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Characteristics of hand tremor time series

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Abstract. Tremor is classified into physiological, essential, and parkinsonian tremor by means of clinical criteria. The aim of our work was to extract quantitative features from the measurements of the acceleration of human postural hand tremor. Different mathematical methods were adopted and modified in order to separate these three types of tremor. Best discrimination between physiological and pathological tremors has been achieved by methods distinguishing nonlinear from linear behavior. On the other hand, methods separating different forms of nonlinear behavior have been found to be superior in discriminating parkinsonian and essential tremor. By these methods physiological and pathological tremors can be separated with an error rate below 20% and essential and parkinsonian tremor with an error rate below 10%. This may help to classify tremor time series by objective mathematical criteria and may increase the understanding of the pathophysiological differences underlying these kinds of tremor.

1 Introduction

Tremor of the hand appears in normal as well as in pathological cases. Among the pathological cases, essential and parkinsonian tremor are the most often observed types. Physiological tremor is usually invisible, and the clinical distinction among pathological tremors can be difficult. It is based on further information including clinical assessment and medical history of the patients. There have been attempts to separate these tremors with objective techniques by measuring the acceleration of the hand tremor (Randell 1973; Rietz and Stiles 1974) and by calculating amplitude and frequency characteristics of these time series. No clear-cut separation has been found with these methods (Elble and Koller 1990). With conventional spectral analysis of the data the amplitude and peak frequency of these time series can be taken into account. As with clinical examination, the amplitude provides, by definition, a good criterion to distinguish between physiological (normal) and pathological forms of tremor, but neither amplitude nor peak frequency helps to decide reliably which kind of pathological tremor is present (Deuschl et al. 1992). Previous work (Gantert et al. 1992) showed that physiological tremor can be regarded as a linear stochastic process consistent with the interpretation of the mechanical system of the hand as a damped linear oscillator driven by the uncorrelated firing of motoneurons (Randall 1973; Rietz and Stiles 1974; Elble and Koller 1990). In the case of pathological tremors nonlinear effects (up to chaos) have been established (Gantert et al. 1992). These results suggested that features, especially of nonlinear dynamics, may lead to a good classification of the different kinds of tremor. The present study was undertaken to establish mathematical tests toward an objective classification of tremors on the basis of waveform characteristics of accelerometer recordings. The results indicate that features other than amplitude and peak frequency can provide better criteria to classify the various kinds of postural hand tremors.

2 Patients and recording procedure

Normal subjects and patients with clinically definite essential tremor and parkinsonian tremor according to the criteria defined by Elble and Koller (1990) were investigated. The recording technique has been described in detail elsewhere (Deuschl et al. 1991). Briefly, the subjects' hand tremor was investigated by light-weight piezoresistive accelerometers. The forearms were fixed in a special arm mold, and all the recordings which have been analyzed here were obtained during voluntarily outstretched posture. Thus, only postural hand tremor was investigated. The acceleration signals were filtered (1–150 Hz) and digitized with a rate of 300 Hz. The length of each record is about 35 s so that 10240 data points were obtained. The time series have been normalized to equal variance. Thus, no information about the amplitude entered our analysis. A data pool consisting of 100 time series of physiological, 25 time series of parkinsonian, and 15 time series of essential tremor was used to judge

