1 BGG+GK Model Skelton

1.1 Parameter Settings

1.1.1 Calibrated Parameters

Discount factor: $\beta = 0.995$
Depreciation rate: $\delta = 0.025$
Capital share: $\alpha = 0.33$
Survival rate of entrepreneurs in steady state: $\gamma_{ss}^{E} = 0.972$
Survival rate of bankers in steady state: $\gamma_{ss}^{F} = 0.972$
Bank's participation constraint parameter: $\lambda = 0.383$
Wage markup: $\psi^{w} = 0.05$
Elasticity of substitution of intermediate goods: $\epsilon = 11$
New entrepreneur entry rate: $\xi^{E} = 0.003$
New banker entry rate: $\xi^{F} = 0.003$

1.1.2 Key Steady States

Steady state marginal cost: $mc_{ss} = \frac{\epsilon - 1}{\epsilon}$
Steady state external finance premium: $S_{ss} = 1.0075$
Steady state corporate bollowing rate (real, QPR): $rr_{ss}^{E} = 1.0152$
Steady state bank lending rate (real, QPR, ex-premium): $rr_{ss}^{F} = rr_{ss}^{E}/S_{ss}$
Steady state real interest rate (deposite rate): $rr_{ss} = 1/\beta$
Steady state Nu: $\nu_{ss} = \frac{(1-\gamma_{ss}^{F})\beta(rr_{ss}^{F} - rr_{ss})}{(1/\beta - \gamma_{ss}^{F})}$
Steady state Eta: $\eta_{ss} = \frac{(1-\gamma_{ss}^{F})}{(1-\beta\gamma_{ss}^{F})}$
Steady state leverage ratio of banker: $Lev_{ss}^{F} = \frac{\eta_{ss}}{\lambda - \nu_{ss}}$
Steady state leverage ratio of entrepreneur: $K_{ss}/N_{ss}^{E} = 1.919$
Steady state capital-output ratio: $K_{ss}/Y_{ss} = \frac{\alpha mc_{ss}}{rr_{ss}^{E} - (1-\delta)}$
Steady state investment: $I_{ss}/Y_{ss} = \delta K_{ss}/Y_{ss}$
Steady state government expenditure: $G_{ss}/Y_{ss} = 0.2$
Steady state consumption: $C_{ss}/Y_{ss} = 1 - I_{ss}/Y_{ss} - G_{ss}/Y_{ss}$

1.1.3 Estimated Parameters

Investment adjustment cost: κ
Habit formation: h
Intertemporal elasticity of substitution of consumption: σ^{c}
Inverse Frisch elasticity of labor supply: σ^{L}
Elasticity of premium to leverage ratio: φ
Price indexation: ι^{p}
Wage indexation: ι^{w}

Calvo parameter for goods pricing: θ^p
Calvo parameter for wage setting: θ^w
Monetary policy persistence param.: ρ^R
Taylor coefficient for inflation: μ^π
Taylor coefficient for output gap: μ^Y

Peresistence param. for TFP shock: ρ^A, persistence param. for preference shock: ρ^c, persistence param. for investment tech. shock: ρ^K, persistence param. for Entrepreneur net worth shock: ρ^E, persistence param. for Banker net worth shock: ρ^F, persistence param. for government expenditure shock: ρ^G, persistence param. for labor supply shock: ρ^L

$S.E.(\varepsilon^A)$, $S.E.(\varepsilon^c)$, $S.E.(\varepsilon^E)$, $S.E.(\varepsilon^F)$, $S.E.(\varepsilon^G)$, $S.E.(\varepsilon^K)$, $S.E.(\varepsilon^L)$, $S.E.(\varepsilon^R)$

1.2 Model Description (Log-linearized Version)

1.2.1 Household Sector

1. Consumption Euler equation:

$$\hat{C}_t = \frac{h}{1+h}\hat{C}_{t-1} + \frac{1}{1+h}E_t\hat{C}_{t+1} - \frac{1-h}{(1+h)\sigma^c}\left(\hat{R}_t - E_t\hat{\pi}_{t+1}\right) + \frac{1-h}{(1+h)\sigma^c}(1-\rho^c)\hat{\chi}_t^c \quad (1)$$

