The 2-fair Domination in Extended Supergrid and Planar Graphs

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Abstract. A dominating set in a graph is a subset of vertices where every vertex in the graph either belongs to this subset or is adjacent to at least one vertex in it. A k-fair dominating set extends this concept by requiring that each vertex outside the set is adjacent to exactly k vertices within the set. The domination problem seeks to determine the smallest possible dominating set, while the k-fair domination problem aims to find the smallest k-fair dominating set. The decision versions of these problems ask whether a given graph contains a dominating set or a k-fair dominating set of size at most a given constant η. Previously, we studied the 1-fair domination problem on planar and extended supergrid graphs. In this paper, we extend our investigation to the complexity of the 2-fair domination problem in these graph classes. First, we prove that the problem is NP-complete for both planar and supergrid graphs, where supergrid graphs form a subclass of extended supergrid graphs. Then, we introduce a linear-time algorithm for solving the 2-fair domination problem on rectangular supergrid graphs, a specific subclass of supergrid graphs.

Keywords: Domination · 2-fair domination · Planar graph · Supergrid graph · Rectangular supergrid graph.

1 Introduction

Let G be a graph characterized by its vertex set $V(G)$ and edge set $E(G)$. The vertex set $V(G)$ consists of all the vertices or nodes in the graph, while the edge set $E(G)$ represents the connections or links between these vertices. For any given vertex $v \in V(G)$, the degree of v, denoted as $deg_G(v)$, is defined as the number of vertices in G that are directly adjacent to v, meaning the number of edges that have v as one of their endpoints. In other words, the degree of a vertex quantifies how many other vertices it is directly connected to within the graph. The open neighborhood of v, represented as $N_G(v)$, consists of all vertices $u \in V(G)$ such that $(u, v) \in E(G)$, formally expressed as $N_G(v) = \{u \in V(G) | (u, v) \in E(G)\}$. The closed neighborhood of v, denoted by $N_G[v]$, includes v itself along with its open neighborhood: $N_G[v] = N_G(v) \cup \{v\}$. A path in G is an ordered sequence of vertices $(v_1, v_2, \ldots, v_{k-1}, v_k)$ such that each consecutive pair

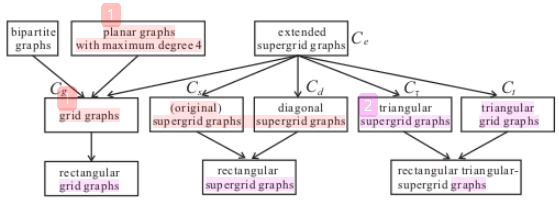

Fig. 1. The hierarchical relationships among various graph classes, including bipartite, planar, extended supergrid, grid, (original) supergrid, diagonal supergrid, triangular supergrid, and triangular grid graphs, are represented by $C \rightarrow C'$, indicating that C is a superclass of C'. [1].

(v_i, v_{i+1}) is connected by an edge in G. This path begins at v_1 and terminates at v_k, and it is referred to as a (v_1, v_k)-path. The vertices along the path are distinct, except when the path forms a cycle, in which case the starting and ending vertices coincide. A path consisting of n vertices is typically represented as P_n.

