

# Analog Circuit Design

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## Introduction

This is the material for an intermediate-level MOSFET circuit design course, held at JKU under course number 336.009 (“KV Analoge Schaltungstechnik”).

The course makes heavy use of circuit simulation, using **Xschem** for schematic entry and **ngspice** for simulation. The 130nm CMOS technology **SG13G2** from IHP Microelectronics is used.

Tools and PDK are integrated in the **IIC-OSIC-TOOLS** Docker image, which will be used during the coursework.

### Note

All course material is made publicly available on GitHub and shared under the Apache-2.0 license.

## IHP's SG13G2 130nm CMOS Technology

SG13G2 is the name of a 130nm CMOS technology (strictly speaking BiCMOS) from IHP Microelectronics. It features low-voltage (thin-oxide) core MOSFET, high-voltage (thick-oxide) I/O MOSFET, various types of linear resistors, and 7 layers of Aluminium metallization (5 thin plus 2 thick metal layers). This PDK is open-source, and the complete process specification can be found at [SG13G2 process specification](#). While we will not do layouts in this course, the layout rules can be found at [SG13G2 layout rules](#).

For our circuit design, the most important parameters of the available devices are summarized in the following:

- **Low-voltage NMOS:** Device `sg13_1v_nmos`; operating voltage nominal  $V_{DD} = 1.5\text{ V}$ ,  $L_{\min} = 0.13\text{ }\mu\text{m}$ ,  $V_{th} \approx 0.5\text{ V}$ ; a triple-well option for the NMOS is available.
- **Low-voltage PMOS:** Device `sg13_1v_pmos`; operating voltage nominal  $V_{DD} = 1.5\text{ V}$ ,  $L_{\min} = 0.13\text{ }\mu\text{m}$ ,  $V_{th} \approx -0.47\text{ V}$ .
- **High-voltage NMOS:** Device `sg13_hv_nmos`; operating voltage nominal  $V_{DD} = 3.3\text{ V}$ ,  $L_{\min} = 0.45\text{ }\mu\text{m}$ ,  $V_{th} \approx 0.7\text{ V}$ ; a triple-well option for the NMOS is available.
- **High-voltage PMOS:** Device `sg13_hv_pmos`; operating voltage nominal  $V_{DD} = 3.3\text{ V}$ ,  $L_{\min} = 0.45\text{ }\mu\text{m}$ ,  $V_{th} \approx -0.65\text{ V}$ .

- **Silicided poly resistor:** Device `rsil`;  $R_{\square} = 7\ \Omega \pm 10\%$ ,  $TC_1 = 3100\ \text{ppm/K}$
- **Poly resistor:** Device `rppd`;  $R_{\square} = 260\ \Omega \pm 10\%$ ,  $TC_1 = 170\ \text{ppm/K}$
- **Poly resistor high:** Device `rhigh`;  $R_{\square} = 1360\ \Omega \pm 15\%$ ,  $TC_1 = -2300\ \text{ppm/K}$
- **MIM capacitor:** Device `cap_cmim`;  $C' = 1.5\ \text{fF}/\mu\text{m}^2 \pm 10\%$ ,  $VC_1 = -26\ \text{ppm/V}$ ,  $TC_1 = 3.6\ \text{ppm/K}$ , breakdown voltage  $> 15\ \text{V}$
- **MOM capacitor:** The metal stack is well-suited for MOM capacitors due to 5 thin metal layers, but no primitive capacitor device is available at this point.

## Schematic Entry Using Xschem

Xschem is an open-source schematic entry tool with emphasis on integrated circuits. For up-to-date information of the many features of Xschem and the basic operation of it please look at the available [online documentation](#). Usage of Xschem will be learned with the first few basic examples, essentially using a single MOSFET. The usage model of Xschem is that the schematic is hierarchically drawn, and the simulation and evaluation statements are contained in the schematics. Further, Xschem offers embedded graphing, which we will mostly use.

## Circuit Simulation Using ngspice

ngspice is an open-source circuit simulator with SPICE dependency (Nagel 1975). Besides the usual simulated types like `op` (operating point), `dc` (dc sweeps), `tran` (time-domain), or `ac` (small-signal frequency sweeps), ngspice offers a script-like control interface, where many different simulation controls and result evaluations can be done. For detailed information please refer to the latest [online manual](#).

## Integrated IC Design Environment (IIC-OSIC-TOOLS)

In order to make use of the various required components (tools like Xschem and ngspice, PDKs like SG13G2) easier, we will use the **IIC-OSIC-TOOLS**. This is a pre-compiled Docker image which allows to do circuit design on a virtual machine on virtually any type of computing equipment (personal PC, Raspberry Pi, cloud server) on various operating systems (Windows, macOS, Linux). For further information like installed tools, how to setup a VM, etc. please look at [IIC-OSIC-TOOLS GitHub page](#).

### Tip

Please make sure to receive information about your personal VM access ahead of the course start.

Experienced users can install this image on their personal computer, for JKU students the IIC will host a VM on our compute cluster and provide personal login credentials.

### **i** Note

In this course, we assume that students have a basic knowledge of Linux and how to operate it using the terminal. If you are not yet familiar with Linux (which is basically a must when doing integrated circuit design as many tools are only available on Linux), then please check out a Linux introductory course or tutorial online, there are many resources available.

## **First Steps**

In this first chapter we will learn to use Xschem for schematic entry, and how to operate the ngspice SPICE simulator for circuit simulations. Further, we will make ourselves familiar with the transistor and other passive components available in the IHP Microelectronics SG13G2 technology. While this is strictly speaking a BiCMOS technology offering MOSFETs as well as SiGe HBTs, we will use it as a pure CMOS technology.

## **The Metal-Oxide-Semiconductor Field-Effect-Transistor (MOSFET)**

In this course, we will not dive into semiconductor physics and derive the device operation bottom-up starting from a fundamental level governed by quantum mechanics. Instead, we will treat the MOSFET as a macroscopic by assuming we have a 4-terminal device, and the performance of this device regarding its terminal voltages and currents we will largely derive from the simulation model.

The circuit symbol that we will use for the n-channel MOSFET is shown in Figure 1, and for the p-channel MOSFET it is shown in Figure 2. A control voltage between gate (“G”) and source (“S”) causes a current to flow between drain (“D”) and source. The MOSFET is a 4-terminal device, so the bulk (“B”) can also control the drain-source current flow. Often, the bulk is connected to source, and then the bulk terminal is not shown to declutter the schematics.

### **i** Note

Strictly speaking is the drain-source current of a MOSFET controlled by the voltage between gate and bulk and the voltage between drain and source. Since bulk is often connected to source anyway, and many circuit designers historically were already familiar with the operation of the bipolar junction transistor, it is common to consider the gate-source voltage (besides the drain-source voltage) as the controlling voltage. This focus on gate-source implies that the source is special compared to the drain. In a typical physical MOSFET, however, the drain and source are constructed exactly the same, and which terminal is drain, and which terminal is source, is only determined by the applied voltage potentials, and can change dynamically during operation (think of a MOSFET operating as a switch... which side is the drain, which side is the source?). Unfortunately, this focus on a “special” source has made its way into some MOSFET

compact models. The model that is used in SG13G2 luckily uses the PSP model, which is formulated symmetrically with regards to drain and source, and is thus very well suited for analog and RF circuit design. For a detailed understanding of the PSP model please refer to the [model documentation](#).

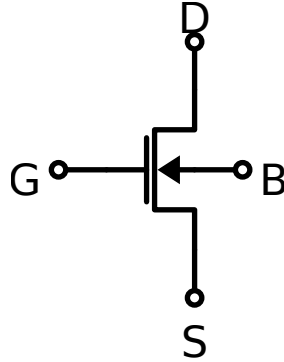


Figure 1: Circuit symbol of n-channel MOSFET.

Source: [Article Notebook](#)

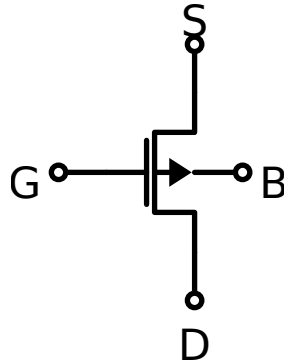


Figure 2: Circuit symbol of p-channel MOSFET.

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For hand calculations and theoretical discussions we will use the following simplified large-signal model, shown in Figure 3. A current source  $I_{DS}$  models the current flow between drain and source, and it is controlled by the three control voltages  $V_{GS}$ ,  $V_{DS}$ , and  $V_{SB}$ . Note that in this way (since  $I_{DS} = f(V_{DS})$ ) also a resistive behavior between D and S can be modelled. In case that B and S are shorted then simply  $V_{SB} = 0$ .

Source: [Article Notebook](#)

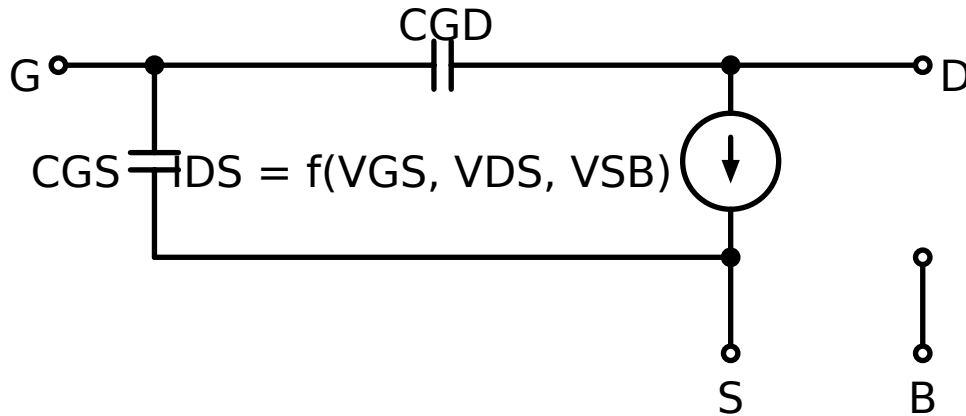


Figure 3: The MOSFET large-signal model.

Source: [Article Notebook](#)

In an ideal MOSFET no dc current is flowing into the gate, the behavior is purely capacitive. We model this by two capacitors:  $C_{GG} = C_{GS} + C_{GD}$  is the total capacitance when looking into the gate of the MOSFET.  $C_{GS}$  is usually the dominant capacitance, and  $C_{GD}$  models the capacitive feedback between D and G, usually induced by a topological overlap capacitance in the physical construction of the MOSFET. This capacitance is often small compared to  $C_{GS}$ , but in situations where we have a large voltage swing at the drain this capacitance will be affected by the [Miller effect](#). In hand calculations we will often set  $C_{GD} = 0$ .

#### **i** Note

The bulk connection in Figure 3 seems floating as we only consider it a control terminal, where the potential difference between source and bulk influences the behaviour of the MOSFET. However, we do not consider resistive or capacitive effects associated with this node, which is of course a gross simplification, but nevertheless one we will make in this course.

Now, as we are skipping the bottom-up approach of deriving the MOSFET large-signal behaviour from basic principles, we need to understand the behaviour of the elements of the large-signal model in Figure 3 by using a circuit simulator and observing what happens. And generally, a first step in any new IC technology should be to investigate basic MOSFET performance, by doing simple dc sweeps of  $V_{GS}$  and  $V_{DS}$  and looking at  $I_{DS}$  and other large- and small-signal parameters.

As a side note, the students who want to understand MOSFET behaviour from a physical angle should consult the MOSFET chapter from the JKU course “Design of Complex Integrated Circuits” (VL 336.048). A great introduction into MOSFET operation and fabrication is given in (Hu 2010), which is available freely [online](#) and is a recommended read. A very detailed description of the MOSFET (leaving usually no question unanswered) is provided in (Tsividis and McAndrew 2011).

Now, in order to get started, basic Xschem testbenches are prepared, and first simple dc sweeps of various voltages and currents will be done. But before that, please look at the import note below!

### ! Important

Throughout this material, we will stick to the following notations:

- A **dc quantity** is shown with an upper-case letter with upper-case subscripts, like  $V_{GS}$ .
- Double-subscripts denote **dc sources**, like  $V_{DD}$  and  $V_{SS}$ .
- An **ac (small-signal) quantity** is a lower-case letter with a lower-case subscript, like  $g_m$ .
- A **total quantity** (dc plus ac) is shown as a lowercase letter with upper-case subscript, like  $i_{DS}$ .
- An upper-case letter with a lower-case subscript is used to denote **RMS quantities**, like  $I_{ds}$ .

## Large-Signal MOSFET Model

We start with an investigation into the large-signal MOSFET model shown in Figure 3 by using the simple testbench for the LV NMOS shown in Figure 4.

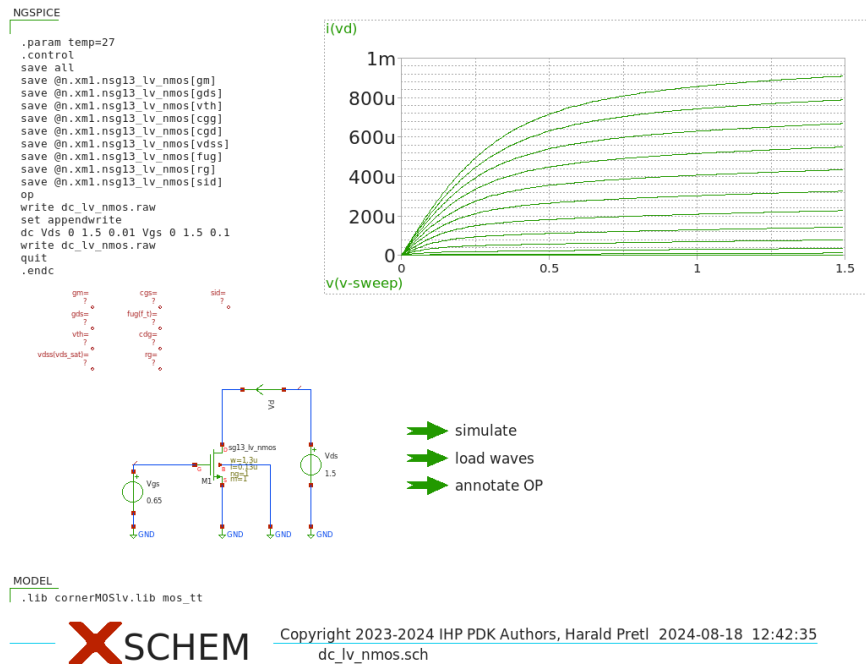


Figure 4: Testbench for NMOS dc sweeps.

