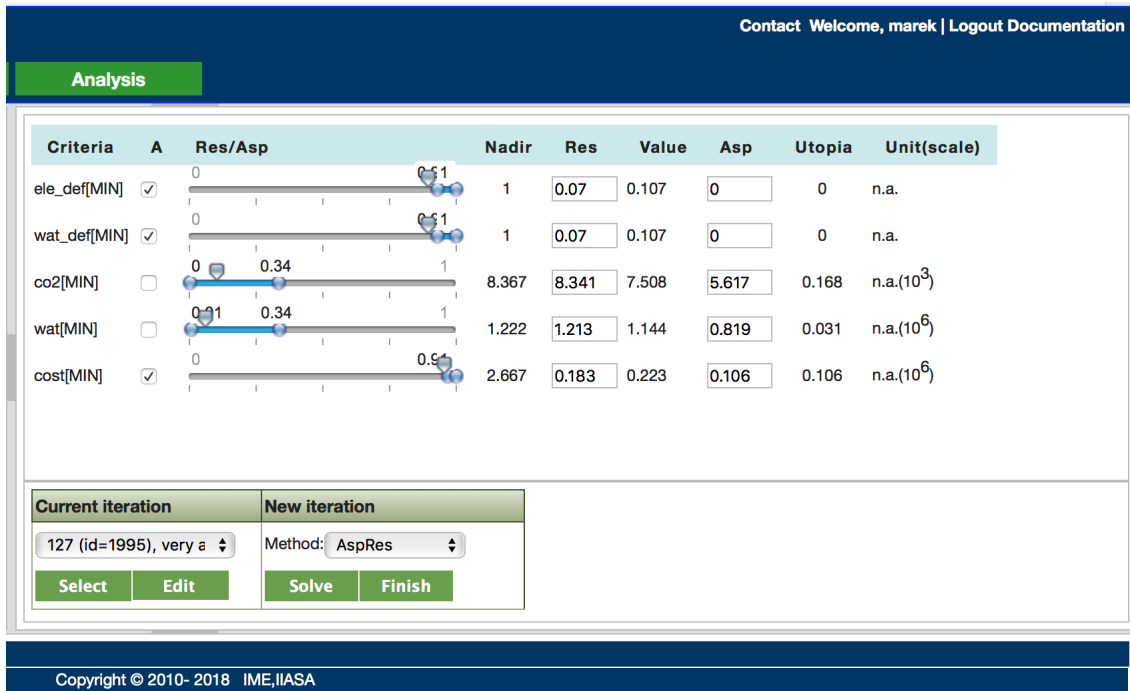


Figure 5: Specification of preferences in terms of criteria reservation and aspiration values.

674 For any given preferences, the multi-criteria problem is represented by an auxiliary parametric
 675 single-objective optimization problem defined through the achievement scalarizing function (6); solution
 676 of the corresponding optimization problem provides a Pareto-solution best fitting the user preferences.
 677 Typically, the MCMA users explore various areas of the Pareto frontier (e.g., cheap and expen-
 678 sive having the corresponding bad and good values of environmental criteria) before deciding which
 679 compromises between the criteria values fit best their preferences. Examples of such exploration and
 680 methodological background on the Pareto set analysis is available e.g., in [6, 11, 7, 9, 1, 12, 10, 4].

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