Games Solved: Now and in the Future

References:

- [vdHUvR02] H. J. van den Herik, J. W. H. M. Uiterwijk, and J. van Rijswijck. Games solved: Now and in the future. Artificial Intelligence, 134:277–311, 2002.

Acknowledgement:

- The slides of this chapter are partly modified from Prof. Hsu's teaching material, under the courtesy of Hsu. http://www.iis.sinica.edu.tw/~tshsu/tcg2007/index.html

The Domain to Discuss in the Paper

- Two-person zero-sum games with perfect information.
  - Zero-sum:
    - one player's loss is exactly the other player's gain, and vice versa.
    - There is no way for both players to win at the same time.
- The aim: the game characters for the solution of a game.
  - It is concluded that decision complexity is more important than state space complexity.
  - There is a trade-off between knowledge-based methods and brute-force methods.
  - There is a clear correlation between the first-player's initiative and the necessary effort to solve a game.

Definitions (I)

- Game-theoretic value of a game: the outcome, i.e.,
  - win, loss or draw, when all participants play optimally.
- Classification of games' solutions according to L.V. Allis in 1994.
  - Ultra-weakly solved: the game-theoretic value of the initial position has been determined.
  - Weakly solved: for the initial position a strategy has been determined to achieve the game-theoretic value against any opponent.
    - The strategy must be efficient and practical in terms of resource usage.
  - Strongly solved: a strategy has been determined for all legal positions.

Definitions (II)

- State-space vs. game-tree Complexity:
  - State-space complexity of a game: the number of all legal positions in a game.
  - Game-tree (or decision) complexity of a game: the number of all leaf nodes in a solution search tree.
    - A solution search tree is a tree where the game-theoretic value of the root position can be decided.
- Convergence vs. Divergence:
  - A convergence game: the size of the state space decreases as the game progress.
    - Start with many pieces on the board and pieces are gradually removed during the course of the game.
  - A divergence game: the size of the state space increases as the game progress.
    - Start with an empty board and pieces are gradually added during the course of the game.
- Initiative: the right to move first

State-Space vs. Game-Tree Complexities

- State-space complexity
  - The number of states or positions of a game.
  - E.g.,
    - Tic-tac-toe has $3^9 = 19,683$.
    - Weichi (Go) has $3^{361} \sim= 10^{172}$.
- Game-tree complexity
  - Viewed as a good indication of a game's decision complexity.
  - E.g.,
    - Tic-tac-toe: $9! = 362,880$.
      - Remove unreachable 355,168.
      - Remove rotations 26,830. ← game tree complexity.
    - Weichi (Go): $361! \sim= 10^{768}$.
      - Game-tree complexity: 10^{360}

Theory of ... ne Solved

State-space complexities and game-tree complexities of various games

Id.	Game	State-space compl.	Game-tree compl.	Reference
1	Awari	10^{12}	10^{32}	[3,7]
2	Checkers	10^{21}	10^{31}	[7,94]
3	Chess	10^{46}	10^{123}	西洋棋一般公認複雜度第四
4	Chinese Chess	10^{48}	10^{150}	象棋一般公認複雜度第三
5	Connect-Four	10^{14}	10^{21}	[2,7]
6	Dakon-6	10^{15}	10^{33}	[62]
7	Domineering (8 × 8)	10^{15}	10^{27}	[20]
8	Draughts	10^{30}	10^{54}	[7]
9	Go (19 × 19)	10^{172}	10^{360}	圍棋一般公認複雜度最高
10	Go-Moku (15 × 15)	10^{105}	10^{70}	[7]
11	Hex (11 × 11)	10^{57}	10^{98}	[90]
12	Kalah(6,4)	10^{13}	10^{18}	[62]
13	Nine Men's Morris	10^{10}	10^{50}	[7,44]
14	Othello	10^{28}	10^{58}	比五子棋複雜度低
15	Pentominoes	10^{12}	10^{18}	[85]
16	Qubic	10^{30}	10^{34}	[7]
17	Renju (15 × 15)	10^{105}	10^{70}	百年前日本即有職業連珠棋院
18	Shogi	10^{71}	10^{226}	日本將棋一般公認複雜度第二高
	Connect6	10^{172}	10^{140} ~ 10^{188}	

資料來源：
Herik的論文
[2002]

Game Space

- A double dichotomy of the game space

↑ log log state-space complexity

Category 3 if solvable at all, then by knowledge-based methods	**Category 4** unsolvable by any method
Category 1 solvable by any method	**Category 2** if solvable at all, then by brute-force methods

log log game-tree complexity →

Fig. 1. A double dichotomy of the game space.

Brute-force vs. Knowledge-based Methods

- Games with both a relative low state-space complexity and a low game-tree complexity have been solved by both methods.
  - Category 1
  - Connect-four and Qubic
- Games with a relative low state-space complexity have mainly been solved with brute-force methods
  - Category 2
  - Namely by constructing endgame database
  - Nine Men's Morris, Triangular Nim.
- Games with a relative low game-tree-complexities have mainly been solved with knowledge-based methods.
  - Category 3
  - Namely, by intelligent (heuristic) searching
  - Sometimes, with the helps of endgame databases
  - Go-Moku, Renju, and k-in-a-row games

The Advantage of the Initiative

- Theorem (or arguments) made by Singmaster in 1981: The first-player has advantages.
  - Two kinds of positions
    - P-positions: the previous player can force a win.
    - N-positions: the next player can force a win.
  - Arguments
    - For the first player to have a forced win, just one of the moves must lead to a P-position.
    - For the second player to have a forced win, all of the moves must lead to N-positions.
    - It is easier for the first player to have a forced win assuming all positions are randomly distributed. (Example: Triangular Nim)
    - Can be easily extended to games with draws.
  - Remarks:
    - One small boards, the second player is able to draw or even to win for certain games.
    - Cannot be applied to the infinite board.

The Advantage of the Initiative in Practice

- Some solutions:
  - Prohibited moves: the first player are not allowed to play some moves.
    - E.g., no double 3 and 4 for Renju.
  - Komi: The first player needs to win by more territory.
    - E.g., Go.
  - Swapping: a player makes the first move, the second player decides the color to play thereafter.
    - E.g., Hex, Renju. Also called Pie rule.
  - The first move places one stone, the rest places two stones.
    - E.g., Connect6.
  - The second player wins when repeating boards.
    - E.g., Shogi.
  - The first player uses less time.
- Must be simple and try to be as fair as possible.

How to Make Use of Initiative

- A potential universal strategy for winning a game:
  - Try to obtain a small initiative
    - The opponent must react adequately on the moves played by the player.
  - To reinforce the initiative the player searches for threats, and even a sequence of threats using an evaluation function E.
- Threat-space search
  - Search for threats only!

Questions to be Researched

- Can perfect knowledge obtained from solved games be translated into rules and strategies which human beings can assimilate?
- Are such rules generic, or do they constitute a multitude of ad-hoc recipes?
- Can methods be transferred between games?
  - More specifically, are there generic methods for all category-*n* games, or is each game in a specific category a law unto itself?

Convergent Games

- Can be possibly solved by the method of endgame databases if we can enumerate and store all possible positions.
- Problems solved:
  - Nine Men's Morris: a total of 7,673,759,269 states.
    - The game theoretic value is proved to be draw in the year 1993.
  - Mancala games
    - Awari: in the year 2002.
    - Kalah: in the year 2000 up to, but not equal, Kalah(6,6)
  - Triangular Nims
  - Checkers: in the year 1994
    - By combining Endgame databases, middle-game databases and verification of opening analysis.
    - Solved the so called 100-year position.
  - Chess/Chinese chess endgames

Nine Men's Morris

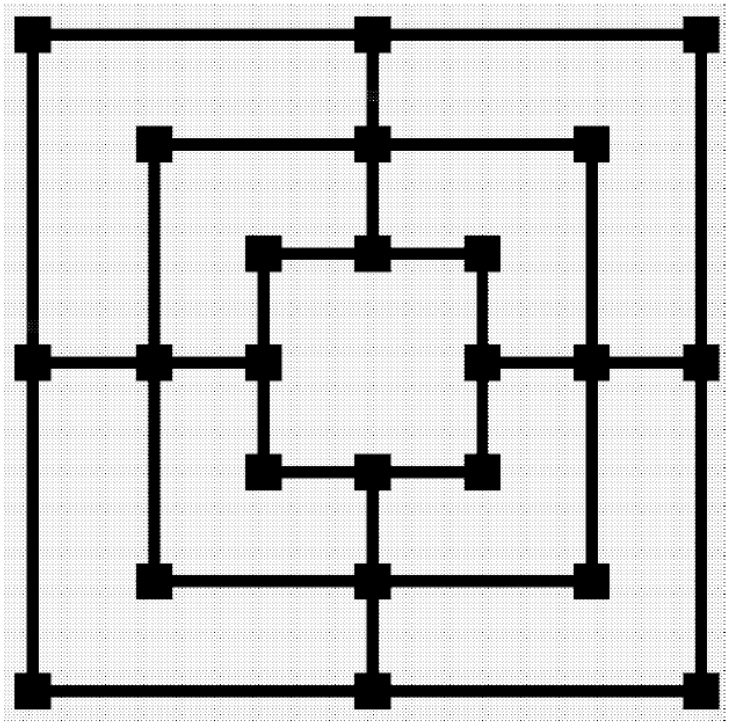


Fig. 2. The Nine-Men's-Morris board.

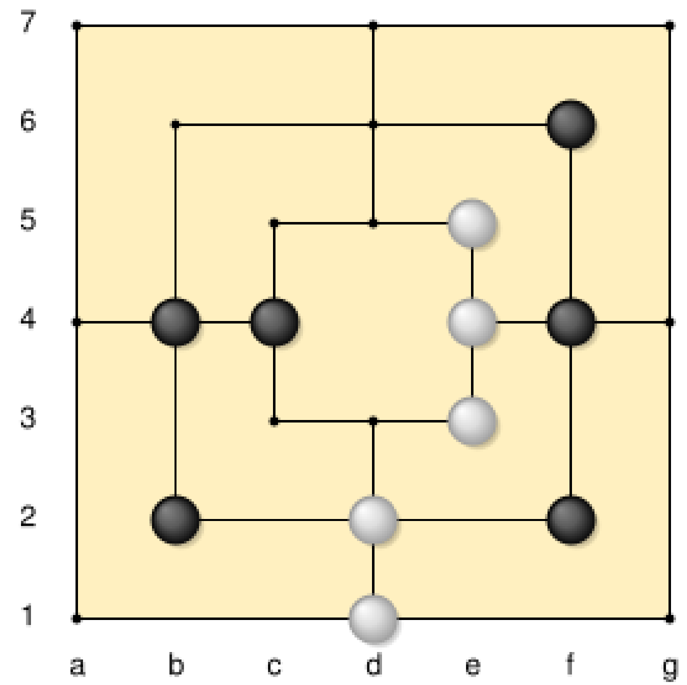


Figure in Wikipedia

Mancala

- Figure from Wikipedia

- Initial position of Awari

North

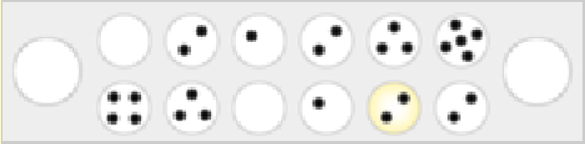
	4	4	4	4	4	4	
	4	4	4	4	4	4	

South

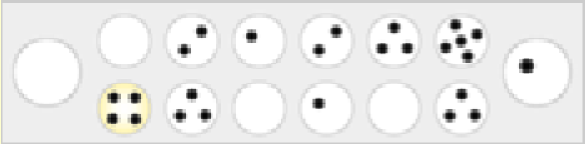
Kalah

- Example:

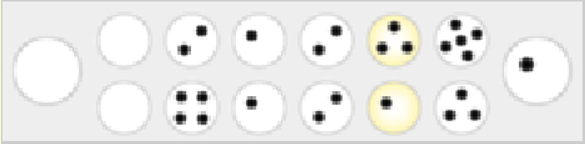
Example turn



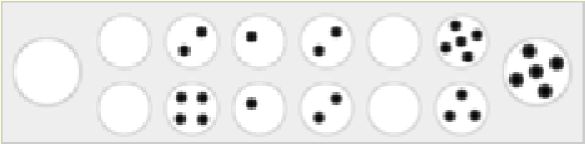
The player begins sowing from the highlighted house.



The last seed falls in the store, so the player receives an extra move.



The last seed falls in an empty house on the player's side, with seeds in the opposite house.



The player captures the 4 seeds and ends his turn.

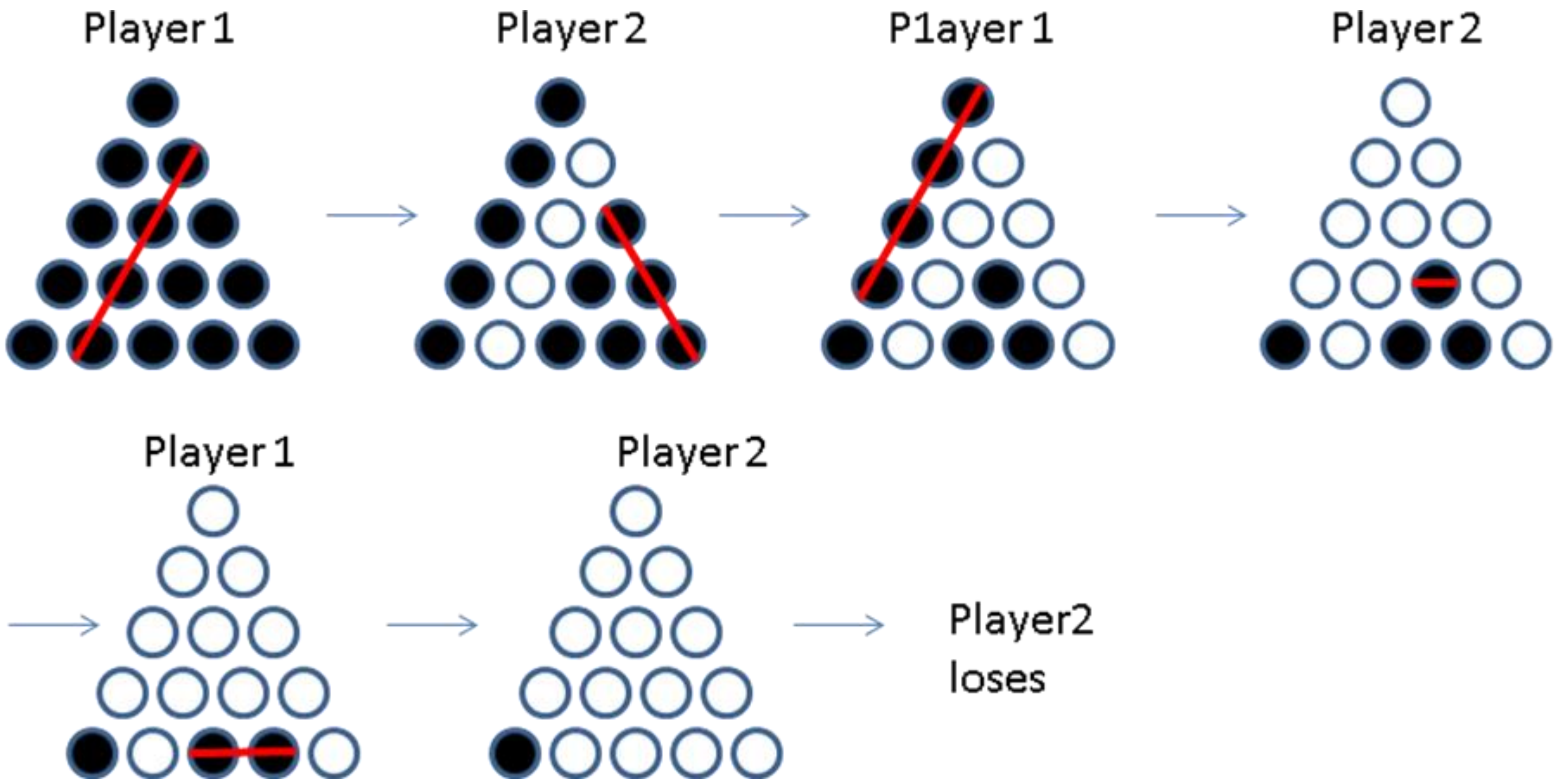
Solved Kalah

Table 2
Game values for Kalah with *m* pits per side and *n* stones per pit

$m \backslash n$	1	2	3	4	5	6
1	D	L	W	L	W	D
2	W	L	L	L	W	W
3	D	W	W	W	W	L
4	W	W	W	W	W	D
5	D	D	W	W	W	W
6	W	W	W	W	W	

Triangular Nim (三角殺棋)

- 5 layer triangular Nim.



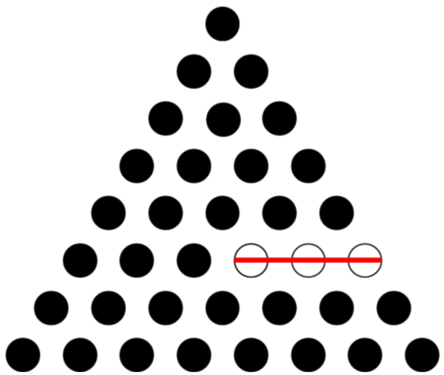
Solved Triangular Nim

- 7 layer: by S.-C. Hsu (許舜欽)
- 8 layer: by H.-H. Lin (林宏軒) & I.-C Wu.
- 9 layer: by Y.-C. Shan (單益章) et al.
- Normal: the one removing the last piece wins.
- Misère: the one removing the last piece loses.