### Exercise

Please try to execute the following steps and answer these questions:

1. Get the LV NMOS testbench (available at [https://github.com/iic-jku/analog-circuit-design/blob/main/xschem/dc\\_lv\\_nmos.sch](https://github.com/iic-jku/analog-circuit-design/blob/main/xschem/dc_lv_nmos.sch)) working in your IIC-OSIC-TOOLS environment.
2. Make yourself familiar with Xschem (change the schematic in various ways, run a simulation, graph the result).
3. Make yourself familiar with ngspice (run various simulations, save nets and parameters, use the embedded Xschem graphing, explore the interactive ngspice shell to look at MOSFET model parameters).
4. Explore the LV NMOS `sg13_lv_nmos`:
  1. How is  $I_{DS}$  affected by  $V_{GS}$  and  $V_{DS}$ ?
  2. Change  $W$  and  $L$  of the MOSFET. What is the impact on the above parameters? Can you explain the variations?
  3. When looking at the model parameters in ngspice, you see that there is a  $C_{GD}$  and a  $C_{DG}$ . Why is this, what could be the difference? Sometimes these capacitors show a negative value, why?
5. Build testbenches in Xschem for the LV PMOS, the HV NMOS, and the HV PMOS. Explore the different results.
  1. For a given  $W$  and  $L$ , which device provides more drain current? How are the capacitances related?
  2. If you would have to size an inverter, what would be the ideal ratio of  $W_p/W_n$ ? Will you exactly design this ratio, or are the reasons to deviate?
  3. There are LV and HV MOSFETs, and you investigated the difference in performance. What is the rationale when designing circuits for selection either an LV type, and when to choose an HV type?
6. Build a test bench to explore the body effect, start with LV NMOS.
  1. What happens when  $V_{BS} \neq 0$ ?

### Small-Signal MOSFET Model

As you have seen in the previous investigations, the large-signal model of Figure 3 describes the behaviour of the MOSFET across a wide range of voltages applied at the MOSFET terminals. Unfortunately, for hand analysis dealing with a nonlinear model is close to impossible, at the very least it is quite tedious.

However, for many practical situations, we bias a MOSFET with a set of dc voltages applied to its terminal, and only apply small signal excursions during operation. If we do this, we can linearize the large-signal model in this dc operating point, and resort to a small-signal model which can be very useful for hand calculations. Many experienced designers analyze their circuits by doing these kind of hand calculations and describing the circuit analytically, which



is a great way to understand fundamental performance limits and relationships between parameters.

We will use the small-signal MOSFET model shown in Figure 5 for this course. The current-source  $i_{ds} = g_m v_{gs}$  models the drain current as a function of  $v_{gs}$ , and the resistor  $g_{ds}$  models the dependency of the drain current by  $v_{ds}$ . The drain current dependency on the source-bulk voltage (the so-called “body effect”) is introduced by the current source  $i_{ds} = g_{mb} v_{sb}$ .

Source: [Article Notebook](#)

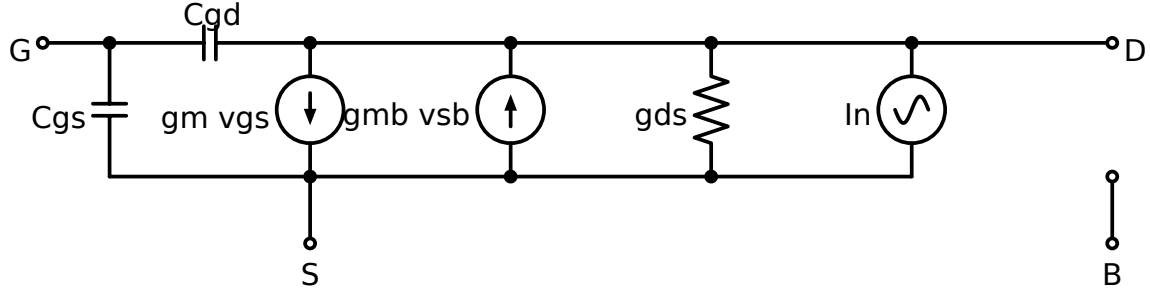


Figure 5: The MOSFET small-signal model.

Source: [Article Notebook](#)

As any electronic device the MOSFET introduces noise into the circuit. In this course we will only consider the drain-source current noise of the MOSFET, given by

$$\overline{I_n^2} = 4kT\gamma g_{d0}, \quad (1)$$

where  $\overline{I_n^2}$  is the power-spectral density of the noise in  $A^2/Hz$ ;  $k$  is the Boltzmann constant;  $T$  is the absolute temperature;  $\gamma$  is a parameter in simplified theory changing between  $\gamma = 2/3$  in saturation and  $\gamma = 1$  for triode operation;  $g_{d0}$  is equal to  $g_m$  in saturation and  $g_{ds}$  in triode).

#### **i** Note

Sometimes we will refer to different operating modes of the MOSFET like “saturation” or “triode”. Generally speaking, when the drain-source voltage is small, then the MOSFET acts as a resistor, and this mode of operation we call “triode” mode. When the drain-source voltage is increased, at some point the drain-source current saturates and is no longer a strong function of the drain-source voltage. This mode is called “saturation” mode. As you can see in the large-signal investigations, these transitions happen gradually, and it is difficult to define a precise point where one operating mode switches to the other one. In this sense we use terms like “triode” and “saturation” only in an approximative sense.

Now we need to see how the small-signal parameters seen in Figure 5 can be investigated and estimated using circuit simulation.

### Exercise

Please try to execute the following steps and answer the following questions:

1. Reuse the LV NMOS testbench (available at [https://github.com/iic-jku/analog-circuit-design/blob/main/xschem/dc\\_lv\\_nmos.sch](https://github.com/iic-jku/analog-circuit-design/blob/main/xschem/dc_lv_nmos.sch)).
2. Explore the LV NMOS `sg13_lv_nmos`:
  1. How are  $g_m$  and  $g_{ds}$  changing when you change the dc node voltages?
  2. What is the ratio of  $g_m$  to  $g_{mb}$ ? What is the physical reason behind this ratio (you might want to revisit MOSFET device physics at this point)?
  3. Take a look at the device capacitances  $C_{gs}$  and  $C_{gd}$ . Why are they important? What is the relation to  $f_T$ ? *Note:  $f_T$  is the transit frequency where the current gain of the MOSFET drops to 1, and can be approximated by  $2\pi f_T = g_m/C_{gg}$ .*
  4. Look at the drain noise current according to the MOSFET model and compare with a hand calculation of the noise. In the noise equation there is the factor  $\gamma$ , which in triode is  $\gamma = 1$  and in saturation is  $\gamma = 2/3$  according to basic text books. Which value of  $\gamma$  are you calculating? Why might it be different?
3. Go back to your testbench for the LVS PMOS `sg13_lv_pmos`:
  1. What is the difference in  $g_m$ ,  $g_{ds}$ , and other parameters between the NMOS and the PMOS? Why could they be different?

## Conclusion

Congratulations for making it thus far! By now you should have a solid grasp of the tool handling of Xschem and ngspice, and you should be familiar with the large- and small-signal operation of both NMOS and PMOS, and the parameters describing these behaviours. If you feel you are not sufficiently fluent in these things, please go back to the beginning of Section and revisit the relevant sections, or dive into further reading about the MOSFET operation, like in (Hu 2010).

## Transistor Sizing Using $g_m/ID$ Methodology

When designing integrated circuits it is an important question how to select various parameters of a MOSFET, like  $W$ ,  $L$ , or the bias current  $I_D$ . In comparison to using discrete components in PCB design, or also compared to a bipolar junction transistor (BJT), we have these degrees of freedom, which make integrated circuit design so interesting.

Often, transistor sizing in entry-level courses is based on the square-law model, where a simple analytical equation for the drain current can be derived. However, in nanometer CMOS, the MOSFET behaviour is much more complex than these simple models. Also, this

highly simplified derivations introduce concepts like the threshold voltage or the overdrive voltage, which are interesting from a theoretical viewpoint, but bear little practical use.

#### **i** Note

One of the many simplifications of the square-law model is that the mobility of the charge carriers is assumed constant (it is not). Further, the existence of a threshold voltage is assumed, but in fact this voltage is just existing given a certain definition, and depending on definition, its value changed. In addition, in nm CMOS, the threshold voltage is a function on many thing, like  $W$  and  $L$ .

An additional shortcoming of the square-law model is that it is only valid in strong inversion, i.e. for large  $V_{GS}$  where the drain current is dominated by the drift current. As soon as the gate-source voltage gets smaller, the square-law model breaks, as the drain current component based on diffusion currents gets dominant. Modern compact MOSFET models (like the PSP model used in SG13G2) use hundreds of parameters and fairly complex equations to somewhat properly describe MOSFET behaviour over a wide range of parameters like  $W$ ,  $L$ , and temperature. A modern approach to MOSFET sizing is thus based on the thought to use exactly these MOSFET models, characterize them, put the resulting data into tables and charts, and thus learn about the complex MOSFET behaviour and use it for MOSFET sizing.

Being a well-established approach we select the  $g_m/I_D$  methodology introduced by P. Jespers and B. Murmann in (Jespers and Murmann 2017). A brief introduction is available [here](#) as well.

The  $g_m/I_D$  methodology has the huge advantage that it catches MOSFET behavior quite accurately over a wide range of operating conditions, and the curves look very similar for pretty much all CMOS technologies, from micrometer bulk CMOS down to nanometer FinFET devices. Of course the absolute values change, but the method applies universally.

## **MOSFET Characterization Testbench**

In order to get the required tabulated data we use a testbench in Xschem which sweeps the terminal voltages, and records various large- and small-signal parameters, which are then stored in large tables. The testbench for the LV NMOS is shown in Figure 6, and the TB for the LV PMOS is shown in Figure 7.

We will use Jupyter notebooks to inspect the resulting data, and interpret some important graphs. This will greatly help to understand the MOSFET behaviour.

## **NMOS Characterization**

First, we will start looking at the LV NMOS. In Section we have the corresponding graphs for the LV PMOS. In this lecture, we will only use the LV MOSFETs. While there are also the HV types available, they are mainly used for high-voltage circuits, like circuits

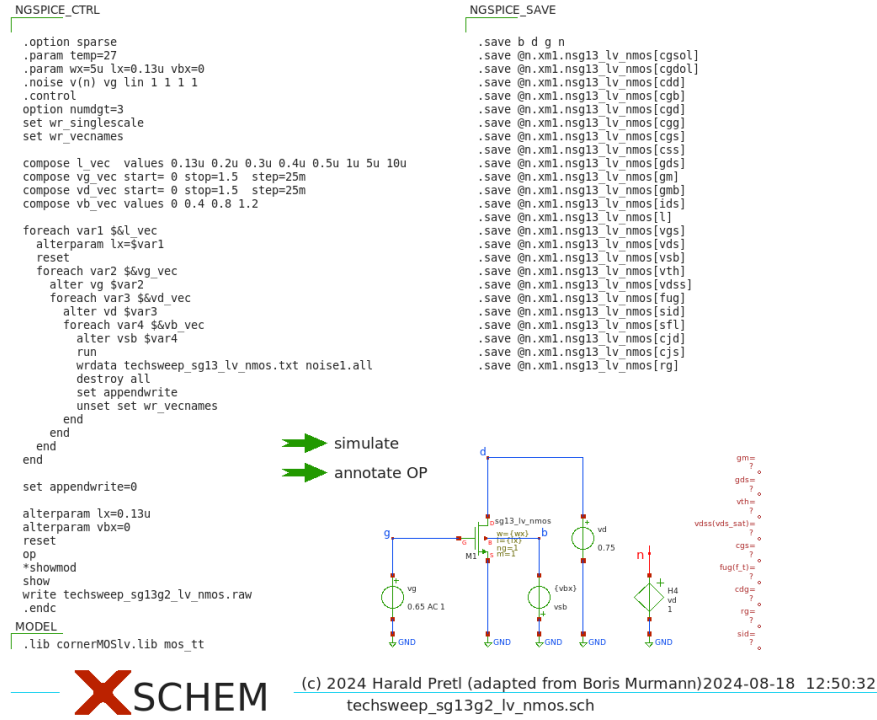


Figure 6: Testbench for LV NMOS  $g_m/I_D$  characterization.

connecting to the outside world. Here, we only will design low-voltage circuits running at a nominal supply voltage of 1.5 V, so only the LV types are of interest to us.

The first import graph is the plot of  $g_m/I_D$  and  $f_T$  versus the gate-source voltage  $V_{GS}$ . First let us answer the question why  $g_m/I_D$  is a good parameter to look at, and actually this is also the central parameter in the  $g_m/I_D$  methodology. In many circuits that are biased in class-A (i.e., with a constant quiescent current that is larger than the largest signal excursion, see [biasing](#)) we want to get a large amplification from a MOSFET, which corresponds to a large  $g_m$ . We want this by spending the minimum biasing current possible (ideally zero), as we always design for minimum power consumption. Thus, a high  $g_m/I_D$  ratio is good.

#### Note

Designing for minimum power consumption is pretty much always mandated. For battery-operated equipment it is a paramount requirement, but also in other equipment electrical energy consumption is a concern, and often severely limited by the cooling capabilities of the electrical system.

However, as can be seen in the below plot, there exists a strong and unfortunate trade-off with device speed, characterized here by the transit frequency  $f_T$ . It would be ideal if there exists a design point where we get high transconductance per bias current concurrently to having the fastest operation, but unfortunately, this is clearly not the case. The  $g_m/I_D$  peaks for  $V_{GS} < 0.3$  V, and the highest speed we get at  $V_{GS} \approx 1.2$  V. The dashed vertical line plots

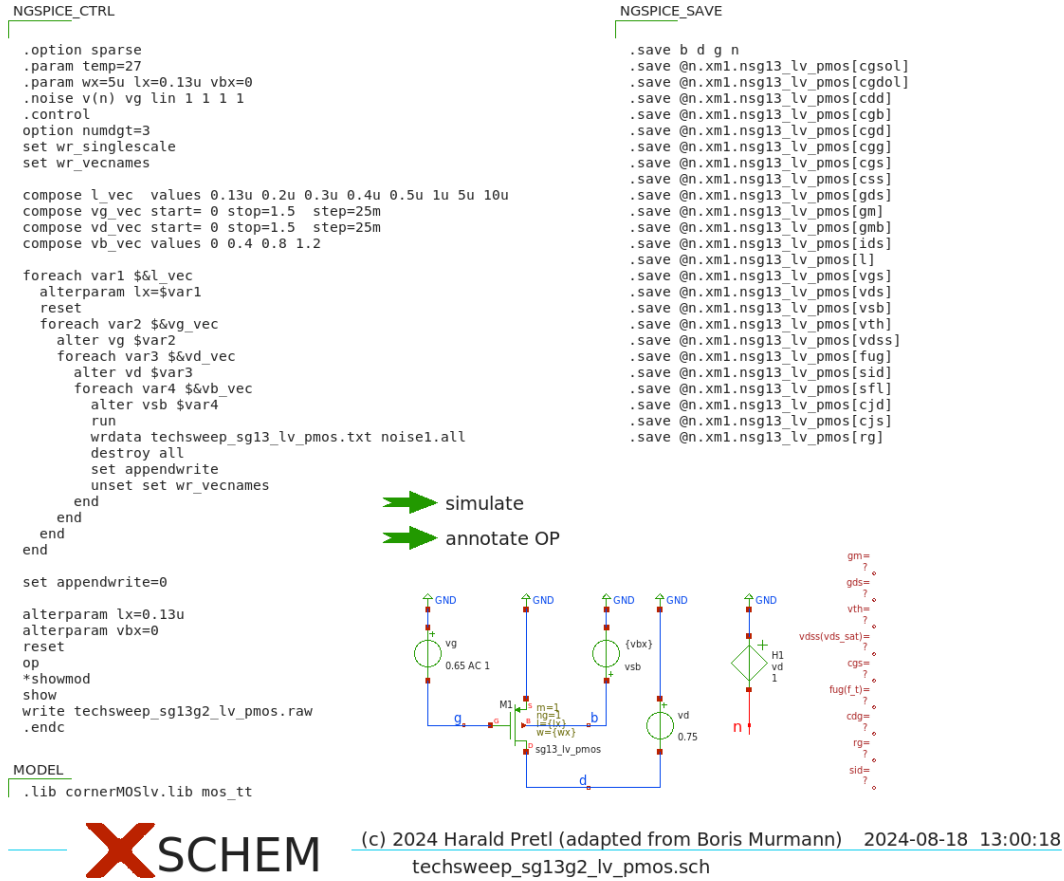


Figure 7: Testbench for LV PMOS  $g_m/I_D$  characterization.

the nominal threshold voltage, as you can see in this continuum of parameter space, it marks not a particularly special point.

