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Variable Spatial Springs for Robot Control Applications

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Abstract

This article presents a passive way to implement varying spatial springs. These are springs with controlling ports which can be used to modify their spatial rest length or spatial properties. These controlling ports have a dual structure which allows to supervise the potential energy injected into the spring by varying its properties. A direct application in telemanipulation using geometric scattering [15] is briefly described.

1 Introduction

A lot of interesting work has been done concerning modeling of spatial springs [7, 2, 11, 5]. The cited work concentrate purely on the modeling of constant stiffnesses and does not address any direct use for specific control purposes. Compliance or impedance control applications using spatial springs can be found in [3, 15].

At the knowledge of the author, only in [12, 15] the use of variable length springs is used in the context of passive grasping. Some recent on going work showed that, using geometric scattering, variable springs can be used in Intrinsically Passive Control (IPC) for, among others, tele-manipulation applications [15].

This last work does not specifically consider a power consistent way to vary the spatial length of a spring together with the position of the center of stiffness and the stiffness values.

It is well known that changing the stiffness value of a spring implemented by means of control, is NOT a passive operation since it changes the energy stored in the spring. By building a proper dual structure this operation can be made passive and the stiffness information can be coded using scattering techniques and sent on a transmission line with delays without compromising the passivity of the total system. This is to the knowledge of the authors never been done.

The paper is structured as follows: in Sec.2 a list of used symbols is introduced. The background knowledge is briefly reviewed in Sec.3. More interested readers can find an extensive treatment in [15]. In Sec.4 a quick review of the major concepts used in spatial springs is presented in order to introduce the major contribution of the paper in Sec.5. Sec.7 will briefly describe some possible applications and Sec.8 will conclude the paper also stating some future developments.

2 Notation

- Ψ_i Right handed orthonormal coordinate frame i.
- H_i^j Homogeneous coordinate transformation from Ψ_i to Ψ_j.
- T_i^j Twist of Ψ_i with respect to Ψ_j.
- $T_i^{k,j}$ Twist of Ψ_i with respect to Ψ_j as a numerical vector expressed in Ψ_k.
- W_i Wrench applied to a mass attached to Ψ_i.
- W_i^k Wrench applied to a mass attached to Ψ_i expressed as a numerical vector expressed in Ψ_k.
- $W_{i,j}$ Wrench applied to a spring element connecting Ψ_i to Ψ_j on the side of Ψ_i.
- $W_{i,j}^k$ Wrench applied to a spring element connecting Ψ_i to Ψ_j on the side of Ψ_i expressed as a numerical vector expressed in Ψ_k.

3 Background

In order to study rigid mechanisms, the theory of Lie groups [10, 4] turns out to be very useful. If we discard important considerations about the intrinsicity of references [14], we can associate to each moving part i of a mechanism a right handed coordinate frame Ψ_i. Some extra information can be found in [15, 8].

3.1 Configurations

Once we have chosen a fixed coordinate frame Ψ_0, we can associate the configuration of each part i with the homogeneous matrix H_i^0 of change of coordinates from Ψ_i to Ψ_0. This homogeneous matrices turn out to belong to a matrix Lie group often indicated[1] with $SE(3)$:

$$SE(3) := \left\{ \begin{pmatrix} R & p \\ 0 & 1 \end{pmatrix} \; s.t. \; R \in SO(3), p \in \mathbb{R}^n \right\} \quad (1)$$

where $SO(3)$ indicates the set of orthonormal matrices with positive determinant, which is also a Lie group.