The graph classes and problems explored in this paper are defined as follows. Consider an infinite graph S^∞, where each vertex has integer coordinates on a Euclidean plane. A vertex v is represented as (v_x, v_y), where v_x and v_y denote its x- and y-coordinates, respectively. This graph S^∞ is known as a two-dimensional integer supergrid. Edges in S^∞ connect vertices u and v if $|u_x - v_x| \leq 1$ and $|u_y - v_y| \leq 1$. These edges are classified as vertical, horizontal, or diagonal, with diagonals further categorized as either l-skewed or r-skewed. A two-dimensional integer grid, G^∞, consists only of horizontal and vertical edges, while a triangular grid, T^∞, includes horizontal, vertical, and r-skewed edges. Extended supergrid graphs are finite, connected subgraphs of S^∞, whereas grid, triangular grid, and supergrid graphs are vertex-induced subgraphs of G^∞, T^∞, and S^∞, respectively. Specific subclasses of extended supergrid graphs, known as diagonal and triangular supergrid graphs, are characterized by the presence of particular diagonal edges. Unless stated otherwise, the terms "supergrid graph" and "original supergrid graph" are used interchangeably. We denote graph classes as follows: C_e for extended supergrid graphs, C_g for grid graphs, C_s for supergrid graphs, C_d for diagonal supergrid graphs, C_τ for triangular supergrid graphs, and C_t for triangular grid graphs. The relationships among these graph classes satisfy $C_g, C_s, C_d, C_\tau, C_t \subseteq C_e$, with specific intersections either empty or non-empty, as illustrated in Fig. 1. Grid graphs are always bipartite and planar, whereas supergrid and triangular grid graphs may not be. Let $G_g \in C_g$, $G_s \in C_s$, and $G_t \in C_t$. Then, for any vertices u, v, and w in G_g, G_s, and G_t, respectively, their degrees are bounded as follows: $deg_{G_g}(u) \leq 4$, $deg_{G_s}(v) \leq 8$, and $deg_{G_t}(w) \leq 6$.

A rectangular grid graph, denoted as $G_{m \times n}$, is constructed as the Cartesian product of the paths P_m and P_n , where each P_i is a simple path with i vertices. Two vertices u and v in $G_{m \times n}$ are adjacent if and only if their Euclidean distance

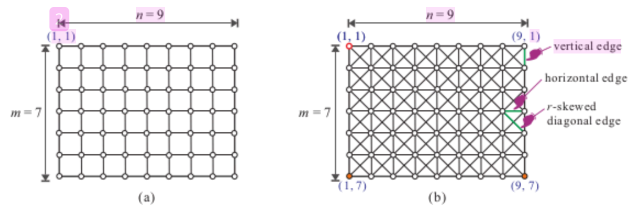

Fig. 2. (a) Rectangular grid graph $G_{7\times 9}$, and (b) rectangular supergrid graph $R_{7\times 9}$.

is 1, i.e., $|u_x - v_x| + |u_y - v_y| = 1$. An example of $G_{7\times 9}$ is illustrated in Fig. 2(a). A rectangular supergrid graph, denoted as $R_{m\times n}$, is formed by the strong product of the paths P_m and P_n. In this graph, two vertices are adjacent if their Euclidean distance is at most $\sqrt{2}$, meaning $0 \le |u_x - v_x| \le 1$ and $0 \le |u_y - v_y| \le 1$. Fig. 2(b) provides an example of $R_{7\times 9}$. The degree of each vertex is bounded as follows: $deg_{G_{m\times n}}(u) \le 4$ and $deg_{R_{m\times n}}(u) \le 8$, for $u \in V(G_{m\times n})$ and $v \in V(R_{m\times n})$. Note that rectangular supergrid graphs are also called king's graphs in the literature. For consistency, the coordinates $(1, 1)$ refer to the top-left vertex in all grid and supergrid graphs. A graph is called planar if it can be embedded in the plane, i.e., it can be drawn in the plane so that no edges intersect except at the vertices, and grid graphs form a subclass of planar graphs with maximum degree 4. This paper focuses on discovering the complexity of the 2-fair domination problem in the previously described graphs including extended supergrid graphs and planar graphs.

Consider a graph G and a subset $D \subseteq V(G)$. The set D is said to *dominate* a vertex $v \in V(G)$ if $N_G[v] \cap D \neq \phi$, meaning at least one vertex in D is either v itself or a neighbor of v. Extending this concept, D dominates a subset $S \subseteq V(G)$ if every vertex in S is dominated by D. A dominating set of G is a vertex subset D such that D dominates the entire vertex set $V(G)$. The domination number, denoted $\gamma(G)$, is the minimum cardinality of a dominating set in G. A minimum dominating set is any dominating set of size $\gamma(G)$, representing the smallest set of vertices that ensures complete domination of the graph. The domination problem seeks to determine the smallest dominating set of a given graph. Its decision variant asks whether there exists a dominating set of size at most η for some given constant η. In computational complexity theory, decision problems of this nature—determining whether a solution of size at most (or at least) η exists—are typically classified as NP-complete while its optimization problem (seeking maximum or minimum set) is known as NP-hard. For clarity, throughout this paper, if the domination problem or any of its variants is referred to as NP-complete, this classification pertains specifically to its decision version. It is well known that the domination problem remains NP-complete for general graphs [2]. A dominating set F of a graph G is called *fair* if every vertex outside