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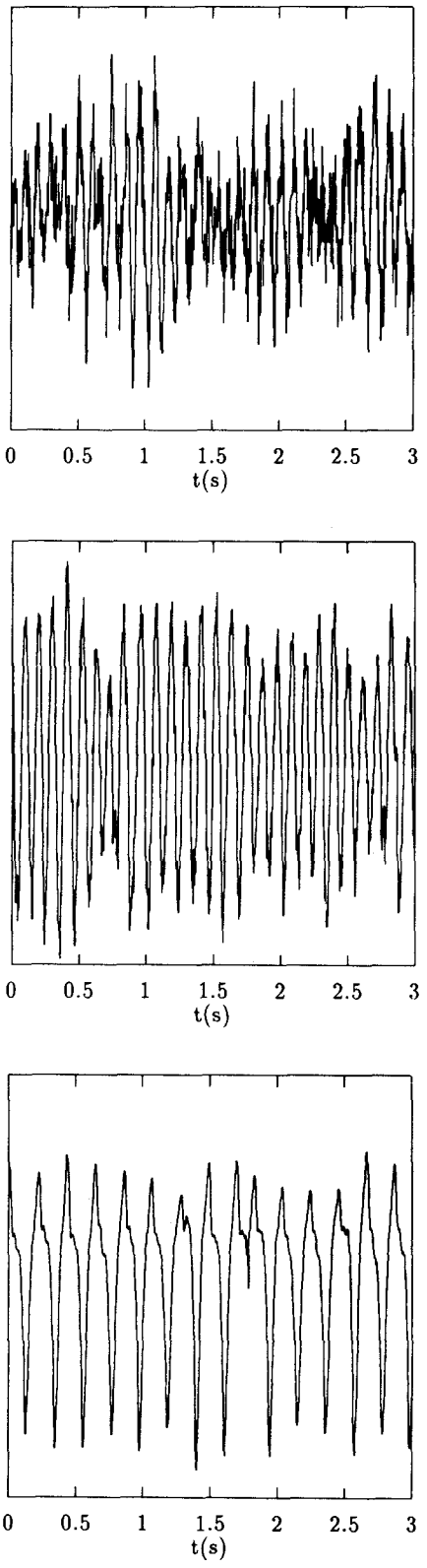

Fig. 1a–c. Acceleration of the hand in physiological tremor (**a**), essential tremor (**b**) and parkinsonian tremor (**c**)

the quality of the features for classification. Examples of the time series analyzed are shown in Fig. 1a–c.

3 Classification in general

The classification of time series requires two steps. The first one consists of extracting characteristic features of the data and the second of assigning these characteristics to the different classes under consideration. For the first step, in general, any mathematical mapping of the data to a number can be used. The second step is a mapping from these features to the three groups of tremor. This mapping called the classificator, may be a classical linear or a nonlinear one like that of the modern neuronal networks. If only one feature is used, linear and nonlinear classificators yield the same result. Differences – and advantages – for the nonlinear classificators may appear in higher-dimensional feature spaces. In this case one needs a larger set of data to obtain a reliable classificator function. With respect to the limited data pool the classification of the features is done by one-dimensional linear standard classificators (i.e., determining the point of maximal distance of the cumulative distributions of the features of the different kinds of tremor). It should be emphasized that the crucial problem is not the classificator function (linear or nonlinear), but the selection of well-discriminating features. In addition, the features should contribute to an understanding of the properties of the various kinds of tremor.

4 Time domain features

Features discriminating in the time domain can be deduced from tests for linearity, from the distribution of the data points, from a measure for time reversal invariance of time series, and from the autocorrelation function. These features will be briefly introduced, and their application will be discussed in Sect. 6.

4.1 Tests for linearity

Linearity tests within the time domain are based on the assumption that the time series can be described by an autoregressive model. These tests are analyzing time structure by third moments. Two such tests have been established in the field of time series analysis: the test of Keenan (1985) and the test of Tsay (1986). They differ with respect to the number of third moments that enter the test statistics. The detailed explanation of these tests is beyond the scope of this paper.