2. Wage setting equation:

$$\begin{aligned}\hat{W}_t = {} & \frac{\beta}{1+\beta}E_t\hat{W}_{t+1} + \frac{1}{1+\beta}\hat{W}_{t-1} + \frac{\beta}{1+\beta}E_t\hat{\pi}_{t+1} - \frac{1+\beta\iota^w}{1+\beta}\hat{\pi}_t + \frac{\iota^w}{1+\beta}\hat{\pi}_{t-1} \\ & - \frac{1}{1+\beta}\Upsilon^w\left[\hat{W}_t - \sigma^L\hat{L}_t - \frac{\sigma^c}{1-h}\left(\hat{C}_t - h\hat{C}_{t-1}\right) - \hat{\chi}_t^L\right] \end{aligned} \quad (2)$$

where $\Upsilon^w = \frac{(1-\beta\theta^w)(1-\theta^w)}{\left(\theta^w + \frac{(1+\psi^w)\theta^w\sigma^L}{\psi^w}\right)}$

1.2.2 Entrepreneur Sector

1. Production function:

$$\hat{Y}_t = \hat{A}_t + \alpha\hat{K}_t + (1-\alpha)\hat{L}_t \quad (3)$$

2. Labor demand:

$$\hat{W}_t = \widehat{mc}_t + \hat{Y}_t - \hat{L}_t \quad (4)$$

3. Capital demand:

$$\hat{R}_t^F + \hat{S}_t - E_t\hat{\pi}_{t+1} = \left(1 - \frac{(1-\delta)}{rr_{ss}^E}\right)\left(E_t\widehat{mc}_{t+1} + E_t\hat{Y}_{t+1} - E_t\hat{K}_{t+1}\right) + \frac{(1-\delta)}{rr_{ss}^E}E_t\hat{Q}_{t+1} - \hat{Q}_t \tag{5}$$

4. Net worth transition of entrepreneur:

$$\hat{N}_t^E = \gamma_{ss}^E rr_{ss}^E\hat{\gamma}_t^E + rr_{ss}^E\gamma_{ss}^E\left[\frac{K_{ss}}{N_{ss}^E}\hat{R}_t^K - \left(\frac{K_{ss}}{N_{ss}^E} - 1\right)\left(\hat{R}_{t-1}^F + \hat{S}_{t-1} - \hat{\pi}_t\right)\right] + rr_{ss}^E\left(\gamma_{ss}^E + \xi^E\right)\hat{N}_{t-1}^E \tag{6}$$

5. Realized marginal return of capital:

$$\hat{R}_t^K = \left(1 - \frac{(1-\delta)}{rr_{ss}^E}\right)\left(\widehat{mc}_t + \hat{Y}_t - \hat{K}_t\right) + \frac{(1-\delta)}{rr_{ss}^E}\hat{Q}_t - \hat{Q}_{t-1} \tag{7}$$

6. Entrepreneur's balance sheet identity:

$$\hat{B}_t^E = \frac{K_{ss}/N_{ss}^E}{(K_{ss}/N_{ss}^E) - 1}\left(\hat{Q}_t + \hat{K}_t\right) - \frac{1}{(K_{ss}/N_{ss}^E) - 1}\hat{N}_t^E \tag{8}$$

7. External finance premium:

$$\hat{S}_t = \varphi\left(\hat{Q}_t + \hat{K}_t - \hat{N}_t^E\right) \tag{9}$$

1.2.3 Financial Intermediary Sector (Banking Sector)

1. Time-varying coefficent ν:

$$\begin{aligned}\hat{\nu}_t = {} & \frac{rr_{ss}^F(1-\beta\gamma_{ss}^F)}{rr_{ss}^F - rr_{ss}}\left(\hat{R}_t^F - E_t\hat{\pi}_{t+1}\right) - \frac{rr_{ss}(1-\beta\gamma_{ss}^F)}{rr_{ss}^F - rr_{ss}}\left(\hat{R}_t - E_t\hat{\pi}_{t+1}\right) \\ & - \frac{\gamma_{ss}^F(1-\beta)}{1-\gamma_{ss}^F}\rho^F\hat{\gamma}_t^F + \beta\gamma_{ss}^F\left(E_t\hat{B}_{t+1}^E - \hat{B}_t^E + E_t\hat{\nu}_{t+1}\right)\end{aligned} \tag{10}$$

