Topic: Tutorial 5 M&S Date:

Probability distributions : discrete

A random variable is a variable whose values are determined by the outcome of a random experiment. It is also called stochastic variable.

Probability distribution: If all possible values of a random variable can be written along their associated probabilities, then the distribution is called probability distribution.

x:	x_1	x_2	x_3	---	x_n
$f(x)$:	$f(x_1)$	$f(x_2)$	$f x_3$	---	$f(x_n)$

* mandatory conditions for probability distribution:

1) $f(x_i) \geq 0$
2) $\sum_{i=1}^{n} f(x_i) = 1$

Types of Probability distribution:

- discrete probability distribution
  - Binomial probability distribution
  - poisson probability distribution
- continuous probability distribution
  - normal probability distribution
  - exponential probability distribution

* Binomial distribution

The word binomial means 2 numbers. A binomial distribution for a random variable X (known as binomial variate) is one in which there are only 2 outcomes, success or failure, for a finite number of trials.

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However the success and failure, the two events must be mutually exclusive and complementary i.e. they mustn't occur at same time and the sum of their probabilities is 100%. (complementary).

- P(success) = p
- P(failure) = q = 1 − p
- n = fixed number of trials
- p = probability of success for any one trial
- q = probability of failure for any one trial

Formula:

$$P(r) = {}^nC_r \; p^r q^{n-r} \quad \text{where } n = 0, 1, 2, 3 \dots$$

r = number of successes

$$\text{frequency } (f_r) = N \, P(r)$$

N → repetition of experiment (n)

example 1: A fair coin is tossed 7 times. Find probabilities of obtaining various number of heads.

Soln,

n = 7

p = probability of getting head in one trial = 1/2

q = probability of getting tail in one trial = 1/2

Probability of getting 0 head (P(0))

$$= {}^7C_0 \cdot p^0 q^7 = 1 \times \left(\frac{1}{2}\right)^0 \times \left(\frac{1}{2}\right)^7$$

$$= 1/128$$

$$P(1\ \text{head}) = {}^7C_1 \cdot p^1 q^6 = \left(\frac{7}{1}\right)\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^6 = \frac{7}{128}$$

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$P(2\ head) = {}^7C_2 \cdot p^2 q^{7-2} = {}^7C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^5$

$= \frac{21}{128}$

$P(3\ head) = {}^7C_3 \cdot p^3 q^4 = {}^7C_3 \cdot \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^4 = \frac{35}{128}$

$P(4\ head) = {}^7C_4\ p^4 q^3 = \frac{35}{128}$

$P(5\ head) = {}^7C_5\ p^5 q^2 = \frac{21}{128}$

$P(6\ head) = {}^7C_6\ p^6 q^1 = \frac{7}{128}$

$P(7\ head) = P(7) = {}^7C_7\ p^7 q^0 = \frac{1}{128}$

Binomial distribution is a Discrete probability distribution used when there are only 2 possible outcomes for a ~~poss~~ random variable: success and failure, which must be exclusive.

example 2: in a college 20% students are girls. In a random sample of 5 students, find probability that there are at most 2 girls?

sol^n:

r = no. of girls (successes)

p = no. of girls (success) probability in one trial

$= 20\% = \frac{20}{100} = 0.2$

q = no. of failures $= 1 - p = 0.8$

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$$P(r \le 2) = P(0) + P(1) + P(2)$$
$$= {}^5C_0\, p^0 q^5 + {}^5C_1\, p^1 q^4 + {}^5C_2\, p^2 q^3$$
$$= 0.32768 + 0.4096 + 0.2048$$
$$= 0.94208$$

examples: an event has $p = \frac{3}{8}$. Find complete binomial distribution for $n = 5$ trials.

sol^n:

$$p = \frac{3}{8},\ q = 1 - p = \frac{5}{8},\ n = 5$$

The complete binomial distribution in $(p+q)^n$ terms i.e. $\left(\frac{3}{8} + \frac{5}{8}\right)^5$ is.

$$\left[{}^nC_0\, p^0 q^n + {}^nC_1\, p^1 q^{n-1} + \cdots\cdots + {}^nC_n\, p^0 q^n\right]$$

$$= \left[{}^5C_0 \left(\frac{3}{8}\right)^0 \left(\frac{5}{8}\right)^5 + {}^5C_1 \left(\frac{3}{8}\right)^1 \left(\frac{5}{8}\right)^4 + {}^5C_2 \left(\frac{3}{8}\right)^2 \left(\frac{5}{8}\right)^3\right.$$
$$\left. + {}^5C_3 \left(\frac{3}{8}\right)^3 \left(\frac{5}{8}\right)^2 + {}^5C_4 \left(\frac{3}{8}\right)^4 \left(\frac{5}{8}\right)^1 + {}^5C_5 \left(\frac{3}{8}\right)^5 \left(\frac{5}{8}\right)^0\right]$$

$$= 0.0954 + 0.2861 + 0.3433 + 0.2060 + 0.0618 + 0.0074$$

<u>Probability table</u>

x	0	1	2	3	4	5
$P(x)$	0.0954	0.2861	0.3433	0.2060	0.0618	0.0074

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* Poisson distribution formula/probability

It is used to find the probability of an independent event that is occuring in a fixed interval of time and has constant mean rate. A Poisson random variable will relatively describe a phenomenon if there are few successes over many trials. It is used as the limiting case of Binomial distribution where trials are large indefinitely. It has only one parameter (λ) which is mean number of events.

Poisson random variable 'x' defines the no. of successes in the experiment. Poisson distribution is used under certain conditions. They are:

- The number of trials "n" tends to infinity
- probability of success tends to zero
- $np = \lambda$ is finite

Formula : $p(x) = \frac{(e^{-\lambda} \lambda^{x})}{x!}$

e = exponential
x = poisson random variable
λ = average rate of value

$$\boxed{P(r\ success) = \frac{e^{-\lambda} \lambda^{r}}{r!}}$$

example 1: The probability that an individual suffers from a disease is 0.001. What is the probability that out of 2000 individuals, more than 2 will suffer from disease?

Sol^n

$p = 0.001,\ n = 2000,\ \lambda = np = 2$

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$$P(\geq 2) = 1 - P(0) - P(1) - P(2)$$
$$= 1 - \left[\frac{e^{-2} 2^0}{0!}\right] - \left[\frac{e^{-2} 2^1}{1!}\right] - \left[\frac{e^{-2} 2^2}{2!}\right]$$
$$= 1 - [0.1353] - [0.2706] - [0.27]$$
$$= 0.5941 - 0.27$$
$$= 0.323$$
$$= 32.3\%$$

example 2: The no. of accidents occuring in a plant in a month follows poisson distribution with a mean as 5.2. The probability of occurence of less than 2 accidents in the plant during randomly selected month is?

sol^n

$\lambda = 5.2$, $P(<2) = P(0) + P(1) +$

$$= e^{-5.2}\left[\frac{5.2^0}{0!} + \frac{5.2^1}{1!}\right]$$
$$= 6.2\, e^{-5.2}$$
$$= 0.034$$

example 3: you go to a party with 500 guests. What is probability that exactly one other guest has same birthday as you?

sol^n

p = same birthday of 1 person as me $= \frac{1}{365}$

= success (same birthday)

$n = 500$ guests

$\lambda = 500 \times \frac{1}{365} = 1.36 \leq 5$ so poisson

$$P(1) = \frac{e^{-\lambda}(\lambda)^1}{1!} = \frac{e^{-1.37}(1.37)}{1} = 0.348$$ #

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