The model equations

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1 DSGE model

1.1 Parameters for the DSGE Model

β	Discount rate
σ	Intertemporal elasticity of substitution
h	Consumption habit parameter
χ	Relative utility weight of labor
φ	Inverse Frisch elasticity of labor supply
α	Capital share
δ	Depreciation rate
η_i	Inverse elasticity of net investment to the price of capital
ζ	Elasticity of depreciation wrt. utilization
G_{ss}	Steady state government consumption
ϵ	Elasticity of substitution between goods
γ	Probability of price change
γ_P	Price indexation parameter
τ_X	Tax on retailers
ρ_i	Interest rate smoothing parameter
κ_π	Inflation coefficient in the monetary policy rule
κ_X	Markup coefficient in the monetary policy rule
ρ_ξ	Persistence of capital quality shock
σ_ξ	Std. dev. of capital quality shock
ρ_a	Persistence of TFP shock
σ_a	Std. dev. of TFP shock

2 Variables

Y_{mt}	Wholesale output
Y_t	Retail output
D_t	Price dispersion
K_t	Capital
L_t	Labor
I_t	Investment
In_t	Net Investment
C_t	Consumption
G_t	Government expenditure
Q_t	Market value of capital
δ_t	Capital depreciation rate
U_t	Capital utilization
ϱ_t	Marginal utility of consumption
Λ_t	Stochastic discount rate
R_{kt}	Capital return
R_t	Gross real interest rate
P_{mt}	Price level of the wholesale good
X_t	Markup
$\Pi_t^* = P_t^*/P_{t-1}$	Optimal price normalized by the last period price level
F_t	Numerator of the optimal normalized price choice
Z_t	Denominator of the optimal normalized price choice
Π_t	Gross inflation rate
i_t	Net nominal interest rate
A_t	Technology level
ξ_t	Capital quality

3 The model equations

Marginal utility of consumption

$$\varrho_t = \left((C_t - hC_{t-1})^{-1} - \beta h E_t(C_{t+1} - hC_t)^{-1}\right) \tag{1}$$

Real stochastic discount factor

$$\Lambda_{t,t+1} = \frac{\varrho_{t+1}}{\varrho_t} \tag{2}$$

Euler equation

$$E_t\left\{\beta \Lambda_{t,t+1} R_{t+1}\right\} = 1 \tag{3}$$

Arbitrage

$$E_t \beta \Lambda_{t,t+1} R_{kt+1} = E_t \beta \Lambda_{t,t+1} R_{t+1} \tag{4}$$

Labor market equation

$$P_{mt}(1-\alpha)\frac{Y_{mt}}{L_t} = \chi \varrho_t^{-1} L^{\varphi} \tag{5}$$

Capital return

$$R_{kt+1} = \frac{\xi_{t+1}\left(P_{mt+1}\alpha\frac{Y_{mt+1}}{\xi_{t+1}K_{t+1}} + Q_{t+1} - \delta(U_{t+1})\right)}{Q_t} \tag{6}$$

Intermediate good production function

$$Y_{mt} = A_t\left(U_t\xi_t K_t\right)^{\alpha} L_t^{1-\alpha} \tag{7}$$

Optimal net investment decision

$$Q_t = 1 + \frac{\eta_i}{2}\left(\frac{I_{nt}+I_{ss}}{I_{nt-1}+I_{ss}} - 1\right)^2 + \eta_i\left(\frac{I_{nt}+I_{ss}}{I_{nt-1}+I_{ss}} - 1\right)\frac{I_{nt}+I_{ss}}{I_{nt-1}+I_{ss}} - E_t\beta\Lambda_{t,t+1}\eta_i\left(\frac{I_{nt+1}+I_{ss}}{I_{nt}+I_{ss}} - 1\right)\left(\frac{I_{nt+1}+I_{ss}}{I_{nt}+I_{ss}}\right)^2 \tag{8}$$

Depreciation rate

$$\delta(U_t) = \delta_c + \frac{b}{1+\zeta}U_t^{1+\zeta} \tag{9}$$

Optimal capacity utilization

$$P_{mt}\alpha\frac{Y_{mt}}{U_t} = \delta'(U_t)\xi_t K_t = bU_t^{\zeta}\xi_t K_t \tag{10}$$

Net investment

$$I_{nt} = I_t - \delta(U_t)\xi_t K_t \tag{11}$$

Capital accumulation equation

$$K_{t+1} = \xi_t K_t + I_{nt} \tag{12}$$

Aggregate resource constraint

$$Y_t = C_t + I_t + G_t + \frac{\eta_i}{2}\left(\frac{I_{nt}+I_{ss}}{I_{nt-1}+I_{ss}} - 1\right)^2 (I_{nt}+I_{ss}) \tag{13}$$

