Date..................

METHOD OF PROOF

① DIRECT METHOD

A direct proof of $p \to q$ is logically valid argument in which we start with the assumption p is true and then using p as well as known show directly that q is true.

Q-1 The product of 2 two odd int is odd

p: product of two odd integers

Q: Result is Odd

Let 2 odd integers

$x = 2n+1$

$y = 2m+1$ odd

n & m any positive integers

$xy = (2n+1)(2m+1)$

$= 4mn + 2n + 2m + 1.$

$2(2mn + n + m) + 1$

$\Downarrow$

q

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$2a+1 \longrightarrow$ odd.

Q- Sq of an even no is a Even No

p: sq of Even no.

q: result is even.

Let: $x : 2n$ } Even   n, & m any +ve

$\cancel{y : 2m}$ }   ints.

So $\cancel{xy}$ $x^2 = (2n)^2$

$x^2 = 4n^2$

$= 2(\underbrace{2n^2}_{a}) = 2a$ Even no.

Q-1 Sum of two odd no is Even Nom

p: sum of 2 odd no.

q: Even no.

So $x = 2n+1$ } n & m are

$y \quad 2m+1$ } any +ve int.

$x+y = 2n+1+2m+1$

$= 2(n+m) + 2$

$2[\underbrace{(n+m)}_{a} + 1]$

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$2\,[\underbrace{a+1}_{k}]$

$\therefore$ $2k \rightarrow$ Even.

Thus q is true

Q-1 INDIRECT METHOD OF PROOF

(1) Proof by contraposition.

It say that $p \to q$ is logical equivalence
$\boxed{\sim q \to \sim p}$

✓ Assume that q is false and then show that
p is false

Q-1 Prove that n^2 is odd then n is odd

p: n^2 is odd.

q: n is odd.

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Now

Assume n is even. → n^2 is even

$n : 2x.$

$n^2 = (2x)^2$

$2\underbrace{(2x^2)}_{a} = \boxed{2a.}$ Even

Hence by contrapositive.

n^2 is odd then n is odd.

Q-2 Pe that if $x, y \in Z$ such that xy is odd then both x & y are odd

P: xy is odd.

q: x & y are odd.

$\neg q$ → x and y are even

So

$x : 2n+1$

$y = 2m.$

$xy = 2n \cdot 2m.$

$= 2\underbrace{(2nm)}_{a}$

$= 2a$ Even

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Q-1 If '$3n+2$' is odd, then n is odd

So let $\sim q$: n is even.

$\sim p \rightarrow 3n+2$ is even

As

$n = 2x$.

Now

$\sim p = 3(2x)+2$

$2 \cdot \underbrace{(3x+1)}_{\ell} = 2\ell \rightarrow$ Even.

So

$\sim q \rightarrow \sim p$.

Q-2 Proof By contradiction.

In this we assume that q is False and $\sim q$ is true

where

$\sim q \rightarrow$ a contradiction.

This can happen only when $\sim q$ is False which implies that q must be true

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Let p is : $3n+2$ is odd and q is 'n' is odd

Assume that q is False

$\sim q$ — n is Even

$$3n+2 = 3(2a)+2$$

$$6a+2$$

$$2(3a+1)$$

$$6$$

$$26$$

Even

But p says that '$3n+2$' is odd this is contradiction

which means q is true

Q-1 Show that $\sqrt{2}$ is an irrational number.

Rational

$$\sqrt{2} = \frac{x}{y} \Rightarrow 2 = \frac{x^2}{y^2}$$

$$2y^2 = x^2$$

x^2 is even

x is even

$$x = 2k$$

$$x^2 = 4k^2$$

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$$2y^2 = 4k^2$$

$$y^2 = 2k^2$$

Ily y^2 is even.

y is even.

So our assumption is False as 2 is a common Factor

So $\sqrt{2}$ is irrational.

Q-2 x belongs to real number, is

$x \in R$, if $\underbrace{x^3+4x=0}_{\not\leftarrow P}$, then $\underbrace{x=0}_{\leftarrow q}$.

Let $x \neq 0$ then.

$$x(x^2+4)=0$$

$$x^2+4=0$$

$$x^2=-4$$

mismatch. make our Assumption False.

Hence. Contradiction

So $\boxed{x=0}$

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(3) Mathematical INDUCTION

$p(n)$ is true for every +ve integer consist of 2 steps

(1) Basic. step

(2) Induction step

P(1) is shown to be true on basic value of n.