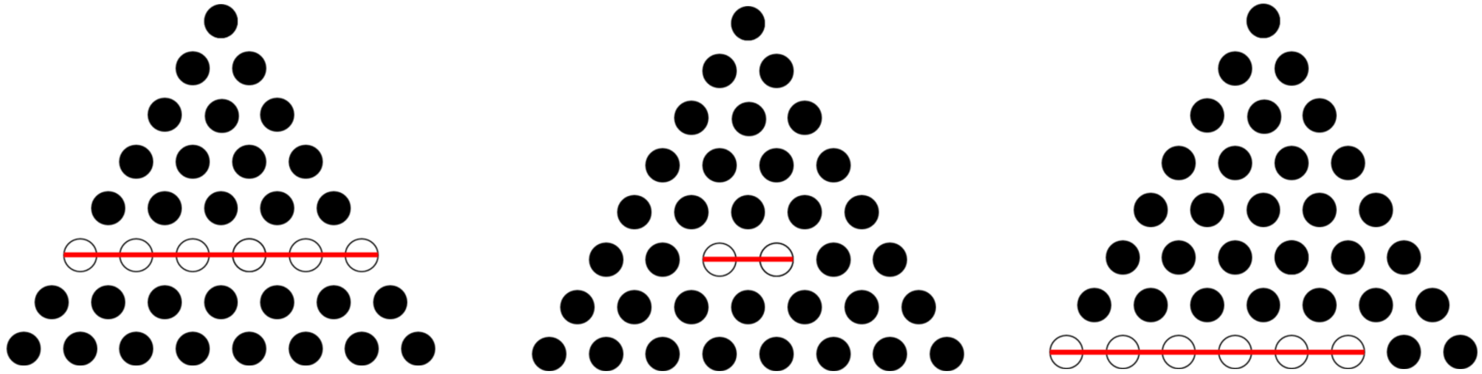
LAYER NUMBER	NORMAL GAME	Misère GAME
1	WIN	LOSS
2	LOSS	WIN
3	WIN	LOSS
4	WIN	WIN
5	WIN	LOSS
6	WIN	WIN
7	WIN	WIN
8	WIN	WIN
9	WIN	WIN

Win the 8 Layer

- Normal:

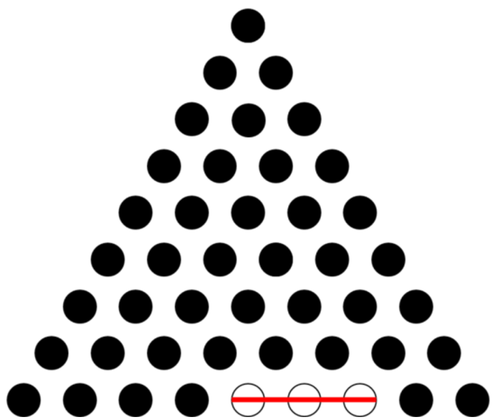


- Misère:

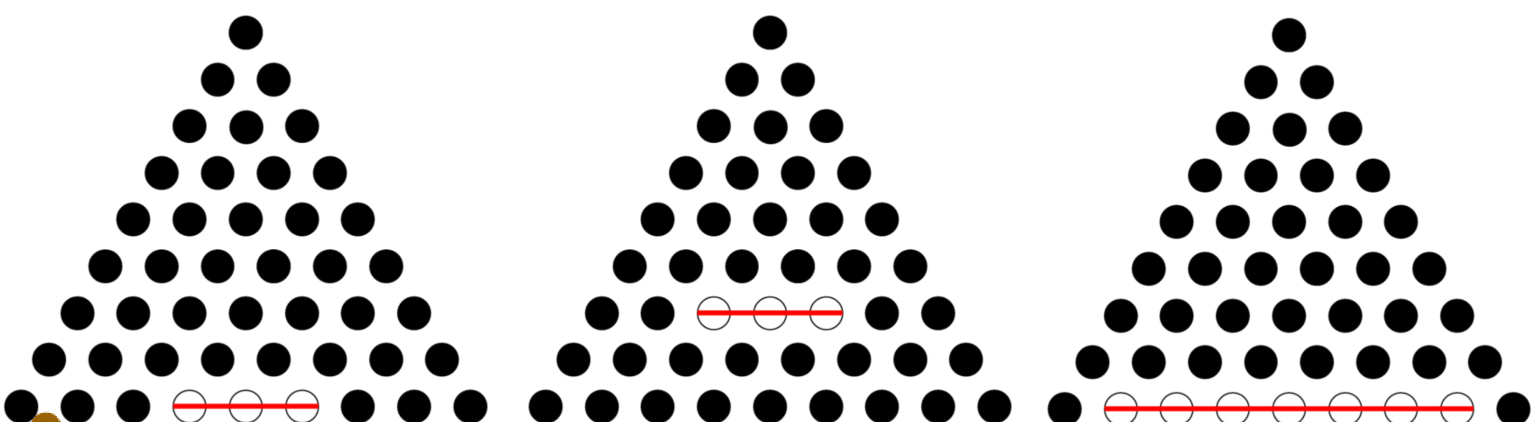


Win the 9 Layer

- Normal:



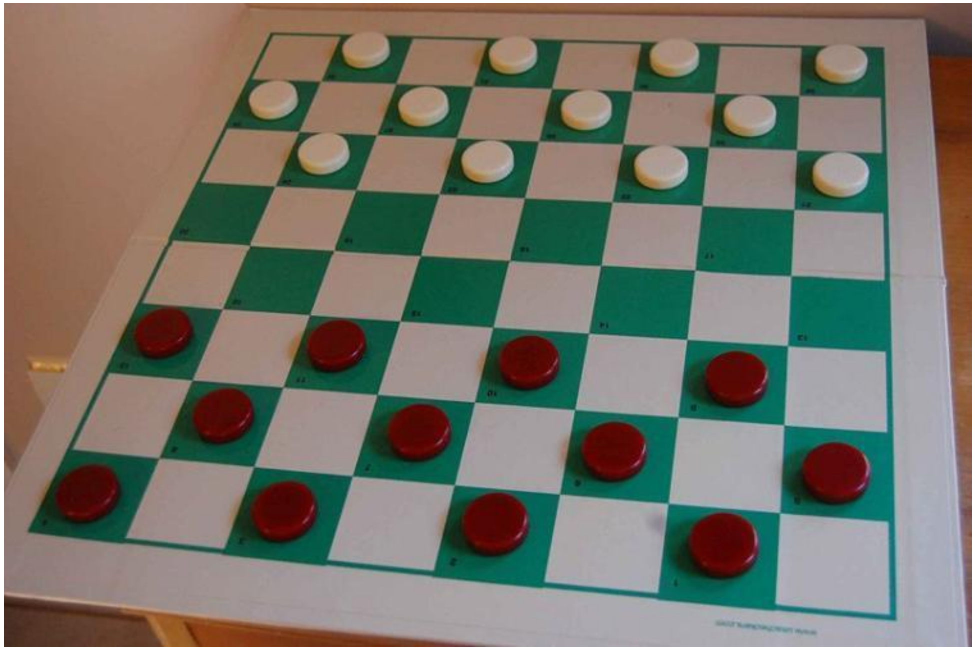
- Misère:



Checkers

American Checkers

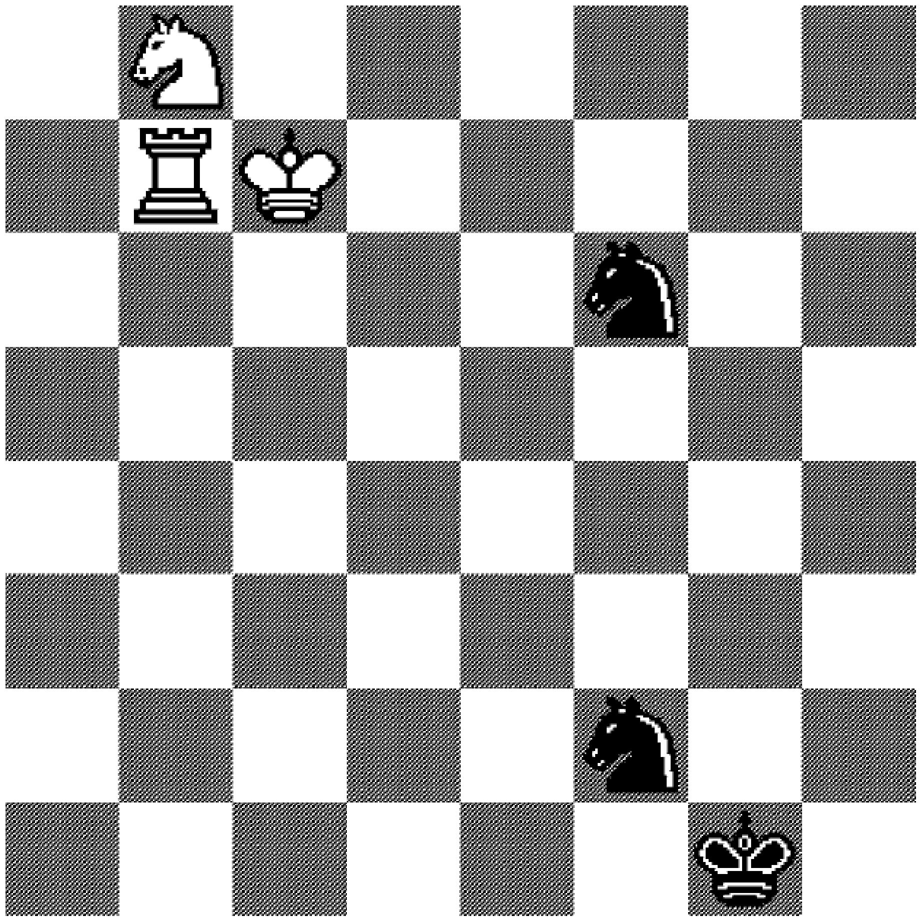
Also called English Draughts



Checkers: Solved

- On July 2007, published in Science Magazine,
  - Chinook's developers announced that the program had been improved to the point where it could not lose a game.
  - If no mistakes were made by either player, the game would always end in a **draw**.

Endgame Chess



The longest (262) chess distance-to-mate position known at present.

Divergent Games

- Connection games
  - Hex (10*10 or 11*11)
  - Connect-four (6*7)
  - Qubic (4*4*4)
  - Gomoku/Renju/Connect6
  - k-in-a-row games
- Polynmino games
  - Pentominoes
  - Domineering

More Divergent Games

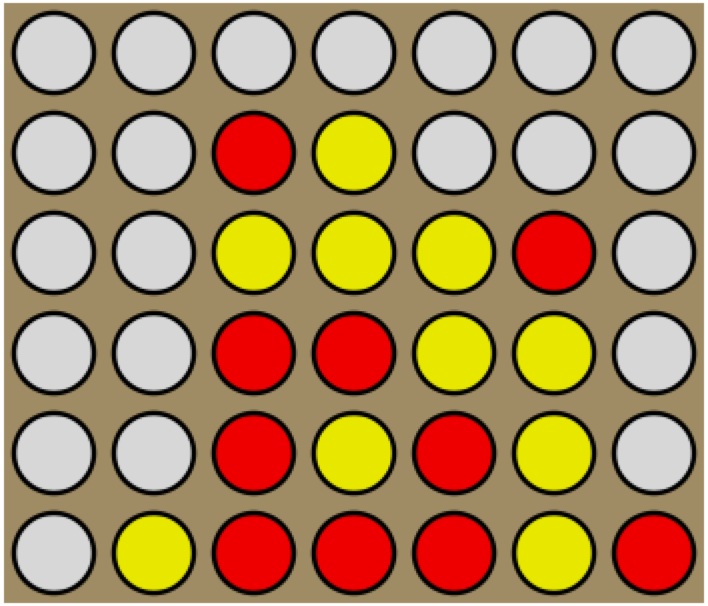
- Othello
  - M. Buro's LOGISTELLO beat the resigning World Champion by 6-0 in 1997.
  - Weakly solved on 6*6 boards by J. Feinstein in 1993.
- Chess
  - DEEP BLUE beat the human World Champion in 1997.
- Chinese chess
  - Still in progress,
- Shogi
  - Still in progress,
- Go
  - Still in progress,

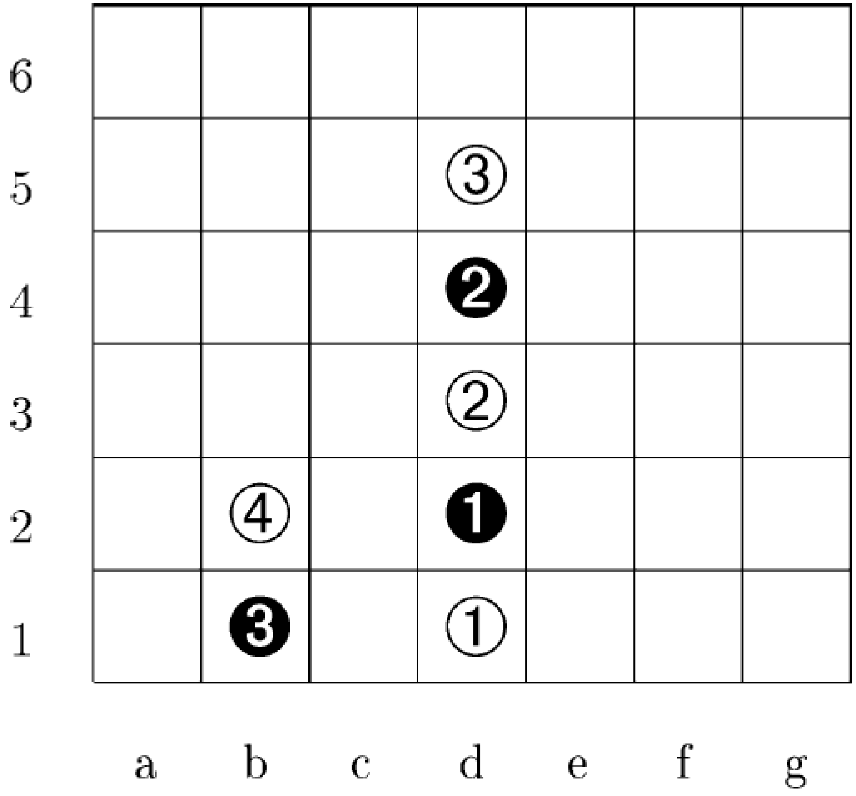
Connection Games (I)

- Connect-four (6*7)
  - Solved by J. Allen in 1989 using a brute-force depth first search with alpha-beta pruning, a transposition table, and killer-move heuristic.
  - Also solved by L.V. Allis in 1988 using a knowledge-based approach by combining 9 strategic rules that identified potential threats of the opponent.
    - Threats are something like forced moved or moves you have little choices.
    - Moves with predictable counter-moves.
  - It is a first-player win.
  - Weakly solved on a SUN 4 workstation using 300+ hours.
- Qubic (4*4*4)
  - A three-dimensional version of Tic-Tac-Toe.
  - Solved in 1980 by O. Patashnik by combining the usual depth-first search with expert knowledge for ordering the moves.

Connect Four

- Drop stone from top.

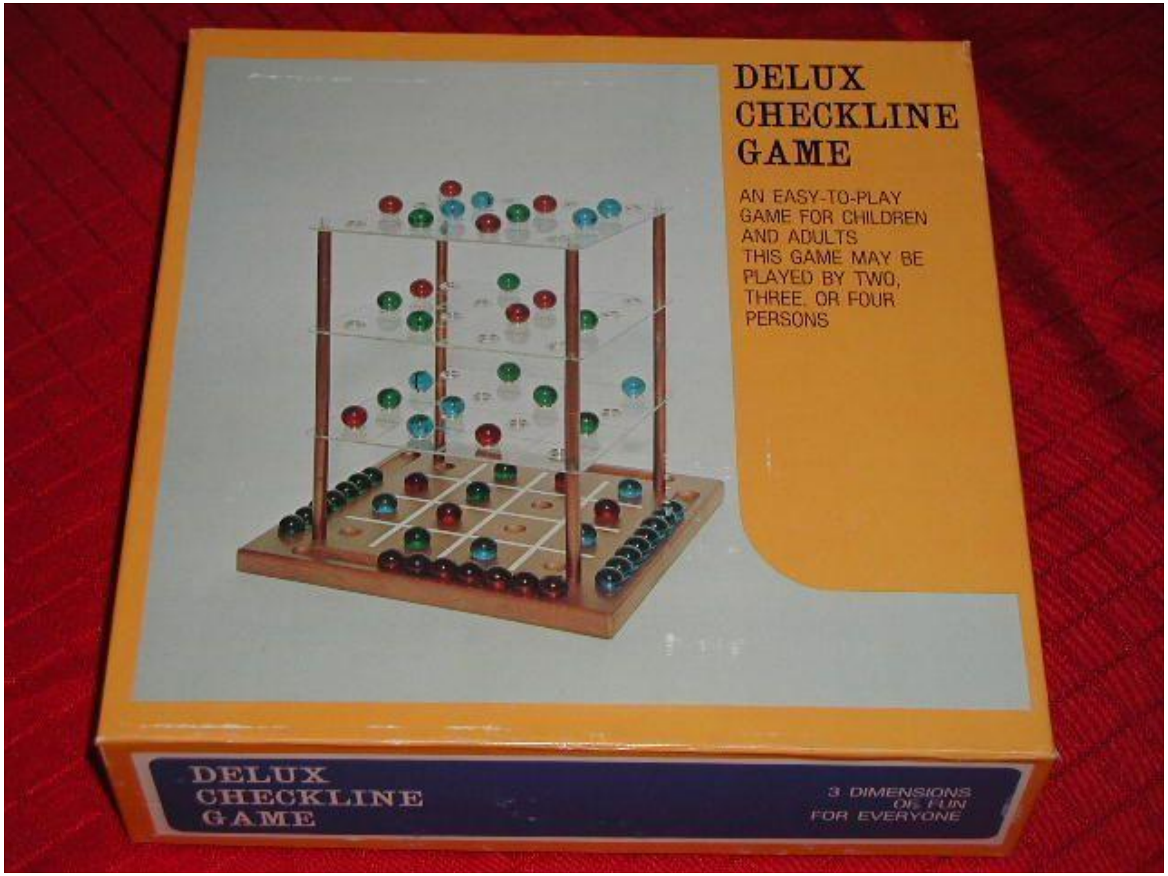




Optimal opening-move sequence in standard Connect-Four.

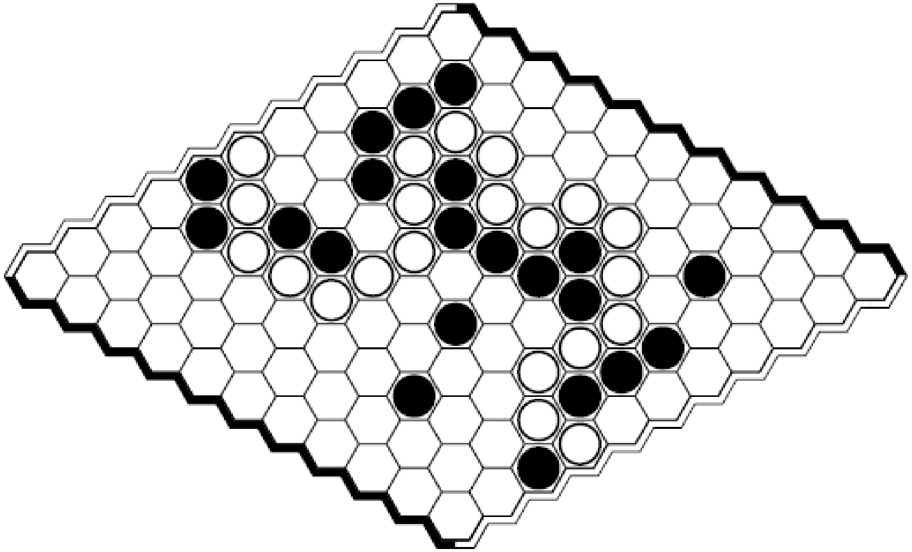
Qubic

- 4x4x4 Connect game.