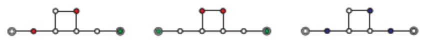

Fig. 3. (a) The minimum 1-fair dominating set of a grid graph, (b) a 1-fair dominating set but is not minimum, and (c) a dominating set but is not a 1-fair dominating set, where solid circles represent the vertices of the fair dominating set or dominating set.

F is dominated by the same number of vertices from F. Formally, for any two vertices $u, v \notin F$, it holds that $|N_G(u) \cap F| = |N_G(v) \cap F|$. When each non-dominating vertex is dominated by exactly k vertices from F, the set F is called a k-fair dominating set. The minimum cardinality of such a set is the k-fair domination number, denoted $\gamma_{k\text{fd}}(G)$, and determining this set constitutes the k-fair domination problem. It is evident that if the maximum vertex degree of G is δ, then a $(\delta+1)$-fair dominating set cannot exist, as no vertex can be dominated by more neighbors than the maximum degree of the graph allows. For example, in grid graphs where $\delta = 4$, a 5-fair dominating set is impossible. Fig. 3(b) illustrates a 1-fair dominating set, though it is not the minimum for the grid graph in Fig. 3(a). In contrast, Fig. 3(c) presents a dominating set that does not satisfy the conditions of a 1-fair dominating set. In the past, we have decided the complexity of the 1-fair domination problem on extended supergrid and planar graphs. In this paper, we will decide the complexity of the 2-fair domination problem on them. In addition, we will provide a linear-time algorithm to solve this problem for rectangular supergrid graphs.

The body of prior research can be elaborated upon as follows: Supergrid graphs were first introduced in [3], where we demonstrated that the Hamiltonian cycle and path problems for these graphs are NP-complete. Additionally, we established that every rectangular supergrid graph contains a Hamiltonian cycle. Given the NP-completeness of Hamiltonian problems in original supergrid graphs, an important research focus is investigating these problems in special cases of supergrid graphs. In [4], we proved that linear-convex supergrid graphs always possess Hamiltonian cycles. In [5], we confirmed the Hamiltonian connectivity of rectangular supergrid graphs. Further, in [6,7], we examined the Hamiltonicity and Hamiltonian connectivity of shaped supergrid graphs and letter supergrid graphs. More recently, we established the Hamiltonicity, Hamiltonian connectivity, and solved the longest path problem for L-shaped, C-shaped, and O-shaped supergrid graphs [8–10]. The domination problem is a well-known NP-complete problem for general graphs [2] and remains NP-complete for specific graph classes, including 4-regular planar graphs [2], cubic planar graphs [11], and grid graphs [12], among others. The fair domination problem was introduced by Caro et al. in 2012 [13], where they explored its properties and established that the 1-fair domination problem is equivalent to the perfect domination problem. Below, we summarize key contributions to the study of fair domination. In 2014, Maravilla et al. [14] examined k-fair dominating sets in graphs derived from operations such as join, corona, composition, and Cartesian product, providing bounds or exact values for their k-fair domination numbers. In 2019, Hajian and Rad determined an upper bound for the fair domination number of cactus graphs, demonstrating that this bound corresponds to the 1-fair domination number for these graphs [15]. That same year, Jayasree and Radha computed the 1-fair domination number for various special graphs, including paths, cycles, Sierpinski graphs, and rectangular supergrid graphs [16]. Hajian et al. further contributed by establishing an upper bound for the fair domination number of outerplanar graphs [17]. In 2022, Sangeetha et al. systematically calculated the fair domination number for several special graphs, including mill graphs, sunlet graphs, crown graphs, ladder graphs, prism graphs, gear graphs, web graphs, and helm graphs [18]. More recently, Alikhani and Safazadeh [19] analyzed the number of fair dominating sets in select special graphs, such as complete bipartite graphs, cycles, paths, sociable friendship graphs, and triangular cactus graphs. In this paper, we undertake a comprehensive study of the 2-fair domination problem, extending our analysis to the classes of planar and extended supergrid graphs.