(2) Induction step — P(n) is true. and $p(n+1)$ is true.

Exp $P(n) = 1+2+3+ - - - - n = \frac{n(n+1)}{2}$

$$P(1) = \frac{1(1+1)}{2} = \frac{1 \cdot \cancel{2}}{\cancel{2}} = 1.$$

Now.

For k.

$$P(k) = \boxed{1+2+3 - - - k = \frac{k(k+1)}{2}} \quad - (1)$$

Now For k+1

$$\boxed{1+2+3 - - - k} + (k+1) = \frac{(k+1)(k+2)}{2}$$

$$\frac{k(k+1)}{2} + (k+1) = (k+1)\left[\frac{k}{2} + 1\right]$$

First time

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Q-1 Sum of n odd integer · ans n^2

So

$$1+3+5+7 - - - - 2n-1 = n^2.$$

Let

For 1 $\quad (1)^2 = 1.$

So

Let

$$1+3+5+7 - - - (2k-1) = k^2.$$

Now

$$\boxed{1+3+5+7 - - - (2k-1)} + (2k+1) = (k+1)^2$$

$$k^2 + 2k + 1.$$

$$(k+1)^2 = (k+1)^2$$

HoP

Q-2

$$1^2 + 2^2 + 3^2 - - - n^2 = \frac{n}{6}(n+1)(2n+1)$$

So.

For 1.

$$1^2 = \frac{1}{6}(1+1)(2+1) = 1$$

$$\boxed{1^2 + 2^2 + 3^2 - - k^2} = \frac{k}{6}(k+1)(2k+1)$$

So

$$\boxed{1^2 + 2^2 - - - k^2} + (k+1)^2 = \frac{(k+1)}{6}(k+2)(2(k+1)+1)$$

$$\frac{k}{6}(k+1)(2k+1) + (k+1)^2$$

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$$(k+1)\left[\frac{k(2k+1)}{6} + (k+1)\right] =$$

$$\left(\frac{2k^2+k+6k+6}{6}\right)$$

$$(k+1)\left(\frac{2k^2+7k+6}{6}\right)$$

$$\frac{2k^2+4k+3k+6}{6}$$

$$\frac{2k(k+2)+3(k+2)}{6}$$

$$\frac{(k+1)(2k+3)(k+2)}{6}$$ so H.P

PRINCIPLE OF COMPLETE INDUCTION

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Let $p(n)$ be statement defined on whole number + ve int such that.

(1) $P(m)$ is true for some $m \in N$

(2). Whenever $P(m), P(m+1) --- P(k) ---$ true --- $P(k+1)$ is true.

where $k \geq m$

$P(1) \wedge P(2) \wedge P(3) --- --P(k) \Rightarrow P(k+1)$

Q-1 Prove that every int $n \geq 2$ is either a prime or product of prime.

Let $p(n)$ be the statement that
— n is prime or Product of Prime.

$P(2) = 2$
$P(3) = 3$
$P(4) = 2\times2$
$P(5) = 5$
$P(6) = 3\times2$

(ii) — Assume that $P(n) \longrightarrow 2 \leq n < k$ Complete Induction

$k+1$

if $(k+1) \rightarrow$ is Prime . After $2 \leq n \leq k$

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then $P(k+1)$ is true.

(2) If $k+1$ is not Prime then,

$k+1 =$ ~~Product of 2 Numbers~~ uv

$2 \leq u \leq k, \quad 2 \leq v \leq k$

~~$P(u)$~~

$P(k+1) = P(u) \cdot P(v)$

• Ex. $n^3 - n =$ Divisible by 3.

So. whenever n is the integer.

~~$n^3 - n$~~

As $(1)^2 - (1) = \frac{0}{3} = 0.$

So Let.

$\frac{k^3 - k}{3} = L. \qquad k^3 - k = 3L$

So.