Note that

$$\frac{g_m}{I_D} = \frac{1}{nV_T} \quad (2)$$

for a MOSFET in weak inversion (i.e., small gate-source voltage).  $n$  is the subthreshold slope, and  $V_T = kT/q$  which is 25.8 mV at 300 K. We thus have  $n \approx 1.38$  for this LV NMOS, which falls nicely into the usual range for  $n$  of 1.3 to 1.5 for bulk CMOS (FinFET have  $n$  very close to 1).

For the classical square-law model of the MOSFET in strong inversion,  $g_m/I_D$  is given as

$$\frac{g_m}{I_D} = \frac{2}{V_{GS} - V_{th}} = \frac{2}{V_{od}} \quad (3)$$

with  $V_{th}$  the threshold voltage and  $V_{od}$  the so-called “overdrive voltage.”

#### **i** Note

Why are we so often using 300 K for a typical condition? As this corresponds to roughly 27°C, this accounts for some self heating compared to otherwise usual room temperatures. Further, engineers like round numbers which are easy to remember, so 300 K is used as a proxy for room temperature.

As we can also see from belows plot, the peak transit frequency of the LV NMOS is about 75 GHz, which allows building radio-frequency circuits up to ca.  $f_T/10 = 7.5$  GHz, which is a respectable number. It is no coincidence, that the transition for RF design in the GHz-range switched from BJT-based technologies to CMOS roughly in the timeframe when 130nm CMOS became available (ca. 2000).



Source: [Article Notebook](#)

The following figure plots  $f_T$  against  $g_m/I_D$  for several different  $L$ . As you can see, device speeds maximizes for a low  $g_m/I_D$  and a short  $L$ . As you can see the drain-source voltage is kept at  $V_{DS} = 0.75 \text{ V} = V_{DD}/2$ , which is a typical value keeping the MOSFET in saturation across the characterization sweeps. Further, the source-bulk voltage is kept at  $V_{SB} = 0 \text{ V}$ , which means bulk and source terminals are connected.



Source: [Article Notebook](#)

The next plot shows the ratio of  $g_m/g_{ds}$  versus  $g_m/I_D$ . The ratio  $g_m/g_{ds}$  is the so-called “**self-gain**” of the MOSFET, and shows the maximum voltage gain we can achieve in a single transistor configuration. As one can see the self gain increases for increasing  $L$ , but this also gives a slower transistor, so again there is a trade-off. This plot allows us to select the proper  $L$  of a MOSFET if we know which amount of self gain we need.





Source: [Article Notebook](#)

The following figure plots the drain current density  $I_D/W$  as a function of  $g_m/I_D$  and  $L$ . With this plot we can find out how to set the  $W$  of a MOSFET once we know the biasing current  $I_D$ , the  $L$  (selected according to self gain,  $f_T$ , and other considerations) and the  $g_m/I_D$  design point we selected. The drain current density  $I_D/W$  is a very useful nomalized metric to use, because the physical action in the MOSFET establishes a charge density in the channel below the gate, and the changing of the  $W$  of the device merely transforms this charge density into an absolute parameter (together with  $L$ ).



Source: [Article Notebook](#)

The following plot shows the minimum drain-source voltage  $V_{ds,sat}$  that we need to establish in order to keep the MOSFET in saturation. As you can see, this value is almost independent of  $L$ , and increases for small  $g_m/I_D$ . So for low-voltage circuits, where headroom is precious, we tend to bias at  $g_m/I_D \geq 10$ , whereas for fast circuits we need to go to small  $g_m/I_D \leq 5$  requiring substantial voltage headroom per MOSFET stage that we stack on top of each other.



Source: [Article Notebook](#)

For analog circuits the noise performance is usually quite important. Thermal noise of a resistor (the Johnson-Nyquist noise) has a flat power-spectral density (PSD) given by  $\overline{V_n^2}/\Delta f = 4kTR$ , where  $k$  is Boltzmann's constant,  $T$  absolute temperature, and  $R$  the value of the resistor (the unit of  $\overline{V_n^2}/\Delta f$  is  $V^2/Hz$ ). This PSD is essentially flat until very high frequencies where [quantum effects](#) start to kick in.

#### **i** Note

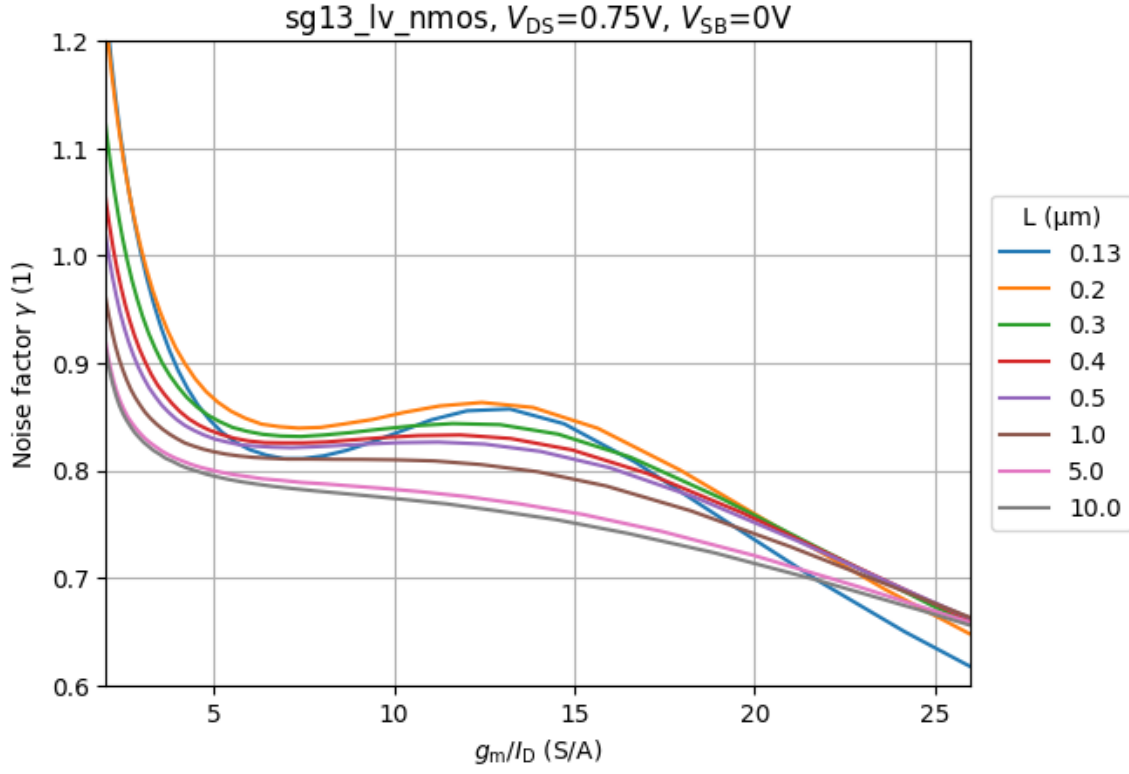
We usually leave the  $\Delta f$  away for a shorter notation, so we write  $\overline{V_n^2}$  when we actually mean  $\overline{V_n^2}/\Delta f$ .

Please also note that the pair of  $kT$  pretty much always shows up together, so when you do a calculation and you miss the one or the other, that is often a sign for miscalculation. Further, when working with PSD there is the usage of a one-sided ( $f$  runs from 0 to  $\infty$ ) or two-sided PSD ( $f$  runs from  $-\infty$  to  $\infty$ ). The default in this lecture is the usage of a **one-sided PSD**.

In this lecture the only MOSFET noise we consider is the drain noise (as discussed in Section ), showing up as a current noise between drain and source. For a for realistic MOSFET noise model, also a (correlated) gate noise component and the thermal noise of the gate resistance needs to be considered.

The factor  $\gamma$  (Equation 1) is a function of many things (in classical theory,  $\gamma = 2/3$  in

saturation and  $\gamma = 1$  in triode), and it is characterized in the following plot as a function of  $g_m/I_D$  and  $L$ . So when calculating MOSFET noise we can lookup  $\gamma$  in the below plot, and use Equation 1 to calculate the effective drain current noise.



Source: [Article Notebook](#)

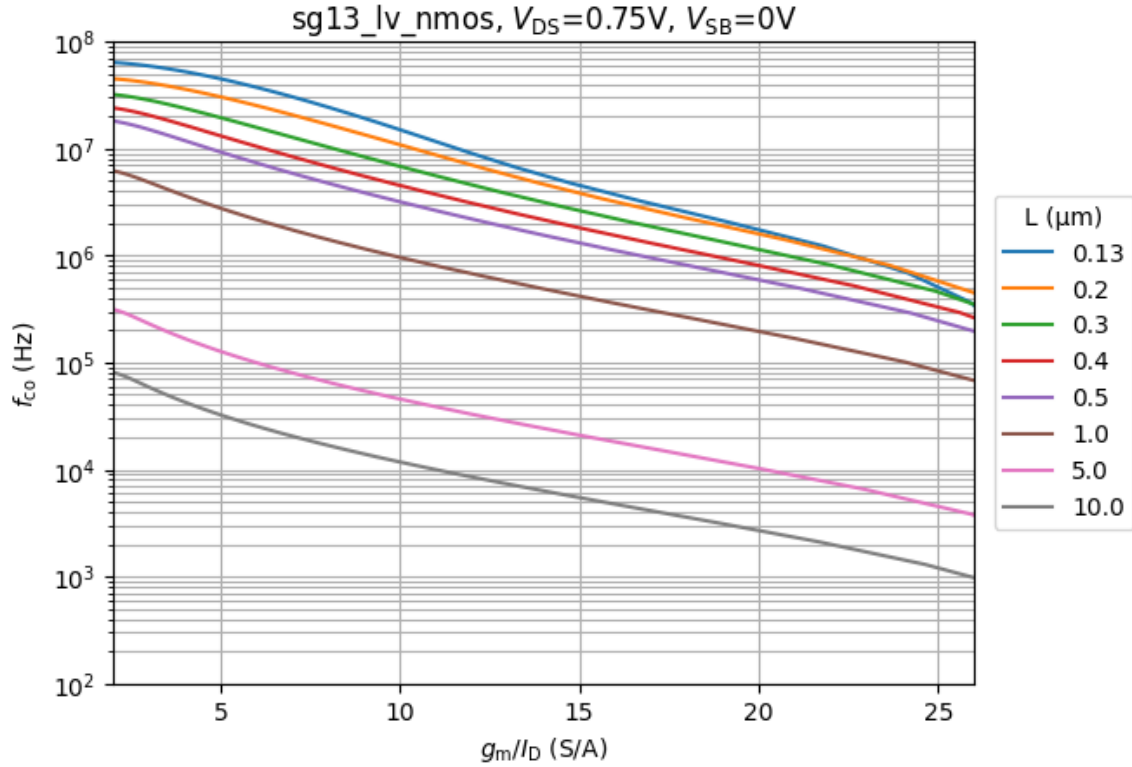
In a MOSFET, unfortunately, besides the thermal noise according to Equation 1, there is also a substantial low-frequency excess noise, called “flicker noise” due to its characteristic  $\overline{I_{d,nf}^2} = K_f/f$  behaviour (this means that this noise PSD decreases versus frequency). In order to characterize this flicker noise the following plot shows the cross-over frequency  $f_{co}$ , where the flicker noise is as large as the thermal noise. As can be seen in the below plot, this frequency is a strong function of  $L$  and  $g_m/I_D$ . Generally, the flicker noise is proportional to  $(WL)^{-1}$ , so the larger the device is, the lower the flicker noise. The parameter  $g_m/I_D$  largely stays constant when we keep  $W/L$  constant, so for a given  $g_m/I_D$  flicker noise is proportional to  $1/L^2$ . However, increasing  $L$  lowers device speed dramatically, so here we have a trade-off between flicker-noise performance and MOSFET speed, and this can have dramatic consequences for high-speed circuits.

#### **i** Note

The physical origin of flicker noise is the crystal interface between silicon (Si) and the silicon dioxide (SiO<sub>2</sub>). Since these are different materials, there are dangling bonds, which can capture charge carriers travelling in the channel. After a random time,

these carriers are released, and flicker noise is the result. The amount of flicker noise is a function of the manufacturing process, and will generally be different between device types and wafer foundries.

As you can see in the following plot,  $f_{co}$  can reach well into the 10's of MHz for short MOSFETs, significantly degrading the noise performance of a circuit.



Source: [Article Notebook](#)

## PMOS Characterization

In the following, we have the same plots as discussed in Section , but now for the PMOS.

### **i** Note

In all PMOS plots we plot positive values for voltages and currents, to have compatible plots to the NMOS. Of course, in a PMOS, voltages and currents have different polarity compared to the NMOS.

$g_m/I_D$  and  $f_T$  versus the gate-source voltage  $V_{GS}$ :



Source: [Article Notebook](#)

$f_T$  against  $g_m/I_D$  for several different  $L$ . One can see significantly lower top speed for the PMOS compared to the NMOS, which means for high-speed circuits the NMOS should be used.