Wholesale, retail output

$$Y_t = Y_{mt}D_t \tag{14}$$

Price dispersion

$$D_t = \gamma D_{t-1}\Pi_{t-1}^{-\gamma_P\varepsilon}\Pi_t^{\varepsilon} + (1-\gamma)\left(\frac{1-\gamma\Pi_{t-1}^{\gamma_P(1-\gamma)}\Pi_t^{\gamma-1}}{1-\gamma}\right)^{-\frac{\varepsilon}{1-\gamma}} \tag{15}$$

Markup ($\Pi_t = P_t/P_{t-1}$)

$$P_{mt} = \frac{1}{X_t}, \tag{16}$$

Recursive formulation of optimal price choice ($\Pi_t^* = P_t^*/P_{t-1}$)

$$F_t = Y_t P_{mt} + E_t \left[\beta\gamma\Lambda_{t,t+1} \frac{\Pi_t^{-(\gamma_P \varepsilon)}}{\Pi_{t+1}^{-\varepsilon}} F_{t+1} \right], \tag{17}$$

$$Z_t = Y_t + E_t \left[\beta\gamma\Lambda_{t,t+1} \frac{\Pi_t^{\gamma_P(1-\varepsilon)}}{\Pi_{t+1}^{(1-\varepsilon)}} Z_{t+1} \right]. \tag{18}$$

$$\Pi_t^* = \frac{1}{\tau_X} \frac{\varepsilon}{\varepsilon - 1} \frac{F_t}{Z_t} \Pi_t, \tag{19}$$

Inflation development

$$\Pi_t^{(1-\varepsilon)} = \gamma \Pi_{t-1}^{\gamma_P(1-\varepsilon)} + (1-\gamma)\Pi_t^{*1-\varepsilon}, \tag{20}$$

Fisher equation

$$i_t = \log R_{t+1} + E_t \pi_{t+1}, \tag{21}$$

Monetary policy rule

$$i_t = \rho_i i_{t-1} + (1-\rho_i)(R + \kappa_\pi \pi_t + \kappa_X \log X_t) + e_{i,t} \tag{22}$$

Shocks

$$G_t = G_{ss} * e^{g_t} \tag{23}$$

$$A_t = e^{a_t} \tag{24}$$

$$\xi_t = e^{\hat{\xi}_t} \tag{25}$$

$$a_t = \rho_a a_{t-1} + e_a \tag{26}$$

$$\hat{\xi}_t = \rho_\xi \hat{\xi}_{t-1} + e_\xi \tag{27}$$

$$g_t = \rho_g g_{t-1} + e_g \tag{28}$$

4 Financial Accelerator

4.1 Parameters for the Financial Accelerator Model

β	Discount rate
σ	Intertemporal elasticity of substitution
h	Consumption habit parameter
χ	Relative utility weight of labor
φ	Inverse Frisch elasticity of labor supply
λ	Fraction of capital that can be diverted by the bank
ω	Proportional transfer to the entering local bankers
θ	Survival rate of bankers
α	Capital share
δ	Depreciation rate
η_i	Inverse elasticity of net investment to the price of capital
ζ	Elasticity of depreciation wrt. utilization
G_{ss}	Steady state government consumption
ϵ	Elasticity of substitution between goods
γ	Probability of price change
γ_P	Price indexation parameter
τ_X	Tax on retailers
ρ_i	Interest rate smoothing parameter
κ_π	Inflation coefficient in the monetary policy rule
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5 Variables

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D_t	Price dispersion
K_t	Capital
L_t	Labor
I_t	Investment
In_t	Net Investment
C_t	Consumption
G_t	Government expenditure
Q_t	Market value of capital
δ_t	Capital depreciation rate
U_t	Capital utilization
ϱ_t	Marginal utility of consumption
Λ_t	Stochastic discount rate
N_t	Net wealth
N_{et}	Surviving bankers' net wealth
N_{nt}	New bankers' net wealth
R_{kt}	Capital return
R_t	Gross real interest rate
v	Value of banks' capital
η_t	Value of bank's net wealth
ϕ_t	Leverage
$z_{t,t+1}$	Growth rate of banks' capital
$x_{t,t+1}$	Growth rate of banks' net wealth
P_{mt}	Price level of the wholesale good
X_t	Markup
$\Pi_t^* = P_t^*/P_{t-1}$	Optimal price normalized by the last period price level
F_t	Numerator of the optimal normalized price choice
Z_t	Denominator of the optimal normalized price choice
Π_t	Gross inflation rate
i_t	Net nominal interest rate
A_t	Technology level
ξ_t	Capital quality

6 The model equations

Marginal utility of consumption

$$\varrho_t = \left((C_t - hC_{t-1})^{-1} - \beta h E_t(C_{t+1} - hC_t)^{-1}\right) \tag{29}$$

Real stochastic discount factor

$$\Lambda_{t,t+1} = \frac{\varrho_{t+1}}{\varrho_t} \tag{30}$$

Euler equation

$$E_t\left\{\beta \Lambda_{t,t+1} R_{t+1}\right\} = 1 \tag{31}$$

Labor market equation

$$P_{mt}(1-\alpha)\frac{Y_{mt}}{L_t} = \chi\varrho_t^{-1}L^{\varphi} \tag{32}$$

Capital return

$$R_{kt+1} = \frac{\xi_{t+1}\left(P_{mt+1}\alpha\frac{Y_{mt+1}}{\xi_{t+1}K_{t+1}} + Q_{t+1} - \delta(U_{t+1})\right)}{Q_t} \tag{33}$$