$(k+1)^3 - (k+1)$

$k^3 + 3k^2 + 3k - k \cancel{+} 1.$

~~$(k^2 + 2k)$~~ $\qquad k^3 + 3k^2 + 2k - 1.$

$(k^3 - \cancel{k}) + 3k^2 + 3k - 1.$

$(3L) + 3\left[k^2 + k - \frac{1}{3} + 1\right.$

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So

$n^3 - n$ is divisible By 3

So

$P(1) = 1^3 - 1 = 0.$

$P(2) = 8 - 2 = \frac{6}{3} = 2$

$k^3 - k = 3m.$

$P(k+1)$

$(k+1)^3 - (k+1) = k^3 + 3k + 3k^2 + \cancel{1} - k \cancel{-1}$

$(k^3 - k) + 3(k + k^2)$

$3m + 3\underbrace{(k+k^2)}_{l}$

$\frac{3[m+l]}{3} = 0$ ✓

RECURRANCE RELATION

$1, 5, 5^2, 5^3 - - - - -$

Growth term

$$a_n = 5 \cdot a_{n-1} \quad n \geq 1$$

$$a_1 = 5 \cdot a_{1-1}$$

$$\boxed{a_1 = 5 \cdot a_0}$$

now

$$a_0 = 5 \cdot a_{-1}$$

let $a_0 = 1$

A sequence which can be defined by a general formula for its n^{th} term.

An Alternative approach is to write the sequence by finding the relationship among its items such a relationship is called recurrence relation

Q-1 $\quad S = \{5, 8, 11, 14, 17 - - \}$

$$a_n = a_{n-1} + 3$$

$$a_0 = 5$$

Q-2 Find the first four term of recurrence relation

$$a_k = 2a_{k-1} + k \qquad a_1 = 1$$

$$k \geq 2$$

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$a_2 = 2(1) + 2 = 4$

$a_3 = 2(4) + 3 = 11$

$a_4 = 2(11) + 4 = 26$

$a_5 = 2(26) + 5 = 57$

Q-1 An Explicit formula that satisfy recurrence relation with initial condition solution to recurrence Relation

1. SOLUTION

(1) Substitution or Iteration

(2) Characteristic Equation

(3) Generating function.

FIRST ORDER LINEAR RECURENCE Relation

$a_{n+1} = 2a_n$ $\quad a_0 = 1$

Procedure

LINEAR EQN

$\Downarrow$

Find root (r)

$\Downarrow$

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$a_n = A(r^n)$ — Find A using initial Condition.

(1) $r^{n+1} = 2 \cdot r^n$

(2) Divide this equation by lowest Power of r

$$\frac{r^{n+1}}{r^n} = 2 \cdot \cancel{\frac{r^n}{r^n}}$$

$$\boxed{r = 2}$$

(3) Solution.

constant

$$a_n = A(2^n)$$

$$1 = A(2^0)$$

$$\boxed{A = \frac{1}{2}}$$

$$a_n = \frac{1}{2}(2^n)$$

$$\boxed{a_n = 2^{n+} n.}$$

$a_n = 2^n$

Q-1 $a_n = 7a_{n-1}$ where $a_2 = 98$

r^n

$r^n = 7r^{n-1}$

$\boxed{r = 7}$

So.

$a_n = A(r^n)$

$a_n = A(7^n)$

$a_2 = A(7^2)$

$\frac{\cancel{98}^{2}}{\cancel{49}} = A$

$\boxed{A = 2}$

$\boxed{a_n = 2(7^n)}$

If Then it will be Non-Homo

Second order Linear Homogenous R.R

$B a_{n+2} + C a_{n+1} + D.a_n = 0$ If $= f(n)$

3 Types
- Distinct real roots
- Repeated " "
- complex " "

SOLHRR — $B.r^{n+2} + C r^{n+1} + D r^n = 0$

Divide by r^n, lowest

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$$\boxed{Bx^2 + Cx + D = 0}$$

If Distinct Real Root $\quad a_n = A(x_1)^n + B(x_2)^n$

2 Initial condition.

Q-1

$$A_n = 5a_{n-1} - 6a_{n-2} \qquad a_0 = 1, \quad a_1 = 1$$

$$A_n - 5a_{n-1} + 6a_{n-2} = 0.$$

$$x^n - 5x^{n-1} + 6x^{n-2} = 0 \qquad x = 2, x = 3$$

$$x^2 - 5x + 6 = 0.$$

$$x^2 - 2x - 3x + 6 = 0$$

$$(x-2)(x-3) = 0$$

$$x_1 = 2$$

$$x_2 = 3$$

$$a_n = A(2)^n + B(3)^n$$

$$a_0 = A(1) + B(1)$$

$$2 \times (\boxed{A + B = 1})$$

$$2A + 3B = 1.$$

$$B = 1$$

$$\boxed{B = -1.} \qquad \boxed{A = 2}$$

$$\boxed{a_n = 2.(2)^n - 1(3)^n}$$

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Q-2 Date........................