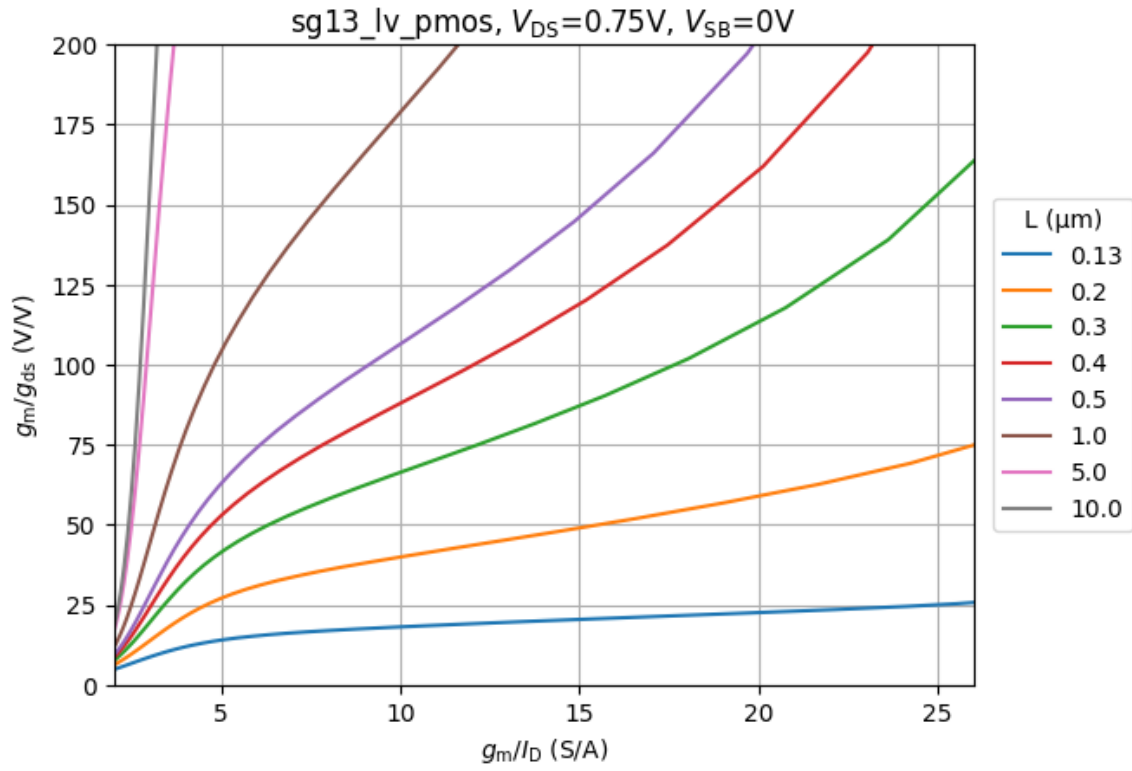


Source: [Article Notebook](#)

$g_m/g_{ds}$  versus  $g_m/I_D$ . Unfortunately, one can see a modelling error for the PMOS in this plot. The self gain  $g_m/g_{ds}$  reaches non-physical values, which indicates an issue with the  $g_{ds}$  modelling for the PMOS. We can not use these values for our circuit sizing, so we will use the respective NMOS plots also for the PMOS.

### ! Important

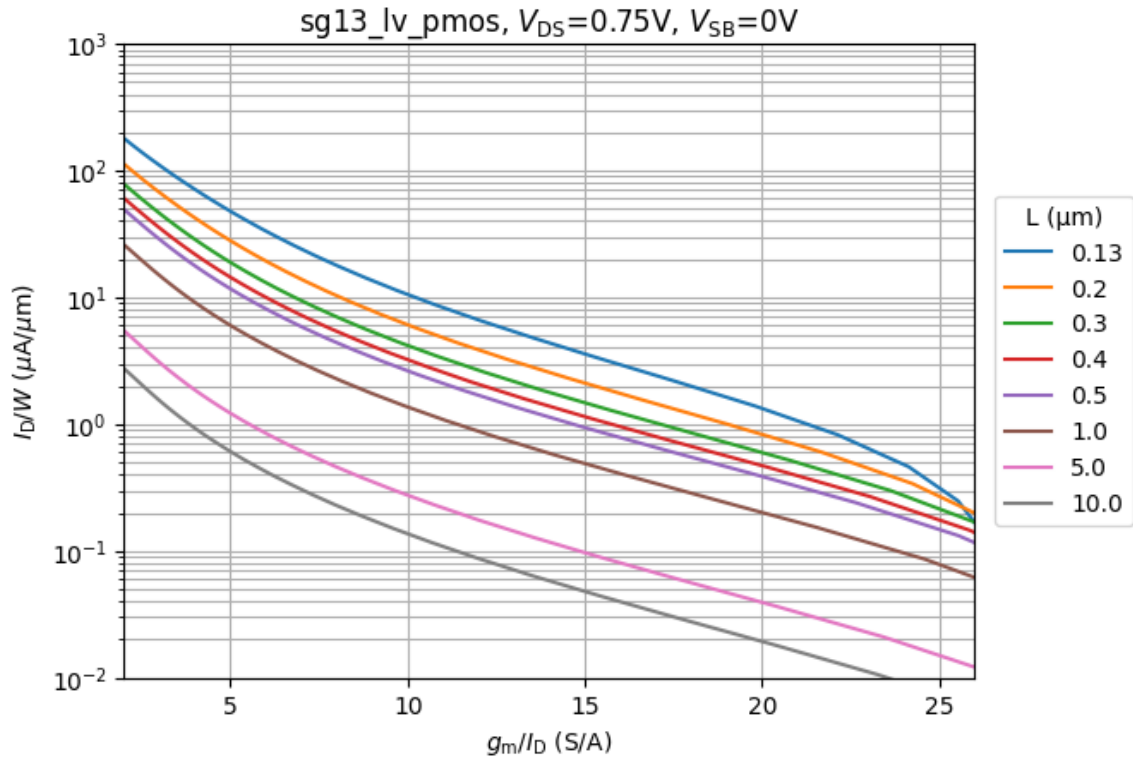
This example shows how important it is to benchmark the device models when starting to use a new technology. Modelling artifacts like the one shown are quite often happening, as setting up the device compact models and parametrizing them according to measurement data is a very complex task. In any case, just be aware that modelling issues could exist in whatever PDK you are using!



Source: [Article Notebook](#)

Drain current density  $I_D/W$  as a function of  $g_m/I_D$  and  $L$ :





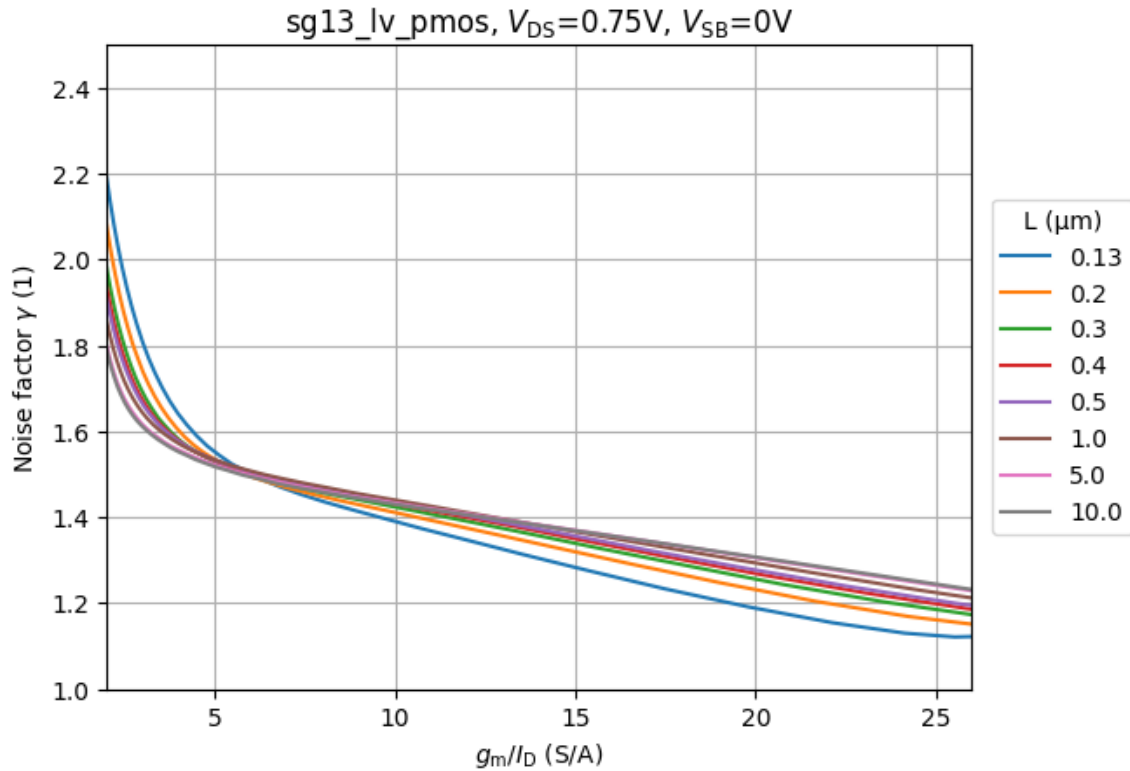
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Minimum drain-source voltage  $V_{ds,sat}$  versus  $g_m/I_D$  and  $L$ :



Source: [Article Notebook](#)

Noise factor  $\gamma$  versus  $g_m/I_D$  and  $L$ :



Source: [Article Notebook](#)

Flicker noise corner frequency  $f_{co}$  versus  $g_m/I_D$  and  $L$ . If you compare this figure carefully with the NMOS figure you can see that for some operating points the flicker noise for the PMOS is lower than for the NMOS. This is often true for CMOS technologies, so it can be an advantage to use a PMOS transistor in places where flicker noise is critical, like an OTA input stage. Using PMOS has the further advantage that the bulk node can be tied to source (which for NMOS is only possible in a triple-well technology, which is often not available), which gets rid of the [body effect](#).



Source: [Article Notebook](#)

## First Circuit: MOSFET Diode

The first (simple) circuit we will investigate is a MOSFET, where the gate is shorted with a drain, a so-called MOSFET “diode”, which is shown in Figure 8. This diode is one half of a current mirror, which we will investigate in a future section.

Source: [Article Notebook](#)

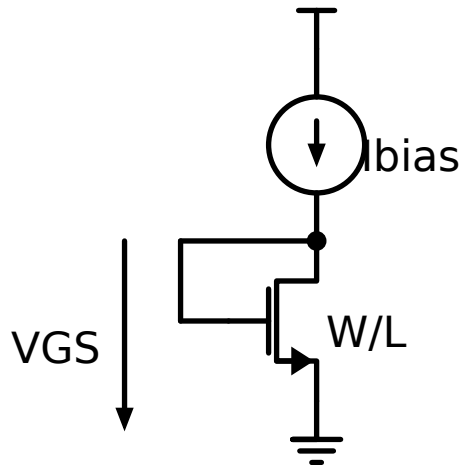


Figure 8: A MOSFET connected as a diode.

Source: [Article Notebook](#)

Why looking at a single-transistor circuit at all? By starting with the simplest possible circuit we can develop important skills in circuit analysis (setting up and calculating a small-signal model, calculating open-loop gain, calculate noise) and Xschem/ngspice simulation testbench creation. We safely assume that also the Mona Lisa was not Leonardo da Vinci's first painting, so let's start slow.

This diode is usually biased by a current source, shown as  $I_{\text{bias}}$  in the figure. Depending on MOSFET sizing with  $W$  and  $L$ , a certain gate-source voltage  $V_{\text{GS}}$  will develop. This voltage can be used as a biasing voltage for other circuit parts, for example.

#### **i** Note

It is important to realize that this configuration essentially employs a feedback loop for operation. The voltage at the drain of the MOSFET is sensed by the gate, and the gate voltage changes until the  $I_{\text{D}}$  is exactly equal to  $I_{\text{bias}}$ . In this sense this is probably the smallest feedback circuit one can build.

## MOSFET Diode Sizing

We will now build this circuit in Xschem. For sizing the MOSFET we will use the  $g_{\text{m}}/I_{\text{D}}$  methodology introduced in Section .

#### **💡** Exercise

Please build a MOSFET diode circuit in Xschem where you use an LV NMOS, set  $I_{\text{bias}} = 20 \mu\text{A}$ ,  $L = 0.13 \mu\text{m}$ , and we want to use  $g_{\text{m}}/I_{\text{D}} = 10$  (often a suitable compromise between transistor speed and  $g_{\text{m}}$  efficiency).

1. Use the figures in Section to find out the proper value for  $W$ .
2. What is  $f_T$  for this MOSFET? What is the value for  $g_m$  and  $g_{ds}$ ?
3. Draw the circuit in Xschem, and simulate the operating point. Do the values match to the values found out before during circuit sizing?

Before continuing, please finish the previous exercise. Once you are done, compare with the below provided solution.

#### Solution

1. Using the fact that  $I_{\text{bias}} = I_D = 20 \mu\text{A}$  and  $g_m/I_D = 10$  directly provides  $g_m = 0.2 \text{ mS}$ .
2. Using the self-gain plot, we see that  $g_m/g_{ds} \approx 21$ , so  $g_{ds} \approx 9.5 \mu\text{S}$ . The  $f_T$  can easily be found in the respective plot to be  $f_T = 23 \text{ GHz}$ .
3. The  $W$  of the MOSFET we find using the drain current density plot and the given bias current. Rounding to half-microns results in  $W = 1 \mu\text{m}$ .
4. Since we are looking at the graphs, we further find  $\gamma = 0.84$ ,  $V_{\text{ds,sat}} = 0.18 \text{ V}$ , and  $f_{\text{co}} \approx 15 \text{ MHz}$ .
5. In addition, we expect  $V_{\text{GS}} \approx 0.6 \text{ V}$ .

An example Jupyter notebook to extract these values accurately you can find [here](#). An Xschem schematic for this exercise is provide [as well](#).

## MOSFET Diode Large-Signal Behaviour

As discussed above, the MOSFET diode configuration is essentially a feedback loop. Before we will analyse this loop in small-signal, we want to investigate how this loop settles in the time domain, and by doing this we can observe the large-signal settling behaviour. To simulate this, we change the dc bias source from the previous example to a transient current source, which we will turn on after some ns. The resulting Xschem testbench is shown in Figure 9.

In Figure 9 another interesting effect can be observed: While the turn-on happens quite rapidly (essentially the bias current source charges the gate capacitance, until the gate-source voltage is large enough that the drain current counteracts the bias current), the turn-off shows a very long settling tail. This is due to the fact that as the gate capacitance is discharged by the drain current the  $V_{\text{GS}}$  drops, which in turn reduces the drain current, which will make the discharge even slower. We have an effect similar to the capacitor discharge by a diode (Hellen 2003).

It is thus generally a good idea to add power-down switches to the circuits to disable the circuit quickly by pulling floating nodes to a defined potential (usually  $V_{\text{DD}}$  or  $V_{\text{SS}}$ ) and to avoid long intermediate states during power down. This will also allow a turn-on from a well-defined off-state.