3.2 Twist and Wrenches

If we consider two frames Ψ_i and Ψ_j moving with respect to each other as a function of time, we can use a trajectory in $SE(3)$ to describe this motion, namely $H_i^j(t) \in SE(3)$. For reasons which are explained in details in [15], a much better description of the instantaneous relative motion of the two bodies, can be achieved using the following matrices which happen to belong to $se(3)$, the Lie algebra corresponding to the Lie group $SE(3)$:

$$T_i^{j,j} = H_i^j \dot{H}_j^i \quad \text{and} \quad T_i^{i,j} = \dot{H}_j^i H_i^j. \qquad (2)$$

The first twist $T_i^{j,j}$ is a geometrical representation of the motion of Ψ_i with respect to Ψ_j expressed in the frame Ψ_j and the second $T_i^{i,j}$ the same motion, but expressed in frame Ψ_i. We can represent a twist either as a 4×4 matrix of the previous form or as a 6 dimensional numerical vector. The form of the matrix representation turns out to be:

$$\tilde{T} = \begin{pmatrix} \tilde{\omega} & v \\ 0 & 0 \end{pmatrix}$$

where $v \in \mathbb{R}^3$, $\omega \in \mathbb{R}^3$ and $\tilde{\omega} \in \mathbb{R}^{3\times3}$ is a skew-symmetric matrix such that $\tilde{\omega}x = \omega x, \forall x \in \mathbb{R}^3$. It is then possible to consider as a vector representation of a twist the following vector:

$$T := \begin{pmatrix} \omega \\ v \end{pmatrix}.$$

From now on we will not make distinctions in the notation between the vector and matrix representation since it will be always clear by the context. The change of coordinates of twists can be calculated with the adjoint map which is function of the relative position of the frames:

$$T_i^{l,j} = Ad_{H_k^l} T_i^{k,j}$$

[1] It is important to realize that $SE(3)$ is actually the set of positive isometries within a three dimensional space and not its matrix representation.

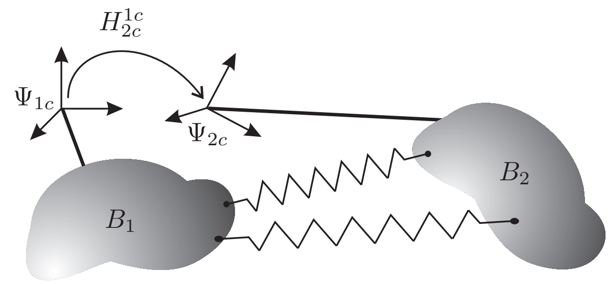

Figure 1: A spatial spring and the center co stiffness

where

$$Ad_{H_k^l} := \begin{pmatrix} R_k^l & 0 \\ \tilde{p}_k^l R_k^l & R_k^l \end{pmatrix} \quad \text{where} H_k^l := \begin{pmatrix} R_k^l & p_k^l \\ 0 & 1 \end{pmatrix}$$

Since $se(3)$ is a vector space, we can consider its dual $se^*(3)$ [15] corresponding to the linear operators on $se(3)$. This dual vector space corresponds to the space of wrenches which are the six dimensional generalization of forces [15]. Also for wrenches there is a matrix and vector representation which are respectively:

$$\tilde{W} = \begin{pmatrix} \tilde{f} & m \\ 0 & 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} m \\ f \end{pmatrix}.$$

Once again, from now on we will not make distinction in the notation. Purely for energetical reasons, it is easy to see that the map of wrenches can be represented as follows:

$$W_i^k = Ad_{H_k^l}^T W_i^l.$$

3.3 Power Ports and Interconnections

A basic concept used in this paper is the one of power port [15]. A power port is the entity which describes the media by means of which subsystems can mutually exchange physical energy. Analytically, a power port can be defined by the Cartesian product of a vector space V and its dual space V^*:

$$P := V \times V^*$$

Therefore, power ports are pairs $(e, f) \in P$. The values of both e and f (*effort* and *flow* variables) change in time and these values are shared by the two subsystems which are exchanging power through the considered port. The power exchanged at a certain time is equal to the intrinsic dual product:

$$\text{Power} = \langle e | f \rangle.$$

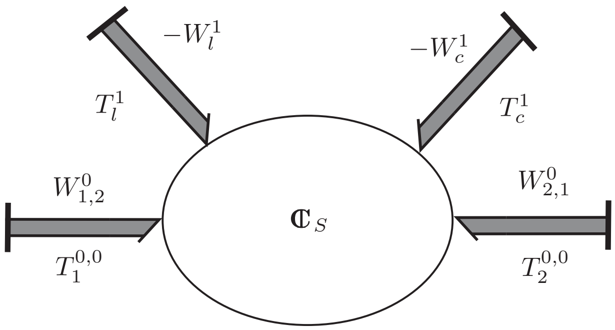

Figure 2: A bond graph of the variable spring.