The paper is structured as follows: Section 2 establishes the NP-completeness of the 2-fair domination problem for various graph classes, including grid graphs, planar graphs with a maximum degree of 3, diagonal supergrid graphs, and supergrid graphs. It further extends this NP-completeness result to cover extended supergrid graphs. Section 3 focuses on the analytical computation of the 2-fair domination number, denoted as $\gamma_{2\mathrm{fd}}(R_{m\times n})$, for rectangular supergrid graphs $R_{m\times n}$. Finally, Section 4 provides a summary of the key findings and insights derived from the study.

2 NP-completeness results

In this section, our first objective is to demonstrate that the 2-fair domination problem for grid graphs is NP-complete. This claim builds upon the foundational work of Clark et al. [12], which established the NP-completeness of the domination problem on grid graphs. Our goal is to extend this result to the 2-fair domination problem. Previous research by Hung et al. [1] has already confirmed the NP-completeness of the restrained domination problem and its various extensions on grid graphs, diagonal supergrid graphs, and supergrid graphs. Inspired by these findings, we employ similar techniques to demonstrate that the 2-fair domination problem remains NP-complete for grid graphs. Moreover, this proof extends beyond grid graphs and can be adapted to other graph classes with minor modifications. The core of our proof lies in constructing a polynomial-time reduction from the domination problem on grid graphs.

Theorem 1. *(See [12].) Determining whether there exists a dominating set of a grid graph with size $\leq k$ is NP-complete.*

Our strategy for verifying the NP-completeness of the 2-fair domination problem on grid graphs is to reduce the domination problem on grid graphs to this problem. The key idea is to take an arbitrary grid graph G_g and construct a new grid graph, denoted $G_g^{2\text{fd}}$, in such a way that the original graph G_g obtains a dominating set D of size $|D| \leq k$ if and only if the constructed graph $G_g^{2\text{fd}}$ contains a 2-fair dominating set $D_{2\text{fd}}^g$ of size $|D_{2\text{fd}}^g| \leq 4|V(G_g)|+2|E(G_g)|+k+d(D)$, where $d(D) = \sum_{v\in D}(deg_{G_g}(v) - 1)$. This relationship between G_g and $G_g^{2\text{fd}}$ serves as the foundation for our reduction. By constructing $G_g^{2\text{fd}}$ in such a way, we ensure that any solution to the 2-fair domination problem on $G_g^{2\text{fd}}$ corresponds directly to a solution of the original domination problem on G_g, thereby proving that the 2-fair domination problem on grid graphs is at least as hard as the general domination problem, which is known to be NP-complete. The process of constructing $G_g^{2\text{fd}}$ from G_g involves carefully adding additional vertices and edges to transform the graph in a way that preserves the essential properties of domination while adapting them to the constraints of the 2-fair domination problem. Specifically, for each vertex in G_g, we introduce structures that enforce the 2-fair domination criteria—requiring that each vertex in $G_g^{2\text{fd}}$ is either dominated by exactly two other vertices or is part of a set that satisfies this condition. This construction process is key to our proof, as it not only establishes the complexity of the problem but also provides a pathway for analyzing similar domination problems in other types of graphs, such as diagonal supergrid, original supergrid, and planar graphs. By leveraging this reduction technique, we can extend our analysis beyond grid graphs, offering insights into the broader class of 2-fair domination problems across various graph structures. The construction of $G_g^{2\text{fd}}$ from G_g proceeds as follows:

Step 1: For each vertex u in the input grid graph G_g, replace it with a structure called a *square cluster*, denoted as C_u, as shown in Fig. 4(b).
Step 2: For each edge (u, v) in the input grid graph G_g, replace it with a horizontal or vertical path $P(u, v)$ consisting of 7 vertices (see Fig. 5), referred to as a *tentacle path*, where each edge in a horizontal (resp., vertical) path is a horizontal (resp. vertical) edge.
Step 3: The graph constructed by the above steps forms a grid graph $G_g^{2\text{fd}}$. For instance, Fig. 4(d) illustrates the grid graph $G_g^{2\text{fd}}$ built from the grid graph G_g depicted in Fig. 4(a).

