Assignment 2

Q. Explain Depth-first and breadth first search algorithm with example.

Q. Explain in order, preorder & post order tree traversal algorithms with example.

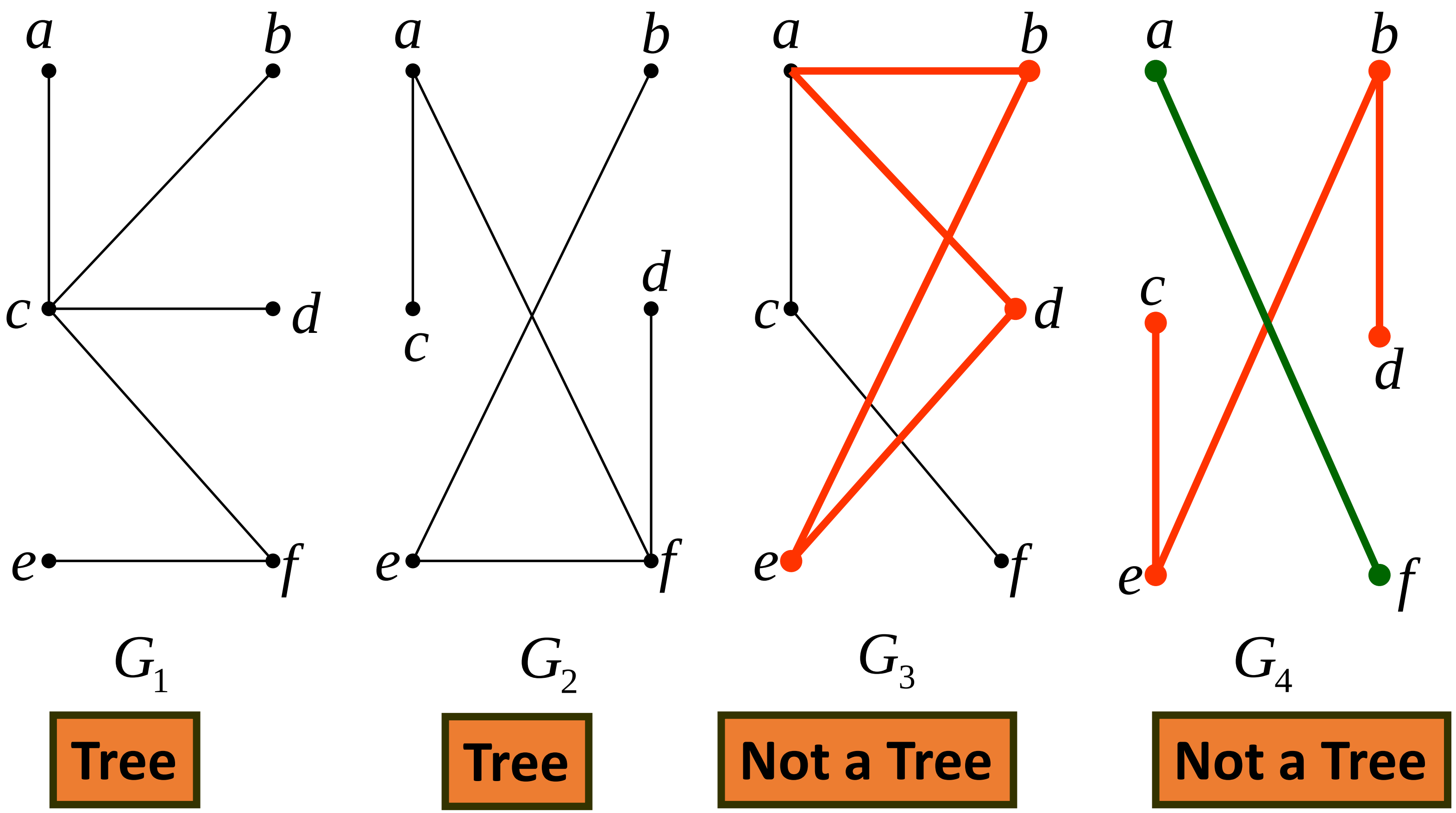
Q. Explain Shortest path & minimal spanning trees with example.

Q. Explain Warshall's algorithms with example.

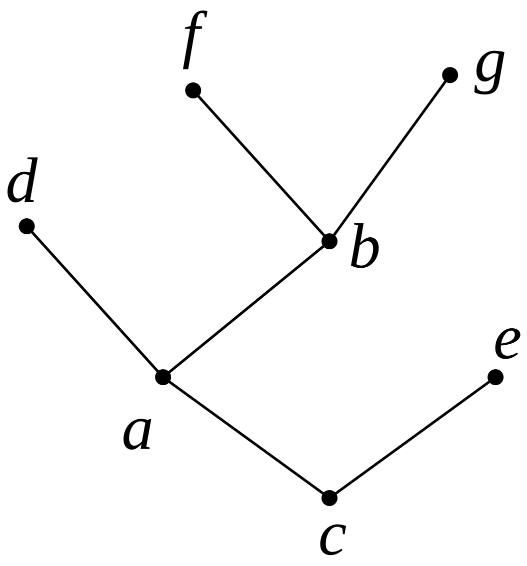
Definition of Tree

- A tree is a finite set of one or more nodes such that:
- There is a specially designated node called the root.
- The remaining nodes are partitioned into $n>=0$ disjoint sets $T_1, ..., T_n$, where each of these sets is a tree.
- We call $T_1, ..., T_n$ the subtrees of the root.

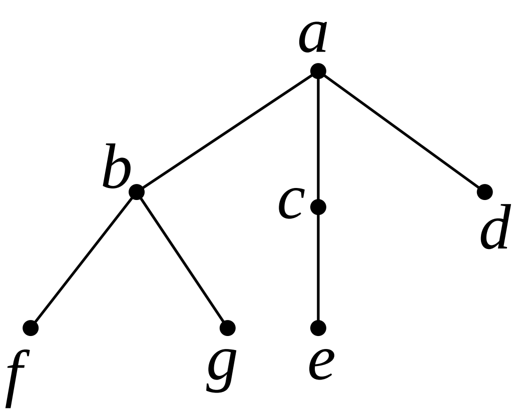
EXAMPLE: Which of the graphs shown below are trees ?



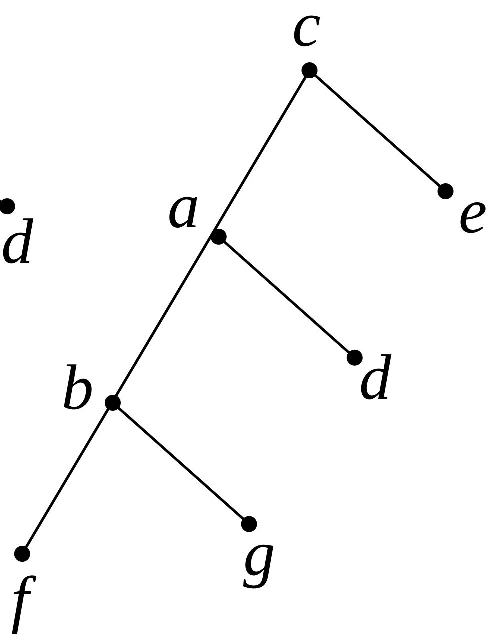
- **Def :** A *rooted tree* is a tree in which one vertex has been designated as the root and every edge is directed away from the root.