This dual product is intrinsic in the sense that elements of V^* are linear operators from V to $\mathbb{R}$, and therefore, to express the operation, we do not need any additional structure than the vector space structure of V.

In this work, the space V will be often the space of twists (flows) $se(3)$.

3.4 Scattering

It is shown in [15] that we can associate to each power port a new representation which is called *scattering* representation of the port. This representation is not unique, but it depends on a choice of an impedance. Once this impedance is chosen, we can in a geometrical way associate uniquely to a port (T, W) a pair of scattering variables (s_-, s_+) in such a way that:

$$\langle W|T\rangle = \frac{1}{2}||s_+||_+^2 - \frac{1}{2}||s_-||_-^2$$

where $\langle W|T\rangle$ represents the dual product corresponding to the power passing through the power port and the norms are properly induced norms. This is fundamental because it makes possible to interpret the power passing through the port as the superposition of two waves, s_+ and s_- going in opposite directions. This fact makes possible to extend the seminal work [9, 1] to a geometrical coordinate free treatment as it is shown in [15].

4 Spatial Springs

A spring is a storage of potential energy and its behavior is completely characterized by an energy function which associates to the relative position of its extremes a corresponding stored energy. In a spatial spring, the relative position of its extremes has a very specific structure, it is namely topologically homeomorphic to the Lie group $SE(3)$. Spatial springs where firstly introduced and deeply studied by [6] and then used for modeling in [3]. They are then used for geometrical control in grasping and tele-manipulation in [15].

Lončarić showed that under certain circumstances, there is a point in space called center of stiffness in which the expression of the stiffness tensor[2] is such that it maximally decouples rotations and translations.

With reference to Fig.1, we can define a spatial spring connecting two bodies B_1 and B_2 in the following way.

First define two references Ψ_1 and Ψ_2 rigidly connected respectively to B_1 and B_2 (not shown in the picture for clarity). Then consider the relative position in which the spring has its minimum potential energy which must exist for passivity. Chose than a point in space representing the center of stiffness and define two coincident frames Ψ_{1c} and Ψ_{2c} respectively rigidly connected to Ψ_1 and Ψ_2. By construction, independently of the configuration of the spring, the matrix of changes of coordinates H_1^{1c} and H_2^{2c} are constant.

Furthermore, by construction, the stored energy function defining the spring can be represented by:

$$V : SE(3) \to \mathbb{R}; H_{1c}^{2c} \mapsto V(H_{1c}^{2c}) \tag{3}$$

in which with $SE(3)$ we indicated the Special Euclidean matrix group and for which $V(I)$ is a minimum.

It is possible then to define based on Eq.(3) [3, 15] or directly [2], a mapping which associates to a certain position of the spring H_{2c}^{1c} the wrench B_1 applies to the spring indicated with $W_{1c,2c}^{1c}$ (expressed in Ψ_{1c}) and the one applied by B_2 indicated with $W_{2c,1c}^{2c}$ (expressed in Ψ_{2c}). In one of the possible models introduced in [3] and explained in detail in [15], these wrenches take the following form:

$$W_{1c,2c}^{1c} = \begin{pmatrix} m \\ f \end{pmatrix} \quad \text{and} \quad W_{2c,1c}^{2c} = -Ad_{H_{2c}^{1c}}^T W_{1c,2c}^{1c} \tag{4}$$