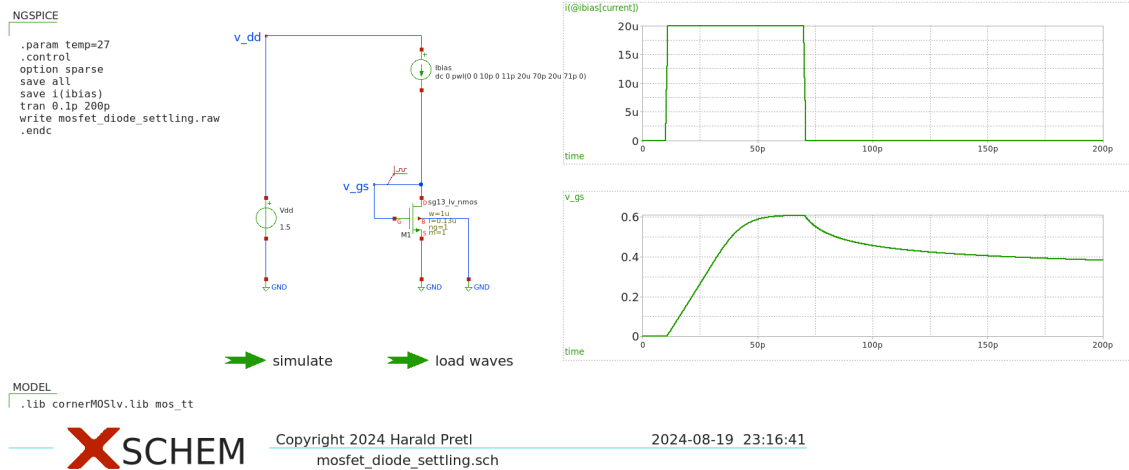


Figure 9: Testbench for MOSFET diode transient settling.

## MOSFET Diode Small-Signal Analysis

We now want to investigate the small-signal behaviour of the MOSFET diode. Based on the small-signal model of the MOSFET in Figure 5 we realize that gate and drain are shorted, and we also connect bulk to source. We can thus simplify the circuit to the one shown in Figure 10.

Source: [Article Notebook](#)

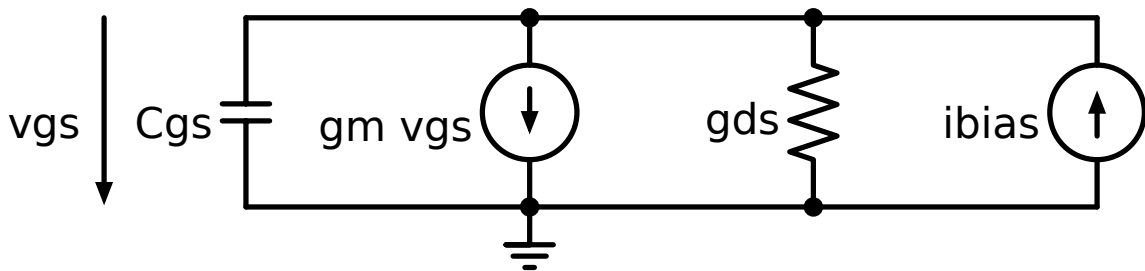


Figure 10: The MOSFET diode small-signal model.

Source: [Article Notebook](#)

### **i** Note

For small-signal analysis we would not need to declare one node as the ground potential. However, when doing so, and selecting the ground node strategically, we can simplify the analysis, as we usually do not formulate KCL for the ground node (as we have only  $N - 1$  independent KCL equations,  $N$  being the number of nodes of a circuit), and the potential difference equations are simpler if one node is at  $0\text{ V}$ .

For calculating the small-signal impedance of the MOSFET diode we formulate KCL at the top node to get

$$i_{\text{bias}} - sC_{\text{gs}}v_{\text{gs}} - g_{\text{m}}v_{\text{gs}} - g_{\text{ds}}v_{\text{gs}} = 0.$$

It follows that

$$Z_{\text{diode}}(s) = \frac{v_{\text{gs}}}{i_{\text{bias}}} = \frac{1}{g_{\text{m}} + g_{\text{ds}} + sC_{\text{gs}}}. \quad (4)$$

When neglecting  $g_{\text{ds}}$  and at dc we get  $Z_{\text{diode}} = 1/g_{\text{m}}$ , which is an important result and should be memorized.

### **!** Important

In circuit analysis it is often algebraically easier to work with conductances instead of impedances, so please remember that Ohm's law for a conductance is  $I = GV$ , and for a capacitance is  $I = sCV$ . When writing equations, it is also practical to keep  $sC$  together, so we will strive to sort terms accordingly.

Looking at Equation 4 we see that for low frequencies, the diode impedance is resistive, and for high frequencies it becomes capacitive as the gate-source capacitance starts to dominate. The corner frequency of this low-pass can be calculated as

$$\omega_{\text{c}} = \frac{g_{\text{m}} + g_{\text{ds}}}{C_{\text{gs}}} \approx \omega_{\text{T}}$$

which is pretty much the transit frequency of the MOSFET!

## **MOSFET Diode Stability Analysis**

The diode-connected MOSFET forms a feedback loop. What is the open-loop gain? For calculating it, we are breaking the loop, and apply a dummy  $C_{\text{gs}}^*$  at the right side to keep the impedances correct. A circuit diagram is shown in Figure 11, we break the loop at the dotted connection. As we can see in this example, it is critically important when breaking up a loop for analysis (also for simulation!) to keep the terminal impedances the same. Only in special cases where the load impedance is very high or the driving impedance is very low is it acceptable to disregard loading effects!

Source: [Article Notebook](#)



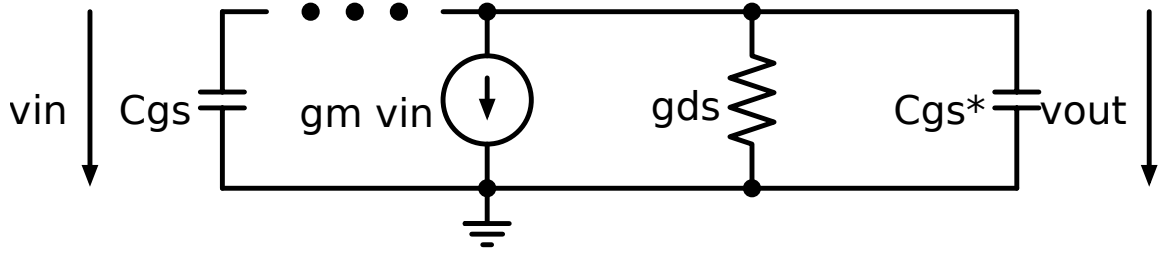


Figure 11: The MOSFET diode small-signal circuit for open-loop analysis.

Source: [Article Notebook](#)

By inspecting Figure 11 we see that

$$v_{\text{out}} = -g_m v_{\text{in}} \frac{1}{g_{\text{ds}} + sC_{\text{gs}}}.$$

The open-loop gain  $H_{\text{ol}}(s)$  is thus

$$H_{\text{ol}}(s) = \frac{v_{\text{out}}}{v_{\text{in}}} = -\frac{g_m}{g_{\text{ds}} + sC_{\text{gs}}}. \quad (5)$$

Inspecting Equation 5 we realize that

1. the dc gain  $g_m/g_{\text{ds}}$  is the self-gain of the MOSFET, so  $20 \log(0.2 \cdot 10^{-3}/9.6 \cdot 10^{-6}) = 26.4 \text{ dB}$ , and
2. there is a pole at  $\omega_p = -g_{\text{ds}}/C_{\text{gs}}$ , which is at  $9.6 \cdot 10^{-6}/(2\pi \cdot 1.4 \cdot 10^{-15}) = 1.1 \text{ GHz}$ .

With this single pole location in  $H_{\text{ol}}(s)$  this loop is perfectly stable at under all conditions.

The question is now how to simulate this open-loop gain, and how to break the loop open in simulation? In general there are various methods, as we can use artificially large (ideal) inductors and capacitors to break loops open and still establish the correct dc operating points for the ac loop analysis. However, mimicking the correct loading can be an issue, and requires a lot of careful consideration.

There is an alternative method which breaks the loop open only by adding an ac voltage source in series (thus keeps the dc operating point intact), or injects current using a current source. Based on both measurements the open-loop gain can be calculated. This is called **Middlebrook's method** (Middlebrook 1975) which is based on double injection, and we will use it for our loop simulations. This method is detailed in Section .

We now want to simulate the open-loop transfer function  $H_{\text{ol}}(s)$  by using Middlebrook's method and confirm our analysis above.

## Exercise

Please build a simulation testbench in Xschem to simulate the open-loop transfer function of the MOSFET diode. Confirm the dc gain and pole location as given by Equation 5.

If you are getting stuck you can look at this Xschem [testbench](#), shown in Figure 12.

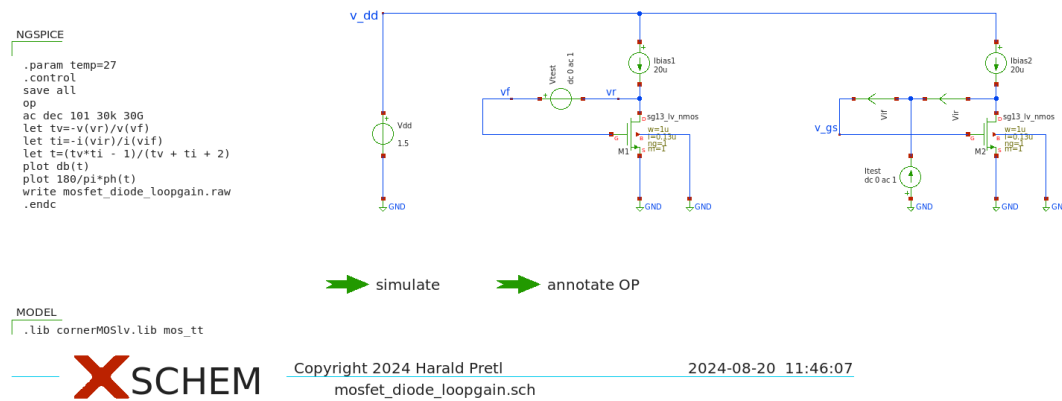


Figure 12: Testbench for MOSFET diode stability analysis.

From simulation we see that the open-loop gain is 24.9 dB at low frequencies, which matches quite well our prediction of 26.4 dB. In the Bode plot we see a low-pass with a  $-3$  dB corner frequency of 1.4 GHz, which again is fairly close to our prediction of 1.1 GHz.

## MOSFET Diode Noise Calculation

As a final exercise on the MOSFET diode circuit we want to calculate the output noise when we consider  $V_{GS}$  the output reference voltage which is created when passing a bias current through the MOSFET diode. The bias current we will assume noiseless.

We will use the small-signal circuit shown in Figure 13.

Source: [Article Notebook](#)

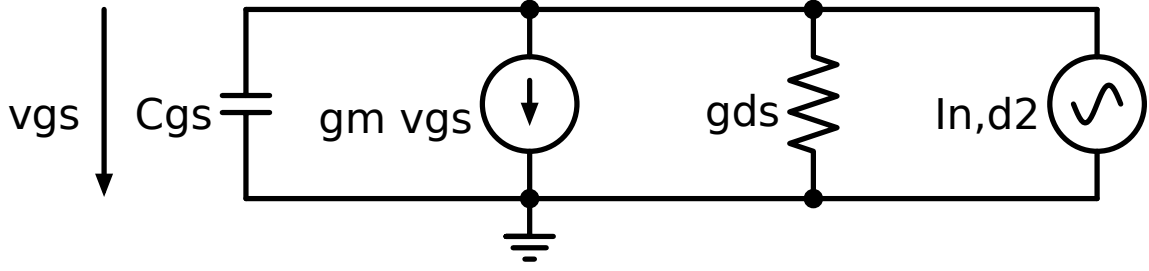


Figure 13: The MOSFET diode small-signal model with drain noise source.

Source: [Article Notebook](#)

As we have already calculated the small-signal diode impedance in Equation 4 we will use this result, and just note that the drain current noise of the MOSFET flows through this impedance. The noise voltage at  $v_{gs}$  is thus given as

$$\overline{V_n^2} = Z_{\text{diode}}^2 \overline{I_{n,d}^2}.$$

The drain current noise of the MOSFET is given as (introduced in Section )

$$\overline{I_{n,d}^2} = 4kT\gamma g_m.$$

For low frequencies (ignoring  $g_{ds}$  and  $C_{gs}$ ) we get

$$\overline{V_n^2} = Z_{\text{diode}}^2 \overline{I_{n,d}^2} = \frac{1}{g_m^2} 4kT\gamma g_m = \frac{4kT\gamma}{g_m}$$

which is the thermal noise of a resistor of value  $1/g_m$  enhanced by the factor  $\gamma$ .

We now calculate the full equation, and after a bit of algebra arrive at

$$\overline{V_n^2}(f) = \frac{4kT\gamma g_m}{(g_m + g_{ds})^2 + (2\pi f C_{gs})^2}. \quad (6)$$

If we are interested in the PSD of the noise then Equation 6 gives us the result. If we are interested in the rms value (the total noise) we need to integrate this equation, using the following identity:

**i** Useful Integral for Noise Calculations

$$\int_0^\infty \frac{a}{b^2 + c^2 f^2} df = \frac{\pi}{2} \frac{a}{b \cdot c} \quad (7)$$

Using the integral help in Equation 7, we can easily transform Equation 6 to

$$V_{n,\text{rms}}^2 = \int_0^\infty \overline{V_n^2}(f) df = \frac{kT\gamma g_m}{(g_m + g_{ds})C_{gs}}. \quad (8)$$

The form of Equation 8 is the exact solution, but we gain additional insight if we assume that  $g_m + g_{ds} \approx g_m$  and then

$$V_{n,\text{rms}}^2 = \frac{kT\gamma}{C_{gs}}. \quad (9)$$

Inspecting Equation 9 we see our familiar  $kT/C$  noise enhanced by the factor  $\gamma$ ! Calculating this value for our MOSFET diode we get  $\sqrt{V_{n,\text{rms}}^2} = \sqrt{1.38 \cdot 10^{-23} \cdot 300 \cdot 0.84 / 1.4 \cdot 10^{-15}} = 1.58 \text{ mV}$ , which is a sizeable value! We run circuits in this technology at  $V_{DD} = 1.5 \text{ V}$ , which leaves us with a signal swing of ca.  $1.1 \text{ V}_{pp}$ , resulting in a dynamic range in this case of  $20 \log(1.58 \cdot 10^{-3} / 0.39) \approx -48 \text{ dB}$ .

#### ! Important

Large BW circuits can integrate noise over a wide bandwidth resulting in considerable rms noise.

#### 💡 Exercise

Please build a simulation testbench in Xschem to simulate the noise performance of the MOSFET diode, and confirm the rms noise value that we just calculated. Look at the rms value and the PSD of the noise, and play around with the integration limits. What is the effect? Can you see the flicker noise in the PSD? How much is its contribution to the rms noise?

If you are getting stuck you can look at this Xschem [testbench](#), shown in Figure 14.