Hex

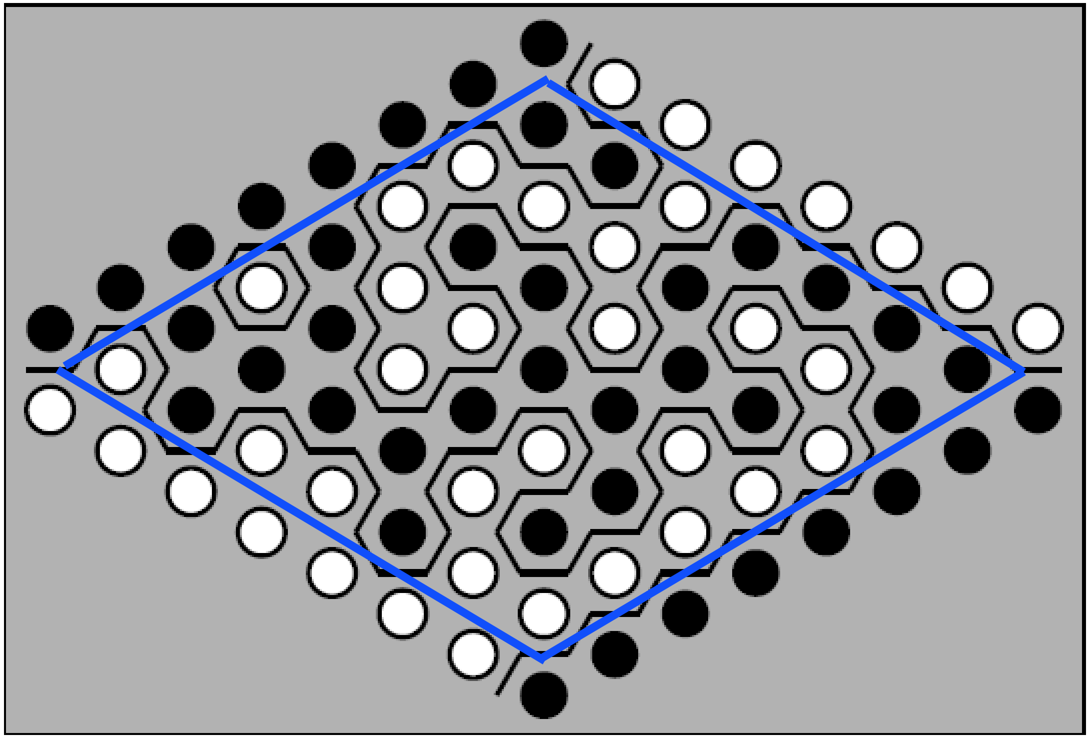
- Both players place a stone alternatively.
- Black wins if
  - connect from the lower left edge to the upper right edge.
- White wins if
  - connect from the lower left edge to the upper right edge.



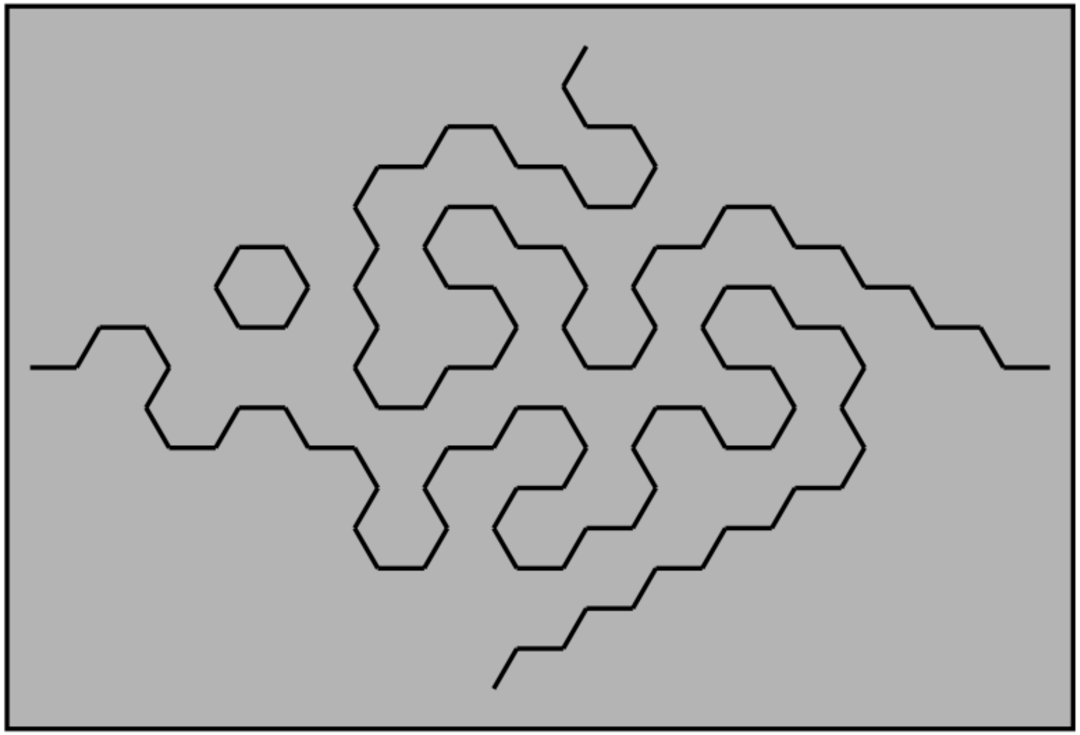
No Draw for Hex

- Theorem: Exactly one of the players can win.
- Proof:
  - Initial condition:
    - True for Hex 2*2
  - Induction hypothesis:
    - it is true for any Hex *i***j*, where $i < n$ or $j < m$.
  - Induction step: try to prove this is true on Hex *n***m*
    - Delete the first row or the last row
      - Give you two white chains w_1 and w_2 (between top and bottom), respectively. Otherwise, Black has black chains (Success).
    - Delete the first column or the last column
      - Give you two black chains b_1 and b_2 (between left and right), respectively. Otherwise, White has white chains (Success).
    - Delete first row, last row, first column and last column
      - Give you either a white chain w_3 or a black chain b_3.
    - Either w_3 intersects with b_1 or b_2, or b_3 intersects with w_1 or w_2;
      - both are contradicting statements.

Another Proof



Another Proof



Strategy-Stealing Argument

- Not a win for SECOND.
  - Using the "strategy-stealing" argument made by John Nash in 1949.
    - If there is a winning strategy for SECOND, FIRST can still win by making an arbitrary first move and using the SECOND's strategy.
    - If using the SECOND's strategy requires playing the chosen first move or any move played before, then make another arbitrary move.
    - An arbitrary extra move can never be a disadvantage in Hex.
      - This is not true for every games.
      - Not a constructive proof.
  - This argument works for any symmetry games when an arbitrary extra move can never be a disadvantage.
- Pie-Rule:
  - The *one-move-equalization* rule:
    - one player plays an opening move and the other player then has to decide which color to play for the reminder of the game.

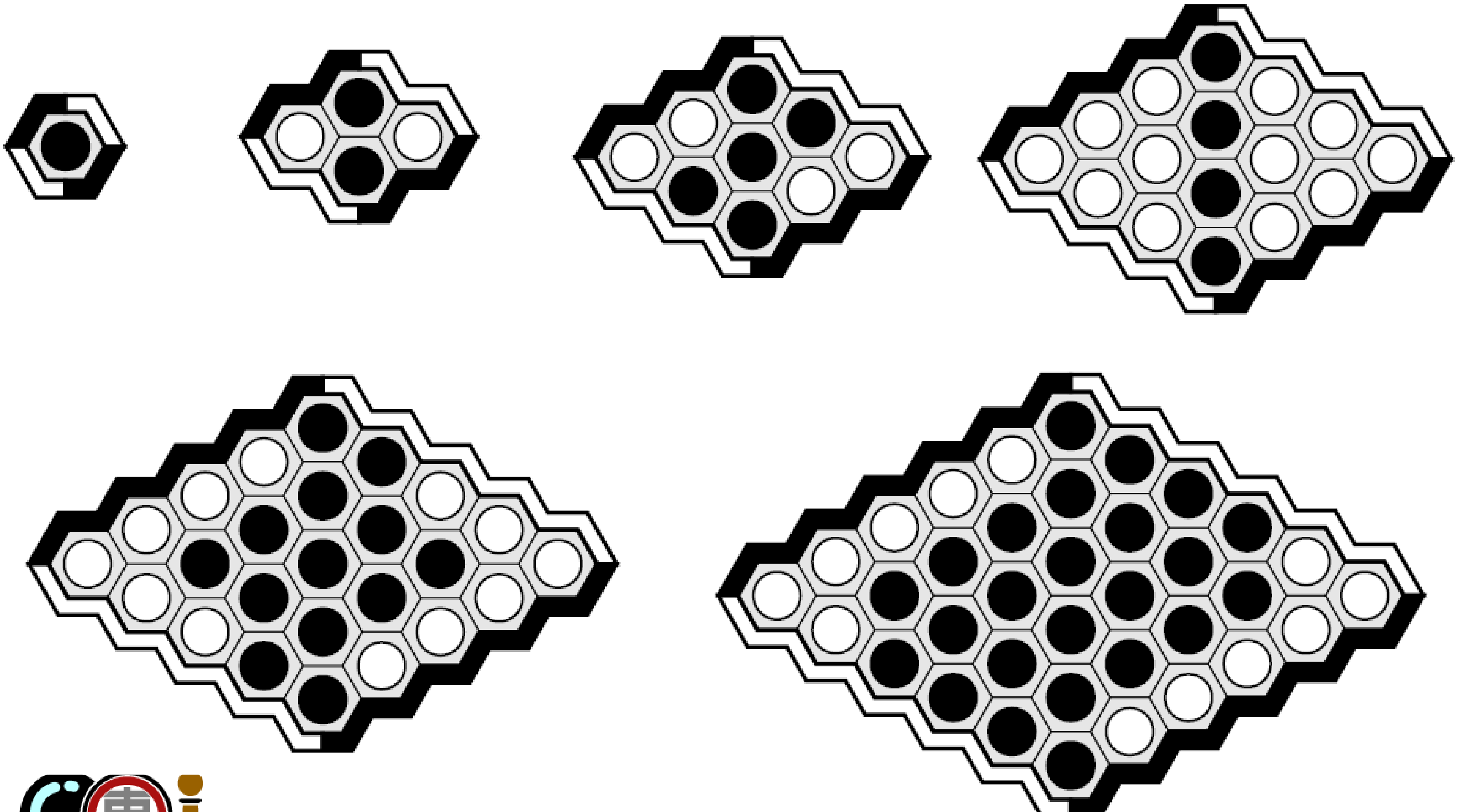
Theoretical Values of Hex

- The first player wins.
  - By the above theorem, there is no draw in this game.
  - By Strategy-stealing argument, the first player does not lose.
- The current rule: Use pie-rule
  - The second player can change color.

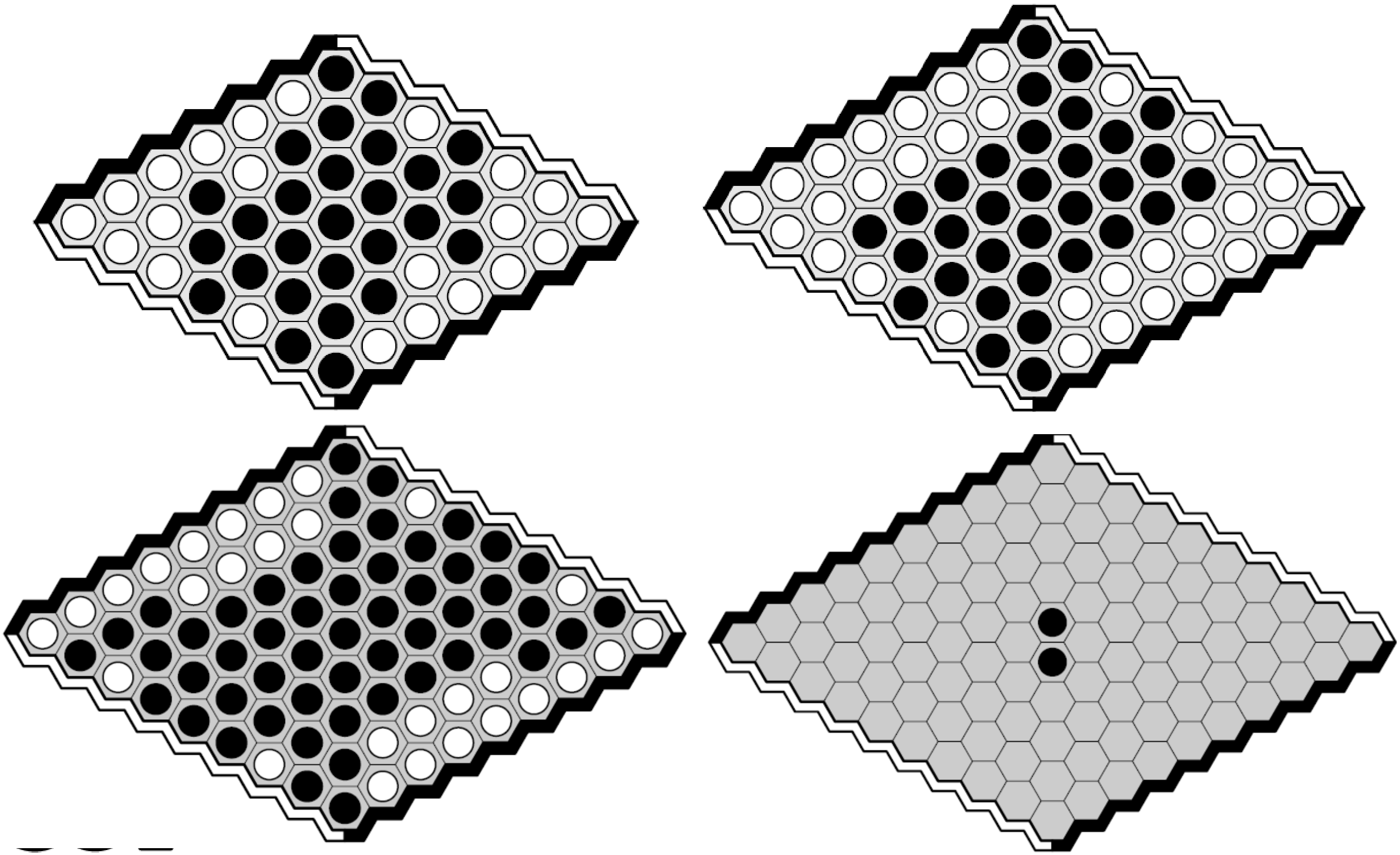
Solutions to HEX

- Hex exhibits considerable mathematical structure.
- Hex has been proved to be PSPACE-complete by Even and Tarjan in 1976 by converting it to a Shannon switching game.
- The state-space and decision complexities are comparable to those of Go on equally-sized boards.
- The results at the time the paper was published.
  - Weakly or strongly solved on 6*6 boards in 1994.
  - Maybe possible to solve the 7*7 case.
  - Not likely to solve the 8*8 version without fundamental breakthroughs.
- The latest results
  - Strongly solve all 9x9 openings. [Pawlewicz & Hayward 2012]
  - Weakly solve 10x10 openings at the center. [Pawlewicz & Hayward 2013]

Solved Hex (I)

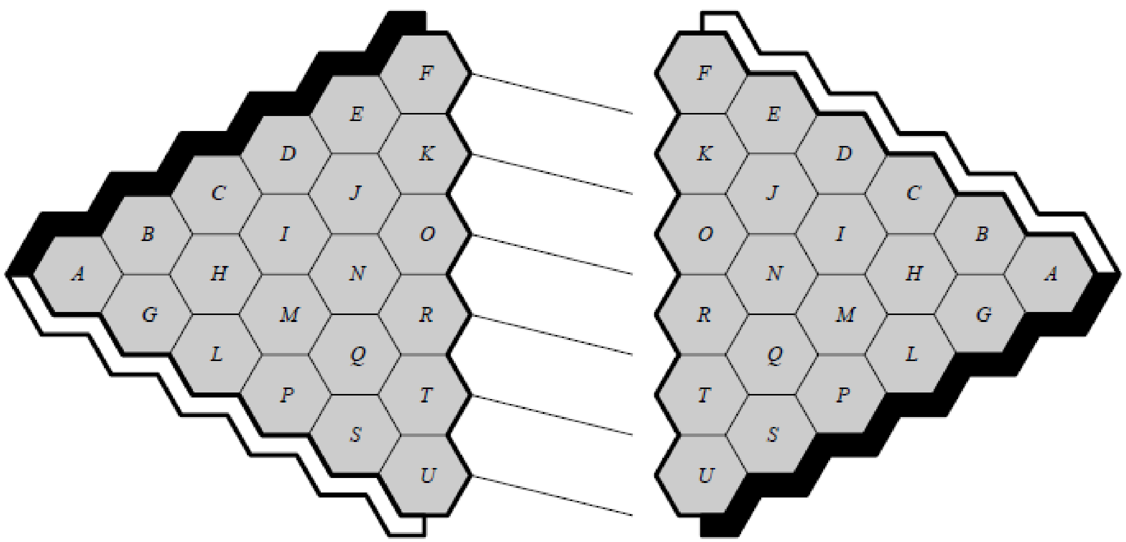


Solved Hex (II)



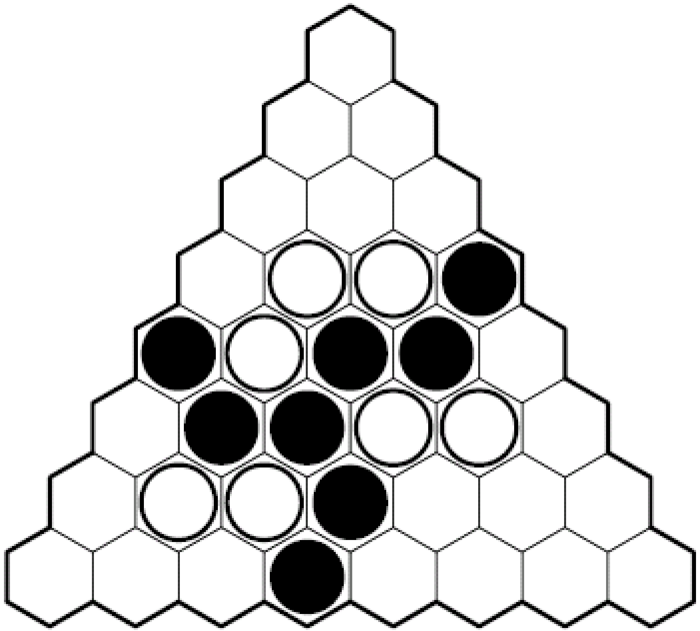
Another Theorem

- Longer-side wins:
- Proof:
  - Simply consider Nx(N+1) Hex.
  - Black simply plays the same character
  - Then, Black wins, Why?