where

$$\begin{aligned} \tilde{m} &= 2\,\mathrm{as}(G_o R_{1c}^{2c}) + \mathrm{as}(G_t R_{2c}^{1c} \tilde{p}_{1c}^{2c} \tilde{p}_{1c}^{2c} R_{1c}^{2c}) \\ &\quad + 2\,\mathrm{as}(G_c \tilde{p}_{ic}^{2c} R_{1c}^{2c}) \end{aligned} \tag{5}$$

$$\begin{aligned} \tilde{f} &= R_{2c}^{1c}\,\mathrm{as}(G_t \tilde{p}_{1c}^{2c}) R_{1c}^{2c} \\ &\quad + \mathrm{as}(G_t R_{2c}^{1c} \tilde{p}_{1c}^{2c} R_{1c}^{2c}) + 2\,\mathrm{as}(G_c R_{1c}^{2c}), \end{aligned} \tag{6}$$

as() is an operator which takes the skew-symmetric part of a matrix, R_{1c}^{2c} and p_{1c}^{2c} are the subparts of the matrix

$$H_{1c}^{2c} = \begin{pmatrix} R_{1c}^{2c} & p_{1c}^{2c} \\ 0 & 1 \end{pmatrix}$$

and G_o, G_t, G_c are called respectively orientational, translational and coupling *co-stiffnesses* of the spring and have been introduced in [3]. More details about the choices and calculations can be found in [15].

[2]It is well known that the stiffness matrix is a tensor only at equilibrium.

Figure 3: A simple model of a varying spring

5 Variable Spatial Springs

In this section we will show how it is possible to define power ports that can be used to modify both geometric and parametric properties in the spatial spring.

5.1 Changing the geometric properties

The proposed spring which is depicted in Fig.2 as a bond graph element, has four power ports:

- $(T_1^{0,0}, W_{1,2}^0)$, $(T_2^{0,0}, W_{2,1}^0)$ corresponding to the hinge points where the bodies connected to the spring can be attached
- $(T_l^1, -W_l^1)$ corresponding to the port which can be used to modify the rest configuration of the spring, namely expressed by H_2^1 in the case in which $H_{2c}^{1c} = I$
- $(T_c^1, -W_c^1)$ corresponding to the port which can be used to modify the position of the center of stiffness and the principle axis of the spatial stiffness.

In Fig.3 a simple model of a variable spring is reported which can be used to follow the discussion hereafter. The spring is composed of 4 massless parts which can be moved with respect to each other. The springs hinge points where the two bodies will be connected correspond to the frames Ψ_1 and Ψ_2. The relative position between Ψ_1 and Ψ_{1c} and between Ψ_2 and Ψ_{2c} can be modified by controlling ports as it will be explained hereafter. The spatial spring connecting Ψ_{1c} to Ψ_{2c} is a geometrical spring with the feature to be at rest when Ψ_{1c} and Ψ_{2c} are aligned (zero displacement). This spring is called the *internal spring* and it is characterized by an energy function of the form reported in Eq.(3) with a minimum at the identity. For the moment we consider the parameters of the spring constant and only analyse how it is possible to vary the length and the position of the center of stiffness. In Sec.5.2 this will be extended with the possibility to vary also the stiffness parameters. First of all, it is possible to relate the controlling ports to the relative variations of H_{1c}^1 and H_{2c}^2 in the following way. First of all the rest length of the spring is as said previously H_2^1 when the energy stored in the spring is zero and therefore per construction when $H_{2c}^{1c} = I$. Clearly the following relations can be chosen:

$$T_{1c}^{1,1} = \frac{1}{2} T_l^1 + T_c^1 \tag{7}$$

$$T_2^{1,2c} = \frac{1}{2} T_l^1 - T_c^1 \tag{8}$$

It is possible to see that if the center of stiffness is not changed $T_c^1 = 0$, the change of the length of the spring corresponds indeed to T_l^1 and the center of stiffness will symmetrically remain between the two extremes of the spring. In the other extreme, if the rest length is not changed ($T_l^1 = 0$), the frames Ψ_{1c} and Ψ_{2c} will move within Ψ_1 and Ψ_2 changing the effective location of the center of stiffness and its principal axis.

