Total No. of Pages 2

III SEMESTER

Roll No. ....

END SEMESTER EXAMINATION

B.Tech.

IT-205 DISCRETE STRUCTURES

Nov/Dec-2019

Time: 3:00 Hours

Max. Marks: 50

Note: Answer all question by Selecting any two parts from each questions.
All questions carry equal marks.
Assume suitable missing data, if any.

Q.1 [a] i) Over the Universe of animals. Let

P(x): x is a whale.

Q(x): x is a fish.

R(x): x lives in water

Translate the following into English-

I. $\exists x(\neg R(x))$

II. $\exists x(Q(x) \wedge \sim P(x))$

III. $\forall x(P(x) \wedge R(x)) \rightarrow Q(x)$ [3]

ii) Find the disjunctive normal form for the proposition $p \rightarrow q$. [2]

[b] i) Write the converse, contrapositive and negation of the following statement: For every integer n, if n is divisible by 3 then n^2 is divisible by 3. [3]

ii) Use the logical equivalences above to show that $\neg(p \vee \neg(p \wedge q))$ is a contradiction. [2]

[c] Show that the hypotheses "It is not sunny this afternoon and it is colder than yesterday," "We will go swimming only if it is sunny this afternoon," "If we do not go swimming, then we will take a canoe trip," and "If we take a canoe trip, then we will be at home by sunset" lead to the conclusion "We will be home by sunset."[5]

Q.2[a] Calculate the time complexity of Quick Sort algorithm in terms of recurrence relation. Sort the list X = {64, 25, 12, 22, 11} using quick sort. [5]

[b] i) Prove by Contradiction that $\sqrt{10}$ is irrational. [2.5]

ii) Let A = {1, 2, 3, 4} and R = {(a, b): a+b>4} be a relation on A. Draw the graph of relation R. [2.5]

[c] i) Prove for finite sets A and B; $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ [2.5]

ii) In a class of 50 students, 15 play Tennis, 20 play Cricket, 20 play Hockey, 3 play Tennis and Cricket, 6 play Cricket and Hockey, and 5 play Tennis and Hockey, 7 play no game at all. How many play Cricket, Tennis and Hockey? [2.5]

Q.3 [a] i) What is closure of relations? Let A = {1, 2, 3, 4} and R = {(1, 2), (2,3), (3, 4)} be a relation in A. Find its reflexive closure, symmetric closure and transitive closure. [3]

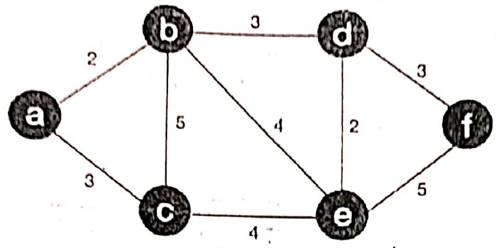
ii) Minimize the following Boolean function using K Map:

$$F(A, B, C) = A'BC + A'BC' + AB'C' + AB'C$$ [2]

[b] Define Equivalence Relation. If R and S be two equivalence relations in a set A, then prove that R U S is also an equivalence relation in A. Also, give suitable example. [5]

[c] Let f: R → R be a function defined as f(x) = 2x-1 and g: R → R be a function defined as $g(x) = \frac{1}{2}x+4$. Find $f^{-1}(x)$, $g^{-1}(x)$, $(f \circ g)^{-1}(x)$ and $(g^{-1} \circ f^{-1})(x)$. What can you conclude? [5]

Q.4[a] Define Spanning Tree and Minimal Spanning Tree. Find Two spanning trees of following graph. Also, Find Minimal Spanning Tree of following graph using Prim's algorithm. [5]



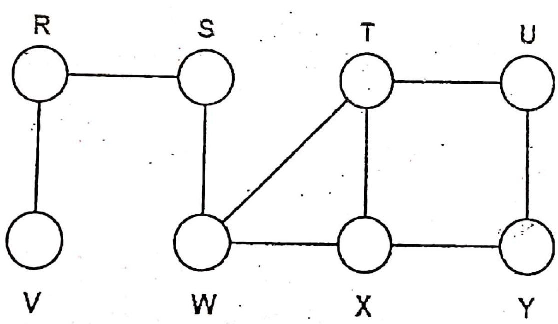
[b] Let x = {1, 3, 5, 7, 15, 21, 35, 105} and R be the relation '/' (divides) on the set x then x is the Poset. Draw the Hasse diagram of the given Poset. Determine the following:

i) LUB of 3 and 7.
ii) GLB of 15 and 35.
iii) Greatest and Least element of x.
iv) Is x a Lattice? [5]

[c] Define 'Join' and 'Meet' in terms of Boolean Matrices. Compute Join and Meet of following Boolean Matrices:

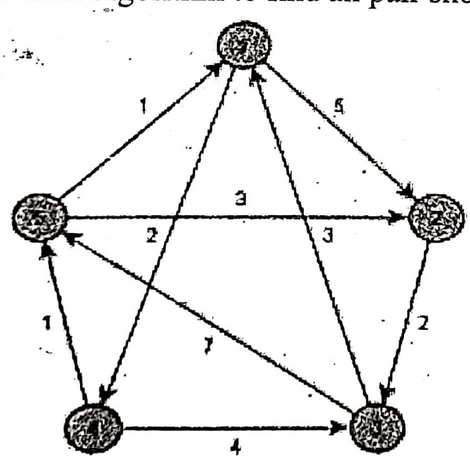
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$ [5]

Q.5[a] What is difference between Breadth First and Depth First Graph Traversals? Apply Breadth First Search to explore all the vertices from the vertex S of the graph given in figure and find the Breadth-first search tree.



[5]

[b] Apply Floyd Warshall algorithm to find all pair shortest path in the following graph.



[5]

[c] Explain Euler's Formula with Proof in Graph Theory. Let G be a graph that has: 21 edges and 7 vertices of degree 1 each; 3 vertices of degree 2 each; 7 vertices of degree 3 each; x vertices of degree 4 each. Compute how many vertices are in G. [2+3]