2. Time-varying coefficient η:

$$\hat{\eta}_t = \beta\gamma_{ss}^F\left(E_t\hat{N}_{t+1}^F - \hat{N}_t^F + E_t\hat{\eta}_{t+1}\right) - \frac{\gamma_{ss}^F(1-\beta)}{1-\gamma_{ss}^F}\rho^F\hat{\gamma}_t^F \tag{11}$$

3. Net worth transition of financial intermediary:

$$\begin{aligned}\hat{N}_t^F = {} & \gamma_{ss}^F\left[(rr_{ss}^F - rr_{ss})Lev_{ss}^F + rr_{ss}\right]\hat{\gamma}_t^F + \gamma_{ss}^F rr_{ss}^F Lev_{ss}^F\left(\hat{R}_{t-1}^F - \hat{\pi}_t\right) \\ & - \gamma_{ss}^F(rr_{ss}^F - rr_{ss})Lev_{ss}^F\left(\hat{R}_{t-1} - \hat{\pi}_t\right) + \gamma_{ss}^F(rr_{ss}^F - rr_{ss})Lev_{ss}^F\hat{B}_{t-1}^E + \left(\gamma_{ss}^F rr_{ss} + \xi^F\right)\hat{N}_{t-1}^F\end{aligned} \tag{12}$$

4. Leverage ratio:

$$\widehat{Lev}_t^F = \hat{\eta}_t + \frac{\nu_{ss}}{\lambda - \nu_{ss}}\hat{\nu}_t \tag{13}$$

5. Financial intermediary's balance sheet identity:

$$\widehat{Lev}_t^F = \hat{B}_t^E - \hat{N}_t^F \tag{14}$$

1.2.4 Capital Producer Sector

1. Investment Euler equation:

$$\hat{I}_t^K = \frac{1}{1+\beta}\hat{I}_{t-1}^K + \frac{\beta}{1+\beta}E_t\hat{I}_{t+1}^K + \frac{\kappa}{1+\beta}\hat{Q}_t + \hat{A}_t^K \tag{15}$$

2. Capital accumulation equation:

$$\hat{K}_t = \delta\hat{I}_{t-1}^K + (1-\delta)\hat{K}_{t-1} \tag{16}$$

1.2.5 Retail Sector

1. Hybrid New Keynesian Phillips curve:

$$(1+\beta\iota^p)\,\hat{\pi}_t = \iota^p\hat{\pi}_{t-1} + \beta E_t\hat{\pi}_{t+1} + \frac{(1-\beta\theta^p)(1-\theta^p)}{\theta^p}\widehat{mc}_t \tag{17}$$

1.2.6 Monetary Policy and Resource Constraint

1. Monetary policy:

$$\hat{R}_t = \rho^R\hat{R}_{t-1} + (1-\rho^R)\left(\mu^\pi\hat{\pi}_t + \mu^Y\hat{Y}_t\right) + \varepsilon_t^R \tag{18}$$

2. Resource constraint:

$$\hat{Y}_t = \frac{C_{ss}}{Y_{ss}}\hat{C}_t + \frac{I_{ss}^K}{Y_{ss}}\hat{I}_t^K + \frac{G_{ss}}{Y_{ss}}\hat{G}_t \tag{19}$$