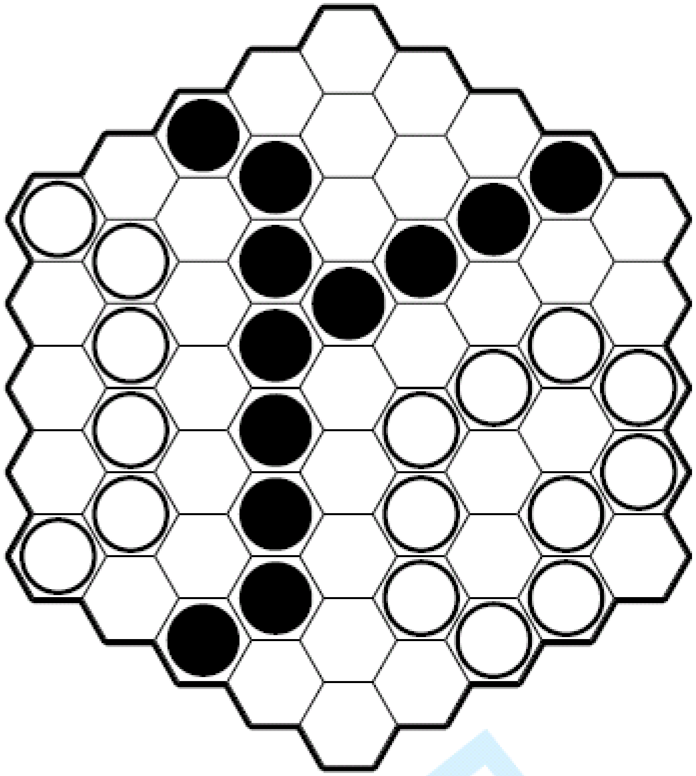
Y

- 雙方輪流下
- 勝利條件
  - 連到三邊



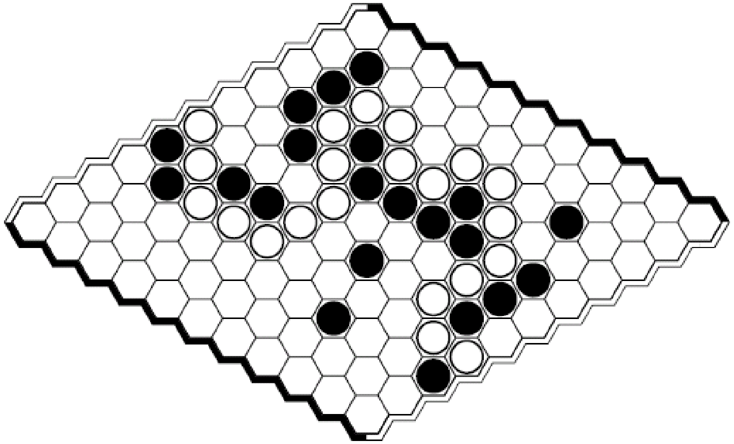
Havannah

- Rules to win:
  - Connect three sides.
  - Connect two corners.
  - Form a circle.

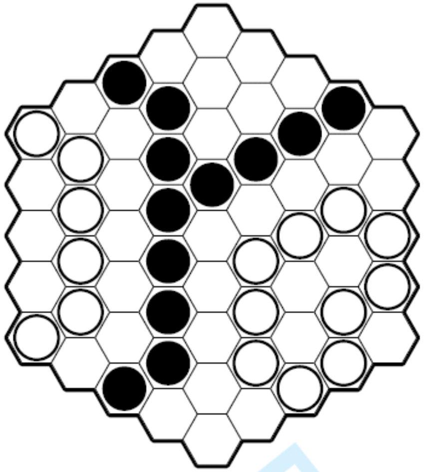


Features

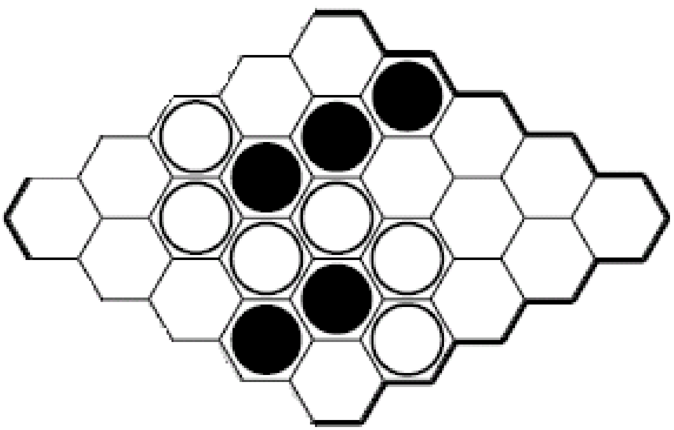
- Hex、Y
  - One must win.

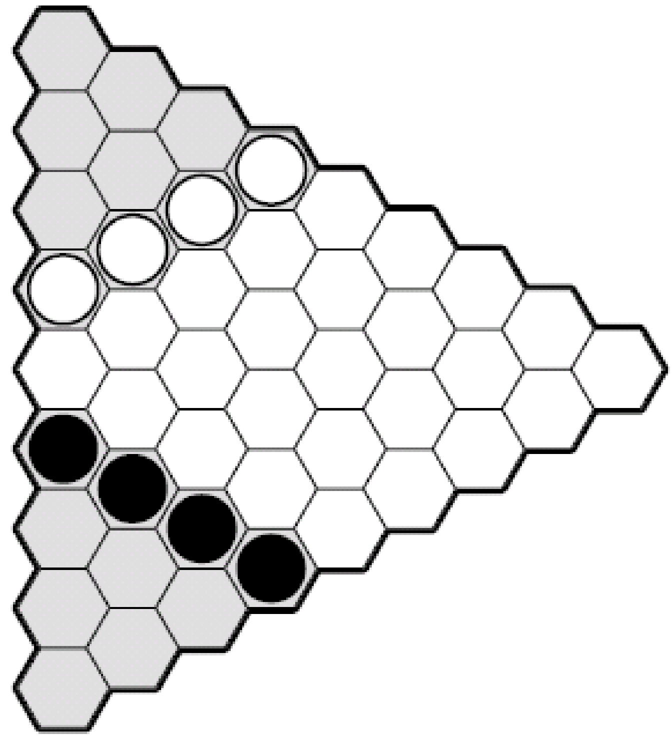


- Havannah
  - Not sure to win for one.



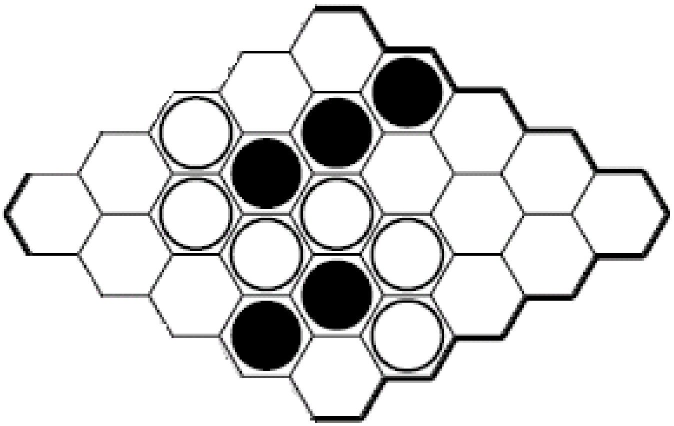
Hex → Y

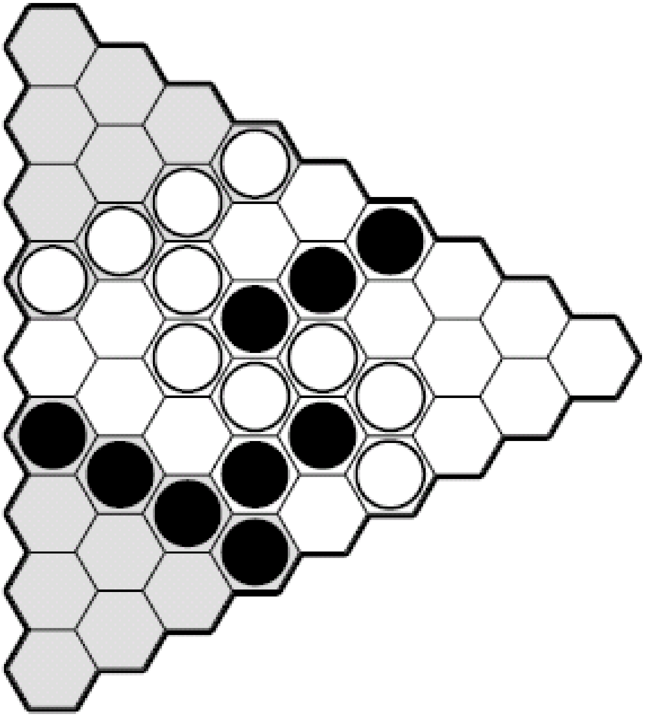




Hex → Y

- Hex is a special case of Y.





Connection Games (II)

- Go-Moku (15*15)
  - First-player win
  - Weakly solved by L.V. Allis in 1995 using a combination of threat-space search and database construction.
- Renju
  - Does not allow the first player to play certain moves.
  - An asymmetric game.
  - Weakly solved by W´agner and Vir´aag in 2000 by combining search and knowledge.
    - It is still first-player win.
  - Took advantage of an iterative-deepening search based on threat sequences up to 17 plies.

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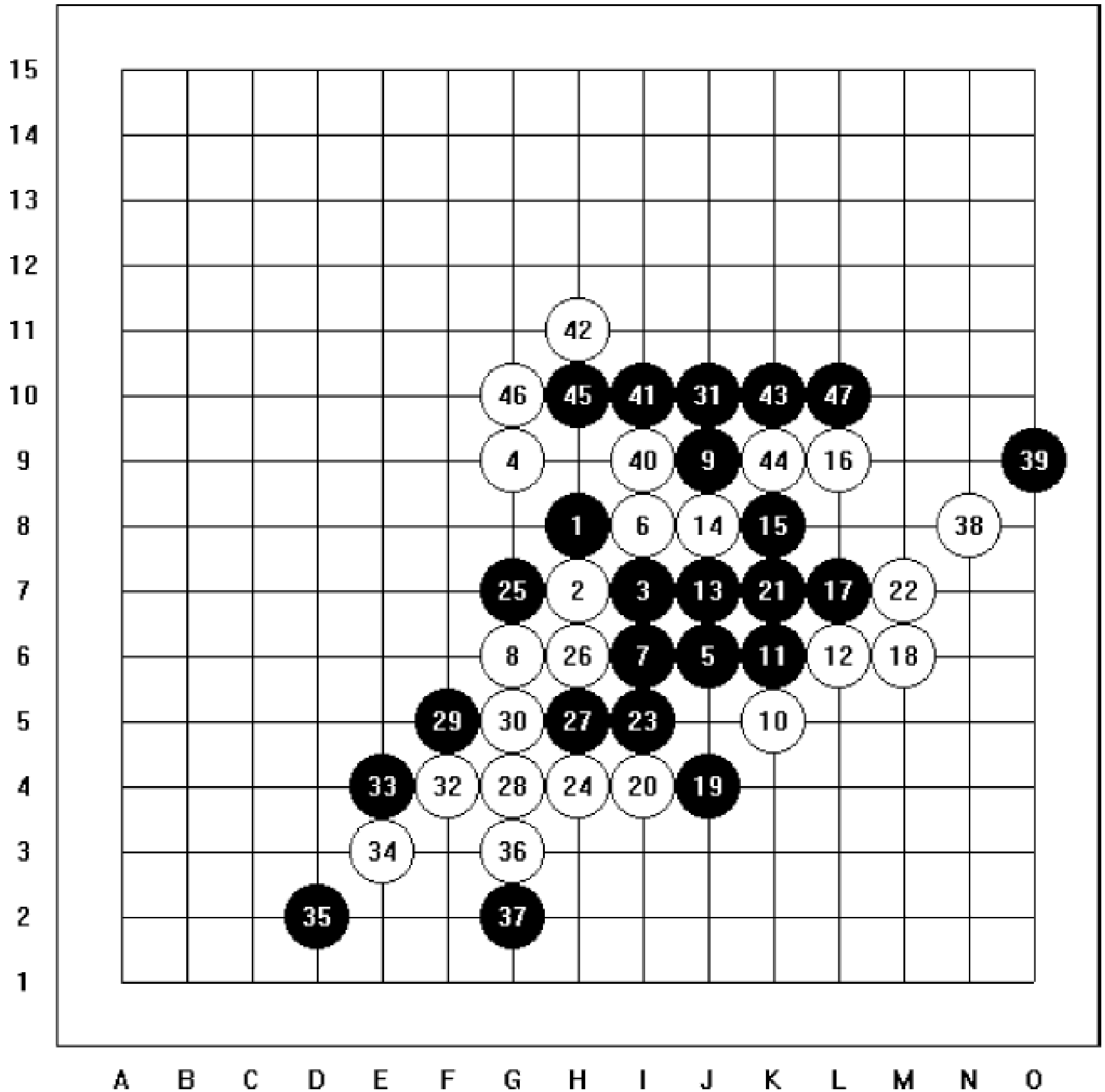


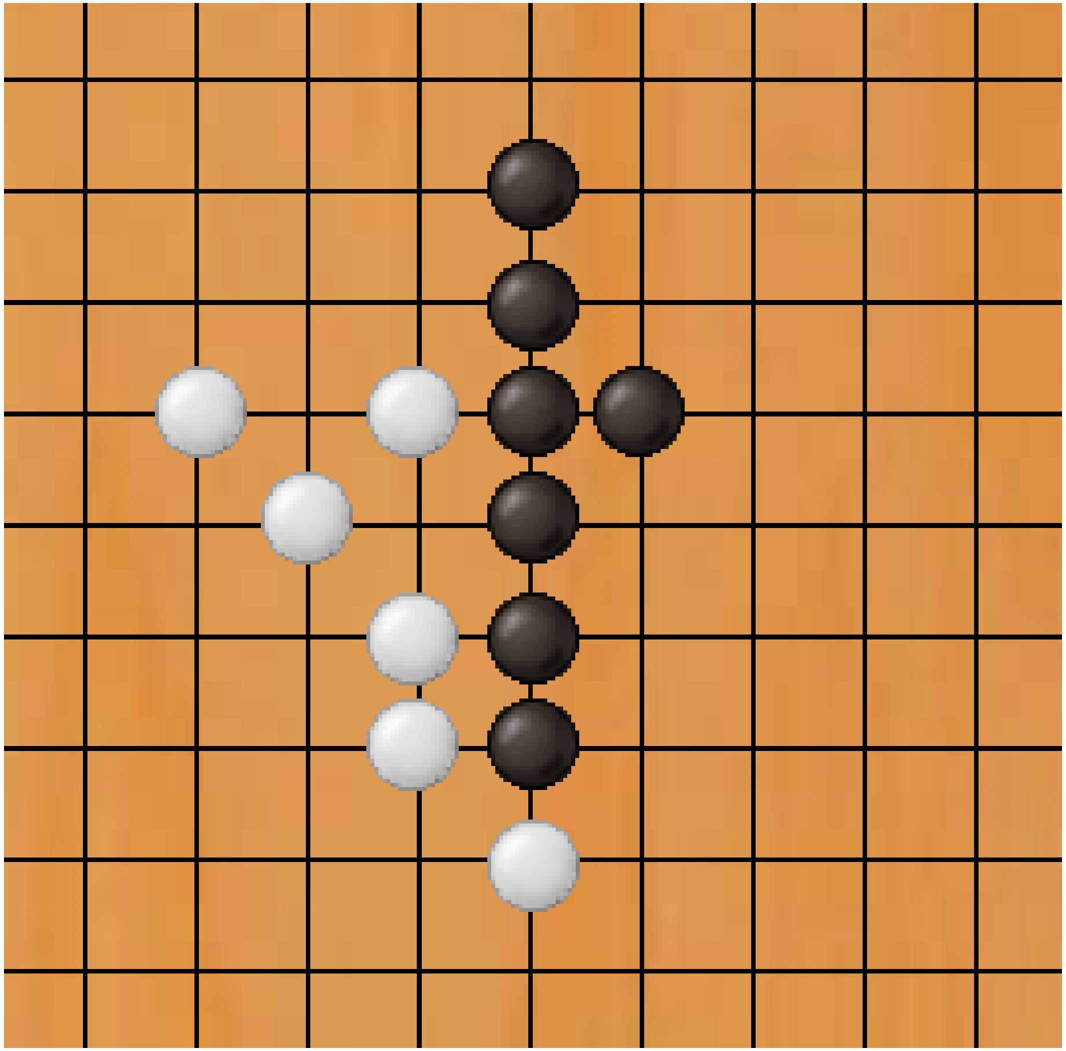
Fig. 8. A first-player win in Renju.

Connection Games (II)

- *k*-in-a-row games
  - *mnk*-Game: a game playing on a board of *m* rows and *n* columns with the goal of obtaining a straight line of length *k*.
  - *Connect(m,n,k,p,q)*: extension by [WH05]
    - Play on a board of *m* rows and *n* columns
    - Win by a straight line of length *k*.
    - Place *q* stones for the first move.
    - Place *p* stones for each of the rest moves.
  - Example:
    - Tic-tac-toe: Connect(3,3,3,1,1)
    - Gomoku: Connect(15,15,5,1,1)
    - Connect6: Connect(19,19,6,2,1)
      ➔ Balance the advantage of the initiative!