A Tree



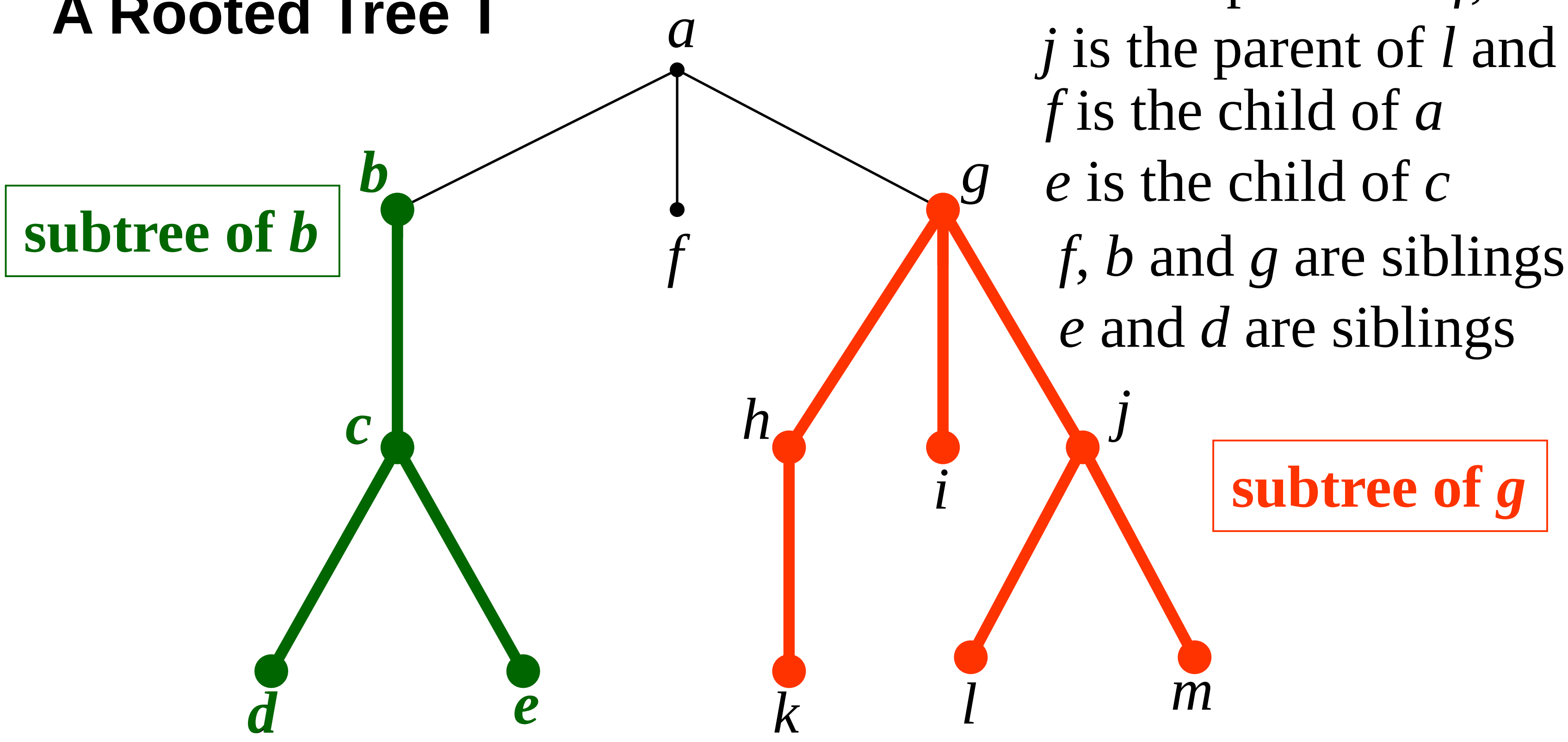
A root tree with root a



A root tree with root c

Terminology

A Rooted Tree T



a is the parent of f, b and g

j is the parent of l and m

f is the child of a

e is the child of c

f, b and g are siblings

e and d are siblings

a, b are ancestors of c

e, c are descendants of b

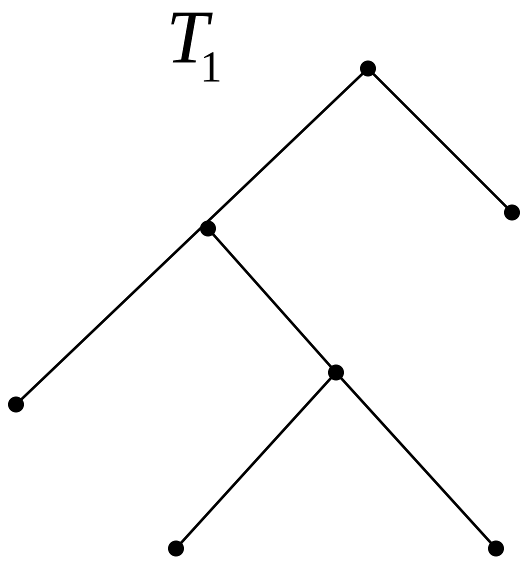
leaves: d, e, f, k, i, l, m

internal vertices: others

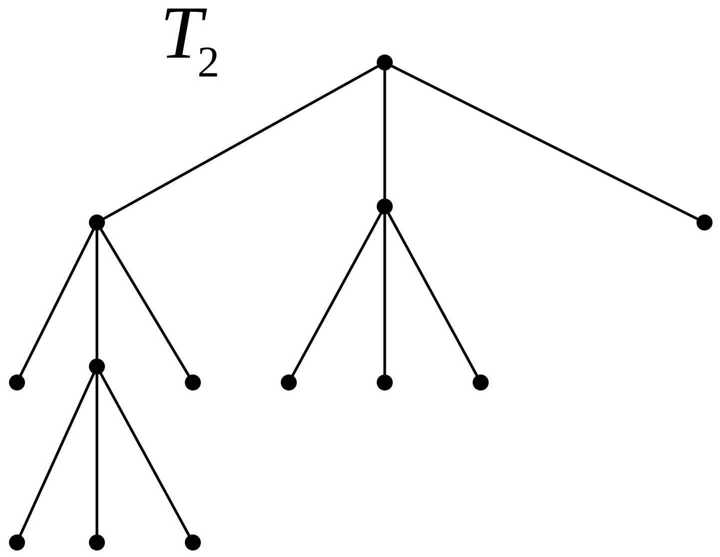
Def:

- A rooted tree is called an *m-ary* tree if every interval vertex has no more than *m* children.
- The tree is called a *full m-ary* tree if every interval vertex has exactly *m* children.
- An *m-ary* tree with $m = 2$ is called a *binary tree*.

Example 3

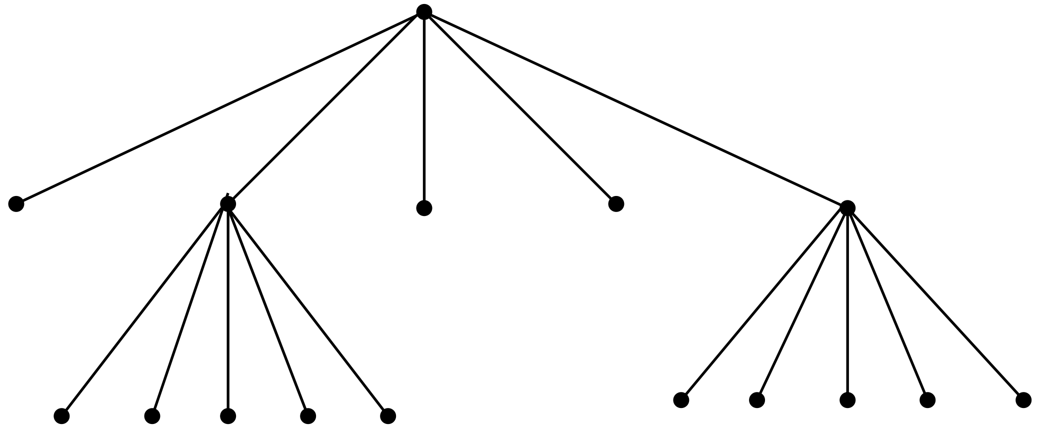


A full binary tree.



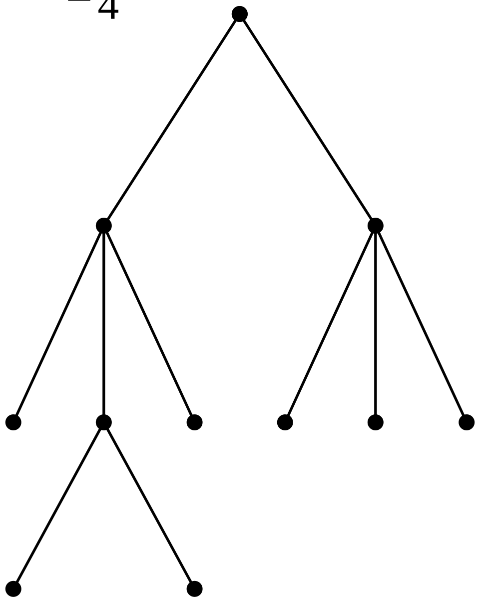
A full 3-ary tree.

T_3



A full 5-ary tree.

