We use KVL from the output node down to ground to find an expression for v_{gs2}.

$$v_{\mathrm{gs2}} = v_{\mathrm{out}} + v_{\mathrm{gs1}} \tag{37}$$

KCL at the tail node:

$$g_{\mathrm{m1}} v_{\mathrm{gs1}} + g_{\mathrm{m2}} v_{\mathrm{gs2}} + g_{\mathrm{ds2}} v_{\mathrm{gs2}} + g_{\mathrm{ds5}} v_{\mathrm{gs2}} = 0 \tag{38}$$

Using Equation 37 to substitute v_{gs2} in {#eq-app-vbufzout-kcl-vtail-cl} we find an equation for v_{gs1}.

$$v_{\mathrm{gs1}} = -\frac{g_{\mathrm{m2}} + g_{\mathrm{ds2}}}{g_{\mathrm{m1}} + g_{\mathrm{m2}} + g_{\mathrm{ds2}} + g_{\mathrm{ds5}}} v_{\mathrm{out}} \tag{39}$$

Again, we derive the output conductance by plugging Equation 37, Equation 30 and Equation 39 step by step into Equation 36. First, we use Equation 37 to eliminate v_{gs2}.

$$i_{\mathrm{out}} - \left(g_{\mathrm{ds4}} + g_{\mathrm{ds2}} + g_{\mathrm{m2}}\right) v_{\mathrm{out}} - g_{\mathrm{m34}} v_{\mathrm{gs34}} - \left(g_{\mathrm{ds2}} + g_{\mathrm{m2}}\right) v_{\mathrm{gs1}} = 0$$

Second, Equation 30 also holds for the closed-loop case and lets us eliminate v_{gs34}.

$$i_{\mathrm{out}} - \left(g_{\mathrm{ds4}} + g_{\mathrm{ds2}} + g_{\mathrm{m2}}\right) v_{\mathrm{out}} - \left(g_{\mathrm{ds2}} + g_{\mathrm{m2}} - g_{\mathrm{m1}}\right) v_{\mathrm{gs1}} = 0$$

Third, we use Equation 39 to eliminate the remaining unknown v_{gs1}.

$$i_{\mathrm{out}} - \left(g_{\mathrm{ds4}} + g_{\mathrm{ds2}} + g_{\mathrm{m2}}\right) v_{\mathrm{out}} + \left(g_{\mathrm{ds2}} + g_{\mathrm{m2}} - g_{\mathrm{m1}}\right) \frac{g_{\mathrm{m2}} + g_{\mathrm{ds2}}}{g_{\mathrm{m1}} + g_{\mathrm{m2}} + g_{\mathrm{ds2}} + g_{\mathrm{ds5}}} v_{\mathrm{out}} = 0$$

A more simpler result can be obtained, if we neglect g_{ds2} and g_{ds5} in Equation 39 first ($g_{\mathrm{m}} \gg g_{\mathrm{ds}}$) and then plug it into our main equation. Additionally, we use $g_{\mathrm{m12}} = g_{\mathrm{m1}} = g_{\mathrm{m2}}$ to further simplify the equation.

$$i_{\mathrm{out}} - \left(g_{\mathrm{ds4}} + \frac{3}{2} g_{\mathrm{ds2}} + g_{\mathrm{m12}}\right) v_{\mathrm{out}} \approx 0$$

If we apply $g_{\mathrm{m}} \gg g_{\mathrm{ds}}$ again, we arrive at the same result which was used for the noise calculation in Section 7.3, compare the expression for Y'_{load} given by Equation 20 .

$$i_{\mathrm{out}} - \left(g_{\mathrm{m12}}\right) v_{\mathrm{out}} \approx 0$$

17 Appendix: ngspice Cheatsheet

Here is an unsorted list of useful ngspice settings and command:

17.1 Commands

- `ac dec|lin points fstart fstop` performs a small-signal ac analysis with either linear or decade sweep
- `dc sourcename vstart vstop vincr [src2 start2 stop2 incr2]` runs a dc-sweep, optionally across two variables
- `display` shows the available data vectors in the current plot
- `echo` can be used to display text, `$variable` or `$&vector`, can be useful for debugging
- `let name = expr` to create a new vector; `unlet vector` deletes a specified vector; access vector data with `$&vec`
- `linearize vec` linearizes a vector on an equidistant time scale, do this before an FFT; with `set specwindow=windowtype` a proper windowing function can be set
- `meas` can be used for various evaluations of measurement results (see ngspice manual for details)
- `noise v(output <ref>) src (dec|lin) pts fstart fstop` runs a small-signal noise analysis
- `op` calculates the operating point, useful for checking bias points and device parameters
- `plot expr vs scale` to plot something
- `print expr` to print it, use `print all` to print everything
- `remzerovec` can be useful to remove vectors with zero length, which otherwise cause issues when saving or plotting data
- `rusage` plot information about resource usage like memory
- `save all` or `save signal` specifies which data is saved during simulation; this lowers RAM usage during simulation and size of RAW file; do save before the actual simulation statement
- `setplot` show a list of available plots
- `set var = value` to set the value of a variable; use variable with `$var`; `unset var` removes a variable
- `set enable_noisy_r` to enable noise of behavioral resistors; usually, this is a good idea
- `shell cmd` to run a shell command
- `show : param`, like `show : gm` shows the g_m of all devices after running an operating point with `op`
- `spec` plots a spectrum (i.e. frequency domain plot)
- `status` shows the saved parameters and nodes
- `tf` runs a transfer function analysis, returning transfer function, input and output resistance
- `tran tstep tstop <tstart <tmax>>` runs a transient analysis until `tstop`, reporting results with `tstep` step size, starting to plot at `tstart` and performs time steps not larger then `tmax`
- `wrdata` writes data into a file in a tabular ASCII format; easy to further process
- `write` writes simulation data (the saved nodes) into a RAW file; default is binary, can be changed to ASCII with `set filetype=ascii`; with `set appendwrite` data is added to an existing file

17.2 Options

Use `option option=val option=val` to set various options; important ones are:

- `abstol` sets the absolute current error tolerance (default is 1pA)
- `gmin` is the conductance applied at every node for convergence improvement (default is 1e-12); this can be critical for very high impedance circuits
- `klu` sets the KLU matrix solver
- `list` print the summary listing of the input data
- `maxord` sets the numerical order of the integration method (default is 2 for Gear)
- `method` set the numerical integration method to `gear` or `trap` (default is `trap`)
- `node` prints the node table
- `opts` prints the option values
- `temp` sets the simulation temperature
- `reltol` set the relative error tolerance (default is 0.001 = 0.1%)
- `savecurrents` saves the terminal currents of all devices
- `sparse` sets the sparse matrix solver, which can run noise analysis, but is slower than `klu`
- `vntol` sets the absolute voltage error tolerance (default is 1µV)
- `warn` enables the printing of the SOA warning messages

17.3 Convergence Helper

- `option gmin` can be used to increase the conductance applied at every node
- `option method=gear` can lead to improved convergence
- `.nodeset` can be used to specify initial node voltage guesses
- `.ic` can be used to set initial conditions

18 Appendix: Xschem Cheatsheet

When opening Xschem, using `Help -> Keys` a pop-up windows comes up with many useful shortcuts. The most useful are:

18.0.0.1 Moving around in a schematic:

- `Cursor keys` to move around
- `Ctrl-e` to go back to parent schematic
- `e` to descend into selected symbol
- `f` full zoom on schematic
- `Shift-z` to zoom in
- `Ctrl-z` to zoom out