Connect6

- Introduced by I-Chen Wu [2005]



Solved Connect Games

- More discussed later.

Table 3
Game values of mnk-games

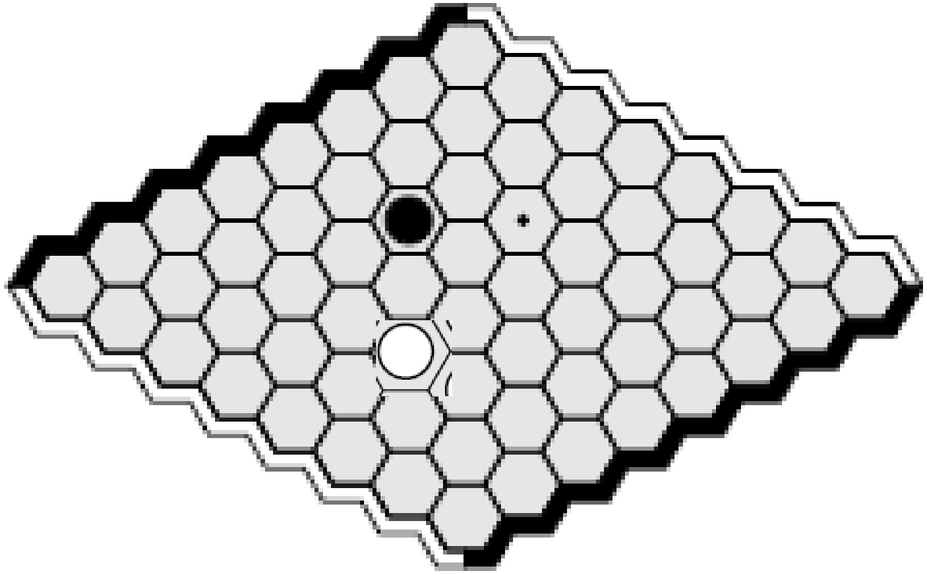
mnk-games ($k = 1, 2$)	W
333-game (Tic-Tac-Toe)	D
$mn3$-games ($m \geqslant 4, n \geqslant 3$)	W
$m44$-games ($m \leqslant 8$)	D
$mn4$-games ($m \leqslant 5, n \leqslant 5$)	D
$mn4$-games ($m \geqslant 6, n \geqslant 5$)	W
$mn5$-games ($m \leqslant 6, n \leqslant 6$)	D
15,15,5-game (Go-Moku)	W
mnk-games ($k \geqslant 8$)	D

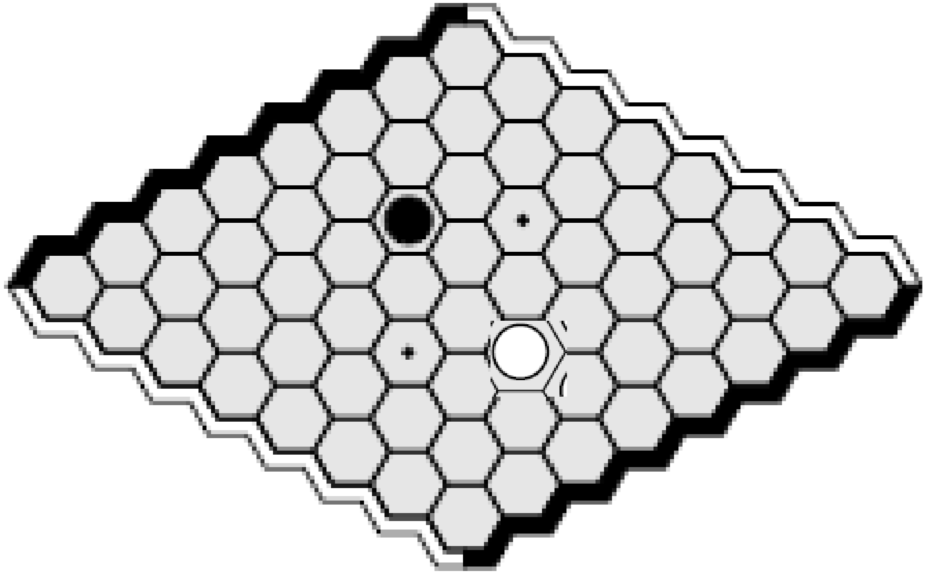
Results from Strategy-Stealing Argument

- Hex: FIRST wins.
- Hex (with pie rule): SECOND wins.
- Gomoku (no prohibit rules): FIRST wins.
- *Connect(m,n,k,p,p)*: FIRST does not lose.
- *Connect(m,n,k,p,q)*: The higher *q* is, the higher chances FIRST wins.
  - For FIRST, Connect(m,n,k,p,q+1) is better than Connect(m,n,k,p,q).
  - Why? Exercise!

More Examples

- What about the following two?





Methods Developed for Solving Games

- Brute-force methods
  - Retrograde analysis
  - Enhanced transposition-table methods
- Knowledge-based methods
  - Threat-space search and λ-search
  - Proof-number search
  - Depth-first proof-number search
  - Pattern search
    - To search for threat patterns, which are collections of cells in a position P.
    - A threat pattern can be thought of as representing the relevant area on the board, an area that human players commonly identify when analyzing a position.

On Fairness

- Herik et al. 2002：
  - A game is considered *fair* if it is a draw and both players have a roughly equal probability of making a mistake.”
  - “一個遊戲是公平的話，那麼它必須是個平手的遊戲，且雙方有相同的犯錯機率。”
- Problem:
  - hard to have a perfect model for calculating the probability of making a mistake
- On the contrary, it is relatively easy and possible to show when a game is *unfair*.

Unfairness

- *Definitely unfair*,
  - if it has been proved that some player wins the game.
  - For example, Go-Moku (in the free style)
- *Monotonically unfair*,
  - if it has been proved that one player does not win the game.
  - For example, for *Connect*(*k*,*p*,*p*) or *Connect*(*m*,*n*,*k*,*p*,*p*), based on the so-called *strategy-stealing argument.*
- *Empirically unfair*,
  - if most players, in particular professionals, have claimed that the game favours some player.
  - For example, before Go-Moku was solved, Go-Moku was empirically unfair.

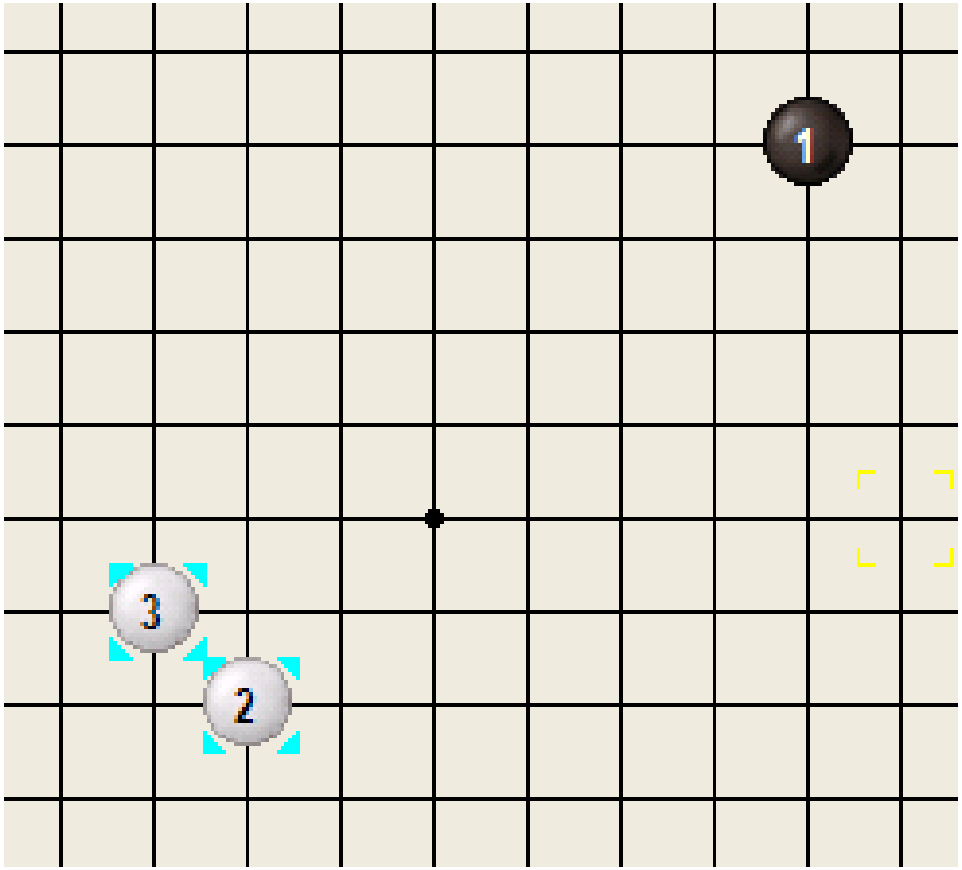
Potential Fairness

- *Potentially fair*,
  - if it has not yet been shown or claimed to be definitely unfair, monotonically unfair, or empirically unfair.
- Properties:
  - A potentially fair game for the time being may not remain potentially fair in the future.
  - If a game remains potentially fair any longer, it could have a higher chance to be fair.

Breakaway and Fairness

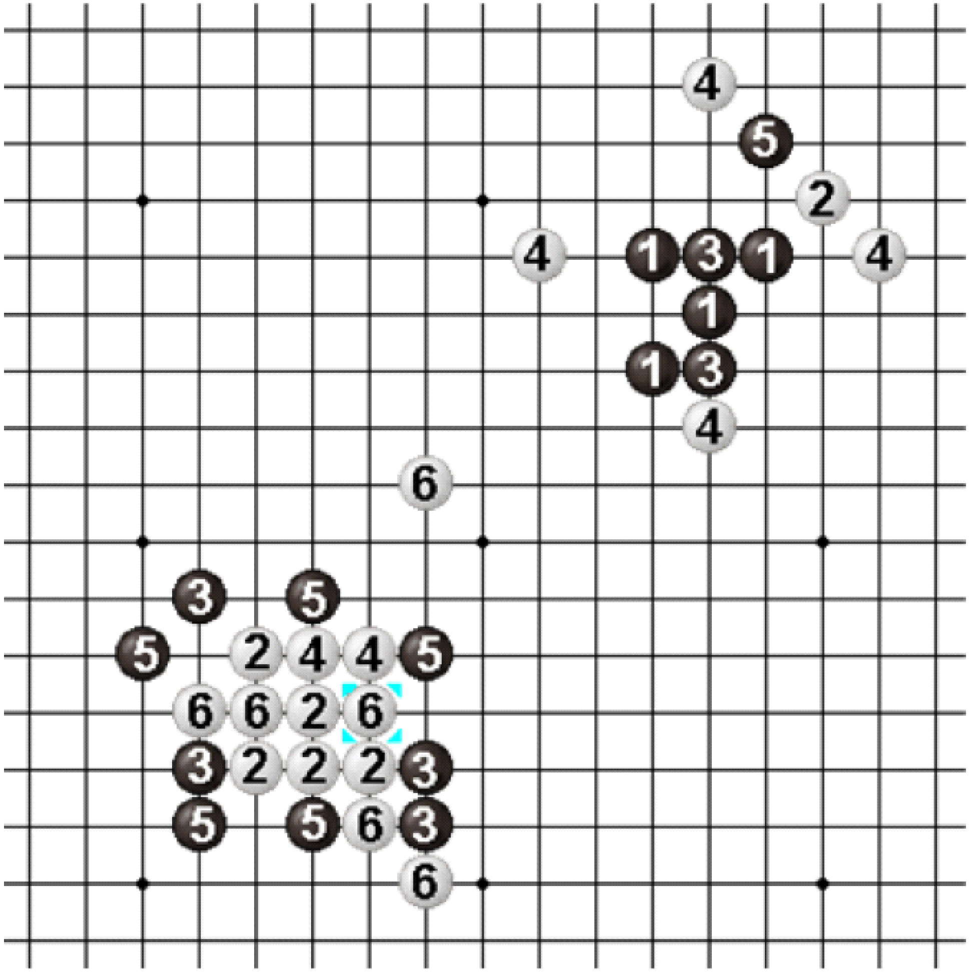
- A *breakaway move*: (脫離戰場)
  - place stones far away from the major battle field
- An *initial breakaway move*:
  - The first move by W (after the first move by B) is also a breakaway move.
- Fairness and Breakaway:
  - If W makes an initial breakaway move without a penalty, then the game is played like *Connect*(k,p,p) with W playing first.
    ➔ Such games are monotonically unfair or

Breakaway of Connect6



Connect(9,6,4)

- Breakaway



Fairness of Connect Games

- Connect(m,n,k,1,1)
- Unfair Connect(m,n,k,p,q)
- Tied Connect(m,n,k,p,q)

Connect(6,5,4,1,1)

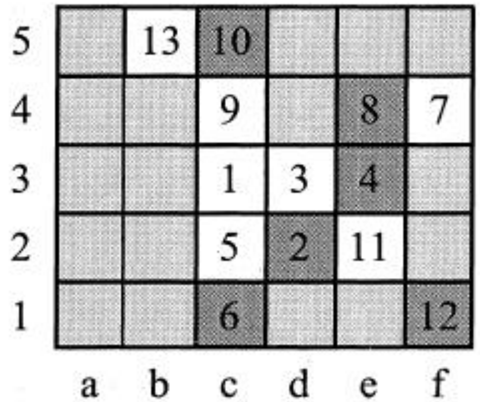


Fig. 1. An optimal variation in the 654-game.

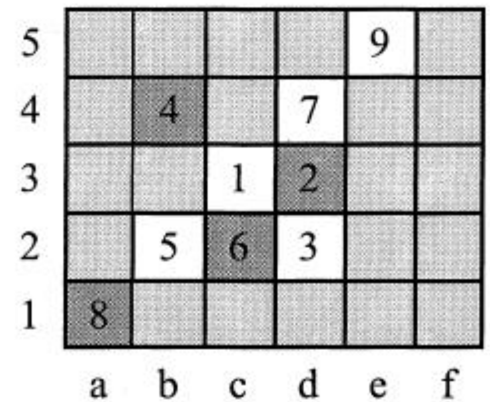


Fig. 2. Another optimal variation in the 654-game.

- Connect(m,n,4,1,1) $m \geqq 6$, $n \geqq 5$ ➔ B wins.

Connect(m,n,5,1,1)

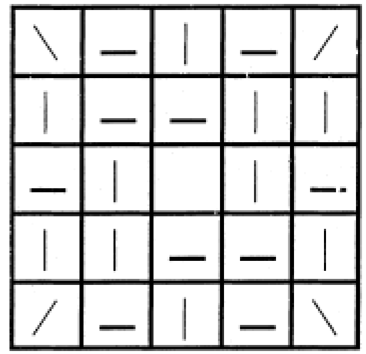


Fig. 3. A Hales-Jewett pairing for the 555-game.

- Connect(m,n,5,1,1) m$\leqq$ 6 , n$\leqq$6 ➔ both ties.
- Connect(15,15,5,1,1) ➔ B wins.

Hales-Jewett Pairing for Connect(9,1,1)

- Connect(9,1,1)
  - Draw
- Connect(8,1,1)
  - Draw?

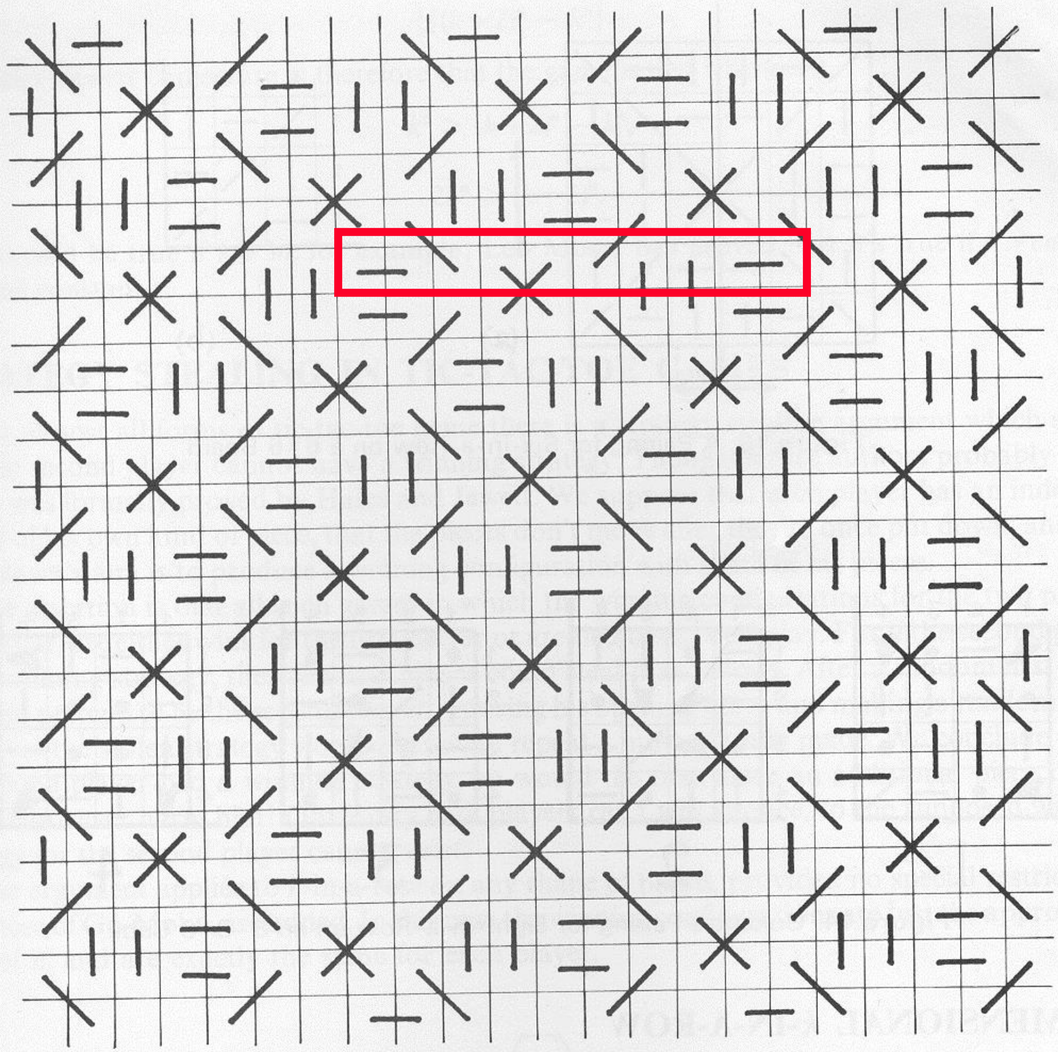
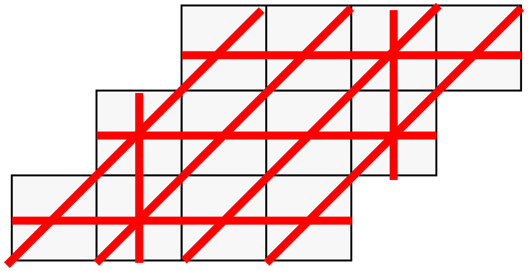