T_4

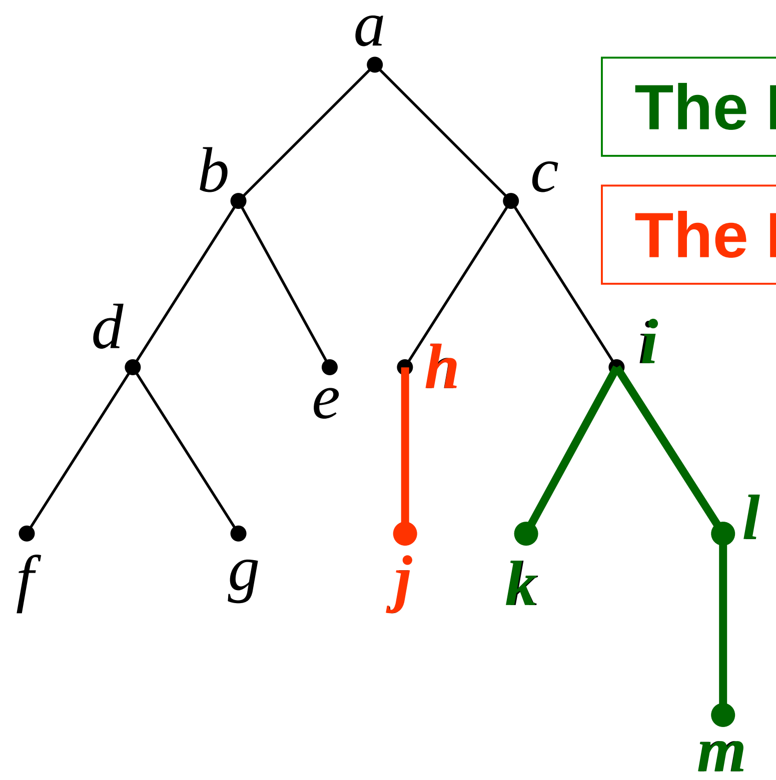


A *m*-ary tree. ($m \geq 3$)
Not a full *m*-ary tree

More Terminology

- An **ordered rooted tree** is a rooted tree where the children of each internal vertex are ordered (from left to right).
- In an ordered binary tree (usually called just a binary tree), if an internal vertex has two children, the first child is called the **left child** and the second one is called the **right child**.
- The tree rooted at the left (right) child of a vertex is called the **left (right) subtree** of this vertex.

Example 4



The Right Subtree of the Vertex *c*.

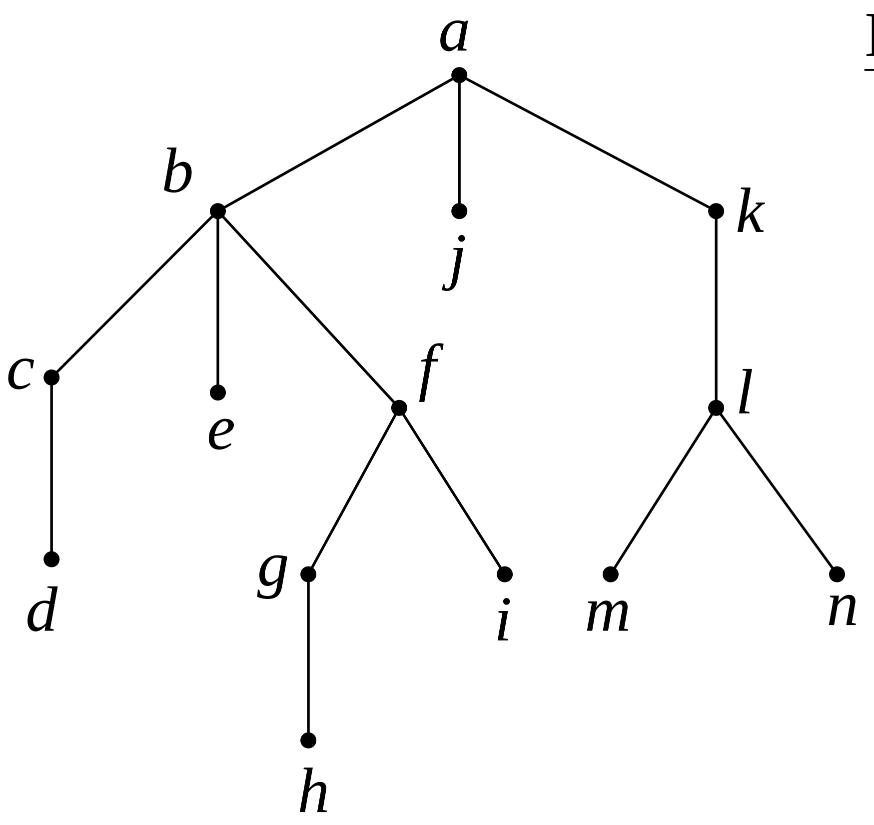
The Left Subtree of the Vertex *c*.

A Binary Tree T

More Terminology

- The **level** of a vertex v in a rooted tree is the length of the unique path from the root to this vertex v.
  - The level of the root is defined to be zero.
- The **height** of a rooted tree is the maximum of the levels of vertices.
  - That is, the height is the length of the longest path from the root to any vertex.
- A rooted m-ary tree of height h is **balanced** if all leaves are at levels h or $h-1$.

Example



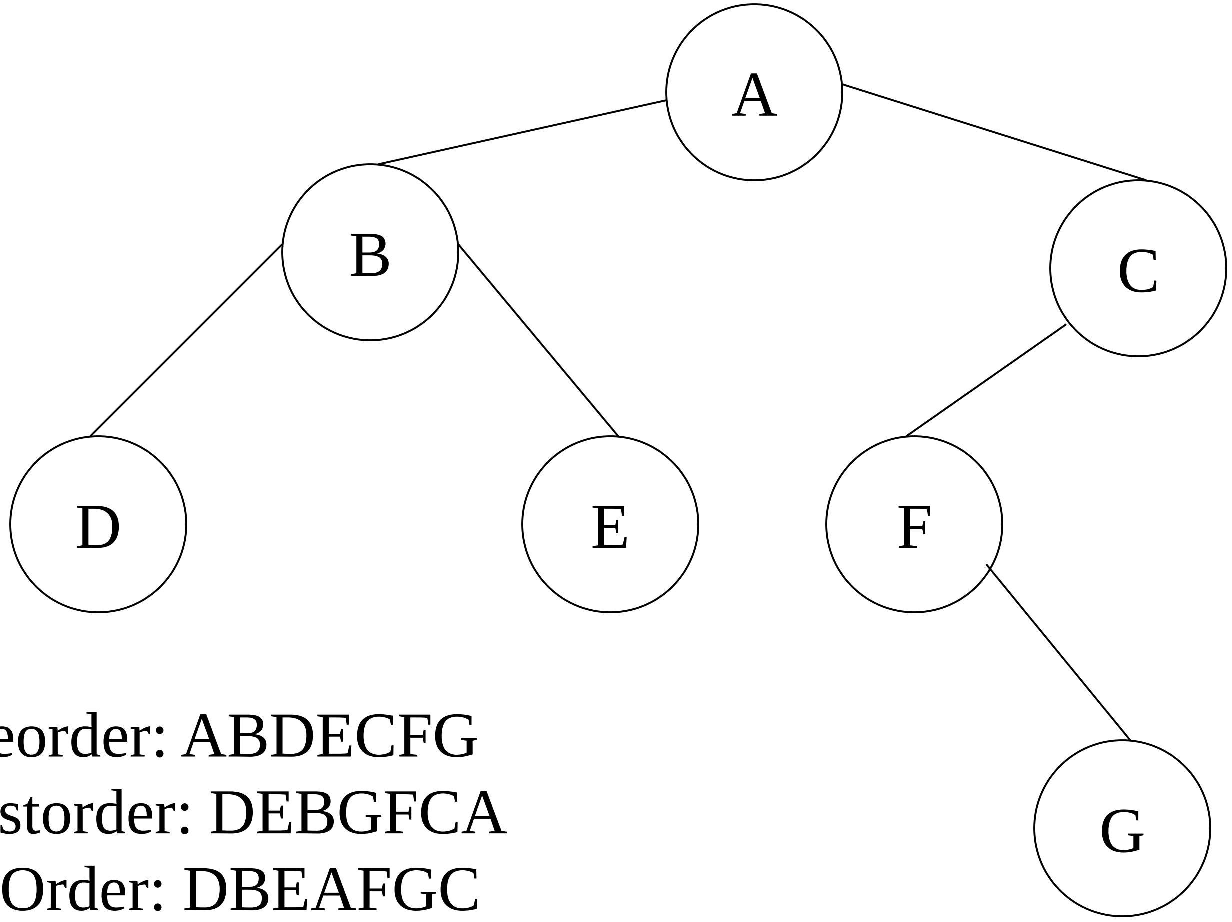
Level	Vertices
0	a
1	b, j, k
2	c, e, f, l
3	d, g, i, m, n
4	h

The height of this rooted tree is 4.