1.2.7 Auxiliary Variables

1. Spread between bank rate and deposit rate:

$$Spread_t^F = \hat{R}_t^F - \hat{R}_t \tag{20}$$

2. Total spread (spread between corporate borrowing rate and deposit rate):

$$Spread_t^T = \hat{R}_t^F + \hat{S}_t - \hat{R}_t \tag{21}$$

3. Entrepreneur's leverage ratio (matched to observed data):

$$\widehat{Lev}_t^E = \hat{Q}_t + \hat{K}_t - \hat{N}_t^E \tag{22}$$

4. Corporate borrowing rate (nominal rate, matched to observed data):

$$\hat{R}_t^E = \hat{R}_t^F + \hat{S}_t \tag{23}$$

1.2.8 Exogenous AR(1) Shocks

1. TFP shock: $\hat{A}_t = \rho^A \hat{A}_{t-1} + \varepsilon_t^A$
2. Preference shock: $\hat{\chi}_t^c = \rho^c \hat{\chi}_{t-1}^c + \varepsilon_t^c$
3. Labor supply shock: $\hat{\chi}_t^L = \rho^L \hat{\chi}_{t-1}^L + \varepsilon_t^L$
4. Investment specific tech. shock: $\hat{A}_t^K = \rho^K \hat{A}_{t-1}^K + \varepsilon_t^K$
5. Entrepreneur net worth shock: $\hat{\gamma}_t^E = \rho^E \hat{\gamma}_{t-1}^E + \varepsilon_t^E$
6. Banks' net worth shock: $\hat{\gamma}_t^F = \rho^F \hat{\gamma}_{t-1}^F + \varepsilon_t^F$
7. Government expenditure shock: $\hat{G}_t = \rho^G \hat{G}_{t-1} + \varepsilon_t^G$

1.2.9 Forecast Errors

1. Consumption forecast error: $\zeta_t^c = \hat{C}_t - E_{t-1}\hat{C}_{t-1}$
2. Inflaton forecast error: $\zeta_t^\pi = \hat{\pi}_t - E_{t-1}\hat{\pi}_t$
3. Output forecast error: $\zeta_t^Y = \hat{Y}_t - E_{t-1}\hat{Y}_t$
4. Capital forecast error: $\zeta_t^K = \hat{K}_t - E_{t-1}\hat{K}_t$
5. Capital price forecast error: $\zeta_t^Q = \hat{Q}_t - E_{t-1}\hat{Q}_t$
6. Entrep. borrowing forecast error: $\zeta_t^{B^E} = \hat{B}_t^E - E_{t-1}\hat{B}_t^E$
7. Nu forecast error: $\zeta_t^\nu = \hat{\nu}_t - E_{t-1}\hat{\nu}_t$
8. Eta forecast error: $\zeta_t^\eta = \hat{\eta}_t - E_{t-1}\hat{\eta}_t$
9. Bank net worth forecast error: $\zeta_t^{N^F} = \hat{N}_t^F - E_{t-1}\hat{N}_t^F$
10. Investment forecast error: $\zeta_t^I = \hat{I}_t^K - E_{t-1}\hat{I}_t^K$
11. Marginal cost forecast error: $\zeta_t^{mc} = \widehat{mc}_t - E_{t-1}\widehat{mc}_t$
12. Wage forecast error: $\zeta_t^W = \hat{W}_t - E_{t-1}\hat{W}_{t-1}$

1.3 Endogenous and Exogenous Variables

1.3.1 Endogenous Variables

1. $\hat{C}_t$: Consumption
2. $\hat{W}_t$: Real wage
3. $\hat{L}_t$: Labor input

4. $\hat{R}_t$: Nominal interest rate (deposit rate)
5. $\hat{\pi}_t$: Inflation rate
6. $\widehat{mc}_t$: Marginal cost
7. $\hat{Y}_t$: Output
8. $\hat{R}_t^F$: Bank lending rate (nominal, ex-premium)
9. $\hat{S}_t$: External finance premium
10. $\hat{K}_t$: Capital
11. $\hat{Q}_t$: Real capital price
12. $\hat{N}_t^E$: Entrepreneur net worth
13. $\hat{R}_t^k$: Realized marginal return from capital investment
14. $\hat{B}_t^E$: Entrepreneur borrowing, bank lending
15. $\hat{\nu}_t$: Time-varying coefficient Nu
16. $\hat{\eta}_t$: Time-varying coefficient Eta
17. $\hat{N}_t^F$: Banking sector net worth
18. $\widehat{Lev}_t^F$: Banking sector leverage ratio
19. $\hat{I}_t^K$: Physical investment