The construction process described above is formally represented as Algorithm *ConstructGrid*. This algorithm is designed to be executed efficiently within polynomial time constraints. In [1], Hung delineated a specific protocol, referred to as *Rule ST*, which governs the organization of the square tentacles within the graph $G_g^{2\text{fd}}$. This rule ensures that the square tentacles are positioned in such a manner that they remain non-overlapping, with the exception of their connecting points. While the detailed methodology for the arrangement of these tentacles is extensive, it has been omitted here due to the constraints on the length of this paper. Nevertheless, it is important to note that the application of Algorithm ConstructGrid, in conjunction with Rule ST, is a procedure that can be completed within a polynomial time. Consequently, the subsequent lemma holds true.

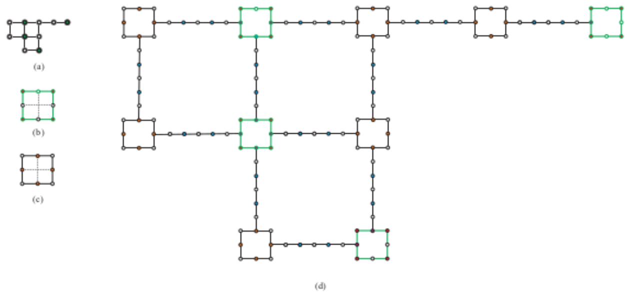

Fig. 4. (a) Grid graph G_g, (b) a black square cluster C_u, (c) a white square cluster C_v, and (d) the grid graph $G_g^{2\mathrm{fd}}$ constructed by connecting the square clusters using tentacle paths, where double circles represent vertices of G_g, solid circles represent vertices in the dominating set of G_g or in the 2-fair dominating set in $G_g^{2\mathrm{fd}}$).

Lemma 1. *Algorithm ConstructGrid constructs a grid graph $G_g^{2\mathrm{fd}}$ from a given grid graph G_g in polynomial time.*

In the following analysis, we aim to establish a proof demonstrating a direct relationship between the existence of a dominating set D in the grid graph G_g and the presence of a 2-fair dominating set in the constructed grid graph $G_g^{2\mathrm{fd}}$. The proof relies on the condition $|D| \le k$, which is essential for ensuring that $G_g^{2\mathrm{fd}}$ contains a 2-fair dominating set, denoted as $D_{2\mathrm{fd}}^g$, satisfying $|D_{2\mathrm{fd}}^g| \le 4|V(G_g)| + 2|E(G_g)| + k + d(D)$, where $d(D) = \sum_{v \in D}(deg_{G_g}(v) - 1)$. To illustrate this concept, we refer to Fig. 4. In this figure, the grid graph G_g is depicted as having a dominating set of size 4. Following the construction process, the resulting grid graph $G_g^{2\mathrm{fd}}$ is shown to contain a 2-fair dominating set with size $4 \times 10 + 2 \times 12 + 4 + (2 + 3 + 0 + 1) = 74$. Suppose D is a dominating set of the grid graph G_g. Given a dominating set D of G_g, the construction of a 2-fair dominating set $D_{2\mathrm{fd}}^g$ for the grid graph $G_g^{2\mathrm{fd}}$ is depicted as the following steps:

Step 1: For $u \in D$, its corresponding square cluster C_u is called a black square cluster, and we construct its 2-fair dominance set, as shown in Fig. 4(b).
Step 2: For $v \notin D$, its corresponding square cluster C_v is called a white square cluster, and we construct its 2-fair dominance set, as shown in Fig. 4(c).
Step 3: For $(u, v) \in E(G_g)$, the 2-fair dominating set of the corresponding tentacle path $P(u, v)$ is a set of 4 vertices, as shown in Fig. 5.

Then, we would like to prove the following theorem. Due to space limitation, the proof of the following theorem is omitted.