Tree Traversal

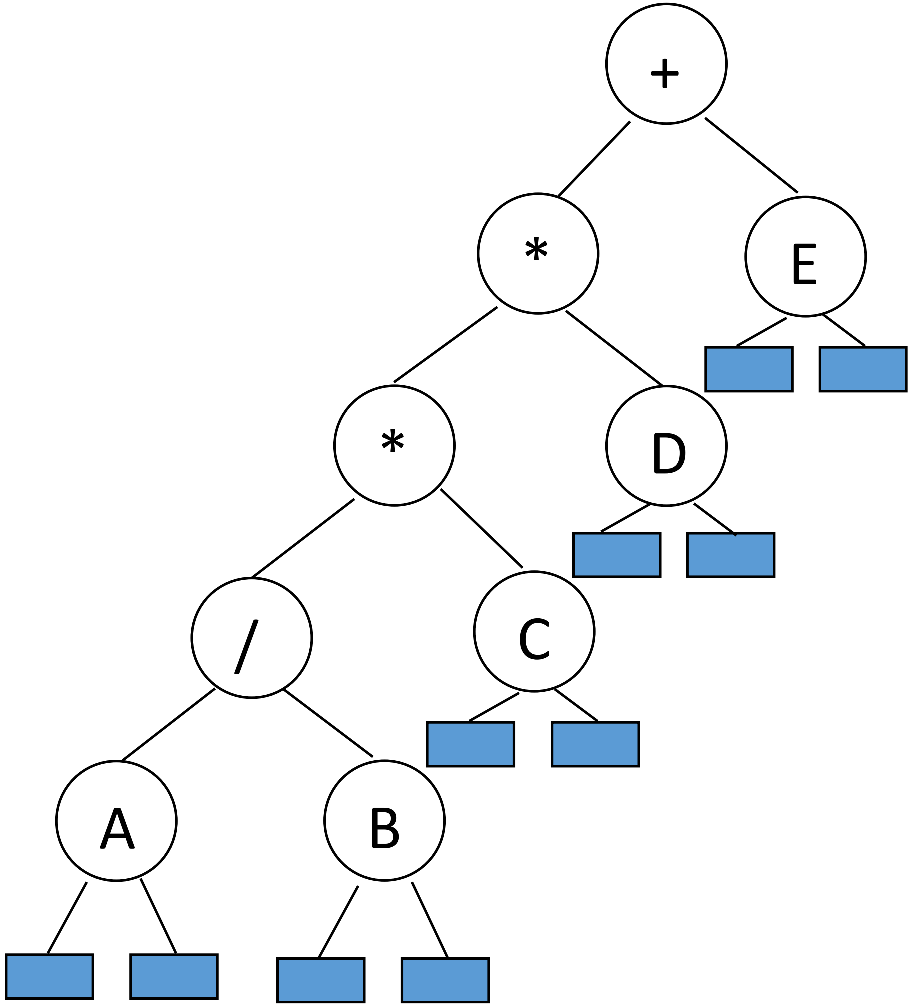
- There are three common ways to traverse a tree:
  - **Preorder**: Visit the root, traverse the left subtree (preorder) and then traverse the right subtree (preorder)
  - **Postorder**: Traverse the left subtree (postorder), traverse the right subtree (postorder) and then visit the root.
  - **Inorder**: Traverse the left subtree (in order), visit the root and the traverse the right subtree (in order).