1.3.2 Auxiliary Variables

20. $Spread_t^F$: Spread between bank lending rate and deposit rate
21. $Spread_t^T$: Spread between corporate borrowing rate and deposit rate
22. $\widehat{Lev}_t^E$: Entrepreneur leverage ratio
23. $\hat{R}_t^E$: Nominal corporate borrowing rate

1.3.3 Expectation Terms

24. $E_t\hat{C}_{t+1}$, 25. $E_t\hat{\pi}_{t+1}$, 26. $E_t\hat{Y}_{t+1}$, 27. $E_t\hat{K}_{t+1}$, 28. $E_t\hat{Q}_{t+1}$, 29. $E_t\hat{B}_{t+1}^E$, 30. $E_t\hat{\nu}_{t+1}$, 31. $E_t\hat{\eta}_{t+1}$, 32. $E_t\hat{N}_{t+1}^F$, 33. $E_t\hat{I}_{t+1}^K$, 34. $E_t\widehat{mc}_{t+1}$, 35. $E_t\hat{W}_{t+1}$

1.3.4 Exogenous AR(1) Terms

36. $\hat{G}_t$: Government expenditure
37. $\hat{A}_t$: Total factor productivity
38. $\hat{A}_t^K$: Investment specific technology shock
39. $\hat{\chi}_t^c$: Preference shock
40. $\hat{\chi}_t^L$: Labor supply shock
41. $\hat{\gamma}_t^E$: Entrepreneur net worth shock (or stochastic survival rate for entrepreneurs)
42. $\hat{\gamma}_t^F$: Banking sector net worth shock (or stochastic survival rate for bankers)

1.3.5 Exogenous IID shocks

1. ε_t^A : IID TFP shock
2. ε_t^c : IID Preference shock
3. ε_t^E : IID Entrepreneur net worth shock
4. ε_t^F : IID Banking sector net worth shock
5. ε_t^G : IID Government expenditure shock
6. ε_t^K : IID Investment specific technology shock
7. ε_t^L : IID Labor supply shock
8. ε_t^R : IID Monetary policy shock

1.3.6 Forecast Errors

1. ζ_t^c, 2. ζ_t^π, 3. ζ_t^Y, 4. ζ_t^K, 5. ζ_t^Q, 6. $\zeta_t^{B^E}$, 7. ζ_t^ν, 8. ζ_t^η, 9. $\zeta_t^{N^F}$, 10. ζ_t^I, 11. ζ_t^{mc}, 12. ζ_t^W
(see next page for companion form)

1.4 Companion Form

$$\Gamma_0 \begin{bmatrix} Endog \\ Auxil \\ Expect \\ ExogAR \end{bmatrix} = \Gamma_1 \begin{bmatrix} Endog(-1) \\ Auxil(-1) \\ Expect(-1) \\ ExogAR(-1) \end{bmatrix} + \Psi \begin{bmatrix} \varepsilon_t^A \\ \varepsilon_t^c \\ \varepsilon_t^E \\ \varepsilon_t^F \\ \varepsilon_t^G \\ \varepsilon_t^K \\ \varepsilon_t^L \\ \varepsilon_t^R \end{bmatrix} + \Pi \begin{bmatrix} \zeta_t^c \\ \zeta_t^\pi \\ \zeta_t^Y \\ \zeta_t^K \\ \zeta_t^Q \\ \zeta_t^{B^E} \\ \zeta_t^\nu \\ \zeta_t^\eta \\ \zeta_t^{N^F} \\ \zeta_t^I \\ \zeta_t^{mc} \\ \zeta_t^W \end{bmatrix}$$