$a_n - a_{n-1} - 6a_{n-2} = 0.$

$x^2 - x - 6 = 0$

~~$x^2 - 3x + 2x - 6 = 0.$~~

$x(x-3) + 2(x-3) = 0$

$\boxed{x = 3, -2.}$

So

$a_n = A(3)^n + B(-2)^n.$

$2\,[\,1 = A + B\,]$

$8 = 3A - 2B$

$10 = 5A$

$A = 2 \quad B = -1$

$\boxed{a_n = 2(3)^n - 1(-2)^n}$

Q-2 $\quad a_n = 6a_{n-1} - 8a_{n-2}$

$x^2 - 6x + 8 = 0. \Big\} \rightarrow$ Characteristic Equation

$x^2 - 4x - 2x + 8 = 0$

$x(x-4) - 2(x-4) = 0$

$\boxed{x = 2, 4.}$

$a_n = A(2)^n + B(4)^n$

$4 = 2\cancel{A} + 4B$

$0 = 2\cancel{A} \; 2B.$

$\cancel{2} = \cancel{2}B$

$B = 2 \qquad A = -2.$

$$\boxed{a_n = -2(2)^n + 2(4)^n.}$$

Q-1 $t_n - 7t_{n-1} + 10t_{n-2} = 0$

$x^2 - 7x + 10 = 0.$

$x^2 - 5x - 2x + 10 = 0$

$x(x-5) - 2(x-5) = 0$

$x = 2, 5$

$a_n = A(2)^n + B(5)^n$

$16 = A(2) + B(5)$

$2(5 = A + B.)$

$10 = 2A + 2B$

$6 = 3B$

$\boxed{B = 2}, \quad A = 3$

$a_n = 3(2)^n + 2(5)^n$

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NON Repeated.

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$a_n = r_1 = r_2$

So

$$\boxed{a_n = A(r_1)^n + Bn(r_1)^n}$$

Solve

$a_n = 6a_{n-1} - 9a_{n-2}$

$r^2 - 6r + 9 = 0$

$r^2 - 3r - 3r + 9 = 0$

$\boxed{r = 3, 3}$

$a_n = A(3)^n + Bn(3)^n$

$\boxed{1 = A}$

$1 = 3A + B(1)(3)$

$\frac{-2}{3} = B$

$$\boxed{a_n = A(3)^n - \frac{2}{3} n(3)^n}$$

$a_n(3)^n + Bn(3)^n$

Q-1 $a_n - 4a_{n-1} + 4a_{n-2} = 0$

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$r^2 - 4r + 4$

$r = 2, 2$

$a_n = A(2)^n + Bn(2)^n.$

$\boxed{1 = A}$

$3 = 2 + 2B$

$\boxed{B = \frac{1}{2}}$

$\boxed{a_n = 1(2)^n + \frac{1}{2}n(2)^n}$

$\boxed{a_n = (n+2)2^{n-1}}$

Q-1 $r^2 - r - 1 = 0.$ FIBBONACCI

$\frac{1 \pm \sqrt{5}}{2}$

$\frac{1+\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2}$

$a_n = A\left(\frac{1+\sqrt{5}}{2}\right)^n + B\left(\frac{1-\sqrt{5}}{2}\right)^n$

$0 = A + B$ 0

$1 = \left(\frac{1}{2} + \frac{\sqrt{5}}{2}\right)A + \left(\frac{1}{2} - \frac{\sqrt{5}}{2}\right)B$

$(n-1-1)$

$n-2-1$

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Q-1

$$\frac{2}{\sqrt{5}} = A - \cancel{B}$$
$$0 = A + \cancel{B}$$

$$\frac{2}{\sqrt{5}} = 2A$$

$A = \frac{1}{\sqrt{5}}$, $B = \frac{-1}{\sqrt{5}}$

3 (1)