Tree Traversals: An Example



Preorder: ABDECFG
Postorder: DEBGFCA
In Order: DBEAFGC

Arithmetic Expression Using BT



inorder traversal

A / B * C * D + E

infix expression

preorder traversal

+ * * / A B C D E

prefix expression

postorder traversal

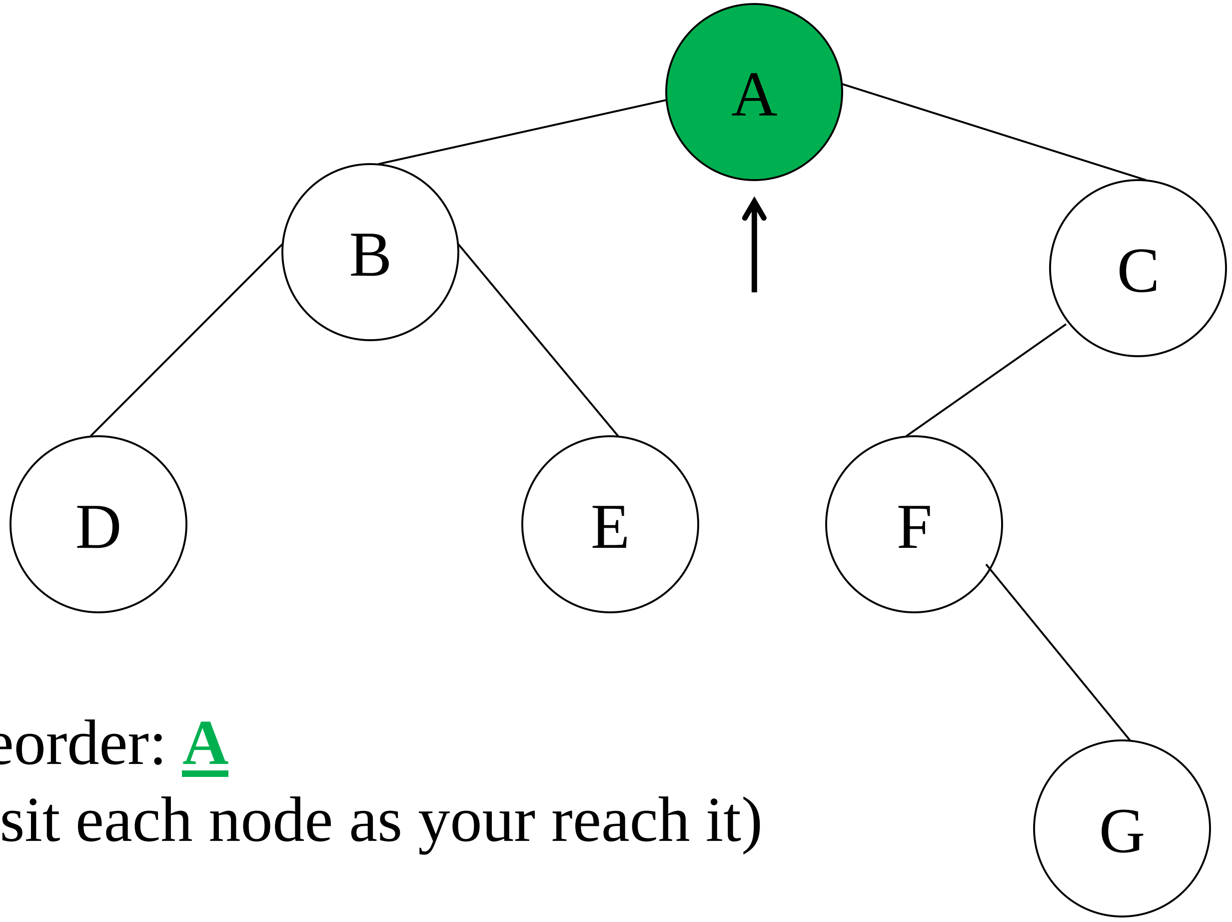
A B / C * D * E +

postfix expression

level order traversal

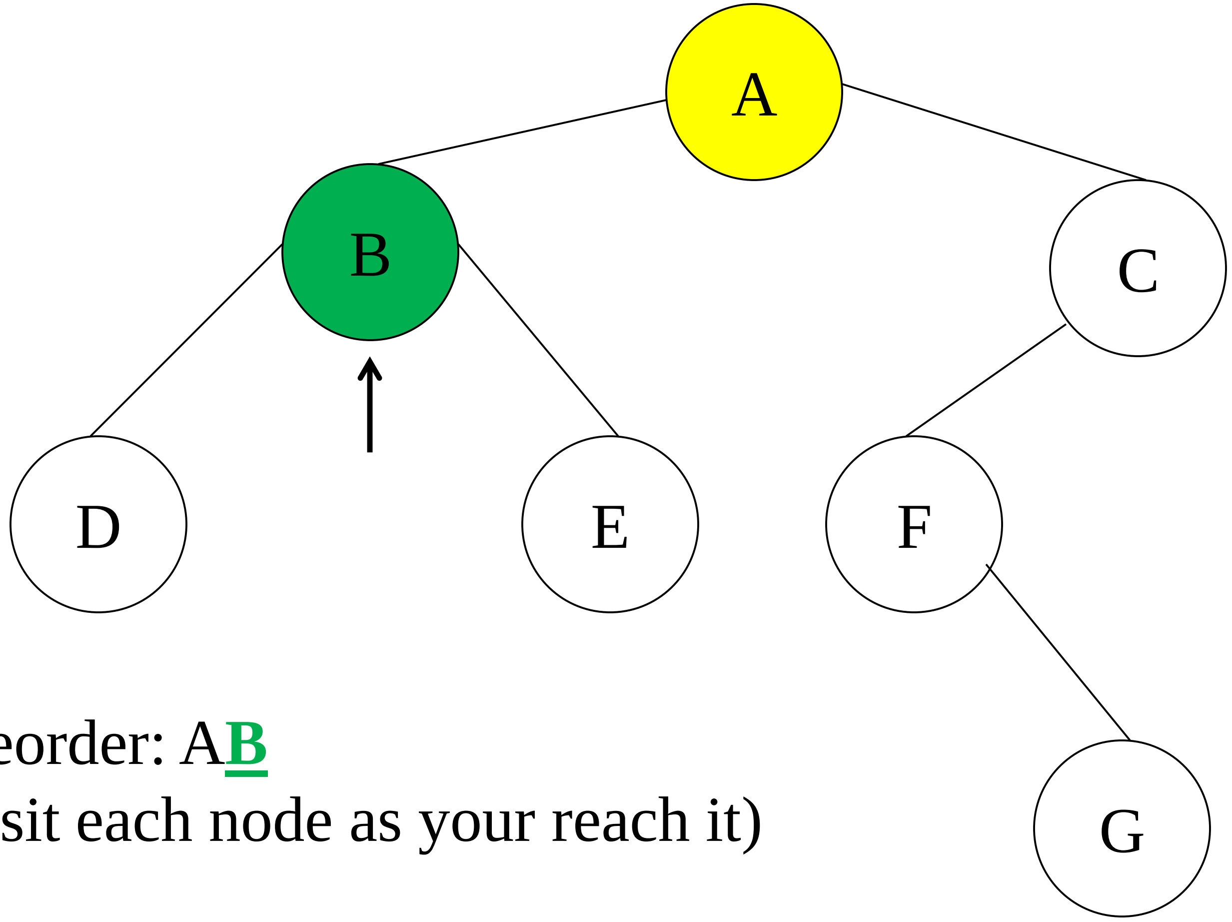
+ * E * D / C A B

Tree Traversals: An Example



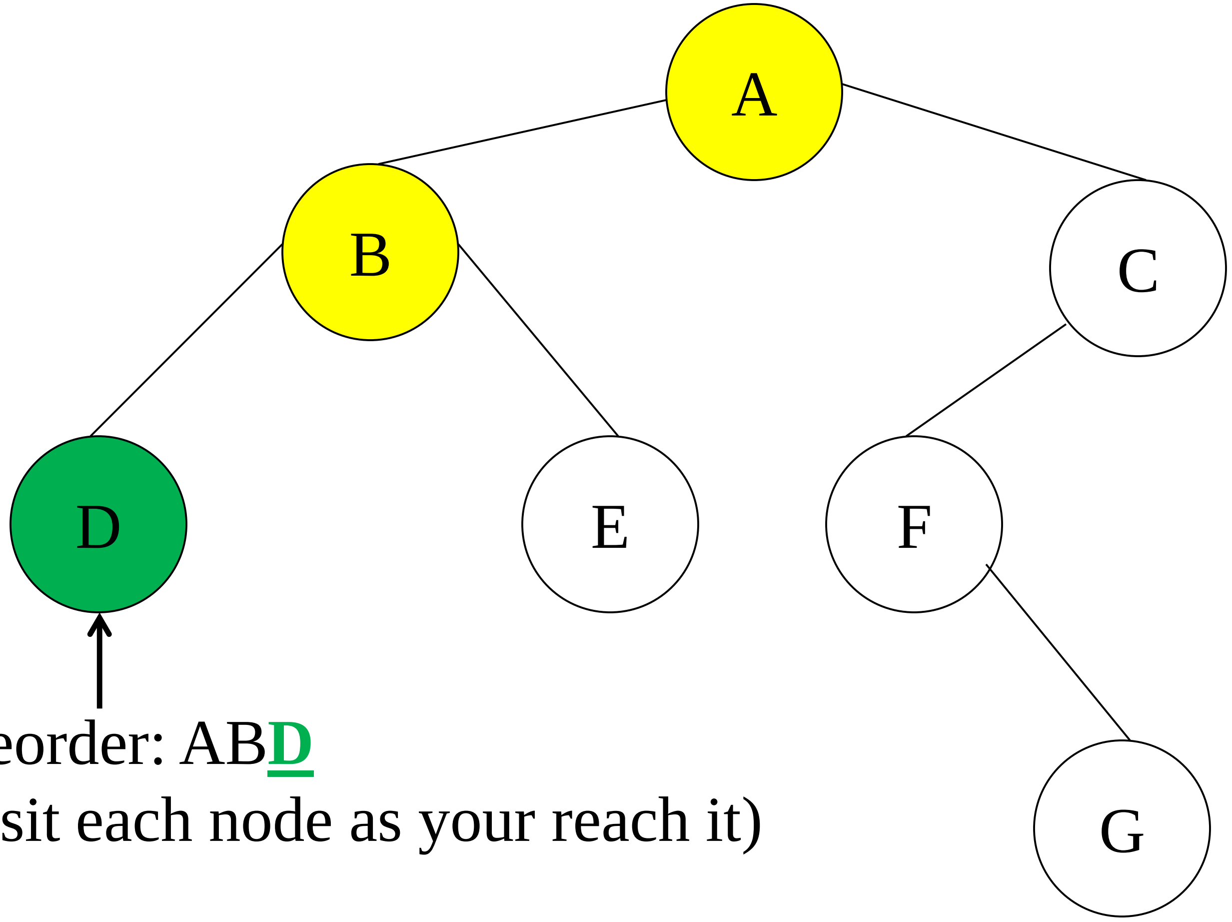
Preorder: **A**
(visit each node as your reach it)

Tree Traversals: An Example



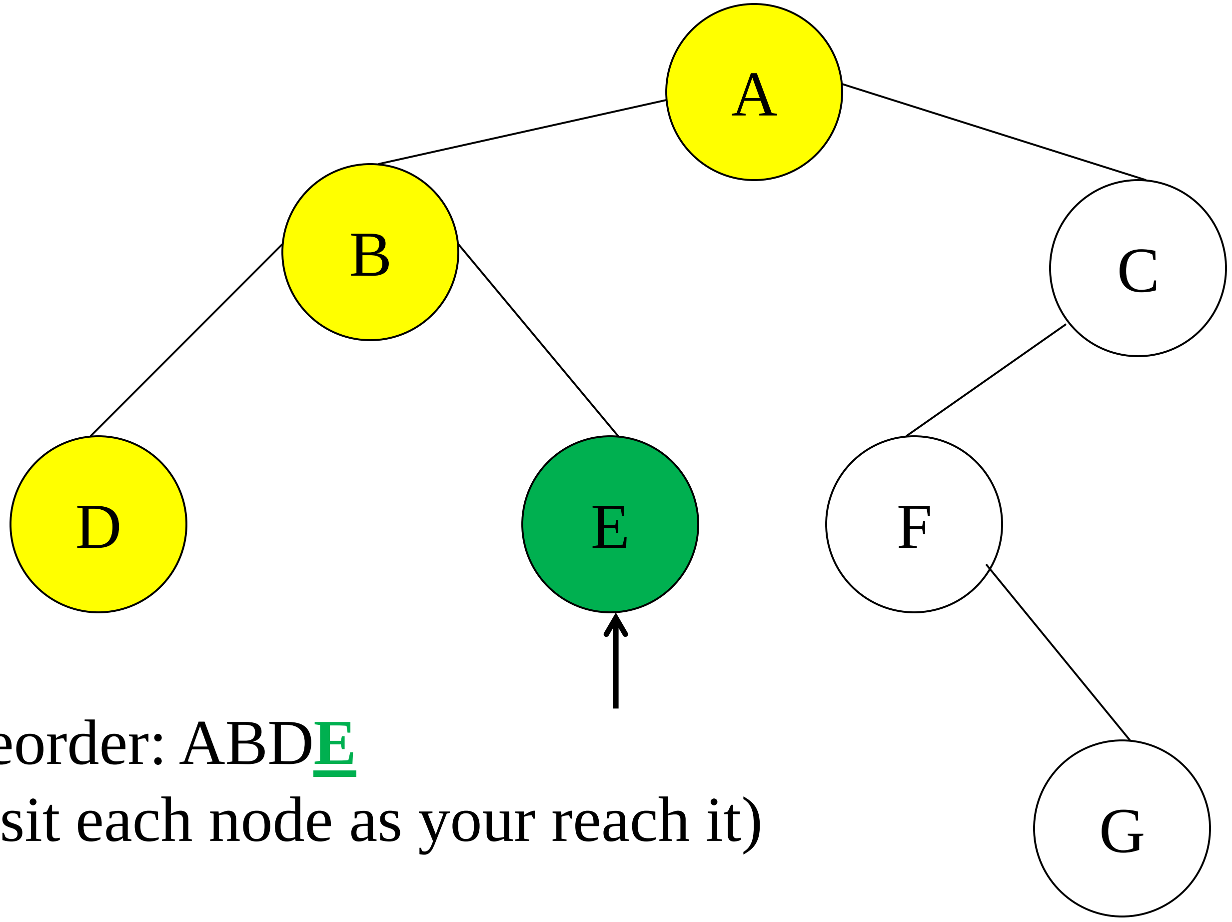
Preorder: A**B**
(visit each node as your reach it)