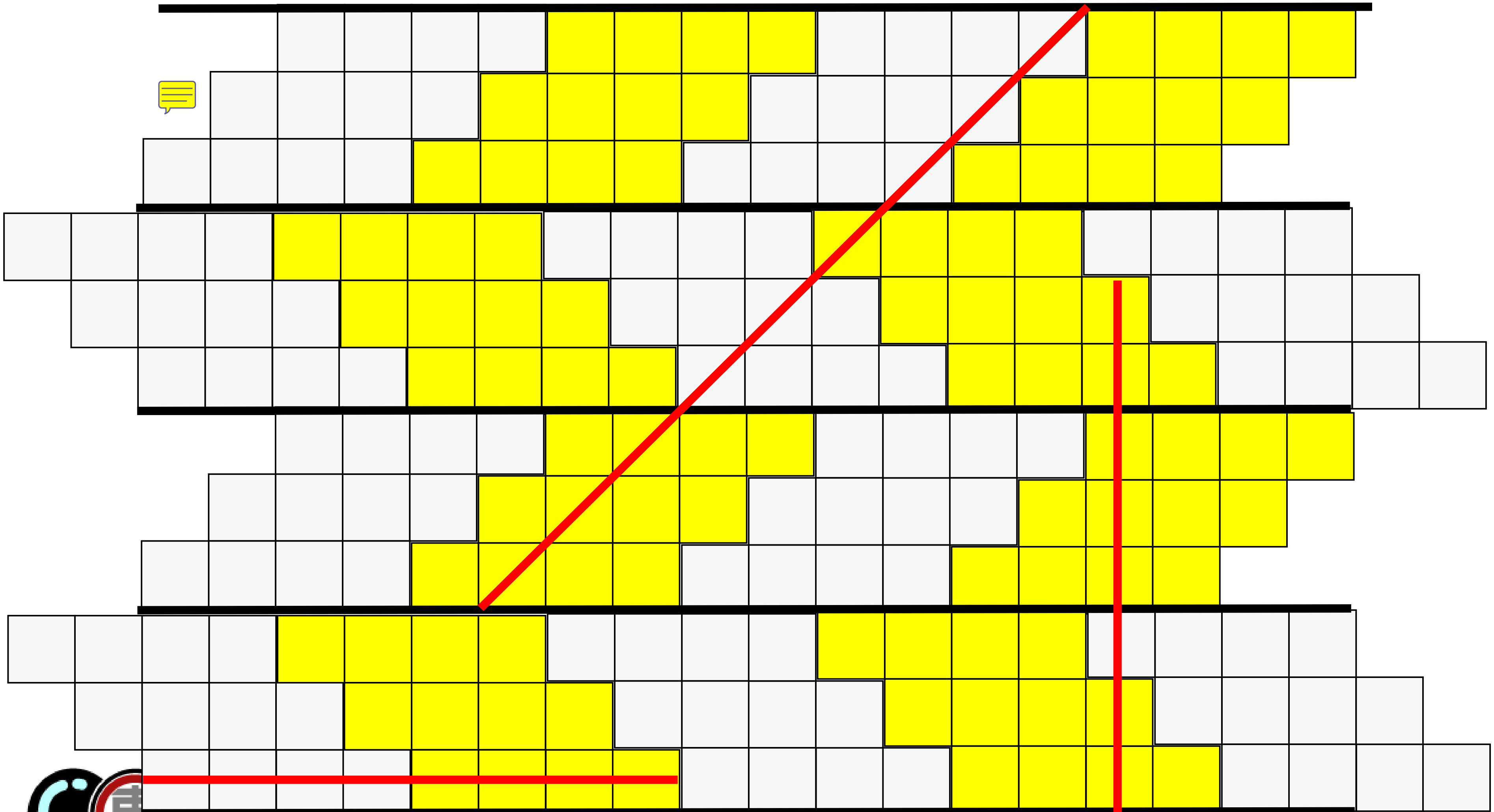
Figure 12. Nine-in-a-Row is a Draw on an Infinite Board.

Claim of the Shape

- Never
  - gets 4 horizontally
  - gets 3 diagonally
  - gets 2 vertically



The Max Line → Connect(8,1,1) Draw

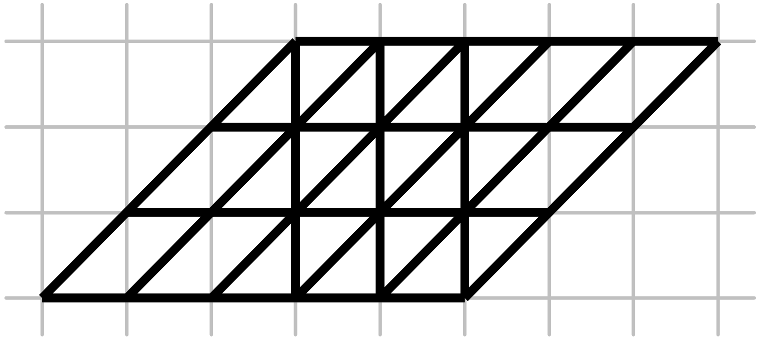


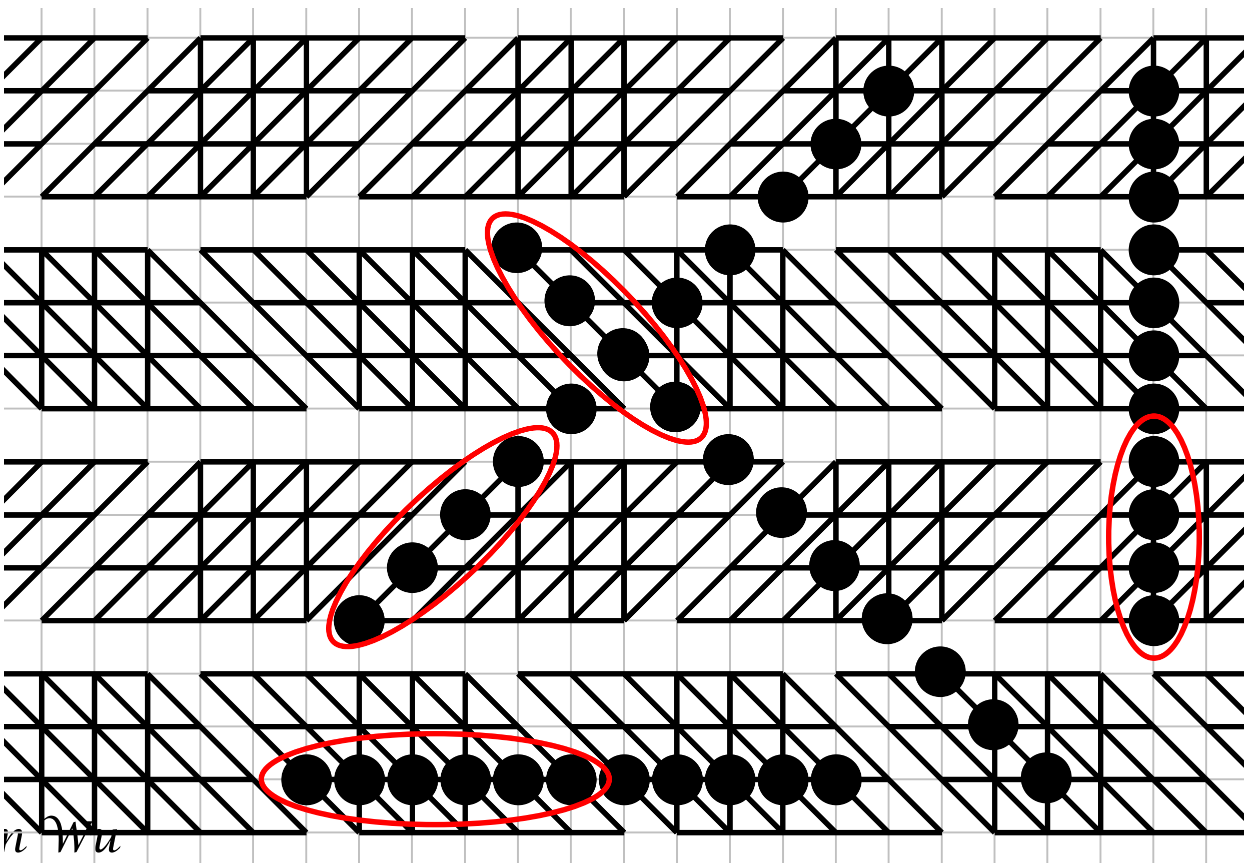
When Two Stones Per Move

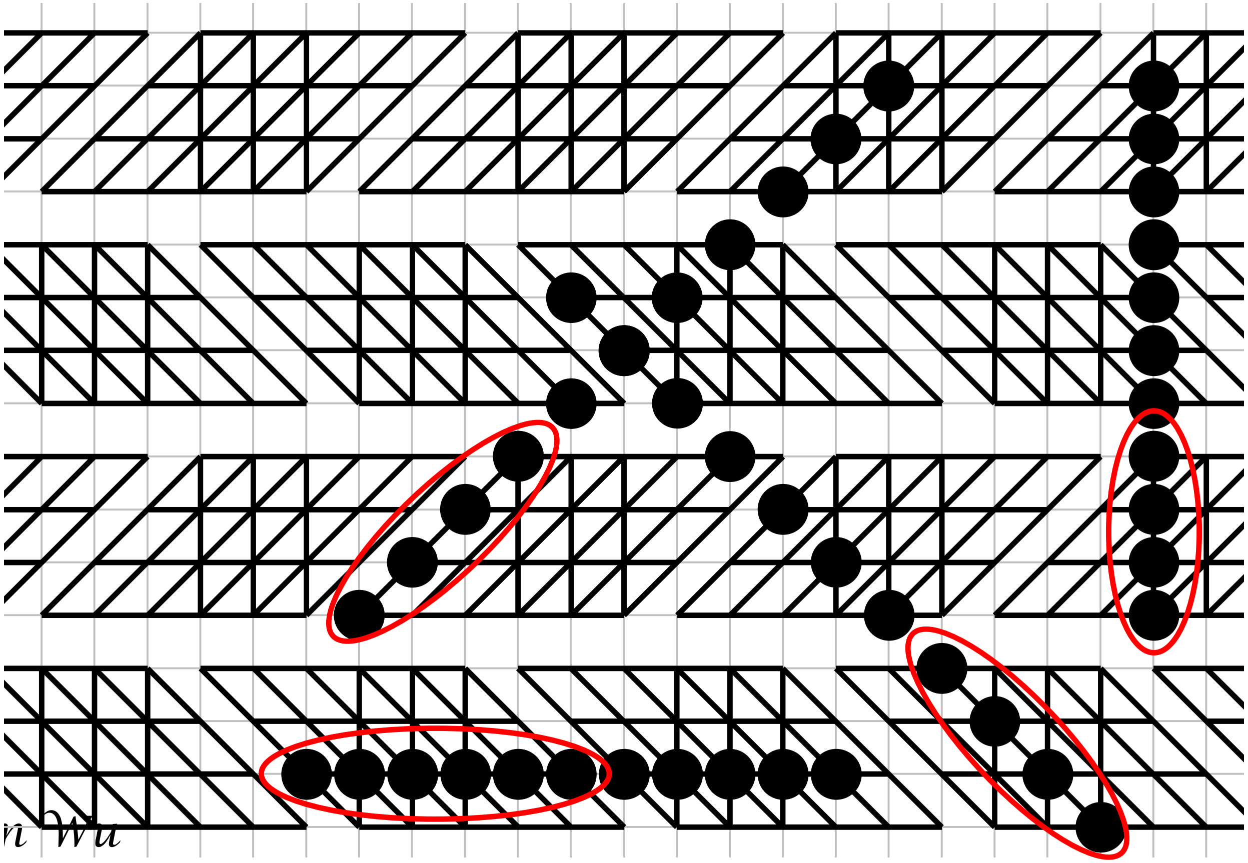
- Connect-15 is a draw.
  - By Tsai's team (蔡錫鈞教授及其研究生)
- Connect-11 is a draw.
  - By Chiang (江盛浩), Wu, Lin
- Results:
  - 獲得臺灣2009年國際科學展覽會 數學科 第一名。
  - 獲得2009年美國英特爾國際科技展覽會二等獎
    - 依據參加國際數理學科奧林匹亞競賽及國際科學展覽成績優良學生升學優待辦法，此科展的二等獎獲頒十萬元，與「國際數理學科奧林匹亞競賽」銀牌獎金相同。
  - 發表於國際會議 the 12th Advances in Computer Games Conference (ACG12), Pamplona, Spain, May 2009.
  - 發表於國際重要期刊雜誌：Theoretical Computer Science.

Sub-Board

- The first player cannot occupy a line.







Connect(m,m,m,1,1)

- Drawn , if m>4, using Hales-Jewett Pairing.

- Reference :
  - Berlekamp, Conway, and Guy, Winning Ways.

Other p & q

- Let $\delta = k-p$.
- B wins when $p < \lfloor q/\delta^2 \rfloor\ (4\delta + 4)$

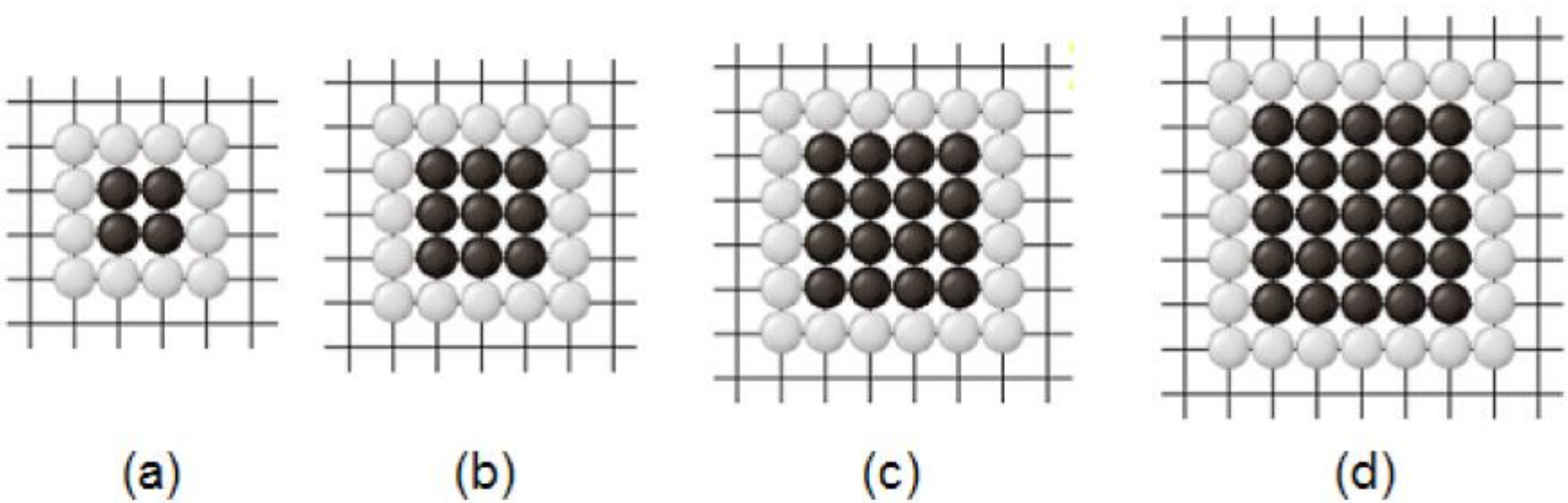
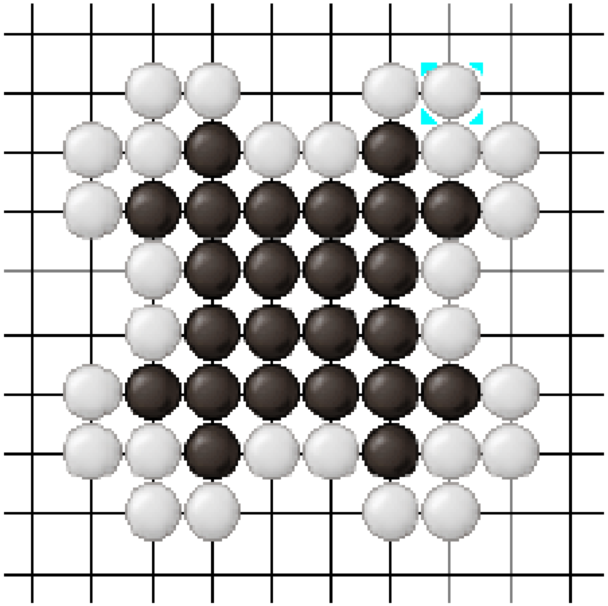


Fig. 1. Putting δ^2 stones when (a) $\delta = 2$, (b)$\delta = 3$, (c)$\delta = 4$, and (d)$\delta = 5$

A Corollary

- Let $\delta = k - p$. For Ren(k,p,q) game, B wins when $p < \lfloor q/\delta^2 \rfloor (4\delta + 4) + \min(q \bmod \delta^2, 8q/\delta^2)$.