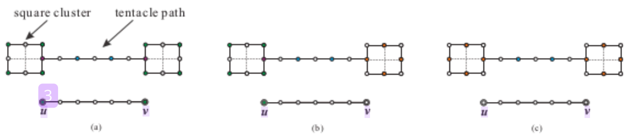

Fig. 5. (A 2-fair dominating set of $C_u \cup P(u, v) \cup C_v$ for (a) $u, v \in D$, (b) $u \in D$ and $v \notin D$, (c) $u, v \notin D$, where D is a dominating set of G_g and the solid circles in the diagram represent the vertices of this 2-fair dominating set.

Theorem 2. *If G_g^{2fd} is the grid graph derived from G_g through Algorithm ConstructGrid and Rule ST, then G_g has a dominating set D of size $\leq k$ if and only if the grid graph G_g^{2fd} encompasses a 2-fair dominating set D_g^{2fd} of size $\leq 4|V(G_g)| + 2|E(G_g)| + k + d(D)$, where $d(D) = \sum_{v \in D}(deg_{G_g}(v) - 1)$.*

It is clear that the 2-fair domination problem pertaining to grid graphs falls within the NP complexity class. By Theorem 1 and Theorem 2, we derive the following theorem:

Theorem 3. *Determining a 2-fair dominating set with size $\leq k$ for a grid graph is an NP-complete problem.*

In a manner analogous to the construction of the grid graph G_g^{2fd} from the input grid graph G_g, we can similarly construct a supergrid graph, denoted as G_s^{2fd}, using the original grid graph G_g as a foundation. This construction proceeds by expanding the vertices and edges of G_g according to specific transformation rules that ensure G_s^{2fd} exhibits a more intricate structure, potentially involving additional layers of connectivity or spatial resolution. The process not only preserves the underlying grid-like arrangement but also enriches the graph's properties, allowing for enhanced modeling of complex systems or multi-dimensional relationships that are otherwise difficult to capture in simpler grid graphs. By carefully selecting the transformation rules, we can tailor G_s^{2fd} to a wide range of applications, from network optimization problems to simulations of physical phenomena, all while maintaining the intuitive geometric framework of the original grid. The construction of the supergrid graph G_s^{2fd} from a grid graph G_g is as follows:

Step 1: For each vertex u in the input grid graph G_g, replace it with a structure called a star cluster denoted as S_u, as shown in Fig. 6(b).
Step 2: For each edge (u, v) in the input grid graph G_g, replace it with a horizontal or vertical path $P(u, v)$ consisting of 7 vertices, referred to as a *tentacle path*.
Step 3: The graph constructed by the above steps forms a supergrid graph G_s^{2fd}. For instance, Fig. 6(d) illustrates the supergrid graph G_s^{2fd} built from the grid

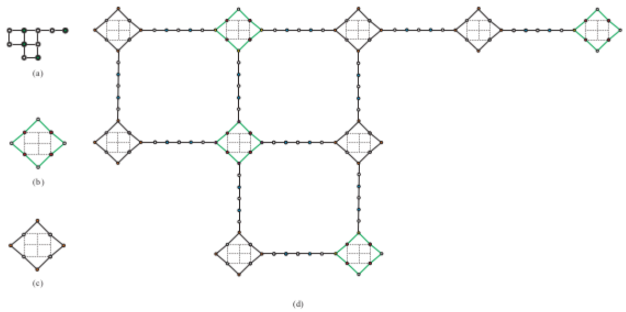

Fig. 6. (a) Grid graph G_g, (b) a black star cluster S_u, (c) a white star cluster S_v, and (d) the supergrid graph $G_s^{2\mathrm{fd}}$ constructed by connecting the star clusters using tentacle paths, where double circles represent vertices of G_g, solid circles represent vertices in the dominating set of G_g or the 2-fair dominating set in $G_s^{2\mathrm{fd}}$.

graph G_g depicted in Fig. 6(a).