It is now possible to calculate the wrenches that the spring generates for the four ports. From Eq.(8), we obtain

$$T_2^{2,2c} = Ad_{H_1^2} \left(\frac{1}{2} T_l^1 - T_c^1 \right). \tag{9}$$

From the definitions of twists given in Sec.3.2, we therefore obtain:

$$\dot{H}_2^{2c} = H_2^{2c} Ad_{H_1^2} \left(\frac{1}{2} T_l^1 - T_c^1 \right) \tag{10}$$

$$\dot{H}_{1c}^1 = H_{1c}^1 \left(\frac{1}{2} T_l^1 + T_c^1 \right) \tag{11}$$

which can be integrated in real time to obtain $H_{1c}^1(t)$ and $H_2^{2c}(t)$. Using the chain rule we can finally calculate the state of the spring as:

$$H_{1c}^{2c} = H_2^{2c} H_0^2 H_1^0 H_{1c}^1,$$

where clearly the configurations of the bodies attached to the springs H_1^0 and H_2^0 are needed. From the constitutive relation of the constant spring attached between Ψ_{1c} and Ψ_{2c}, it is then directly possible to have an expression of the following wrench:

$$\tilde{W} := W_{1c,2c}^{1c}(H_{1c}^{2c}). \tag{12}$$

Eventually, after some kinematic analysis it is possible to calculate the wrenches of the power ports reported in Fig.2 which result:

$$W_{1,2}^0 = -Ad_{H_0^1 c}^T \tilde{W} \tag{13}$$

$$W_{2,1}^0 = Ad_{H_0^1 c}^T \tilde{W} \tag{14}$$

$$W_l^1 = Ad_{H_1^1 c}^T \tilde{W} \tag{15}$$

$$W_c^1 = 0 \tag{16}$$

It is very interesting to notice that the dual wrench of the twist which can be used to control the center

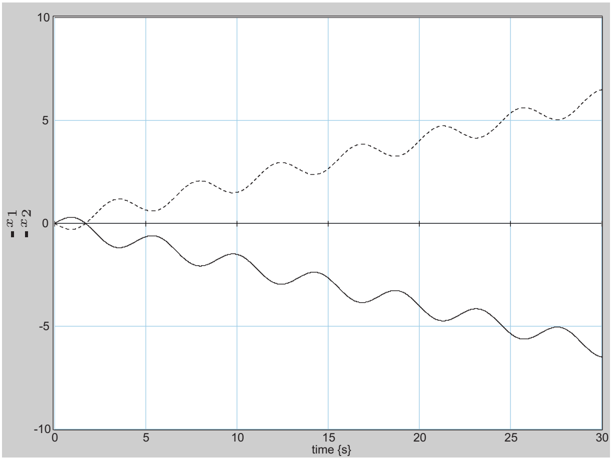

Figure 4: Changing the spring's length.

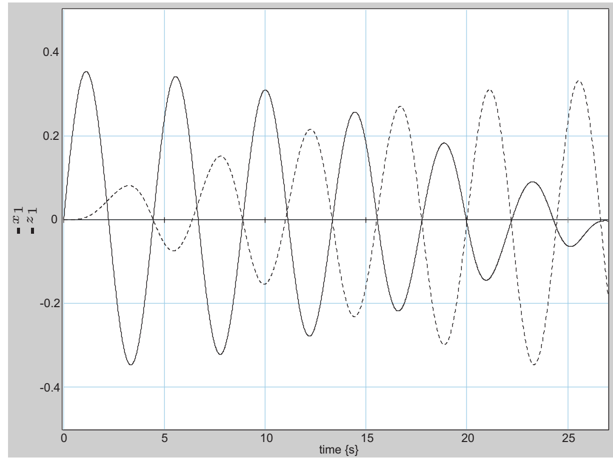

Figure 5: Changing the spring's principal axis.

of stiffness is always zero ! This implies that it is not possible to exchange power with the spring using this port. Nevertheless, with this port it is possible to influence the behaviors of the spring, but without changing its internal energy directly.