4.2 Features of the distribution

Although the dynamics is the fundamental property characterizing the different types of tremor, the pure distributions of the time series points neglecting the time structure might exhibit discriminating features. Because the mean of the time series is set to zero and their

variance is set equal to one, these distributions are determined by their higher moments. The third moment characterizes the skewness of the distribution. An estimator of the third moment is defined by

$$\hat{m}_3 := 1/(N-1) \sum_{t=1}^{N} x(t)^3 \tag{1}$$

A measure of the peakedness of the distribution is the fourth moment estimated by

$$\hat{m}_4 := 1/(N-1) \sum_{t=1}^{N} x(t)^4 \tag{2}$$

This measure of peakedness is obviously very sensitive to outliers in the data. As a more robust measure we used

$$\hat{m}_a := 1/(N-1) \sum_{t=1}^{N} |x(t)| \tag{3}$$

In the case of gaussian distribution the expectation of $\hat{m}_a$ is equal to $\sqrt{2/\pi}$. More peaked distributions show a larger value of $\hat{m}_a$.

A measure of the structure of the distribution $p(x)$ is given by its entropy defined by

$$e := \int p(x) \ln p(x) \, dx \tag{4}$$

The entropy has been estimated via a kernel density estimator using a gaussian kernel. Taking into account the time structure of the process it is possible to estimate the entropy in two dimensions. Therefore, a time-delay embedding of the time series has been constructed to obtain bivariate data:

$$x(t) := [x(t), x(t-\tau)] \tag{5}$$

where τ was chosen to be the first zero-crossing of the autocorrelation function to achieve a good resolution of the two-dimensional density. Calculating entropies in higher dimensions is not possible due to the limited number of data points.

4.3 Time reversal invariance

As described above, the physiological tremor is assumed to be a linear gaussian process. The properties of a time series of such a process are invariant under reversal of the time direction. This invariance is also conserved if the time series is observed by a nonlinear mapping. For nonlinear processes it is well known that the time reversal invariance often does not hold. Thus, properties of the time series that change if the time series is analyzed with reversed time can be used to detect an underlying nonlinear process. We used the difference of the conditional expectation forward and backward as such a measure:

$$E\{x(t+\tau)|x(t) = y\} - E\{x(t-\tau)|x(t) = y\} \tag{6}$$

These conditional expectations can be estimated by kernel regression estimators under weak assumptions on the process $x(t)$ (Robinson 1983). The estimator of $E\{x(t+\tau)|x(t) = y\}$ is defined by

$$\hat{E}(y,\tau) = \frac{\sum_{i=1}^{N} x(i+\tau) \Psi\left[\dfrac{y - x(i)}{h}\right]}{\sum_{i=1}^{N} \Psi\left[\dfrac{y - x(i)}{h}\right]} \tag{7}$$

with

$$\Psi(x) = \begin{cases} 1 - |x| & \text{if } |x| < 1 \\ 0 & \text{if } |x| \geqslant 1 \end{cases} \tag{9}$$

The parameter h fixes the strength of the smoothing. Different reasonable choices for this value did not significantly change the result of the classification. Although the estimator $\hat{E}$ is biased, it should be noted that for processes with time-reversal invariance the estimation of the difference of the conditional expectations is unbiased. This measure of time reversal is still a function of the lag τ and of the value y. As a characteristic feature of the time series we therefore defined the measure

$$\hat{D}(\tau) := \int [\hat{E}(y,\tau) - \hat{E}(y, -\tau)]^2 \, dy \tag{10}$$

$$\hat{D}_m := \max_{\tau} \hat{D}(\tau) \tag{11}$$

Integrating with respect to y gives a measure of the mean deviation of the conditional expectations, and choosing τ to maximize D is a natural way to obtain a scalar value that characterizes the effect of time reversal. Instead of that choice it would also be possible to connect τ to a typical time scale of the time series.

4.4 Asymmetric decay of the autocorrelation function

The autocorrelation function $\gamma(h)$ of a damped linear oscillator with frequency ω shows an exponential decay:

$$\gamma(h) = \cos \omega h \exp(-|h|/\tau) \tag{12}$$

with a characteristic decay time τ.

The absolute values of the extrema of $\gamma(h)$ form a monotone decreasing series. In case of nonlinear oscillations this series can be nonmonotone. Especially in the case of the time series with negative skewness the absolute values of the minima of the autocorrelation function can be less than those of the following maxima.