Tree Traversals: An Example



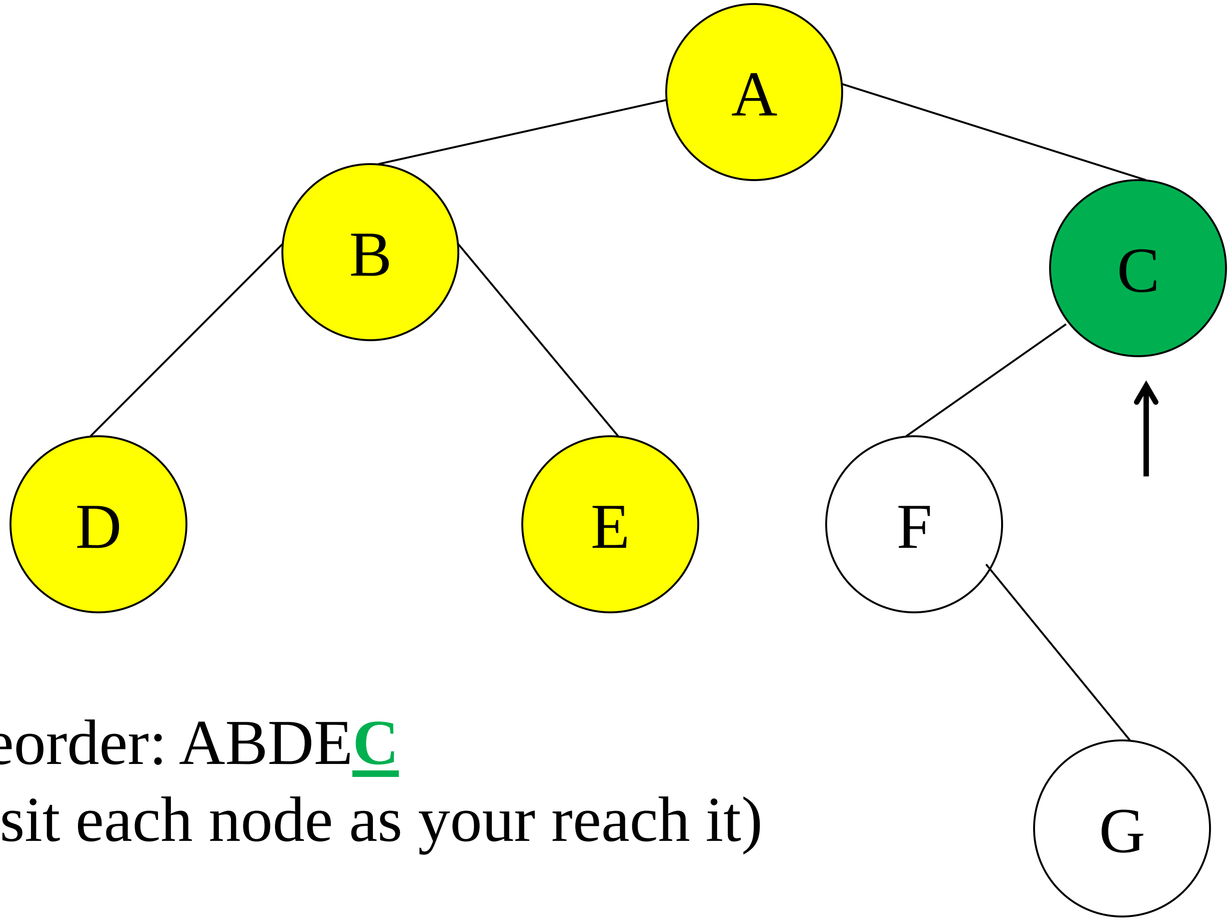
Preorder: AB**D**
(visit each node as your reach it)

Tree Traversals: An Example



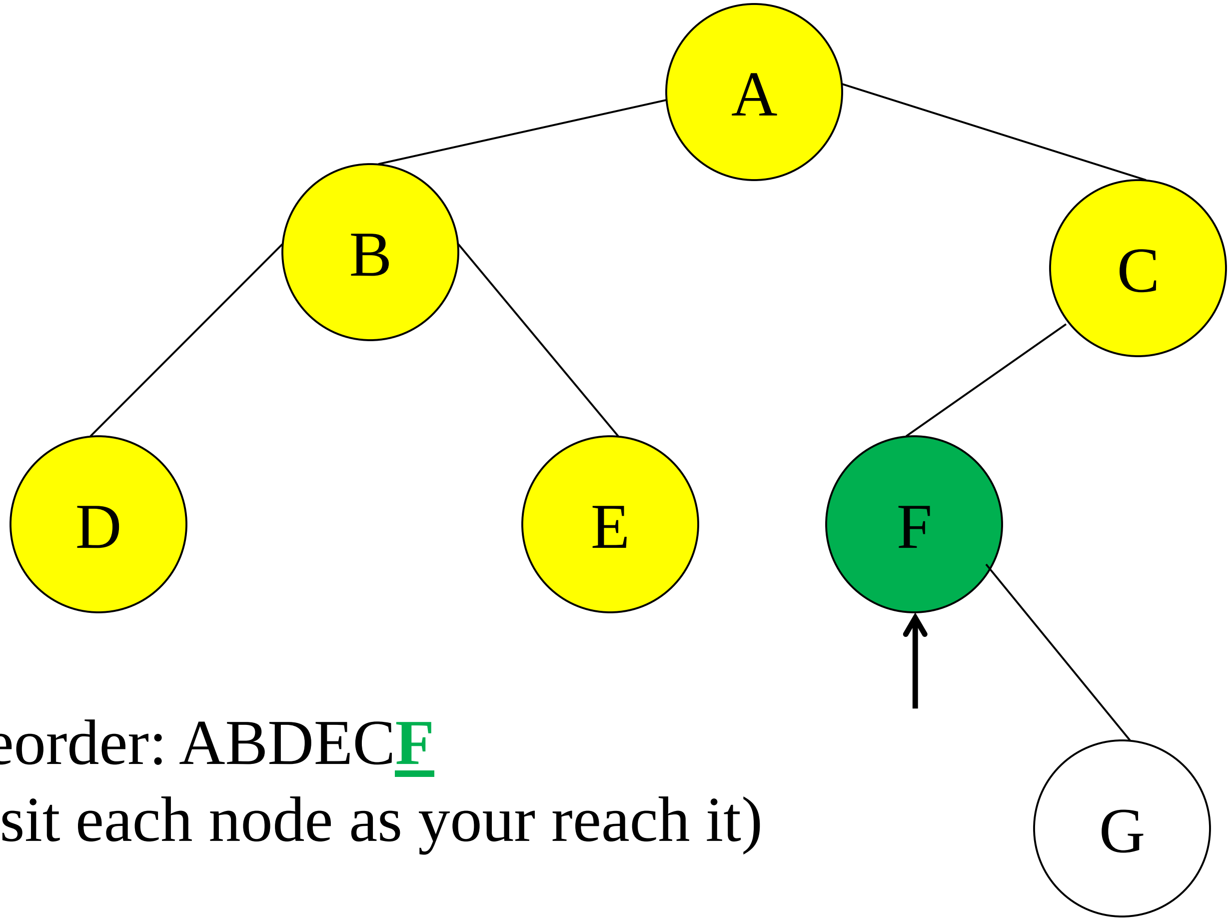
Preorder: ABD**E**
(visit each node as your reach it)