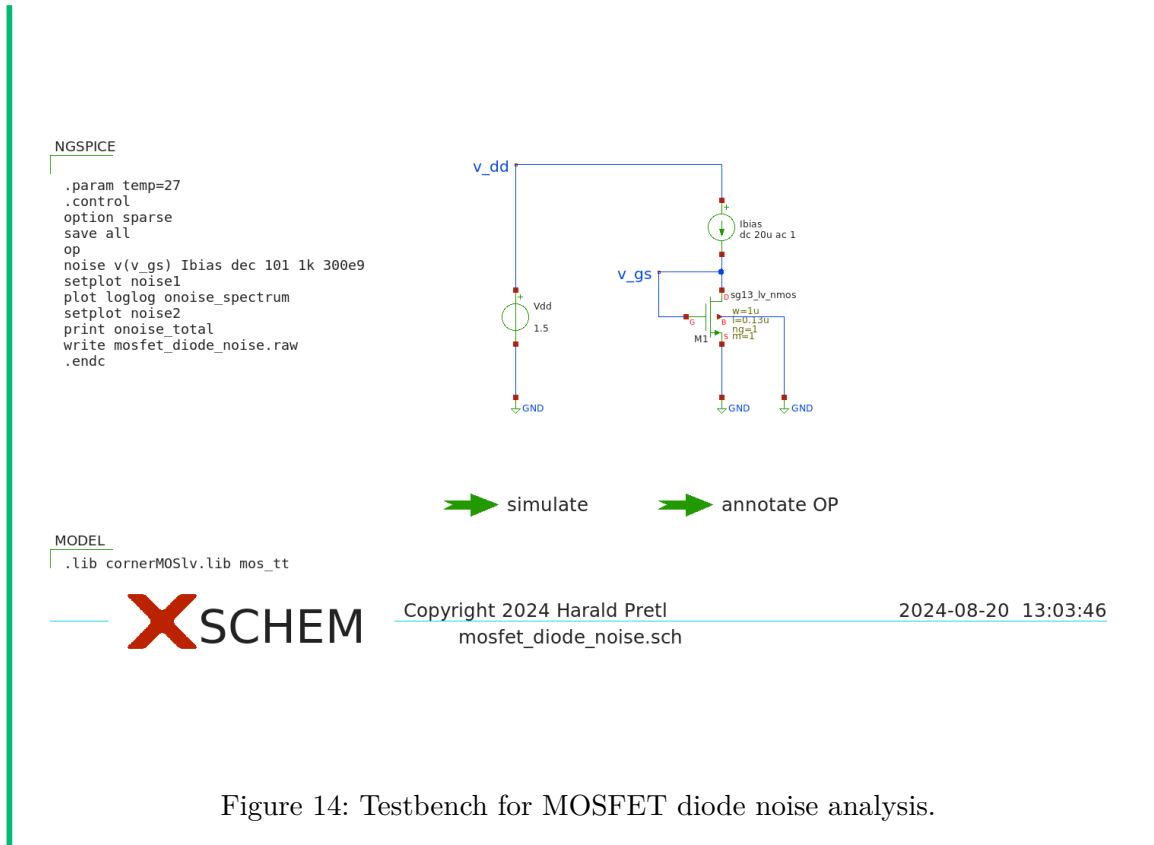


Figure 14: Testbench for MOSFET diode noise analysis.

## Conclusion

In this section we investigated the simple MOSFET-diode circuit. We learned important skills like how to derive a small-signal model, how to calculate important features like noise and open-loop gain for stability analysis. We introduced Middlebrook's method to have a mechanism to open up loops in simulation (and calculation) without disturbing operating points for change loading conditions.

If you feel that you have not yet mastered these topics or are uncertain in the operation of ngspice, please go back to the beginning of the section and read through the theory and redo the exercises.

## Current Mirror

In this section we will look into a fundamental building block which is often used in integrated circuit design, the **current mirror**. A diagram is shown in Figure 15 with one MOSFET diode converting the incoming bias current into a voltage, and two output MOSFETs working as current sources, which are biased from the diode. By properly selecting all  $W$  and  $L$  the input current can be scaled, and multiple copies can be created atn once. Shown in the figure are two output currents, but any number of parallel branches can be realized.

Source: [Article Notebook](#)

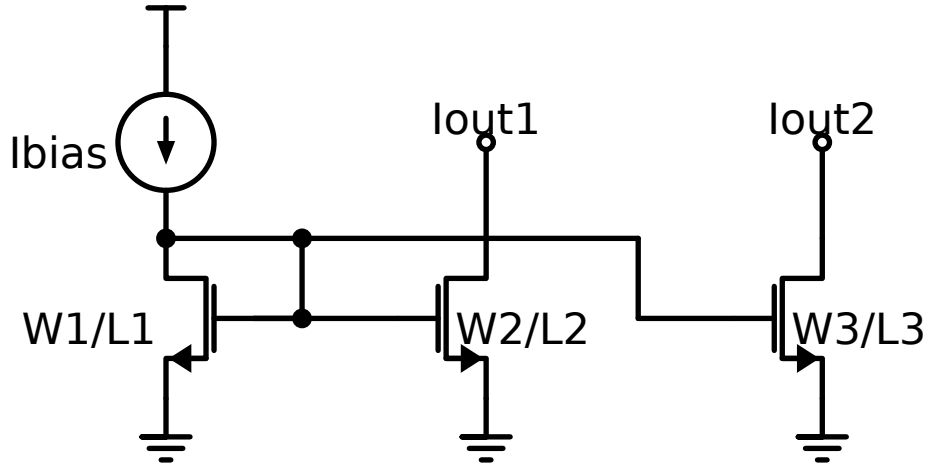


Figure 15: A current mirror with two output branches.

Source: [Article Notebook](#)

The output current  $I_{out1}$  is then given by

$$I_{out1} = I_{bias} \frac{W_2}{L_2} \frac{L_1}{W_1}$$

and the output current  $I_{out2}$  is given by

$$I_{out2} = I_{bias} \frac{W_3}{L_3} \frac{L_1}{W_1}.$$

For good matching in layout care has to be taken that the MOSFET widths and lengths are constructed out of **unit elements** of identical size, where an appropriate amount of these single units are then arranged in series or parallel configuration to arrive at the target  $W$  and  $L$ .

As we know from earlier investigations of the MOSFET performance in Section the drain current of a MOSFET is a function of  $V_{GS}$  and  $V_{DS}$ . As long as the MOSFET stays in saturation (i.e.,  $V_{DS} > V_{ds,dsat}$ ) the drain current is just a mild function of  $V_{DS}$  (essentially the effect of  $g_{ds}$ , which is the output conductance of the MOSFET). A fundamental flaw of the basic current mirror shown in Figure 15 is the mismatch of the  $V_{DS}$  of the MOSFET. The input-side diode has  $V_{GS} = V_{DS}$ , whereas the output current sources have a  $V_{DS}$  depending on the connected circuitry. Improved current mirrors exist (basically fixing this flaw), still, when just a simple current mirror is required this structure is used for its simplicity.

### 💡 Exercise

Please construct a current mirror based on the MOSFET-diode which we sized in Section . The input current  $I_{\text{bias}} = 20 \mu\text{A}$ , and we want three output currents of size  $10 \mu\text{A}$ ,  $20 \mu\text{A}$ , and  $40 \mu\text{A}$ .

Sweep the output voltage of all three current branches and see over which voltage range an acceptable current is created. For which output voltage range is the current departing from its ideal value, and why?

You see that the slope of the output current is quite bad, as  $g_{\text{ds}}$  is too large. We can improve this by changing the length to  $L = 5 \mu\text{m}$  (for motivation, please look at the graphs in Section ). In addition, for a current mirror we are not interested in a high  $g_{\text{m}}/I_{\text{D}}$  value, so we can use  $g_{\text{m}}/I_{\text{D}} = 5$  in this case. Please size the current mirror MOSFETs accordingly (please round the  $W$  to half micron, to keep sizes a bit more practical). Compare this result to the previous one, what changed?

In case you get stuck, here are Xschem schematics for the [original](#) and the [improved](#) current mirrors.

## Differential Pair

Like the current mirror in Section the **differential pair** is an ubiquitous building block often used in integrated circuit design. The fundamental structure is given in Figure 16.

Source: [Article Notebook](#)

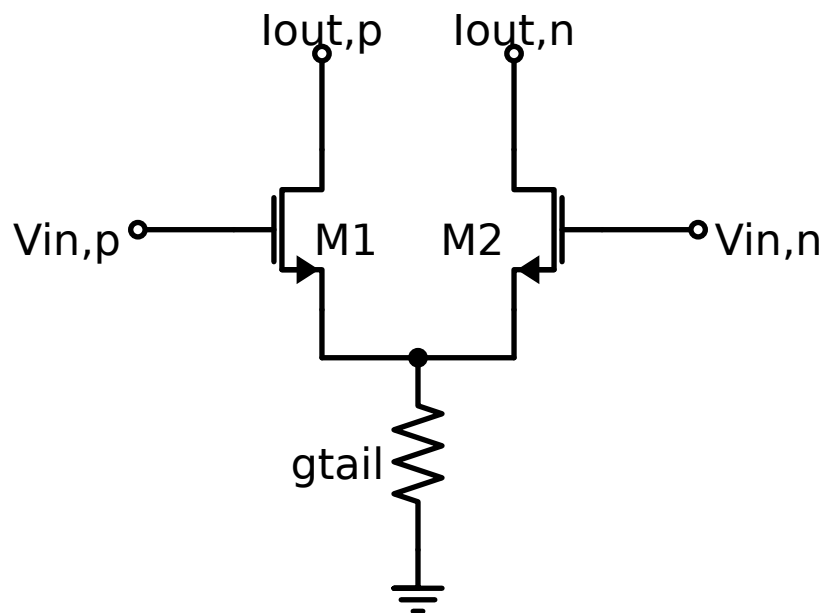


Figure 16: A differential pair.

Source: [Article Notebook](#)

In order to understand its operation it is instructive to separate the input condition into (1) a purely differential voltage, and (2) into a common-mode voltage, and see what the impact on the output currents is.

### Differential Operation of the Diffpair

For a differential mode of operation we assume that the input common mode voltage is constant, i.e.  $V_{in,p} + V_{in,n} = V_{CM}$ . A differential input voltage  $v_{in}$  then results in

$$V_{in,p} = V_{CM} + \frac{v_{in}}{2}$$

and

$$V_{in,n} = V_{CM} - \frac{v_{in}}{2}.$$

For a small-signal differential drive the potential at the tail point stays constant and we can treat it as a virtual ground. The output current on each side is then given by (neglecting  $g_{ds}$  and  $g_{mb}$  of  $M_1$  and  $M_2$ )

$$i_{out,p} = g_{m1} \left( \frac{v_{in}}{2} \right)$$

and

$$i_{out,n} = g_{m2} \left( -\frac{v_{in}}{2} \right).$$

Usually we assume symmetry in the differential pair, so  $g_{m1} = g_{m2} = g_m$ . The differential output current  $i_{out}$  is then given by

$$i_{out} = i_{out,p} - i_{out,n} = g_m v_{in} \quad (10)$$

We see in Equation 10 that the differential output current is simply the differential input voltage multiplied by the  $g_m$  of the individual transistor. We also note that the bottom conductance  $g_{tail}$  plays no role for the small-signal differential operation.

### Common-Mode Operation of the Diffpair

Usually, the source conductance  $g_{tail}$  is realized by a current source and ideally should be  $g_{tail} = 0$ . If this is the case, then the output currents are not a function of the common-mode input voltage, and ( $I_{tail}$  is set by the tail current source)

$$I_{out,p} = I_{out,n} = \frac{I_{tail}}{2}.$$

However, if we assume a realistic tail current source then  $g_{tail} > 0$ . For analysis we can simply look at a half circuit since everything is symmetric. In order to simplify the analysis a bit we remove all capacitors from the MOSFET small-signal model and set  $g_{ds} = g_{mb} = 0$ .



We arrive then at the small-signal equivalent circuit shown in Figure 17 (note that we set  $v_{in,p} = v_{in,n} = v_{in}$  and  $i_{out,p} = i_{out,n} = i_{out}$  under symmetry considerations).

Source: [Article Notebook](#)

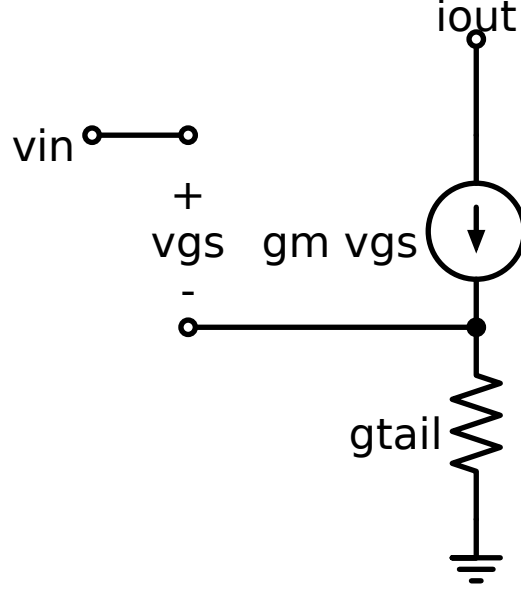


Figure 17: Small-signal model of the differential pair half-circuit in common-mode operation.

Source: [Article Notebook](#)

Formulating KVL for the input-side loop we get

$$v_{in} = v_{gs} + \frac{i_{ds}}{g_{tail}}.$$

With  $i_{out} = i_{ds} = g_m v_{gs}$  we arrive at

$$i_{out} = \frac{g_m g_{tail}}{g_m + g_{tail}} v_{in} \quad (11)$$

Interpreting Equation 11 we can distinguish the following extreme cases:

1. If  $g_{tail} = 0$  (ideal tail current source) then  $i_{out} = 0$ , the common-mode voltage variation from the input is suppressed and does not show up at the common-mode output current (which is constant due to the ideal tail current source). This is usually the case that we want to achieve.
2. If  $g_{tail} = \infty$  then  $i_{out} = g_m v_{in}$ , which means the output current is a function of the MOSFET  $g_m$ . If everything is perfectly matched, then the differential output current is zero, but the common-mode output current changes according to the common-mode input voltage. In special cases this can be a wanted behaviour, this configuration is called a “pseudo-differential pair.”

## A Basic 5-Transistor OTA

Suited with the knowledge of basic transistor operation (Section and Section ) and the working knowledge of the current mirror (Section and Section ) as well as the differential pair (Section ) we can now start to design our first real circuit. A fundamental (simple) circuit that is often used for basic tasks is the 5-transistor operational transconductance amplifier (OTA). A circuit diagram of this 5T-OTA is shown in Figure 18.