Substitution Method

$a_n = a_{n-1} + 3$ $\qquad a_1 = 2$

As $a_{n-1} = a_{n-2} + 3$

$a_n = a_{n-2} + 3 + 3$

$a_{n-2} = a_{n-3} + 3$

$a_n = a_{n-3} + 3 + 3 + 3$

K Times

$$\boxed{a_n = a_{n-k} + 3k}$$

$n - k = 1$

$\boxed{n = k + 1}$ or $\boxed{k = n - 1}$

$a_n = a_1 + 3(n-1)$

$\boxed{a_n = 2 + 3(n-1)}$ Solution

$nk - (1 \cdots k) \quad \frac{k(k+1)}{2}$

Q-1 $\quad H_n = H_{n-1} + (n-1) \quad n \geq 2$

$H_n = H_{n-k} + (n-k-1)$

$(n-1)(n-k-1) \cdots (n$

$H_{n-1} = H_{n-2} + (n-2)$

$$\boxed{H_n = H_{n-k} + (n-k)}$$

$$H_n = H_{n-k} + nk - \frac{k(k+1)}{2}$$

$n-k=1.$

$k = n-1$

$$H_n = H_1 + n(n-1) - \frac{(n-1)(n)}{2}.$$

$$\boxed{H_n = H_1 + \frac{n(n-1)}{2}}$$

$H_1 = 0.$

$$\boxed{H_n = \frac{n(n-1)}{2}}$$

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$\boxed{T_n = T_{n-1} + n}$ $n \geq 1$.

$T_n = T_{n-2} + (n-1) + n$.

$T_n = T_{n-k} + nk - (1 - \cdots - k-1)$

$$T_n = T_{n-k} + nk - \cancel{\frac{(k-1)(k-2)}{2}} \frac{(k-1)k}{2}$$

$n - k = 0$

$\boxed{k = n}$

$$T_n = T_0 + n^2 - \frac{(n-1)n}{2}$$

$$n^2 - \frac{n^2}{2} + \frac{n}{2}$$

$$T_0 + \frac{n^2 + n}{2}$$

$$\boxed{T_n = T_0 + \frac{n(n+1)}{2}}$$

1

Q-1

Q-2

Q-3

How

Q-1 5, 3, 1, -1, -3 - - -

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$(T_n = T_{n-1} - 2)$ $n \geq 1$.

$a_0 = 5$

Q-2 16, 8, 4, 2, 1, 1, 1 - - - - -

$a_{n+1} = 1 \longrightarrow a_n = 1$

$= n/2$ otherwise

Q-3 1, 3, 7, 15, 31, 63 - - - $n \geq 1$ $a_0 = 1$

$\boxed{\cancel{T_n = T_{n-1} + 2n}}$ $\boxed{T_n = 2a_{n-1} + 1.}$ ✓

Homework

substitution

(1) $t_n = t_{n-1} + 4n$ $n \geq 1$.

$= 0$ $n = 0$.

(2) $t_n = 7t_{n-1} + 10t_{n-2}$ → using char.

$t_0 = 5$

$t_1 = 16$

Consider AP sequence $a_0 = 2$, $d = 3$

~~$a_1 = (a_0 + (n-1)3)$~~

(A) - 1^{st} 6 terms

(B) - recurrence Relation

(C) 123^{rd}.

$$\boxed{2, 5, 8, 11, 14, 17}$$

$a_n = a_{n-1} + 3$

$a_n = a_{n-k} + 3k$

$n - k = 0$

$k = n$.

$a_n = a_0 + 3n$.

$$\boxed{a_n = 2 + 3n}$$

$a_{123} = 2 + 3(123)$

$$\boxed{a_{123} = 2 + 369 = 372}$$

Ans 368

Homework

A-1

Homework

A-1 $t_n = 7t_{n-1} - 10t_{n-2}$

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$t_n - 7t_{n-1} + 10t_{n-2} = 0$