Tree Traversals: An Example



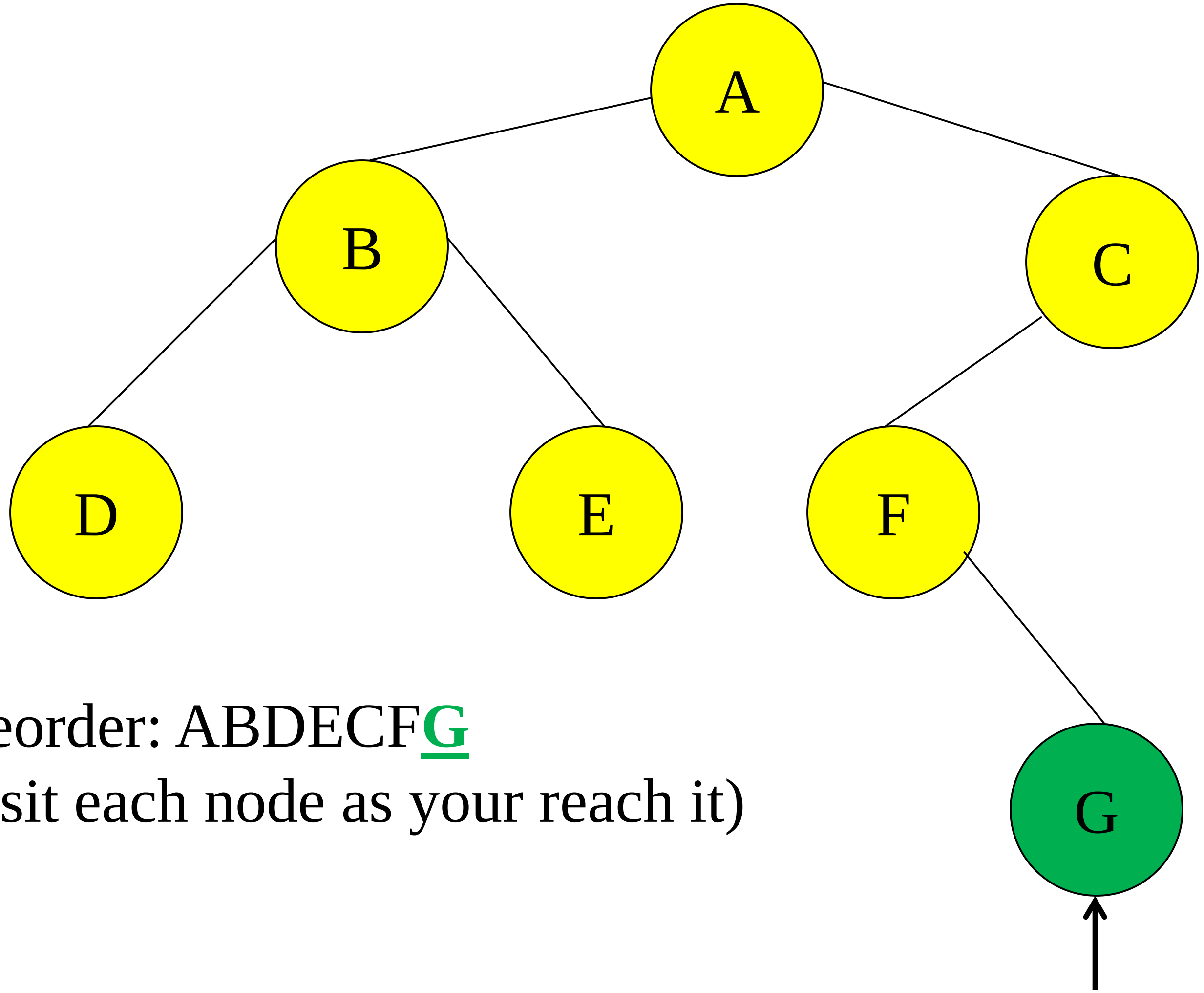
Preorder: ABDE**C**
(visit each node as your reach it)

Tree Traversals: An Example



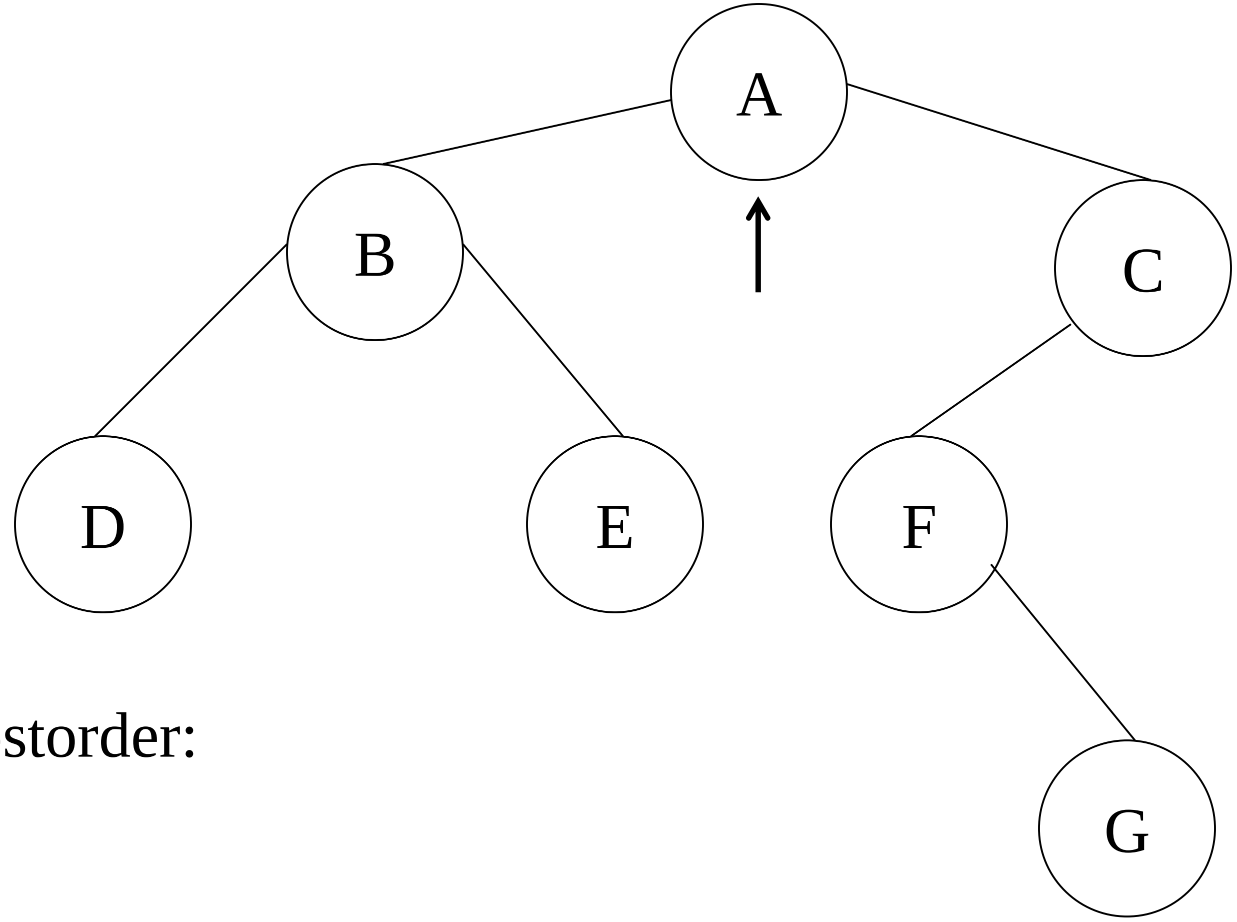
Preorder: ABDEC**F**
(visit each node as your reach it)

Tree Traversals: An Example



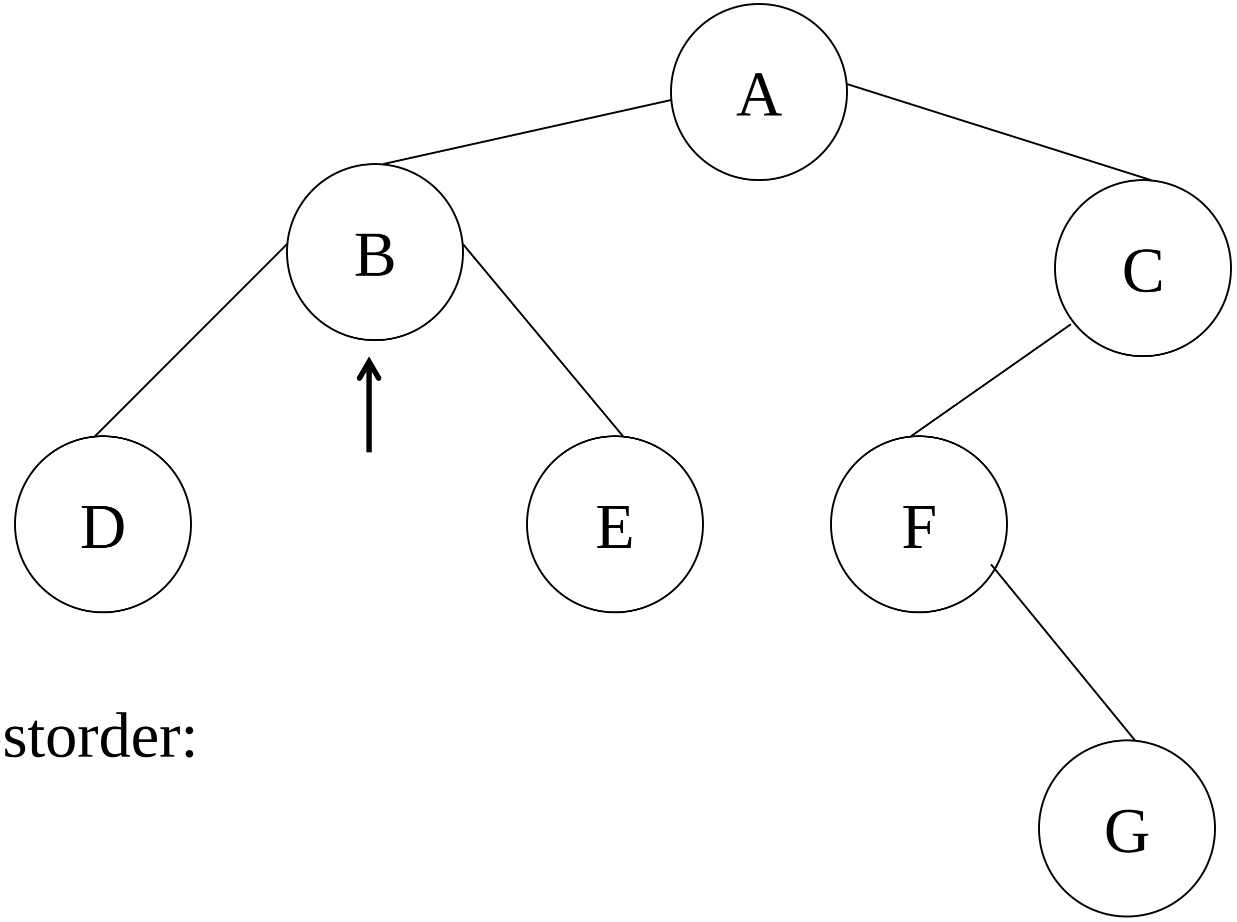
Preorder: ABDECF**G**
(visit each node as your reach it)