As a measure for such an asymmetric decay we calculate the difference γ_d of the absolute values of the first two extrema of the autocorrelation function. In contrast to the third moment m_3, which measures only the skewness of the distribution of the data points, this measure also includes information of the time structure and is more robust since only second moments have to be calculated.

5 Frequency domain features

In the frequency domain, discriminating features were obtained from the bispectrum (Brillinger 1965).

An often-used test for nonlinearities is based on the estimation of the bispectrum. In contrast to the above-mentioned tests for linearity in the time domain these

tests do not depend on special model assumptions of the process.

Since the ordinary spectrum is only related to second moments, it is not sensitive to a possible nonlinearity of the dynamics. As an extension of the power spectrum the bispectrum $f(\omega_1, \omega_2)$ is defined by the Fourier transform of the triple correlations $c(\tau_1, \tau_2)$:

$$c(\tau_1, \tau_2) := E\{x_t x_{t+\tau_1} x_{t+\tau_2}\} \tag{13}$$

$$f(\omega_1, \omega_2) := \frac{1}{2\pi} \sum_{\tau_1=-\infty}^{\infty} \sum_{\tau_2=-\infty}^{\infty} c(\tau_1, \tau_2) \exp[-i(\omega_1 \tau_1 + \omega_2 \tau_2)] \tag{14}$$

In the case of a process with a linear dynamics which is driven by gaussian noise it can be shown that the triple correlations and, hence, the bispectrum are zero.

Subba Rao and Gabr (1980) and Hinich (1982) developed statistical tests that decide whether the estimated bispectrum is consistent with the hypothesis of the bispectrum being zero. The two tests differ merely in the way of normalizing the variance of the bispectrum estimates. We used the estimate of the test statistic as a characterizing feature of the time series.

6 Results

The tests for linearity by Keenan, Tsay, Subba Rao, and Hinnich showed only poor discrimination between physiological and pathological or between pathological tremors, although often recommended in the statistical literature. Their error rate was always larger than 40%. The test of Keenan failed in discriminating different tremors significantly (Fig. 2). The test of Tsay shows evidence for nonlinear structure in the case of pathological tremors, but the distribution of the test statistics is too broad to obtain a reliable classification (Fig. 3). In the frequency domain the test of Hinich gave slightly better results than the one of Subba Rao. Both tests show a similar behavior to that of the test of Tsay, because they detect nonlinear structures in many of the time series of pathological tremors, but they failed in discriminating between the different forms of tremor with a sufficiently low error rate.

6.1 Physiological and pathological tremor

The discrimination between physiological and pathological human hand tremor is possible with some of the above-described methods with an error rate below 20%. This result is less important for clinical application, since the amplitude alone provides a discrimination criterion of better quality because pathological tremors are mainly defined by their increased amplitude. However, it is of general interest to understand features separating these two types of time series. The best discrimination between physiological and pathological tremors was obtained by analyzing the linearity and gaussianity of the tremor dynamics. Physiological tremor was found to have these features. In case of time series from pathological tremors all features under consideration show deviations from the expected result for linear and gaussian processes. These deviations were most significantly measured with the tests based on the distribution of the data points as described in Sect. 4.2. The cumulative distribution of the third moments of the tremor data is shown in Fig. 4. The more-negative skewness of the parkinsonian tremor is in good agreement with the visual impression from various data sets. It is visible that also the physiological tremor time series (thick line) show a slight deviation from zero skewness.