$x^n - 7x^{n-1} + 10x^{n-2} = 0$

$x^2 - 7x + 10 = 0$

$x^2 - 2x - 5x + 10 = 0$

$x(x-2) - 5(x-2) = 0$

$(x-2)(x-2) = 0$

$a_n = A(x_1)^n + B(x_2)^n$

$a_n = A(2)^n + B(5)^n$

$t_0 = 5$

$2 \times \boxed{5 = A + B}$ — ①

$16 = 2A + 5B$

$10 = 2A + 2B$

$6 = 3B$

$\boxed{B = 2}$ $\boxed{A = 3}$

$\boxed{t_n = 3(2)^n + 2(5)^n}$ Ans

A-2 $n \not= 0$

$t_n = t_{n-1} + 4n$

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As $n \geq 1$

$t_{n-1} = t_{n-2} + 4(n-1)$ $n = 0$

$t_{n-2} = t_{n-3} + 4(n-2)$

$t_n = t_{n-k} + 4n + 4(n-1) + 4(n-2) - - - - - 4(n-k)$

$t_n = t_{n-k} + 4\left[n + (n-1) + (n-2) + - - - -(n-k)\right]$

$t_n = t_{n-k} + \cancel{4}^{2}\left[\frac{(k+1)(n+n-k)}{\cancel{2}}\right]$

$$\boxed{t_n = t_0 + \{2((n+1)(n))\}}$$

As $t_0 = 0$

$$t_n = 2(n+1)(n)$$

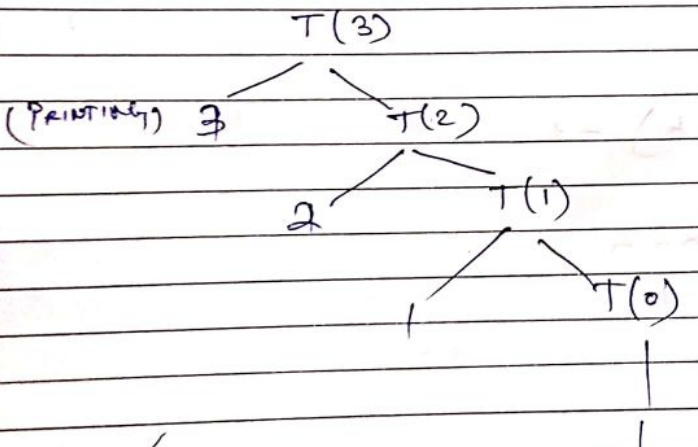
void test (int n) — T(n)

Date..........................

```
{ if (n>0)
  { printf("%d", n);   — 1 (constant)
    test(n-1);         — T(n)
  }
}
```

$T(n) = T(n-1) + 1 \qquad n > 0$

$n = 0$



calling itself

3 → 4 times

~~Printing 3 times~~

n → n+1

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$T(n) = T(n-1) + 1 \qquad n > 0$

$1 \leftrightarrow n = 0$

$T(n-1) = T(n-2) + 1$

$T(n) = \underbrace{T(n-2)}_{} + 1 + 1$

$T(n-3) + 1 + 1 + 1$

k times

$T(n) = T(n-k) + k$

$n - k = 0$

$n = k$

So

$$\boxed{T_n = T_0 + n}$$

Order of 'n'

Q-2 void Test (int n)

Date........................

{ if (n>0)

{ for (i=0; i<n; i++) — n+1

{

printf ("%d", n); — n

}

Test (n-1); — T(n-1)

}

}

$T(n-1) = T(n-2) + (n-1)$

~~$T(n) = T(n-1) + 2n + 2$~~

Ignore

$T(n) = T(n-1) + n + (n-1)$

$T(n) = T(n-2) + n + (n-1)$

$T(n) = T(n-3) + n + (n-1)(n-2)$

|

|

$T(n) = T_{n-k} + n + (n-1)(n-2) \ldots (n-k+1)$

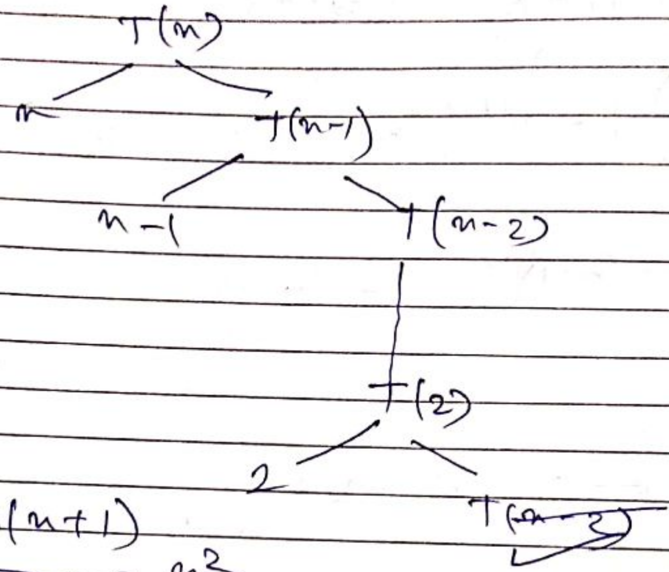
$T(n) = T_{n-k} + \frac{k (n+n-k+1)}{2}$