where

$$Endog \equiv \begin{bmatrix} \hat{C}_t \\ \hat{W}_t \\ \hat{L}_t \\ \hat{R}_t \\ \hat{\pi}_t \\ \widehat{mc}_t \\ \hat{Y}_t \\ \hat{R}_t^F \\ \hat{S}_t \\ \hat{K}_t \\ \hat{Q}_t \\ \hat{N}_t^E \\ \hat{R}_t^k \\ \hat{B}_t^E \\ \hat{\nu}_t \\ \hat{\eta}_t \\ \hat{N}_t^F \\ \widehat{Lev}_t^F \\ \hat{I}_t^K \end{bmatrix}, \; Auxil = \begin{bmatrix} Spread_t^F \\ Spread_t^T \\ \widehat{Lev}_t^E \\ \hat{R}_t^E \end{bmatrix}, \; Expect = \begin{bmatrix} E_t\hat{C}_{t+1} \\ E_t\hat{\pi}_{t+1} \\ E_t\hat{Y}_{t+1} \\ E_t\hat{K}_{t+1} \\ E_t\hat{Q}_{t+1} \\ E_t\hat{B}_{t+1}^E \\ E_t\hat{\nu}_{t+1} \\ E_t\hat{\eta}_{t+1} \\ E_t\hat{N}_{t+1}^F \\ E_t\hat{I}_{t+1}^K \\ E_t\widehat{mc}_{t+1} \\ E_t\hat{W}_{t+1} \end{bmatrix}, \; ExogAR = \begin{bmatrix} \hat{G}_t \\ \hat{A}_t \\ \hat{A}_t^K \\ \hat{\chi}_t^c \\ \hat{\chi}_t^L \\ \hat{\gamma}_t^E \\ \hat{\gamma}_t^F \end{bmatrix}$$

1.5 Measurement Equation (Case A)

1.5.1 Linking Observed data and Endogenous variables

1. Observed Output: $Y_t^{obs} = \hat{Y}_t + m_t^Y$
2. Observed Consumption: $C_t^{obs} = \hat{C}_t + m_t^c$

3. Observed Investment: $I_t^{K,obs} = \hat{I}_t^K + m_t^K$
4. Observed Labor: $L_t^{obs} = \hat{L}_t + m_t^L$
5. Observed Wage: $W_t^{obs} = \hat{W}_t + m_t^W$
6. Observed Inflation: $\pi_t^{obs} = \hat{\pi}_t + m_t^\pi$
7. Observed Nominal Interest Rate: $R_t^{obs} = \hat{R}_t + m_t^R$
8. Observed Corporate Borrwoing Rate: $R_t^{E,obs} = \hat{R}_t^E + m_t^{R^E}$
9. Observed External Premium: $S_t^{obs} = \hat{S}_t + m_t^S$
10. Observed Corporate Leverage: $Lev_t^{E,obs} = \widehat{Lev}_t^E + m_t^{Lev^E}$
11. Observed Bank Leverage: $Lev_t^{F,obs} = \widehat{Lev}_t^F + m_t^{Lev^F}$

1.5.2 Measurement Error Specification

1. Output measurement error: $m_t^Y = \delta^Y m_{t-1}^Y + u_t^Y$
2. Consumption measurement error: $m_t^c = \delta^c m_{t-1}^c + u_t^c$
3. Investment measurement error: $m_t^K = \delta^K m_{t-1}^K + u_t^K$
4. Labor measurement error: $m_t^L = \delta^L m_{t-1}^L + u_t^L$
5. Wage measurement error: $m_t^W = \delta^W m_{t-1}^W + u_t^W$
6. Inflation measurement error: $m_t^\pi = \delta^\pi m_{t-1}^\pi + u_t^\pi$
7. Nominal interest rate measurement error: $m_t^R = \delta^R m_{t-1}^R + u_t^R$
8. Corporate borrowing rate measurement error: $m_t^{R^E} = \delta^{R^E} m_{t-1}^{R^E} + u_t^{R^E}$
9. External premium measurement error: $m_t^S = \delta^S m_{t-1}^S + u_t^S$
10. Corporate leverage measurement error: $m_t^{Lev^E} = \delta^{Lev^E} m_{t-1}^{Lev^E} + u_t^{Lev^E}$
11. Bank leverage measurement error: $m_t^{Lev^F} = \delta^{Lev^F} m_{t-1}^{Lev^F} + u_t^{Lev^F}$