The result from measuring the peakedness with $\hat{m}_a$ is shown in Fig. 5. For physiological tremor this value is in

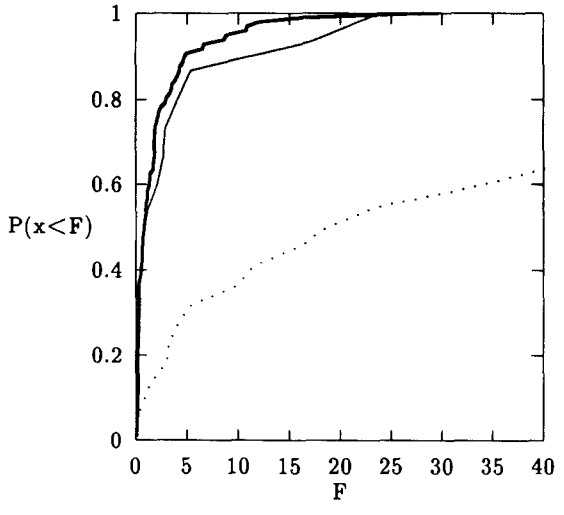

Fig. 2. Cumulative distribution of the test statistics F of the test for linearity from Keenan of physiological tremor (*thick line*), essential tremor (*thin line*), parkinsonian tremor (*dotted line*)

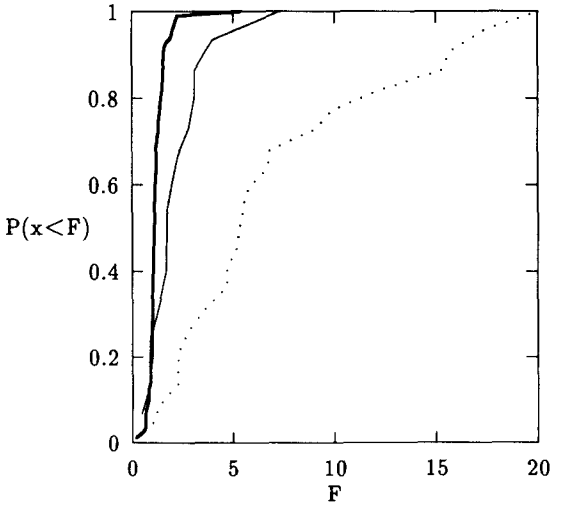

Fig. 3. Cumulative distribution of the test statistics F of the test for linearity from Tsay of physiological tremor (*thick line*), essential tremor (*thin line*), parkinsonian tremor (*dotted line*)

good agreement with the expectation $\sqrt{2/\pi} = 0.798\ldots$ for gaussian data. The larger values of $\hat{m}_a$ for the pathological cases indicate a more bulgy distribution.

The entropy gives a measure of the structure of the distribution. As the gaussian distribution is the distribution with maximal entropy, one obtains lower entropies in the case of pathological tremor (Fig. 6).

The result for the two-dimensional entropy does not differ significantly from the result of the entropy in one dimension. The entropy estimated from the time series of physiological tremor deviates slightly from the expected value for gaussian-distributed data. This is due to the fact that the estimator of the entropy is only asymptotically unbiased. Instead of the entropy any test for normal distribution (e.g., Kolmogorov-Smirnov) can be used but the results are almost the same.

6.2 *Essential and parkinsonian tremor*

Because of its clinical relevance it is more important to provide good features to discriminate between the two pathological types of tremor. Up to now there were no successful tools to separate these two tremors based on the acceleration measurement only (Elble and Koller 1990). Our analysis confirmed that distinction between the pathological forms of tremor is not possible by the above-mentioned features of the distribution of the data points. As both kinds of pathological tremors show

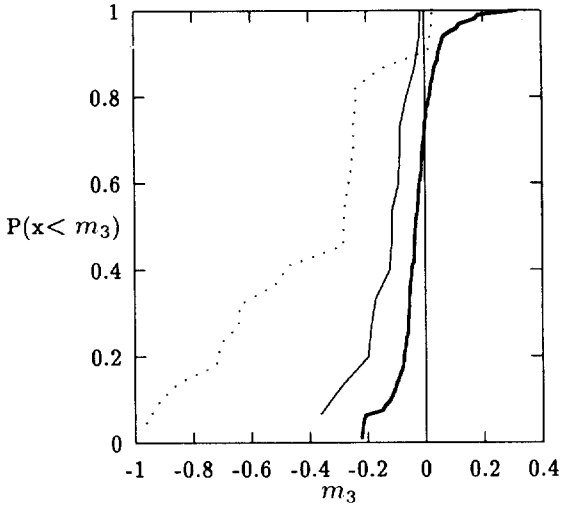

Fig. 4. Cumulative distribution of the third moment $\hat{m}_3$ of physiological tremor (*thick line*), essential tremor (*thin line*), parkinsonian tremor (*dotted line*)