$O(n^2)$

$T_n = T_0 + \frac{(n+1)(n)}{2}$

$$\boxed{T_n = T_0 + \frac{n^2}{2} + \frac{n}{2}}$$

Date........................



$$\frac{n(n+1)}{2} = n^2$$

$(K = (n-1))$

$n - (k-1) + n - (k-2) + \cdots n - 1 + n$

$1 + 2$

$n - 1$

Void test
{
if (
for

T
}
}

```
Void Test (int n)
{
  if (n>0)
  {for (i=1; i<n; i=i*2)
     {
       printf("%d", n);     } log_2 n
     }
   Test(n-1);   ---> T(n-1)
  }
}
```

$2^k = n$

$\boxed{k = \log n}$

$n>0$

$n=0$

$T_n = \underbrace{T(n-1)} + \log_2 n$

$T_n = T(n-2) + \log_2(n-1) + \log_2(n) \cdots$

$T_n = T_{(n-k)} + \log_2(n-k+1) + \log_2(n-k) \cdots$

$\cdots \log_2(n-1)$

$+ \log_2(n)$

$T_0 + \log_2(1) + \log_2(2) + \log_2(3) \cdots$

$\cdots \log_2(n)$

$\log(n!) \longrightarrow \log n^2$

$n \log n$

Scanned by CamScanner

~~Q-1~~

Date........................

$T(n) = T(n-1) + 1$ $\qquad O(n)$

$T(n) = T(n-1) + n$ $\qquad O(n^2)$

$T(n) = T(n-1) \log n$ $\qquad O(n \log n)$

$T(n) = T(n-1) + n^2$ $\qquad O(n^3)$

$T(n) = T(n-2) + 1$ $\qquad \frac{n}{2}$ --- $O(n)$

$T(n) = T(n-100) + n$ $\qquad n^2$

$T(n) = 2T(n-1) + 1$ — $\underline{O(2^n)}$

```
Test (int n)
{ if (n>0)
  {
    printf("%d,n);
    Test(n-1);
    Test(n-1);
  }
}
```

$T(n) = aT(n-a) + f(n)$

Master Theor
for D
func

if
if
if

Master Theorem for Decreasing functions.

Date........................

$a > 0,\ b > 0,\ f(n) = O(n^k)$

if $a = 1$, $O(n^{k+1}) \rightarrow f(n)$ is multiplied with n

if $a > 1$, $O(n^k a^{n/b}) \rightarrow f(n)$ is " " a^n

if $a < 1$ $O(n^k) \rightarrow O(f(n))$

$T(n) = 2T(n-1) + 1$

$\cancel{T(n-1) = 2T(n-2) + 1}$

$2T(n-1) = 4T(n-2) + 2.$

$T(n) = \cancel{2}(2T(n-2)) + 1 + 2$ Homework

$2^k T(n-k) +$.

Architecture

- ~~BINARY Search~~
- ~~Tower of Hanoi~~
- Selection. Sort
- Merge Sort
- QUICK Sort

Time complexity

RECURRENCE Relation.
For Divide and Conquer

Date........................

```
Test (int n)  ——  T(n)
{
  if (n>1)
  {  printf("%d", n);  —— 1
     Test(n/2);  → T(n/2)
  }
}
```

1

RR

$$T(n) = \begin{cases} 1 + T(n/2) & n > 1 \\ 1 & n = 1 \end{cases}$$

$T(n)$

$1 \quad T(n/2)$

$1 \quad T(n/2^2)$

$1 \quad T(n/2^3)$

$1 \quad T(n/2^k)$

1. — — —99. 100 $\quad K + T(n/2^k)$

Date.........................

$100 = 99 + 1$

$T(1)$

$\log_2 n + 1$

$O(\log n)$

$\frac{n}{2^k} = 1$

$k = \log_2 n.$

$\boxed{T(n) = T(n/2) + 1.}$

$T(n/2) = T(n/4) + 1.$

$T(n) = T(n/2^2) + 2.$

$T(n) = T\left(\frac{n}{2^k}\right) + k$

So

$\frac{n}{2^k} = 1$

$n = 2^k$

$\log n = k \log 2$

$\boxed{k = \log_2 n}$

$T(n) = T(1) + \log_2 n$

$\boxed{T(n) = 1 + \log_2 n}$

$\frac{a}{1-r}$

Test (int n) —— T(n)

Date.....................