By similar arguments in proving Theorem 2, we will verify the following theorem:

Theorem 4. *Let G_g be a grid graph, and let $G_s^{2\mathrm{fd}}$ be a supergrid graph constructed from G_g. Then, G_g has a dominating set D of size $\leq k$ if and only if the grid graph $G_s^{2\mathrm{fd}}$ contsins a 2-fair dominating set $D_s^{2\mathrm{fd}}$ with size $|D_s^{2\mathrm{fd}}| \leq 4|V(G_g)| + 2|E(G_g)| + k + d(D)$, where $d(D) = \sum_{v\in D}(deg_{G_g}(v) - 1)$.*

Then, we can conclude the following theorem.

Theorem 5. *Determining a 2-fair dominating set with size $\leq k$ for a supergrid graph is an NP-complete problem.*

In the above construction of the supergrid graph $G_s^{2\mathrm{fd}}$, we can see that $G_s^{2\mathrm{fd}}$ is also a diagonal supergrid graph and is a planar graph with maximum degree 3, so we can obtain the following theorem:

Theorem 6. *The 2-fair domination problem for diagonal supergrid graphs and planar graphs with maximum degree 3 is NP-hard.*

3 The 2-fair domination number of rectangular supergrid graphs

On the positive side, we will demonstrate that the 2-fair domination problem on a rectangular supergrid graph $R_{m\times n}$ is linearly solvable. Consider that $m = 1$

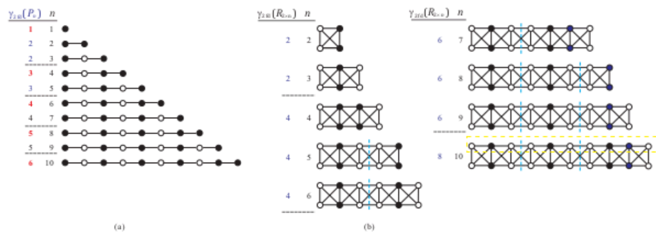

Fig. 7. The minimum 2-fair dominating set for (a) $R_{1\times n}$, and (b) $R_{2\times n}$, where the solid circles represent the vertices of the minimum 2-fair dominating set.

or 2. By observing Fig. 7, we will prove the following lemma. Due to space limitation, the proof is omitted.

Lemma 2. $\gamma_{2\mathrm{fd}}(R_{1\times n}) = \gamma_{2\mathrm{fd}}(P_n) = \lfloor \frac{n}{2} \rfloor + 1$*, and* $\gamma_{2\mathrm{fd}}(R_{2\times n}) = 2 \times \lfloor \frac{n}{3} \rfloor$.

For $n \geq m \geq 3$, we will use mathematical induction to prove the following lemma. Due to space limitation, the proof is omitted.

Lemma 3. $\gamma_{2\mathrm{fd}}(R_{m\times n}) = m \times n$*, where* $n \geq m \geq 3$.

It immediately follows from the above two lemmas that the following theorem holds true.

Theorem 7. *Let* $R_{m\times n}$ *be a rectangular supergrid graph with* $n \geq m$*. Then,*

$$\gamma_{2\mathrm{fd}}(R_{m\times n}) = \begin{cases} \lfloor \frac{n}{2} \rfloor + 1 \text{ , } if\ m = 1; \\ 2 \times \lfloor \frac{n}{3} \rfloor \text{ , } if\ m = 2; \\ m \times n \quad \text{ , } otherwise. \end{cases}$$

4 Concluding remarks

In our previous work, we have studied the 1-fair domination problem on extended supergrid and planar graphs. In this paper, we shift our focus to the 2-fair domination problem in these graph classes. We begin by establishing the computational complexity of the 2-fair domination problem on grid graphs, rigorously proving its NP-completeness. This result extends to several related graph structures, including diagonal and supergrid graphs, as well as planar graphs with a maximum degree of 3, confirming NP-completeness for these categories as well. Furthermore, we introduce a novel linear-time algorithm for solving the 2-fair domination problem specifically on rectangular supergrid graphs, presenting an efficient approach to this challenging problem. Looking ahead, our research will explore the more complex 3-fair and 4-fair domination problems on extended supergrid graphs, with the goal of uncovering new insights and potentially groundbreaking advancements in computational complexity.

Acknowledgments

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