Some More Cases

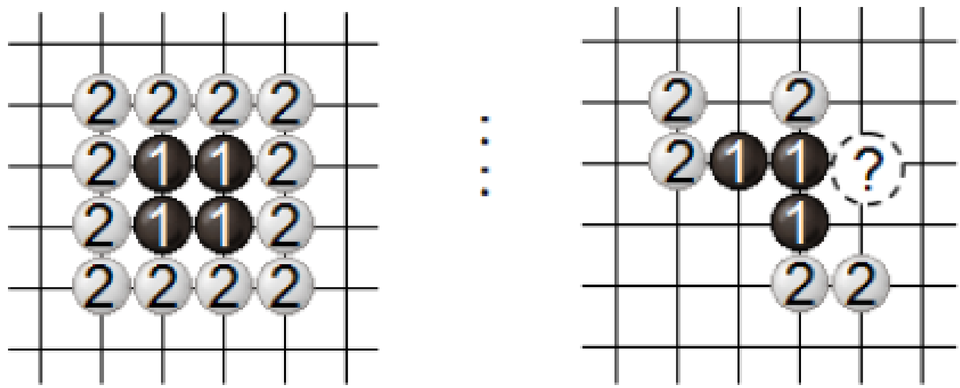


Fig. 3. B's winning strategy for Ren(19,17,7).

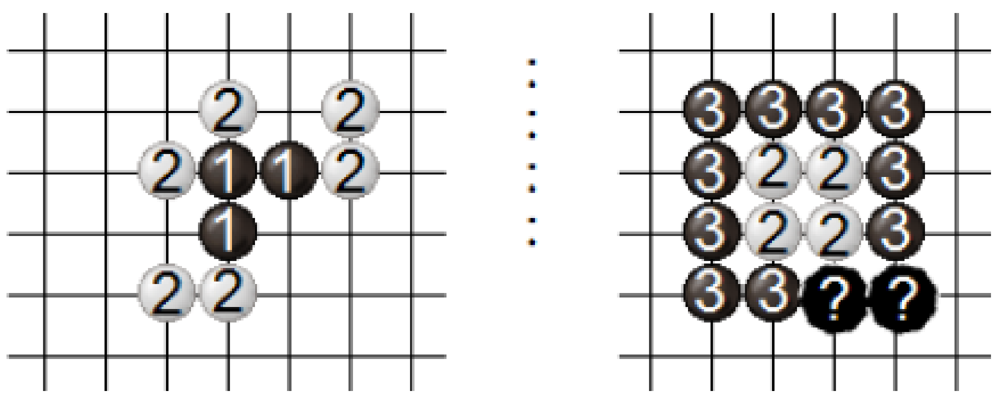


Fig. 4. W's winning strategy for Ren(12,10,3).

Empirical Analysis

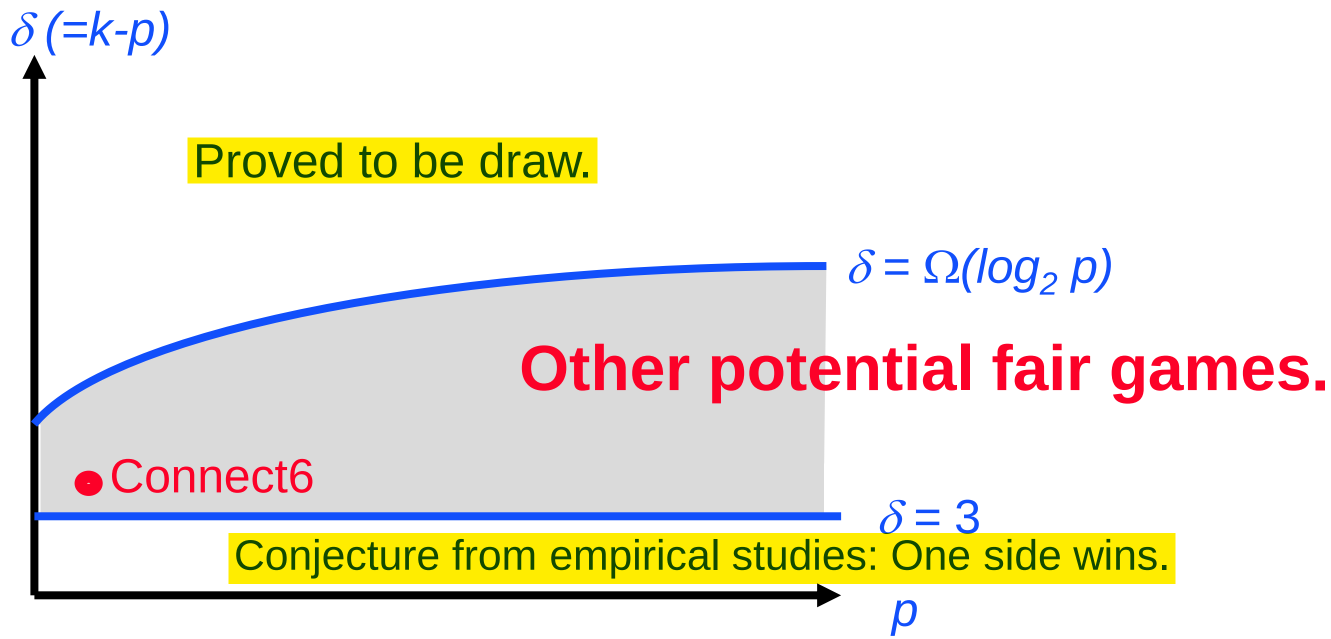
- For most $\delta = k–p = 3$ games, most are empirical unfair according to our experiments.
  - B (W): Informally proved. FB (FW): Favors Black (White)

$q(\leq p)$	$k=4$ $p=1$	$k=5$ $p=2$	$k=6$ $p=3$	$k=7$ $p=4$	$k=8$ $p=5$	$k=9$ $p=6$
1	B	B	W	W	W	W
2		B	W	W	W	W
3			B	FB	FB	FW
4				B	B	FW
5					B	B
6						B

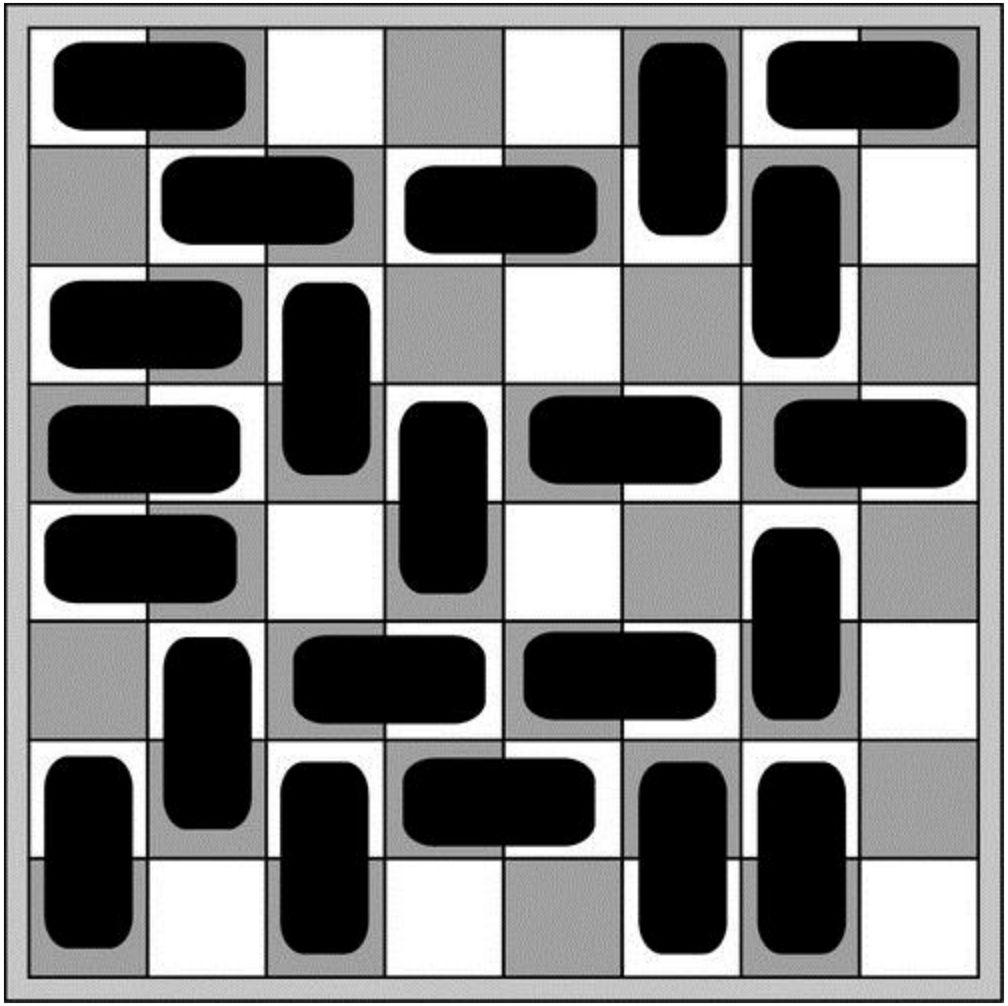
Maker-Breaker

- Strategy stealing argument: [Csirmaz 1980, Pluhar 1994]
  - *Connect(m,n,k,p,p)*, ➔ monotonically unfair (白不會贏)
- So, for combinatorial analysis, some researchers proposed Maker-Breaker Model:
  - W is not allowed to win.
- Theorems:
  - Let k and p satisfy a condition, roughly like $\delta = k - p = O(\log_2 p / \log_2 \log_2 p)$. For all q, $1 \le q \le p$, B wins *Connect(k,p,q)*.
  - Let k and p satisfy a condition, roughly like $\delta = k - p = \Omega(\log_2 p)$. For all q, $1 \le q \le p$, both B and W tie for *Connect(k,p,q)*.

Fairness Analysis for Connect Games



Domineering



Solved Domineering

Table 4

Game-theoretic values of Domineering games on $m \times n$ boards

$m\backslash n$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	L	L	L	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*
2	W	W	W	L	W	W	W	*L*	W	W	W	*L*	L	W	W	*L*	*L*	W	W	*L*	*L*	L	W	*L*	*L*	*L*	W	*L*	*L*	*L*
3	W	W	W	L	L	L	L	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*
4	*W*	W	W	W	*W*	W	W	L	*W*	L	W	L	L	L	L	*L*	L	*L*		*L*		*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*
5	*W*	L	W	*L*	L	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*
6	*W*	W	W	W	*W*	W	W	L	*W*	W	W	L	*W*			*L*				*L*				*L*				*L*		
7	*W*	W	W	L	*W*	L	W	*L*	L	*L*	L	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*
8	*W*	*W*	*W*	W	*W*	W	*W*	W	*W*		*W*		*W*			*L*														
9	*W*	L	*W*	*L*	*W*	*L*	W	*L*	W	*L*		*L*		*L*		*L*		*L*		*L*		*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*	*L*
10	*W*	W	*W*	W	*W*	W	*W*	W	*W*		*W*		*W*							*L*										
11	*W*	W	*W*	L	*W*	W	W	*L*	*W*			*L*				*L*				*L*		*L*		*L*		*L*		*L*		*L*
12	*W*	*W*	*W*	W	*W*	W	*W*		*W*		*W*		*W*											*L*						
13	*W*	L	*W*	*L*	*W*	*L*	*W*	*L*		*L*		*L*		*L*		*L*		*L*		*L*		*L*		*L*		*L*		*L*		*L*
14	*W*	W	*W*	W	*W*	*W*	*W*		*W*		*W*		*W*															*L*		
15	*W*	W	*W*	W	*W*		*W*		*W*		*W*																			*L*
16	*W*	*W*	*W*	*W*	*W*	*W*	*W*		*W*		*W*		*W*																	
17	*W*	*W*	*W*	W	*W*		*W*		*W*		*W*		*W*																	
18	*W*	W	*W*	*W*	*W*	*W*	*W*		*W*		*W*		*W*																	
19	*W*	W	*W*	*W*	*W*	*W*	*W*		*W*		*W*		*W*																	
20	*W*	*W*	*W*	*W*	*W*	*W*	*W*		*W*		*W*		*W*																	
21	*W*	*W*	*W*	*W*	*W*		*W*		*W*		*W*		*W*																	
22	*W*	*W*	*W*	*W*	*W*	*W*	*W*		*W*	*W*	*W*		*W*																	
23	*W*	W	*W*	*W*	*W*	*W*	*W*		*W*		*W*		*W*																	
24	*W*	W	*W*	*W*	*W*	*W*	*W*	*W*	*W*		*W*		*W*																	
25	*W*	*W*	*W*	*W*	*W*		*W*		*W*		*W*		*W*																	
26	*W*	*W*	*W*	*W*	*W*	*W*	*W*		*W*	*W*	*W*		*W*																	
27	*W*	W	*W*	*W*	*W*	*W*	*W*		*W*		*W*		*W*																	
28	*W*	W	*W*	*W*	*W*	*W*	*W*		*W*		*W*		*W*																	
29	*W*	*W*	*W*	*W*	*W*		*W*		*W*		*W*		*W*																	
30	*W*	W	*W*	*W*	*W*	*W*	*W*		*W*		*W*		*W*	*W*																

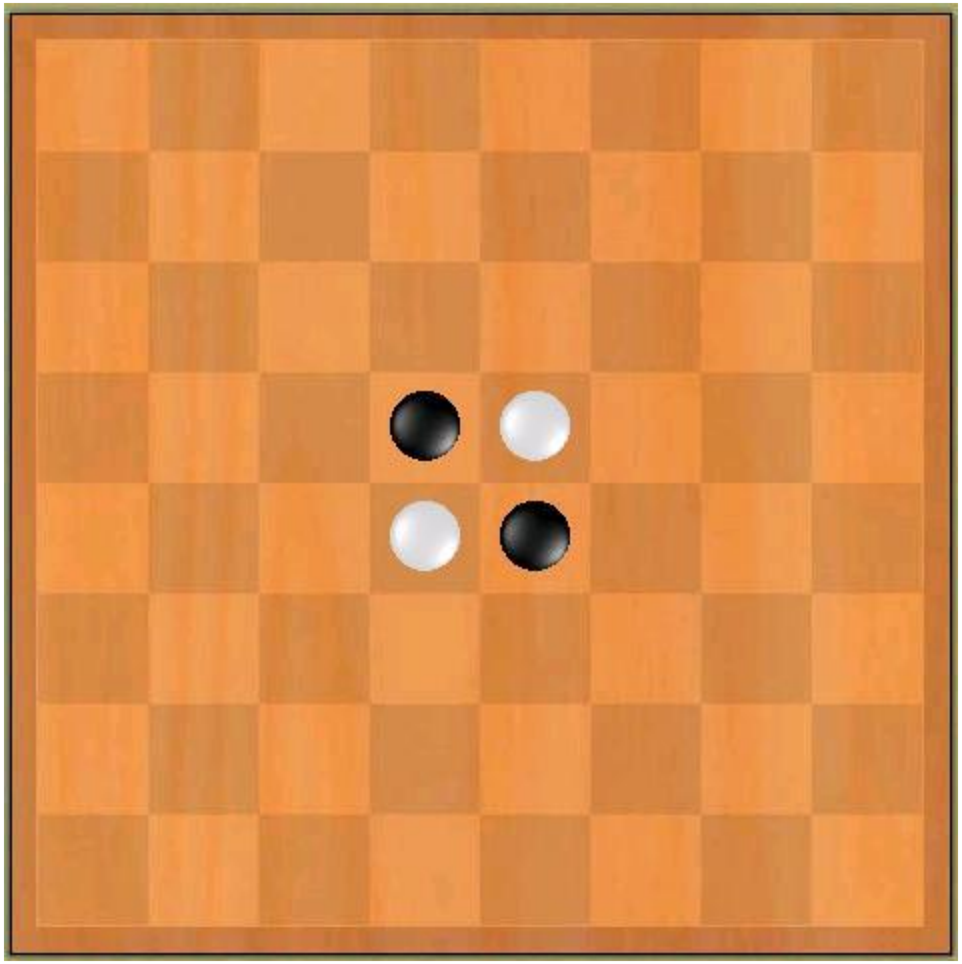
Chilled Domineering (also named XT Domineering)

- Invented by Prof. Kao (高國元)
- Rules: The same as Domineering, except:
  - Allowed to place 1x1, when no 1x2 or 2x1 Dominos in an isolated area.
- Complexity:
  - Very complicated, since more moves are allowed.
- Key contribution:
  - More infinitesimals(無限小的數字).

More Divergent Games

- Othello
  - M. Buro’s LOGISTELLO beat the resigning World Champion by 6-0 in 1997.
  - Weakly solved on 6*6 boards by J. Feinstein in 1993.
- Chess
  - DEEP BLUE beat the human World Champion in 1997.
- Chinese chess
  - Still in progress,
  - Professional 7-dan in 2007.
- Shogi
  - Still in progress,
  - Professional 2-dan in 2007.
- Go
  - Still in progress, Amateur 4 kyu.
  - Note: it makes much difference now when Monte-Carlo methods are used.