{ if (n>1)

$\frac{1}{1-\frac{1}{2}} = 2$

{ for(i=0; i<n; i++)

printf ("%d", n); } n.

Test(n/2); T(n/2)

}

$k = \log_2 n$

$T(n) = T(n/2^k) + \frac{n}{2^{k-1}} + 1$

$1 + n\left[1 + \frac{1}{2} + \frac{1}{2^2} + \cdots \frac{1}{2^{k-1}}\right]$

$1 + 2n$

$O(n)$

$T(n) = T\left(\frac{n}{2}\right) + n$

$T(n/2) = T(n/4) + n/2$

$T(n) = T\left(\frac{n}{2^k}\right) + \frac{n}{2^{k-1}} + 1 \cdots$

$T(n) = T\left(\frac{n}{2^k}\right) + 1 + \sum \frac{n}{2^{k-1}} \cdots$

$\frac{n}{2^k} = 1 \qquad n = 2^k$

$\boxed{\log_2 n = k}$

Date..................

$T(n) = T(1) + 1 + n\left(\frac{1}{1-\frac{1}{2}}\right)$

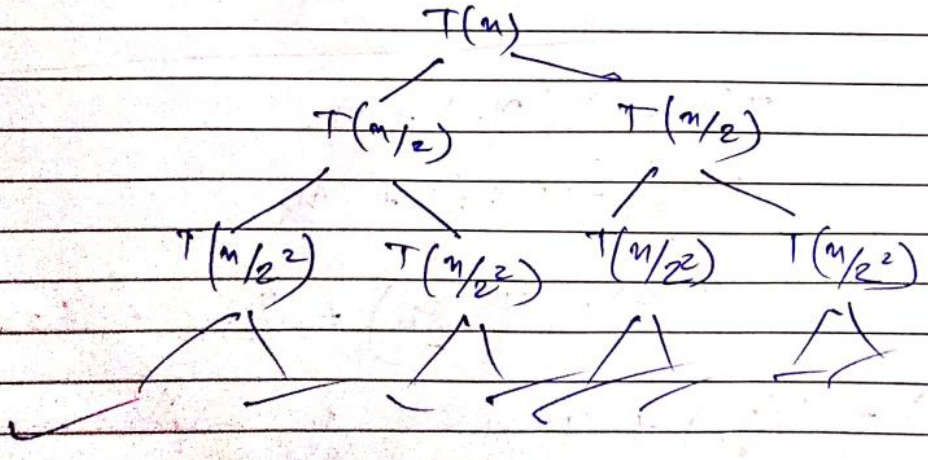
$T(1)+1+2n$

$2+2n \rightarrow n$

Q-1

$O(n \log_2 n)$

```
Test (int n)
{ if (n>1)
  { for(i=0; i<n; i++)
       printf("%d", n);
    Test(n/2);   →   T(n/2)
    Test(n/2);
```



$k = \log n$

$T(n) = 2T(n/2) + n$.

$T(n/2) = 2T(n/2^2) + n/2$.

Date........................

$T(n) = 4T\left(\frac{n}{2^2}\right) + n + n + n \ldots$

$$\boxed{T(n) = 2^k T\left(\frac{n}{2^k}\right) + nk}$$

$\frac{n}{2^k} = 1$

$n = 2^k$.

$\boxed{\log_2 n = k}$

$T(n) = 2^k \; T(1) \quad + n \log_2 n$

$\cancel{2^k} + n \log_2 n$.

$n(1 + \log_2 n)$

$O\boxed{n \log_2 n}$

Quick Sort

$$T(n) = 2T(n/2) + n+1$$

Cost for recursively sorting two half size arrays

Partitioning cost for n elements - n+1 comparisons

$$T(1) = 1$$

MERGE SORT

$$T(n) = 2T(n/2) + n$$

Time taken. Split list into two halves

merging time

$$T(1) = 1$$

$$O(n \log n)$$

Selection Sort

n.

Smallest → n-1

$$T(n) = n + T(n-1) \quad n > 1$$

$$= 1 \quad n = 1$$