Source: [Article Notebook](#)

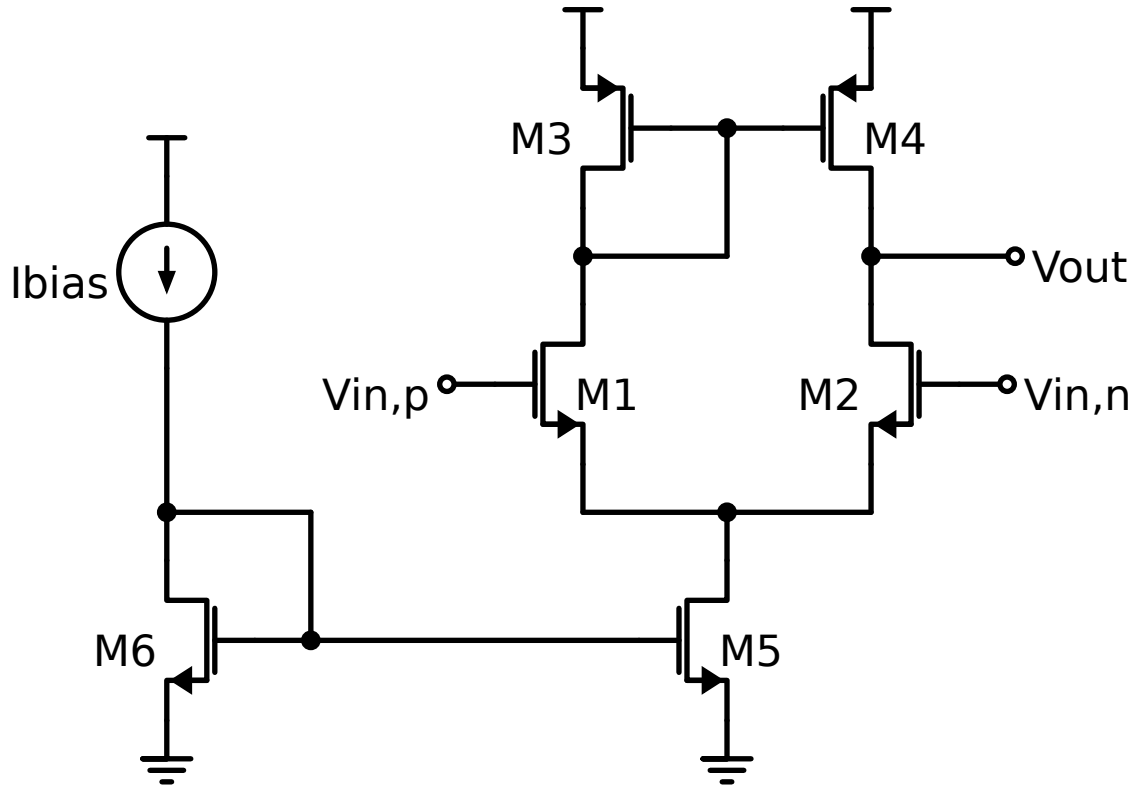


Figure 18: The 5-transistor OTA.

Source: [Article Notebook](#)

The operation is as follows:  $M_{1,2}$  form a differential pair which is biased by the current source  $M_5$ .  $M_{5,6}$  form a current mirror, thus the input bias current  $I_{bias}$  sets the bias current in the OTA. The differential pair  $M_{1,2}$  is loaded by the current mirror  $M_{3,4}$  which mirrors the output current of  $M_1$  to the right side. Here, the currents from  $M_4$  and  $M_2$  are summed, and together with the conductance effective at the output node a voltage builds up.

We note that  $M_{1,2}$  and  $M_{3,4}$  need to be symmetric, thus will have the same  $W&L$  dimensioning.  $M_{5,6}$  we scale accordingly to set the correct bias current in the OTA.

As this is an OTA the output is a current; if the load impedance is high (i.e., purely capacitive, which is often the case in integrated circuits when driving MOSFET inputs) then the voltage gain of the OTA can be high (of course, in this simple OTA it is limited). With a high-impedance loading this OTA can provide a voltage output, and this is actually how OTAs are mostly operated.

### Voltage Buffer with OTA

In order to design an OTA we need an application, and from this we need to derive the circuit specifications. We want to use this OTA to realize a voltage buffer which lightly loads a voltage source and can drive a large capacitive load. Such a configuration is often used to, e.g., buffer a reference voltage that is needed (and thus loaded) by another circuit. The block diagram of this configuration is shown in Figure 19.

Source: [Article Notebook](#)

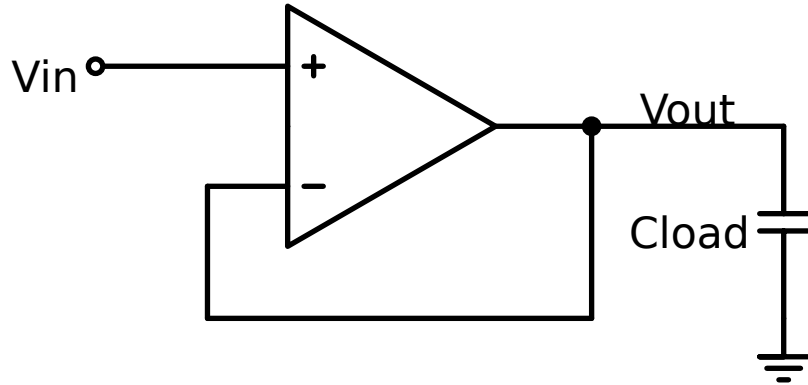


Figure 19: A voltage buffer (based on OTA) driving a capacitive load.

Source: [Article Notebook](#)

If the voltage gain of the OTA in Figure 19 is high, then  $V_{out} \approx V_{in}$ . We now want to design an OTA for this application for the following specification values (see Table 1).

Table 1: Voltage buffer specification

Specification	Value	Unit
Load capacitance $C_{load}$	50	fF
Input voltage range (for buffering bandgap voltage)	1.1—1.3	V
Signal bandwidth (3dB)	>1	MHz
Voltage gain error	<5	%
Total output noise (rms)	<1	mV <sub>rms</sub>
Supply voltage	1.45—1.5—1.55	V

Specification	Value	Unit
Supply current (as low as possible)	<100	$\mu\text{A}$
Stability	stable for rated $C_{\text{load}}$	
Turn-on time (settled to with 1%)	<10	$\mu\text{s}$
Externally provided bias current (nominal)	20	$\mu\text{A}$

## Large-Signal Analysis of the OTA

The first step when receiving a design task is to look at the specifications, and see whether they make sense. Detailed performance of the design will be the result of the circuit simulation, but before we step into sizing we need to do a few simple calculations to (a) allows to do back-of-the-envelope gauging if the specification makes sense, and (b) the derived analytical equations will serve as guide for the sizing procedure.

- In terms of large-signal operation, we will now check whether the input and output voltage range, as well as the settling time can be roughly met.
- When the input is at its maximum of 1.3 V, we see that we need to keep  $M_1$  in saturation. We can calculate that  $V_{\text{DS1}} = V_{\text{DD}} - |V_{\text{GS3}}| + V_{\text{GS1}} - V_{\text{in}} = 1.45 - 0.6 + 0.6 - 1.3 = 0.15 \text{ V}$ , which barely leaves enough margin, so we must keep the gate-source voltages of  $M_1$  and  $M_3$  reasonably small.
- When the input is at its minimum of 1.1 V, we see that the  $V_{\text{DS5}}$  of  $M_5$  is calculated as  $V_{\text{GS5}} = V_{\text{in}} - V_{\text{GS1}} = 1.1 - 0.6 = 0.5 \text{ V}$ , so this leaves plenty of margin.
- For the output voltage, when the output voltage is on the high side, it leaves  $|V_{\text{DS4}}| = V_{\text{DD}} - V_{\text{out}} = 1.45 - 1.3 = 0.15 \text{ V}$ , whis is on the edge.
- When the output voltage is on the low side, it leaves  $V_{\text{DS2}} = V_{\text{out}} - V_{\text{in}} + V_{\text{GS2}} = 1.3 - 1.55 + 0.6 = 0.35 \text{ V}$ , so this should work.

In summary, we think that we can make an NMOS-input OTA like the one in Figure 18 work for the required supply and input- and output voltages. If this would not work out, we need to look for further options, like a PMOS-input OTA, or a NMOS/PMOS-input OTA.

Another large-signal specification item that we can quickly check is the settling time. Under slewing conditions, we complete bias current in the OTA is steered towards the output (try to understand why this is the case), so when the output capacitor is fully discharged, and we assume just a linear ramp due to constant-current charging of the output capacitor, the settling time is

$$T_{\text{slew}} \approx \frac{C_{\text{load}} V_{\text{out}}}{I_{\text{tail}}} = \frac{50 \cdot 10^{-15} \cdot 1.3}{100 \cdot 10^{-6}} = 0.65 \text{ ns}$$

so this leaves plenty of margin for additional slow-signal settling due to the limited bandwidth, as well as reducing the supply current.

The small-signal settling (assuming one pole at the bandwidth corner frequency) leads to an approximate settling time (1% error corresponds to  $\approx 5\tau$ ) of

$$T_{\text{slew}} \approx \frac{5}{2\pi f_c} = \frac{5}{2\pi \cdot 1 \cdot 10^{-6}} = 0.8 \mu\text{s}.$$

which also checks out.

## Small-Signal Analysis of the OTA

In order to size the OTA components we need to see derive how MOSFET parameters define the performance. The important small-signal metrics are

- dc gain  $A_0$
- gain-bandwidth product (GBW)
- output noise

The specification for GBW is given in Table 1, the dc gain we have to calculate from the voltage accuracy specification. For a voltage follower in the configuration shown in Figure 19 the voltage gain is given by

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{A_0}{1 + A_0}.$$

So in order to reach an output voltage accuracy of at least 5% we need a dc gain of  $A_0 > 25.6$  dB. To allow for process and temperature variation we will strive for  $A_0 = 30$  dB

## OTA Small-Signal Transfer Function

In order to derive the governing equations for the OTA we will make a few simplifications:

- We will set  $g_{mb} = 0$  for all MOSFET.
- We will further set  $C_{gd} = 0$  for all MOSFET except for  $M_4$  where we expect a Miller effect on this capacitor, and we could add its effect by increasing the capacitance at the gate node of  $M_{3,4}$ . However, as this does not create a dominant pole in this circuit, we consider this a minor effect (see Equation 14).
- We assume  $g_m \gg g_{ds}$ , so we set  $g_{ds1} = g_{ds3} = 0$ .
- The drain capacitance of  $M_2$  and  $M_4$  we can add to the load capacitance  $C_{\text{load}}$ .

The resulting small-signal equivalent circuit is shown in Figure 20.

### **i** Note

Please review the MOSFET small-signal equivalent model in Figure 5 at this point. For the PMOS just flip the model upside-down.

Source: [Article Notebook](#)

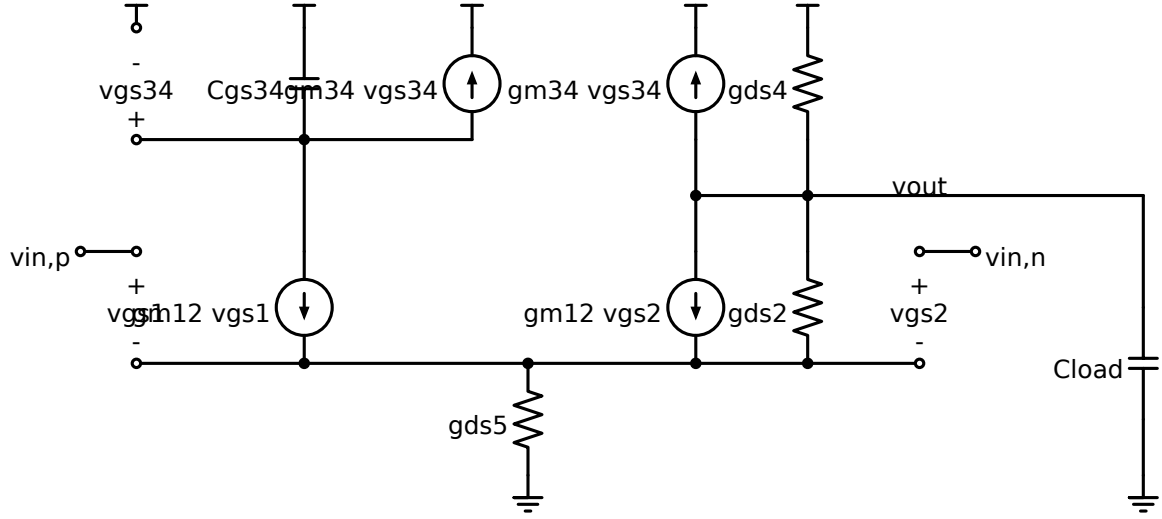


Figure 20: 5-transistor OTA small-signal model.

Source: [Article Notebook](#)

We can further simplify the output side by recognizing that the impedance looking from the output down we have  $g_{ds2}$  in series with  $g_{ds5} + g_{m12}$  (since we treat  $M_1$  as a common-gate stage when looking from the output, and since it is loaded by a low impedance of  $g_{m34}^{-1}$  we can approximate the impedance looking into  $M_1$  with  $g_{m12}^{-1}$ ). With the approximation that  $g_m \gg g_{ds}$  we can move  $g_{ds2} + g_{ds4}$  in parallel to  $C_{load}$ . Further assuming a differential drive with a virtual ground at the tailpoint we can remove  $g_{ds5}$ . The current source  $g_{m34}v_{gs34}$  with replace with the equivalent conductance  $g_{m34}$ . This results in the further simplified equivalent circuit shown in Figure 21.

#### **i** Note

For repetition and as a reference, here the simplified relationships for the common-source, common-gate, and the common-drain single transistor stages:  
 FIXME – to be done

Source: [Article Notebook](#)

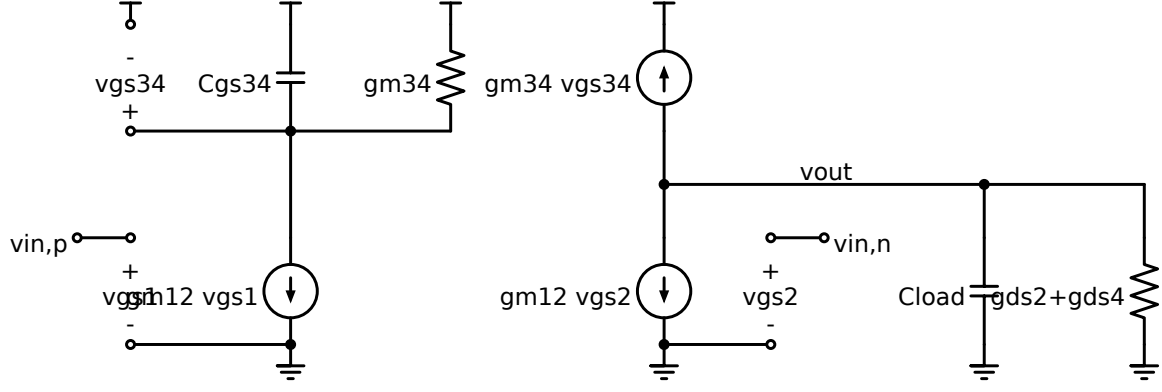


Figure 21: 5-transistor OTA small-signal model with further simplifications.