Intermediate good production function

$$Y_{mt} = A_t\left(U_t\xi_t K_t\right)^{\alpha} L_t^{1-\alpha} \tag{34}$$

Optimal net investment decision

$$Q_t = 1 + \frac{\eta_i}{2}\left(\frac{I_{nt}+I_{ss}}{I_{nt-1}+I_{ss}} - 1\right)^2 + \eta_i\left(\frac{I_{nt}+I_{ss}}{I_{nt-1}+I_{ss}} - 1\right)\frac{I_{nt}+I_{ss}}{I_{nt-1}+I_{ss}} - E_t\beta\Lambda_{t,t+1}\eta_i\left(\frac{I_{nt+1}+I_{ss}}{I_{nt}+I_{ss}} - 1\right)\left(\frac{I_{nt+1}+I_{ss}}{I_{nt}+I_{ss}}\right)^2 \tag{35}$$

Depreciation rate

$$\delta(U_t) = \delta_c + \frac{b}{1+\zeta}U_t^{1+\zeta} \tag{36}$$

Optimal capacity utilization

$$P_{mt}\alpha\frac{Y_{mt}}{U_t} = \delta'(U_t)\xi_t K_t = bU_t^{\zeta}\xi_t K_t \tag{37}$$

Net investment

$$I_{nt} = I_t - \delta(U_t)\xi_t K_t \tag{38}$$

Capital accumulation equation

$$K_{t+1} = \xi_t K_t + I_{nt} \tag{39}$$

Aggregate resource constraint

$$Y_t = C_t + I_t + G_t + \frac{\eta_i}{2}\left(\frac{I_{nt}+I_{ss}}{I_{nt-1}+I_{ss}} - 1\right)^2 (I_{nt}+I_{ss}) \tag{40}$$

Value of firms' capital

$$v_t = E_t\{(1-\theta)\beta\Lambda_{t,t+1}(R_{kt+1} - R_{t+1}) + \beta\Lambda_{t,t+1}\theta x_{t,t+1}v_{t+1}\} \tag{41}$$

Value of firms' net wealth

$$\eta_t = \max E_t\{(1-\theta) + \beta\Lambda_{t,t+1}\theta z_{t,t+1}\eta_{t+1}\} \tag{42}$$

Optimal leverage

$$\phi_t = \frac{\eta_t}{\lambda - v_t} \tag{43}$$

Growth rate of individual firms' capital

$$z_{t,t+1} = (R_{kt+1} - R_{t+1})\phi_t + R_{t+1} \tag{44}$$

Growth rate of individual firms' net wealth

$$x_{t,t+1} = (\phi_{t+1}/\phi_t)z_{t,t+1} \tag{45}$$

Aggregate capital

$$Q_t K_{t+1} = \phi_t N_t \tag{46}$$

Aggregate net wealth

$$N_t = N_{et} + N_{nt} \tag{47}$$

Entrepreneur net wealth accumulation

$$N_{et} = \theta\left[(R_{kt} - R_t)\phi_{t-1} + R_t\right] N_{t-1} e^{e_{Ne}t} \tag{48}$$

New entrepreneurs' net wealth

$$N_{nt} = (1-\theta)\omega Q_t \xi_t K_t \tag{49}$$

Wholesale, retail output

$$Y_t = Y_{mt} D_t \tag{50}$$

Price dispersion

$$D_t = \gamma D_{t-1} \Pi_{t-1}^{-\gamma_P \varepsilon} \Pi_t^{\varepsilon} + (1-\gamma)\left(\frac{1 - \gamma \Pi_{t-1}^{\gamma_P(1-\gamma)} \Pi_t^{\gamma-1}}{1-\gamma}\right)^{-\frac{\varepsilon}{1-\gamma}} \tag{51}$$

Markup ($\Pi_t = P_t/P_{t-1}$)

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$$Z_t = Y_t + E_t\left[\beta\gamma\Lambda_{t,t+1} \frac{\Pi_t^{\gamma_P(1-\varepsilon)}}{\Pi_{t+1}^{(1-\varepsilon)}} Z_{t+1}\right]. \tag{54}$$

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$$\Pi_t^{(1-\varepsilon)} = \gamma \Pi_{t-1}^{\gamma_P(1-\varepsilon)} + (1-\gamma)\Pi_t^{*1-\varepsilon}, \tag{56}$$

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$$i_t = \log R_{t+1} + E_t \pi_{t+1}, \tag{57}$$

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$$a_t = \rho_a a_{t-1} + e_a \tag{62}$$

$$\hat{\xi}_t = \rho_\xi \hat{\xi}_{t-1} + e_\xi \tag{63}$$

$$g_t = \rho_g g_{t-1} + e_g \tag{64}$$