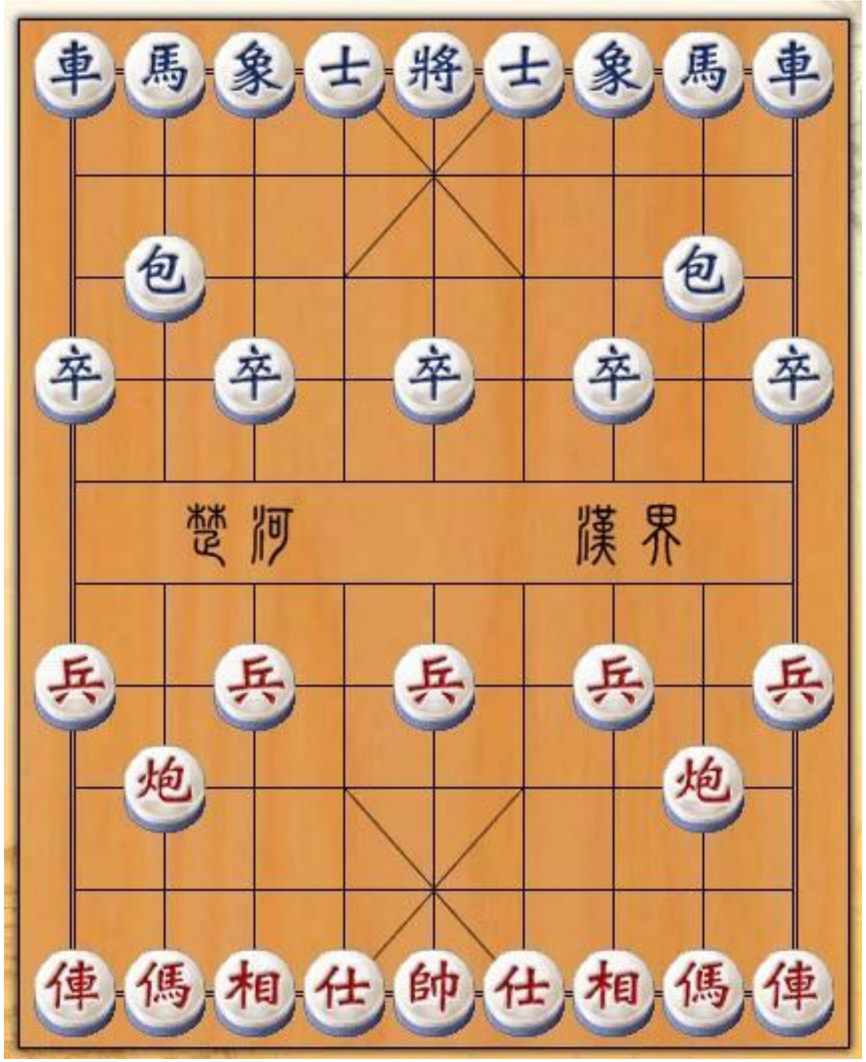
Othello



Chess

(From wikipedia.org)

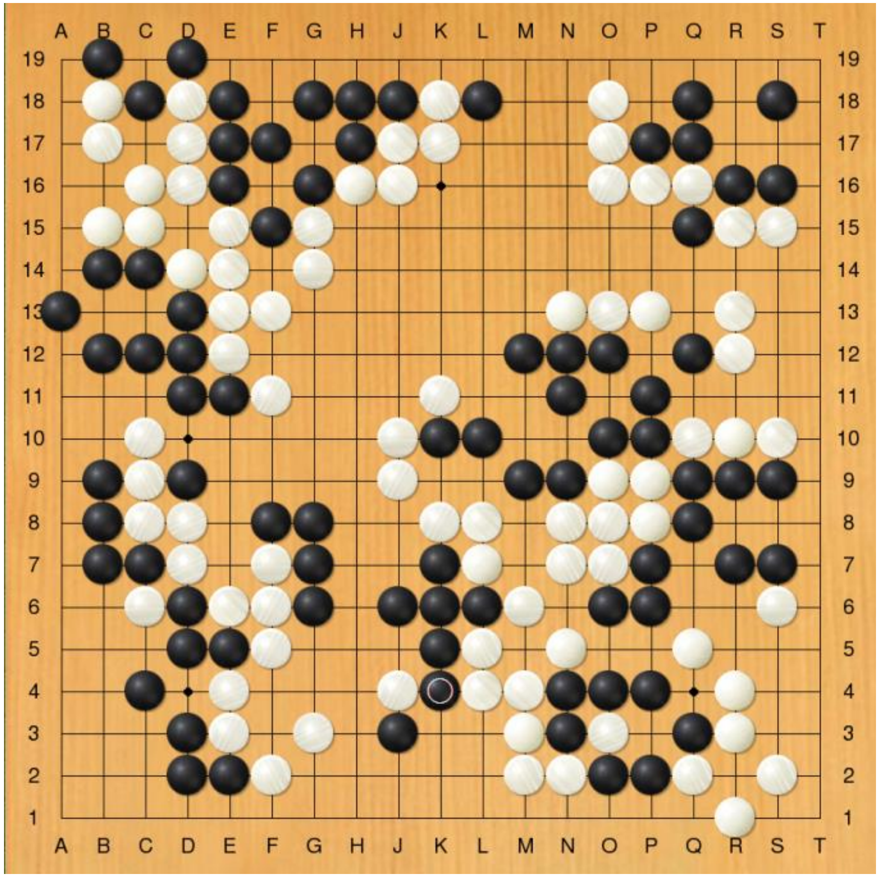
Chinese Chess



Shogi 日本將棋

Fig. 12. Microcosmos (1525 steps).

Go (Weichi)



First-Player Scores for Go

Table 5

First-player scores for Go on $m \times n$ boards

$m \backslash n$	1	2	3	4	5	6	7	8	9	10	11	12	13
1	**0**	**0**	**3**	**4**	**0**	**1**	**2**	**3**	**0**	**1**	**2**	1?	2
2	**0**	**1**	**0**	**8**	**10**	**12**	14	16	18	4			
3	**3**	**0**	**9**	**4**	15	18	5?	24?					
4	**4**	**8**	**4**	**2**	20	1							
5	**0**	**10**	15	20	25	0	9						
6	**1**	**12**	18	1	0	4							
7	**2**	14	5?		9								
8	**3**	16	24?										
9	**0**	18											
10	**1**	4											
11	**2**												
12	1?												
13	2												

NoGo

- All rules are the same as Go, except:
  - The moves to capture opponents' pieces are prohibited.
  - The moves to suicide are prohibited.
- A game developed by combinatorial game theory people.
  - http://mogotw.nutn.edu.tw/chinese/nogo.htm (download)

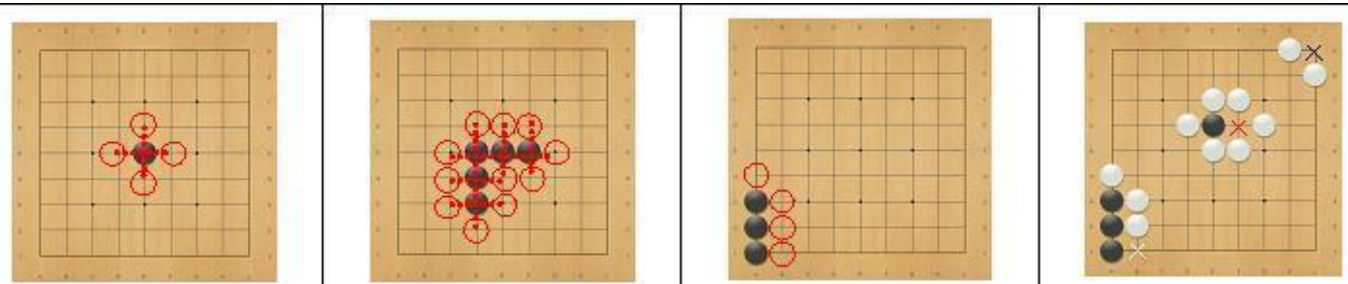


圖 a 圖 b 圖 c 圖 d

Combinatorial Games

(discussed more in the chapter of combinatorial games.)

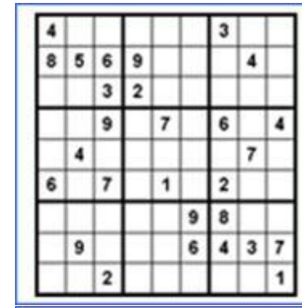
- Nim
- Triangular Nim
- Chilled Domineering
- NoGo

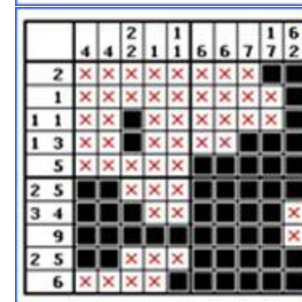
Puzzle Games

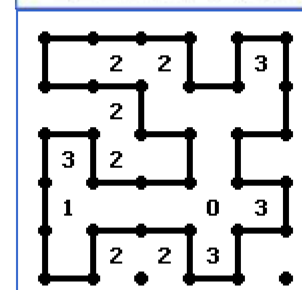
(discussed more in the chapter of puzzle games)

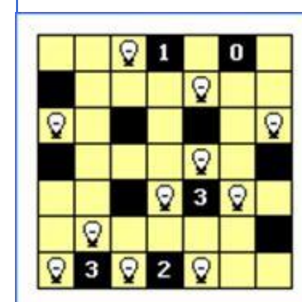
- Sudoku (數獨)
  - Open problem: the minimum Sudoku problem.
- Nonograms
  - http://www.puzzle-nonograms.com/
- Slither Link
- Light Up
- Nurikabe
- Bridge
- Dominosa

Most are NP-complete.

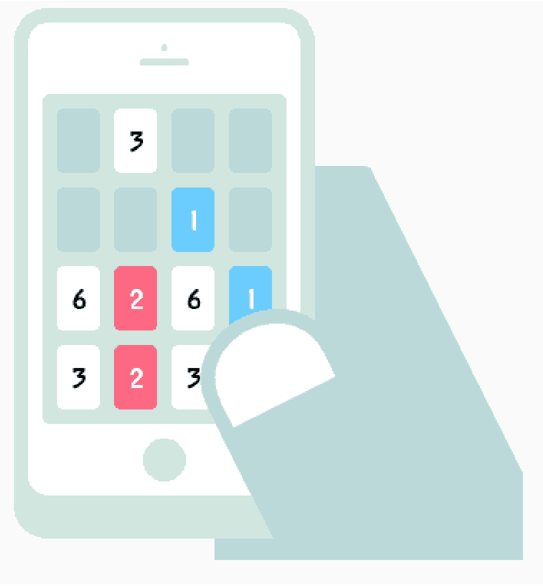






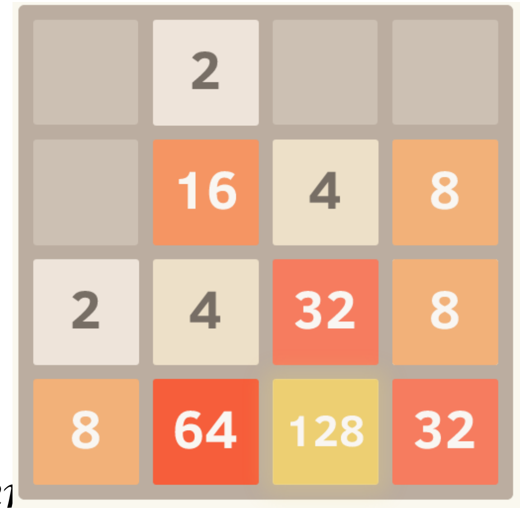


Stochastic Puzzle Games



Single-Player Games

- Threes! (http://asherv.com/threes/)
- 2048 (http://gabrielecirulli.github.io/2048/)
  - Inherit from Threes!.
  - 2-tiles are generated with probability 9/10
  - 4-tiles are generated with probability 1/10



The Best AI Programs (up to 2015)

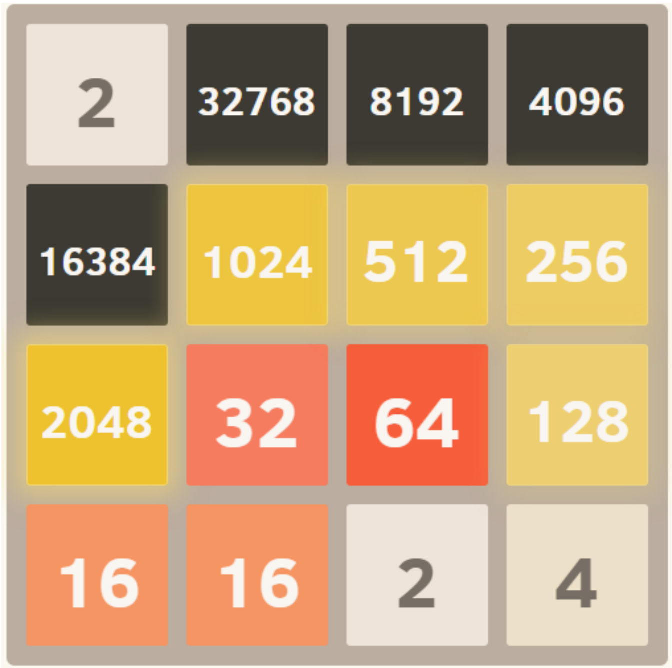
(The best to our knowledge up to 2015)

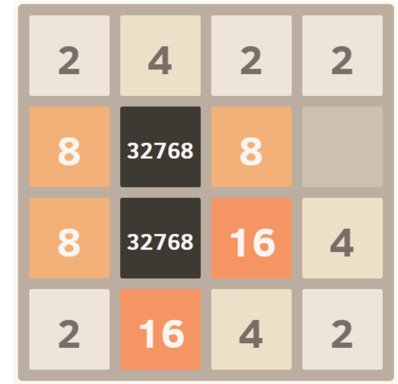
- Threes! – CGI-Threes (from NCTU)
  - 384 — 100%
  - 768 — 100%
  - 1536 — 96.9%
  - 3072 — 68.5%
  - 6144 — 10.1%
  - Max score — 794,250
  - Ave score — 229,834
  - Speed — About 500 moves/sec
- – 2048: CGI-2048 (from NCTU)
  - 2048 — 100.00%
  - 4096 — 100.00%
  - 8192 — 99.50%
  - 16384 — 93.60%
  - **32768 — 33.50%**
  - Max score — 833300
  - Ave score — 446116
  - **Speed — 661 moves/sec**

(The second best to our knowledge)

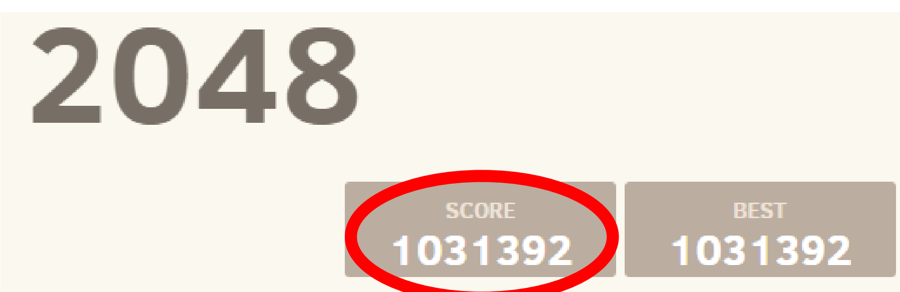
- Threes! (blog.waltdestler.com/2014/04/threesus.html)
  - 384: — 100%
  - 768: — 98%
  - 1536: — 92%
  - 3072: — 27%
  - 6144: — 3%
  - max score — 774,996
  - median score — 89,235
  - Speed — unknown
- 2048 (github.com/nneonneo/2048-ai/pull/27)
  - 2048 — 100.0%
  - 4096 — 100.0%
  - 8192 — 99.0%
  - 16384 — 93.0%
  - **32768 — 32.0%**
  - Max score — 829,300
  - Ave score — 442,419
  - **Speed — About 2-3 moves/sec**

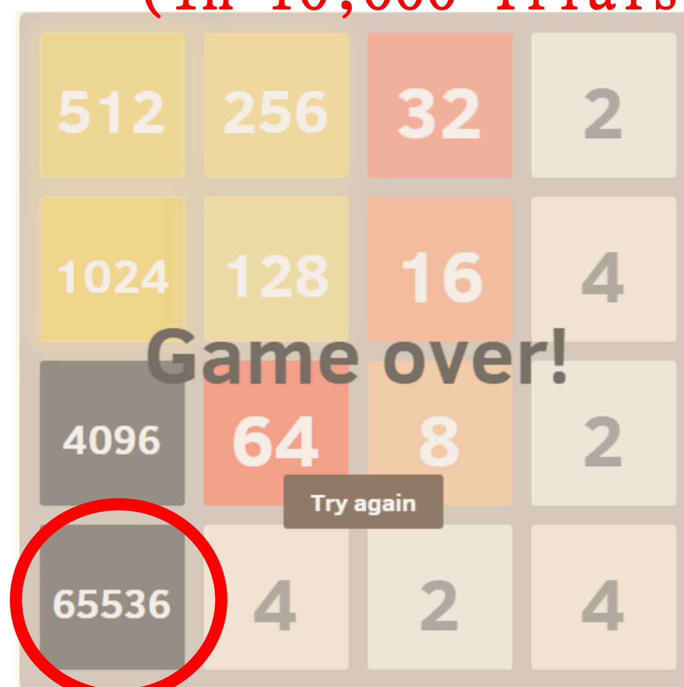
The First Game with 65536 in the World





(In 10,000 Trials)





Conclusion of This Paper

- The knowledge-based methods mostly inform us on the structure of the game, while exhaustive enumeration rarely does.
- Many ad-hoc recipes are produced currently.
- The database can be used as a corrector of strategies formulated by human experts.
- It may be hopeful to use data mining techniques to obtain cross-game methods.
  - Currently not very successful.

Prediction Made in 1990

- Predictions were made at 1990 for the year 2000 concerning the expected playing strength of computer programs.

Table 1
Predicted program strengths for the Computer Olympiad games in the year 2000

Solved or cracked	Over champion	World champion	Grand master	Amateur
Connect-Four	Checkers (8 × 8)	Chess	Go (9 × 9)	Go (19 × 19)
Qubic	Renju	Draughts (10 × 10)	Chinese chess	
Nine Men's Morris	Othello		Bridge	
Go-Moku	Scrabble			
Awari	Backgammon			

Predictions for 2010

- Predictions were made for the year 2010 concerning the
- expected playing strength of computer programs.
- solved over champion world champion grand master amateur

Table 7
Predicted program strengths for the Computer Olympiad games in the year 2010

Solved or cracked	Over champion	World champion	Grand master	Amateur
Awari	Chess	Go (9 × 9)	Bridge	Go (19 × 19)
Othello	Draughts (10 × 10)	Chinese Chess	Shogi	
Checkers (8 × 8)	Scrabble	Hex		
	Backgammon	Amazons		
	Lines of Action			

Status Around 2010

- Chinese chess
  - 象棋特級大師吳貴臨 與 棋天大聖 平手, in 2007.
    - 吳貴臨是台灣象棋九段。
    - 目前全世界象棋特級大師約有20位。
- Connect6
  - NCTU6: 11 wins and 1 loss, 2008.
  - NCTU6: 8 wins and 0 loss, 2009.
  - NCTU6: 5 wins and 3 loss, 2011.
- Shogi
  - Beat a professional player in October, 2010.
  - Beat Miura Hiroyuk (professional 8 dan) in April, 2013.
- 9x9 Go
  - Mogo already beat professional 7-dan player in 2008.
  - Zen beat 周俊勳 twice in 2012. (But, still lost to 9 dan in Japan 2013)
- 19x19 Go
  - Mogo beat Amateur 6 dan with 5-stone handicap in 2008.
  - Zen beat Takuto Ooomote with a 3 stone handicap in 2013.
    - Takuto Ooomote is a 9 dan on the Tygem server.

Any New Predictions?

- None from Herik, but …
  - Computer Shogi beat champions in 2014-6.
  - AlphaGo beat Lee Sedol with 4:1 in 2016.
- In IEEE CIG 2015, a question was raised:
  When to beat Go Grand Master?

Researchers voted for 10-30 years!

The real answer is: 2016!!

- A reasonable conjecture now:

Computer will beat people all eventually!