5.2 Changing parametric properties

In the previous section we have considered the parametric properties of the internal spring constant. It can be useful to define power ports thanks to which it is possible to modify certain parameters as some scalar values of the principal stiffnesses. This can be done in the following way. Consider to generalize Eq.(3) to an energy function of the form:

$$V_K : SE(3) \times K \to \mathbb{R}; (H_{ic}^{jc}, k) \mapsto V_K(H_{ic}^{jc}, k) \quad (17)$$

where K is a parametric space which can for simplicity be considered equal to $\mathbb{R}^n$. It is then possible to consider an additional port defined on $TK \times T^*K$ and equal to

$$P_K := \left(\dot{k}, \frac{\partial V_K}{\partial k}\right).$$

Clearly this port is energetically consistent since the dual product

$$\langle \frac{\partial V_K}{\partial k} | \dot{k} \rangle$$

corresponds to the increase in internal energy due to the change of parameters value $\dot{k}$ supplied.

6 Simulations

In order to test the proposed idea, a 3D model based on screw-bond graphs [15] has been implemented in the simulation package 20-sim[3]. The model consists of two spheres connected by the introduced varying spring. At the start of the simulation the masses are hit along an horizontal line in order to let the system oscillate. In Fig.4 the positions of the two masses x_1 and x_2 are shown when the spring length is increased.

More interesting is the plot shown in Fig.5 where the vertical position z_1 and horizontal position x_1 of only one mass is shown. Here the orientation of the principal axis of the center of compliance is rotated around an axis normal to the screen up to 90^0 and as expected the oscillation energy is moved from the horizontal (continuous line) to a vertical line (dashed line). This shows that without supplying energy, the spatial properties of the spring are changed.

7 Possible Applications

The relevancy of the proposed spring becomes clear namely in two control settings, the first one is the IPC-Supervisor setting explained briefly in [13] or in more details in[15] and the second is telemanipulation. The major idea in this control structure is to split the controller in two parts:

- The Intrinsically Passive Controller (IPC) which is a passive system with two power ports. The first port is connected to the system to be controlled and the second to the supervisor
- The Supervisor which is a higher level of control which can inject energy to the system in order to complete the desired task

The IPC can be nicely designed as an equivalent, spatial geometrical mechanical system and thanks to this passivity is ensured during interaction with the environment. In this setting, elements of the IPC could be the presented variable springs whose

[3] 20-sim is a powerful modeling and simulation package developed by Control Lab Products `http://www.20sim.com`

controlling ports would be part of an extended port which connects the IPC to the supervisor. This has been done for IPC grasping applications [16] and for tele-manipulation applications where the Supervisor is composed of a transmission line and a second IPC and a user on the other side of the line [13].

In this kind of tele-manipulation applications the presented springs are very relevant since geometric and parametric information can be exchanged along a transmission line with delay preserving passivity! This is simply done by scatterizing the corresponding ports as explained in Sec.3.4 and using one scattering as the delayed information coming from the line and sending the other one. Details about this topic can be found in [15].

8 Conclusions

In the present paper a way to model a geometrical spring has been presented. The geometrical and parametrical properties of this spring can be changed using power ports and therefore allowing various applications where passivity is essential like tele-manipulation and interacting tasks.

Three controlling ports have been introduced. The first can be used to change the rest length of the springs and this is relevant for grasping applications as it is shown in [15]. The second can be used to modify the position of the center of stiffness and its principal stiffness axis. The last one can be used to change parametric values of the spring like the value of the stiffness in certain directions.

It has been shown that the dual wrench of the twist which is used to modify the center of stiffness of the spring is always zero and this implies that the geometry of the center of stiffness can be changed without exchanging energy. Nevertheless this will have consequences on the energy exchange between the spring and the parts connected to it.

In future work we will further decompose the control of the principal axis or rotation, translation and coupling and we will test this technique for experimental tele-manipulation applications.

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