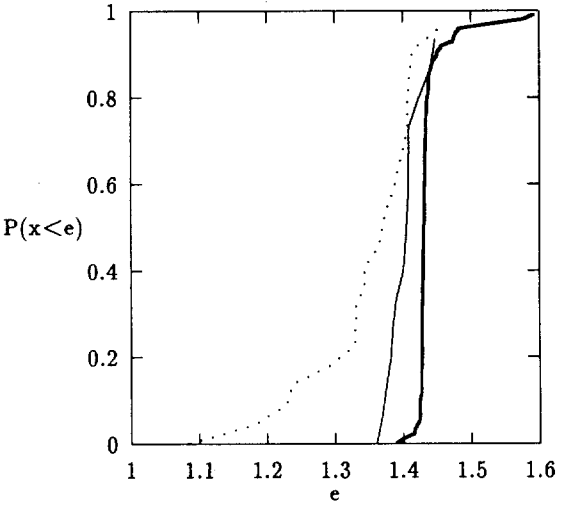

Fig. 6. Cumulative distribution of the entropy of physiological tremor (*thick line*), essential tremor (*thin line*), parkinsonian tremor (*dotted line*). The entropy of a gaussian distribution is equal to 1.419...

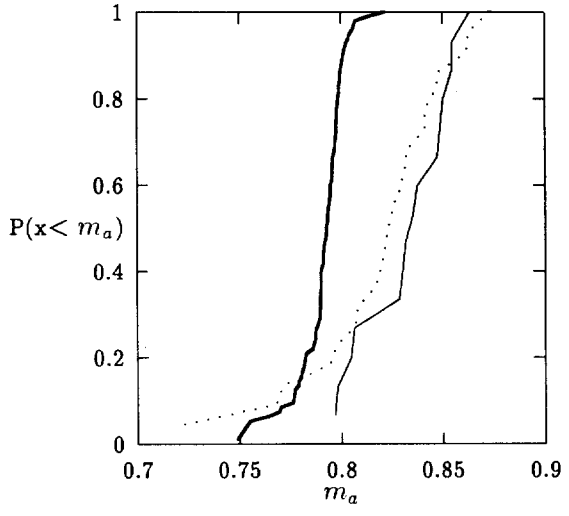

Fig. 5. Cumulative distribution of measure for peakedness $\hat{m}_a$ of physiological tremor (*thick line*), essential tremor (*thin line*), parkinsonian tremor (*dotted line*). In a gaussian distribution m_a is equal to 0.798...

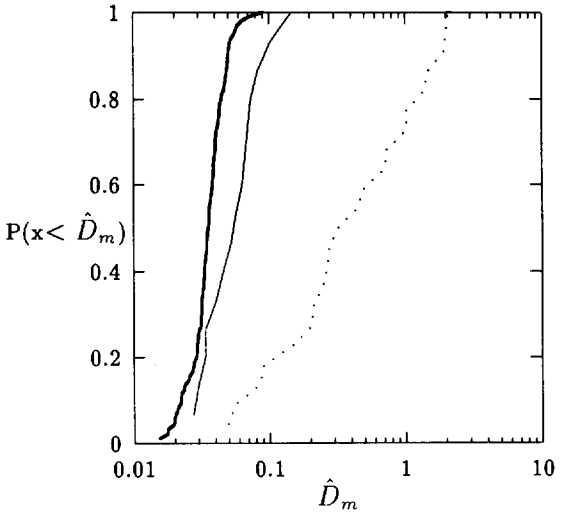

Fig. 7. Cumulative distribution of the measure for time reversal invariance of physiological tremor (*thick line*), essential tremor (*thin line*), parkinsonian tremor (*dotted line*). The smoothing parameter h is equal to 0.02