Source: [Article Notebook](#)

In the simplified circuit model in Figure 21 we can see that we have two poles in the circuit, one at the gate node of  $M_{3,4}$ , and one at the output. Realizing that  $v_{in,p} = v_{in}/2$  and  $v_{in,n} = -v_{in}/2$  we can formulate KCL at the output node to

$$-g_{m34}V_{gs34} - \left(-g_{m12}\frac{V_{in}}{2}\right) - V_{out}(g_{ds2} + g_{ds4} + sC_{load}) = 0. \quad (12)$$

We further realize that

$$V_{gs34} = -g_{m12}\frac{V_{in}}{2}\frac{1}{g_{m34} + sC_{gs34}}. \quad (13)$$

By combining Equation 12 and Equation 13 and after a bit of algebraic manipulation we arrive at

$$A(s) = \frac{V_{out}}{V_{in}} = \frac{g_{m12}}{2} \frac{2g_{m34} + sC_{gs34}}{(g_{m34} + sC_{gs34})(g_{ds2} + g_{ds4} + sC_{load})}. \quad (14)$$

When we now inspect Equation 14 we can see that for low frequencies the gain is

$$A(s \rightarrow 0) = A_0 = \frac{g_{m12}}{g_{ds2} + g_{ds4}}$$

which is plausible, and confirms the requirement of a high impedance at the output node. For very large frequencies we get

$$A(s \rightarrow \infty) = \frac{g_{m12}}{2sC_{load}} \quad (15)$$

which is essentially the behaviour of an integrator, and we can use Equation 15 to calculate the frequency where the gain drops to 1:

$$f_{ug} = \frac{g_{m12}}{4\pi C_{load}}$$

when looking at Equation 14 we see that we have a dominant pole at  $s_p$  and a pole-zero doublet with  $s_{pd}/s_{zd}$ :

$$s_p = -\frac{g_{ds2} + g_{ds4}}{C_{load}}$$

$$s_{pd} = -\frac{g_{m34}}{C_{gs34}}$$

$$s_{zd} = -\frac{2g_{m34}}{C_{gs34}}$$

### OTA Noise

For the noise analysis we ignore the pole-zero doublet due to  $C_{gs34}$  (we assume minor impact due to this) and just consider the dominant pole. For the noise analysis at the output we set the input signal to zero, and thus we arrive at the simplified small-signal circuit shown in Figure 22.

Source: [Article Notebook](#)

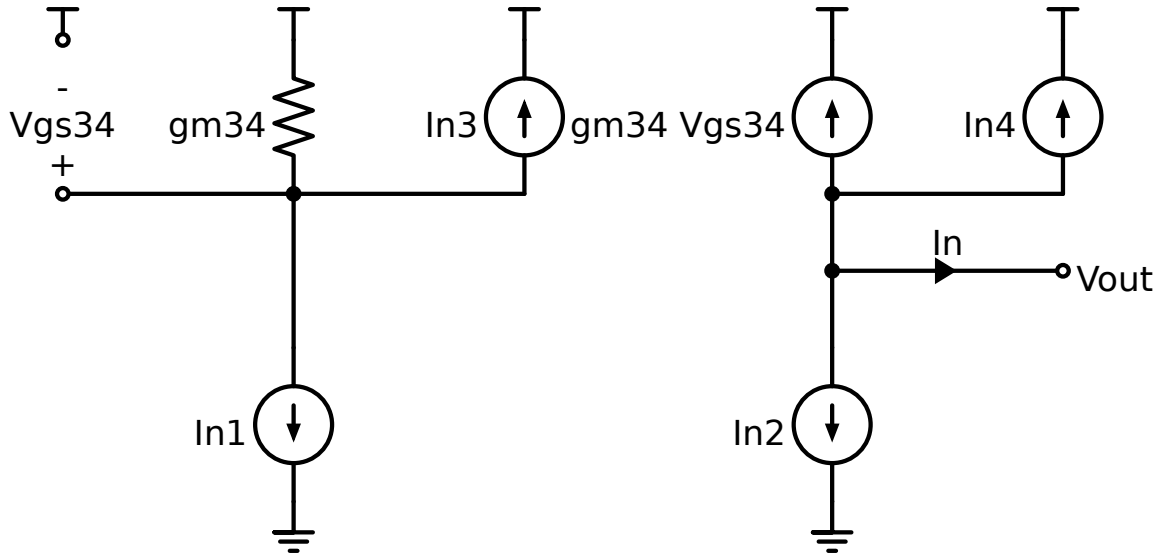


Figure 22: 5-transistor OTA small-signal model for noise calculation.

Source: [Article Notebook](#)

We see that

$$\overline{V_{gs34}^2} = \frac{1}{g_{m34}^2} \left( \overline{I_{n1}^2} + \overline{I_{n3}^2} \right).$$

#### **i** Note

Remember that **uncorrelated** noise quantities need to be power-summed (i.e.,  $I^2 = I_1^2 + I_2^2$ )!

We can then sum the output noise current  $\overline{I_n}$  as

$$\overline{I_n^2} = \overline{I_{n2}^2} + \overline{I_{n4}^2} + g_{m34}^2 \frac{1}{g_{m34}^2} \left( \overline{I_{n1}^2} + \overline{I_{n3}^2} \right) = 2 \left( \overline{I_{n12}^2} + \overline{I_{n34}^2} \right).$$



As a next step, let us rewrite the OTA transfer function  $A(s)$  (see Equation 14) by getting rid of the pole-zero doublet as a simplifying assumption to get

$$A'(s) = \frac{g_{m12}}{g_{ds2} + g_{ds4} + sC_{load}}. \quad (16)$$

Inspecting Equation 16 we can interpret the OTA transfer function as a transconductor  $g_{m12}$  driving a load of  $Y_{load} = g_{ds2} + g_{ds4} + sC_{load}$ . We can thus redraw Figure 19 in the following way, injecting the previously calculated noise current into the output node. The result is shown in Figure 23.

Source: [Article Notebook](#)

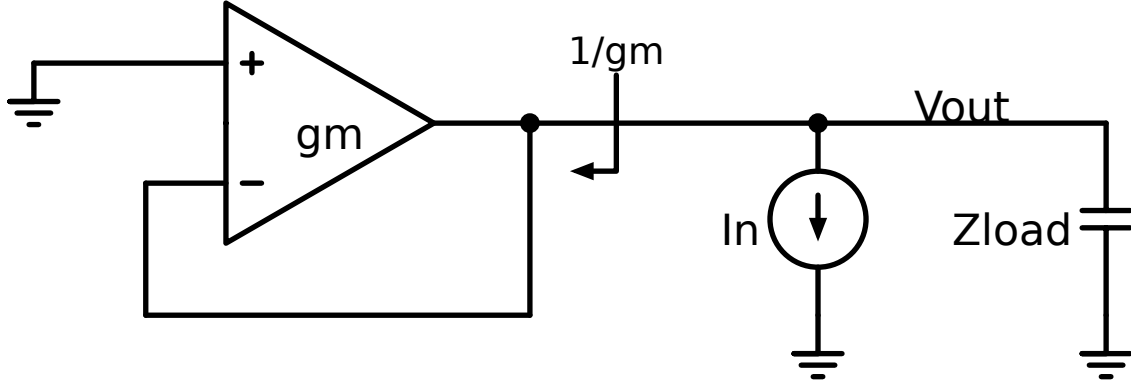


Figure 23: A voltage buffer redrawn for noise analysis.

Source: [Article Notebook](#)

We see that the feedback around the transconductor  $g_m$  creates an impedance of  $1/g_m$ . We can now calculate the effective load conductance of

$$Y'_{load} = g_{ds2} + g_{ds4} + sC_{load} + g_m \approx g_m + sC_{load}.$$

The output noise voltage is then

$$\overline{V_{n,out}^2}(s) = \frac{\overline{I_n^2}}{|g_m + sC_{load}|^2} =$$

## Cascode Stage

## A Fully-Differential OTA

## Biasing the OTA

## An RC-OPAMP Filter

## Summary & Conclusion

## Appendix: Middlebrook's Method

When we want to do a closed-loop gain analysis (for stability or other investigations), we have the need to break the loop at one point, apply a stimulus, and monitor the response on the other end. By doing this we want to keep the loading on both ends similar to the original case. To achieve this, we break the loop at one point by inserting (1) an ac voltage source, and (2) attach an ac current source, as shown in Figure 24 and Figure 25. The derivation of this approach is presented in (Middlebrook 1975), and has the big advantage that loading is not changed, and the bias points are also correct.

Source: [Article Notebook](#)

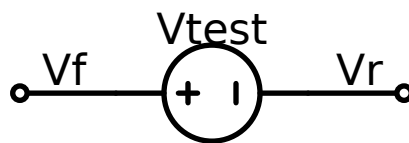


Figure 24: Middlebrook voltage loop gain simulation.

Source: [Article Notebook](#)

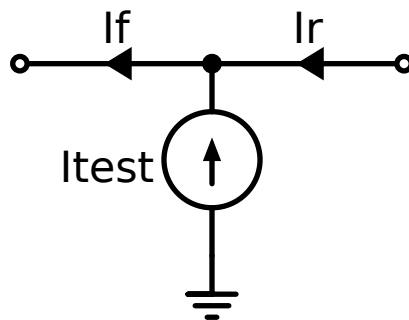


Figure 25: Middlebrook current loop gain simulation.

Source: [Article Notebook](#)

For both cases we do an ac analysis, and find the corresponding transfer functions  $T_v$  and  $T_i$  as

$$T_v = -\frac{V_r}{V_f}$$

and

$$T_i = -\frac{I_r}{I_f}.$$

Then, we can calculate the closed-loop transfer function  $T(s) = H_{ol(s)}$  as

$$T(s) = \frac{T_v T_i - 1}{T_v + T_i + 2}.$$

## Appendix: ngspice Cheatsheet

Here is an unsorted list of useful ngspice settings and command:

### Commands

- `ac dec|lin points fstart fstop` performs a small-signal ac analysis with either linear or decade sweep
- `dc sourcename vstart vstop vincr [src2 start2 stop2 incr2]` runs a dc-sweep, optionally across two variables
- `display` shows the available data vectors in the current plot
- `echo` can be used to display text, `$variable` or `$$vector`, can be useful for debugging
- `let name = expr` to create a new vector; `unlet vector` deletes a specified vector; access vector data with `$$vec`
- `linearize vec` linearizes a vector on an equidistant time scale, do this before an FFT; with `set specwindow=windowtype` a proper windowing function can be set
- `meas` can be used for various evaluations of measurement results (see ngspice manual for details)
- `noise v(output <ref>) src (dec|lin) pts fstart fstop` runs a small-signal noise analysis
- `op` calculates the operating point, useful for checking bias points and device parameters
- `plot expr vs scale` to plot something
- `print expr` to print it, use `print all` to print everything
- `remzerovec` can be useful to remove vectors with zero length, which otherwise cause issues when saving or plotting data
- `rusage` plot information about resource usage like memory
- `save all` or `save signal` specifies which data is saved during simulation; this lowers RAM usage during simulation and size of RAW file; do save before the actual simulation statement
- `setplot` show a list of available plots
- `set var = value` to set the value of a variable; use variable with `$var`; `unset var` removes a variable

- `set enable_noisy_r` to enable noise of behavioral resistors; usually, this is a good idea
- `shell cmd` to run a shell command
- `show : param`, like `show : gm` shows the  $g_m$  of all devices after running an operating point with `op`
- `spec` plots a spectrum (i.e. frequency domain plot)
- `status` shows the saved parameters and nodes
- `tf` runs a transfer function analysis, returning transfer function, input and output resistance
- `tran tstep tstop <tstart <tmax>>` runs a transient analysis until `tstop`, reporting results with `tstep` stepsize, starting to plot at `tstart` and performs time steps not larger than `tmax`
- `wrdata` writes data into a file in a tabular ASCII format; easy to further process
- `write` writes simulation data (the saved nodes) into a RAW file; default is binary, can be changed to ASCII with `set filetype=ascii`; with `set appendwrite` data is added to an existing file

## Options

Use `option option=val option=val` to set various options; important ones are:

- `abstol` sets the absolute current error tolerance (default is 1pA)
- `gmin` is the conductance applied at every node for convergence improvement (default is 1e-12); this can be critical for very high impedance circuits
- `klu` sets the KLU matrix solver
- `list` print the summary listing of the input data
- `maxord` sets the numerical order of the integration method (default is 2 for Gear)
- `method` set the numerical integration method to `gear` or `trap` (default is `trap`)
- `node` prints the node table
- `opts` prints the option values
- `temp` sets the simulation temperature
- `reltol` set the relative error tolerance (default is 0.001 = 0.1%)
- `savecurrents` saves the terminal currents of all devices
- `sparse` sets the sparse matrix solver, which can run noise analysis, but is slower than `klu`
- `vntol` sets the absolute voltage error tolerance (default is 1μV)
- `warn` enables the printing of the SOA warning messages

## Convergence Helper

- `option gmin` can be used to increase the conductance applied at every node
- `option method=gear` can lead to improved convergence
- `.nodeset` can be used to specify initial node voltage guesses
- `.ic` can be used to set initial conditions

## Appendix: Xschem Cheatsheet

When opening Xschem, using **Help -> Keys** a pop-up windows comes up with many useful shortcuts. The most useful are:

### Moving around in a schematic:

- **Cursor** keys to move around
- **Ctrl-e** to go back to parent schematic
- **e** to descend into selected symbol
- **f** full zoom on schematic
- **Shift-z** to zoom in
- **Ctrl-z** to zoom out

### Editing schematics:

- **Del** to delete elements
- **Ins** to insert elements from library
- **Escape** to abort an operation
- **Ctrl-#** to rename components with duplicate names
- **c** to copy elements
- **Alt-Shift-l** to add wire lable
- **Alt-l** to add lable pin
- **m** move selected objects
- **q** to edit properties
- **Ctrl-s** to save schematic
- **t** to place a text
- **Shift-T** to toggle the **ignore** flag on an instance
- **u** to undo an operation
- **w** to draw a wire
- **Shift-W** draw wire and snap to close pin or netpoint
- **&** to join, break, and collapse wires

### Viewing/Simulating Schematics

- **5** to only view probes
- **k** to highlight selected net
- **Shift-K** to unhighlight all nets
- **Shift-o** to toggle light/dark color scheme
- **s** to run a simulation

Source: [Article Notebook](#)

Hellen, Edward H. 2003. "Verifying the diode-capacitor circuit voltage decay." *American Journal of Physics* 71 (8): 797–800. <https://doi.org/10.1119/1.1578070>.

- Hu, Chenming. 2010. *Modern Semiconductor Devices for Integrated Circuits*. Pearson.
- Jespers, Paul G. A., and Boris Murmann. 2017. *Systematic Design of Analog CMOS Circuits: Using Pre-Computed Lookup Tables*. Cambridge University Press.
- Middlebrook, R. D. 1975. "Measurement of loop gain in feedback systems." *International Journal of Electronics* 38 (4): 485–512. <https://doi.org/10.1080/00207217508920421>.
- Nagel, Laurence W. 1975. "SPICE2: A Computer Program to Simulate Semiconductor Circuits." PhD thesis, EECS Department, University of California, Berkeley. <http://www2.eecs.berkeley.edu/Pubs/TechRpts/1975/9602.html>.
- Tsividis, Yannis, and Colin McAndrew. 2011. *Operation and Modeling of the MOS Transistor*. Oxford University Press.