significant nonlinear behavior, we conclude that different types of nonlinear structures have to be separated. In fact, the measure of time reversal invariance and of the asymmetric decay of the autocorrelation provide such a classification with an error rate less than 25%. The violation of time reversal invariance is sometimes visible already in the raw traces in time series of parkinsonian tremor. Almost any time series from parkinsonian tremor exhibits a larger deviation from time reversal invariance than the time series of essential tremor, whereas the deviation between essential and physiological tremor is much smaller (Fig. 7).

The asymmetric decay of the autocorrelation function of a time series of a parkinsonian tremor is shown in contrast to the symmetric decay of the autocorrelation function of a time series of an essential tremor in Fig. 8. By this feature the best separation between the two

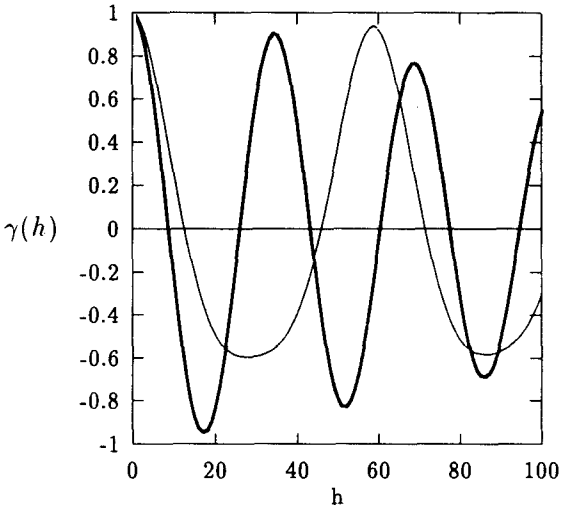

Fig. 8. Autocorrelation function of the time series of an essential tremor (*thick line*) and of the time series of a parkinsonian tremor (*thin line*)

Table 1. Survey of the achieved maximal discrimination rates for the described features

	Maximal discrimination (%)	
	Physiological vs pathological	Essential vs parkinsonian
Keenan	5	55
Tsay	55	60
m_a	70	20
m_3	60	60
Entropy	75	40
2-dimensional entropy	80	45
Time reversal	50	75
Bispectrum (Hinich)	65	55
γ_a	60	90

The maximal discrimination rate of a feature is the maximal distance of the cumulative distributions obtained from the different tremors. The discrimination rate between physiological and pathological tremor is the minimum of the discrimination rates for physiological/essential and physiological/parkinsonian tremors

pathological forms of tremor could be obtained with an error rate of 10% (Table 1).

7 Summary

We found some features hitherto not used for the analysis of tremor time series. Compared with the tests for linearity which are often recommended in the statistical literature, our methods showed a more reliable discrimination between physiological and pathological tremor or between essential and parkinsonian tremor, respectively.

Based only on normalized data corresponding to the waveform without amplitude information, the discrimination between the physiological and pathological kinds of human hand tremor is possible with an error rate below 20%. To separate essential and parkinsonian tremor it is necessary to discriminate between different kinds of nonlinear processes. The measure of time reversal invariance and the asymmetric decay of the autocorrelation function provide classifications with acceptable error rates of 25% and 10%, respectively. These tests provide a new tool which may be used for objective classification of tremors. Future work has to be done to test these criteria prospectively with an enlarged data pool including complicated tremors. This will help to improve the validity of these tests. It should be kept in mind that the criteria to distinguish between normal and pathological tremor and among pathological tremors are different, which may provide additional information to better understand the underlying mechanisms.

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