Tree Traversals: An Example



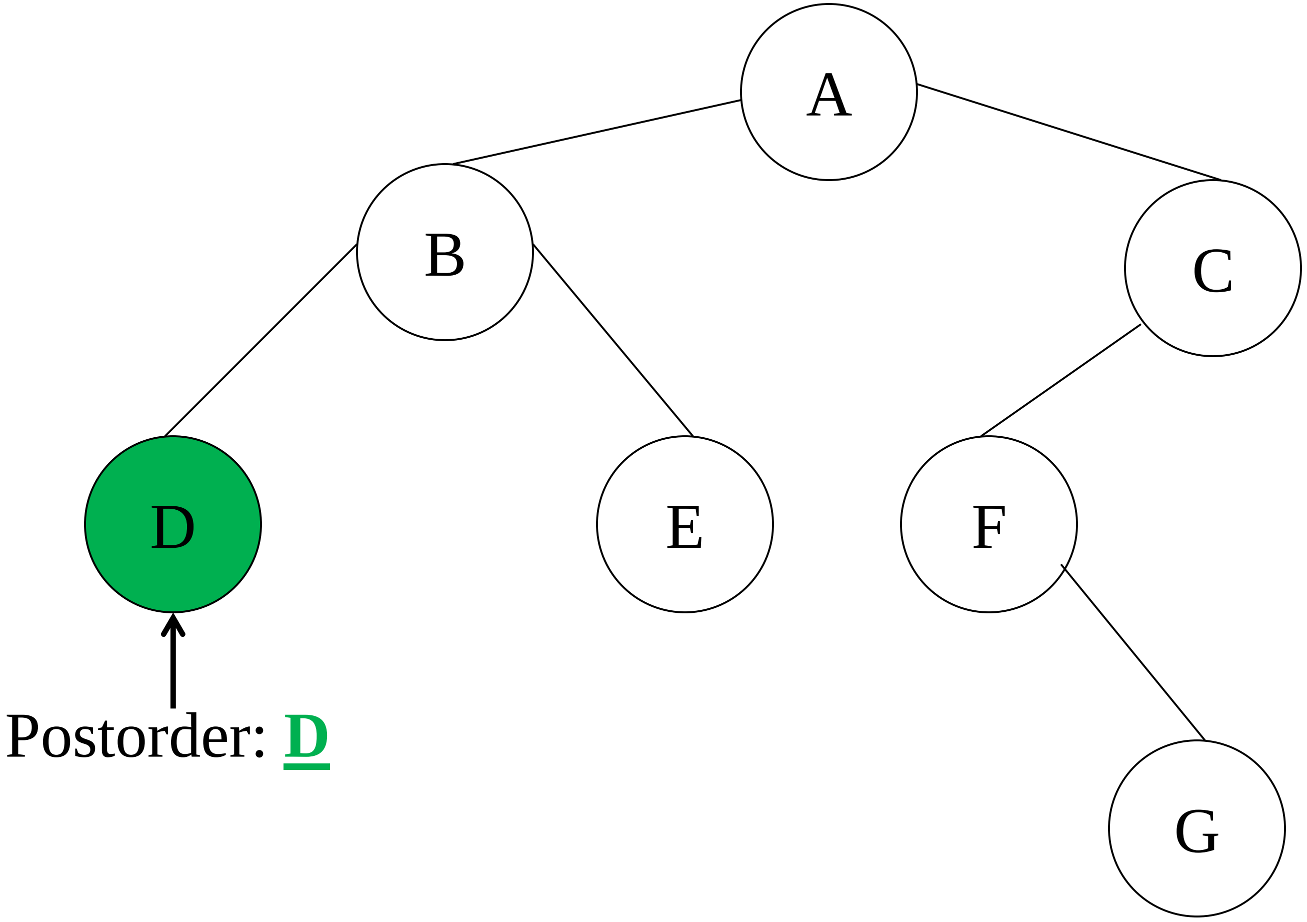
Postorder:

Tree Traversals: An Example

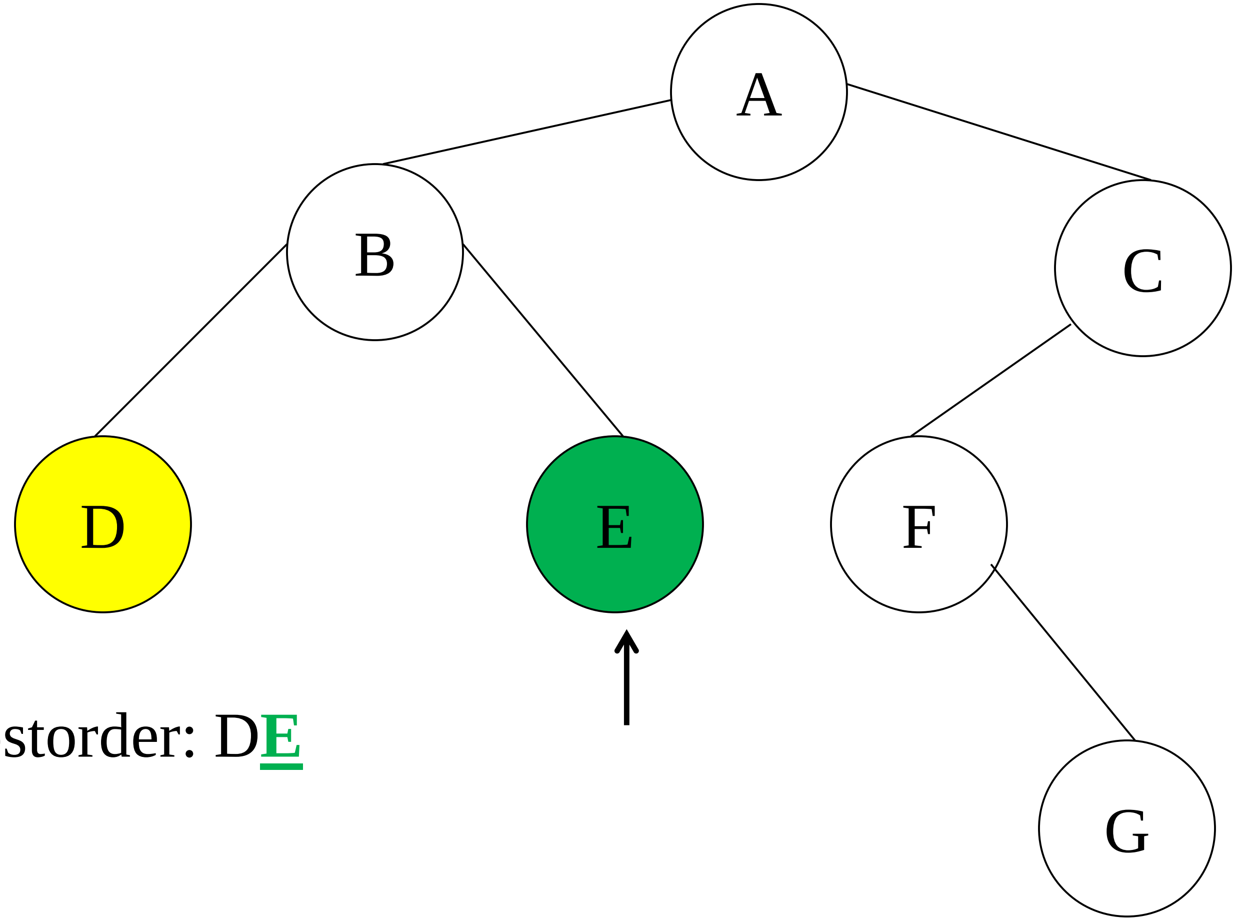


Postorder:

Tree Traversals: An Example

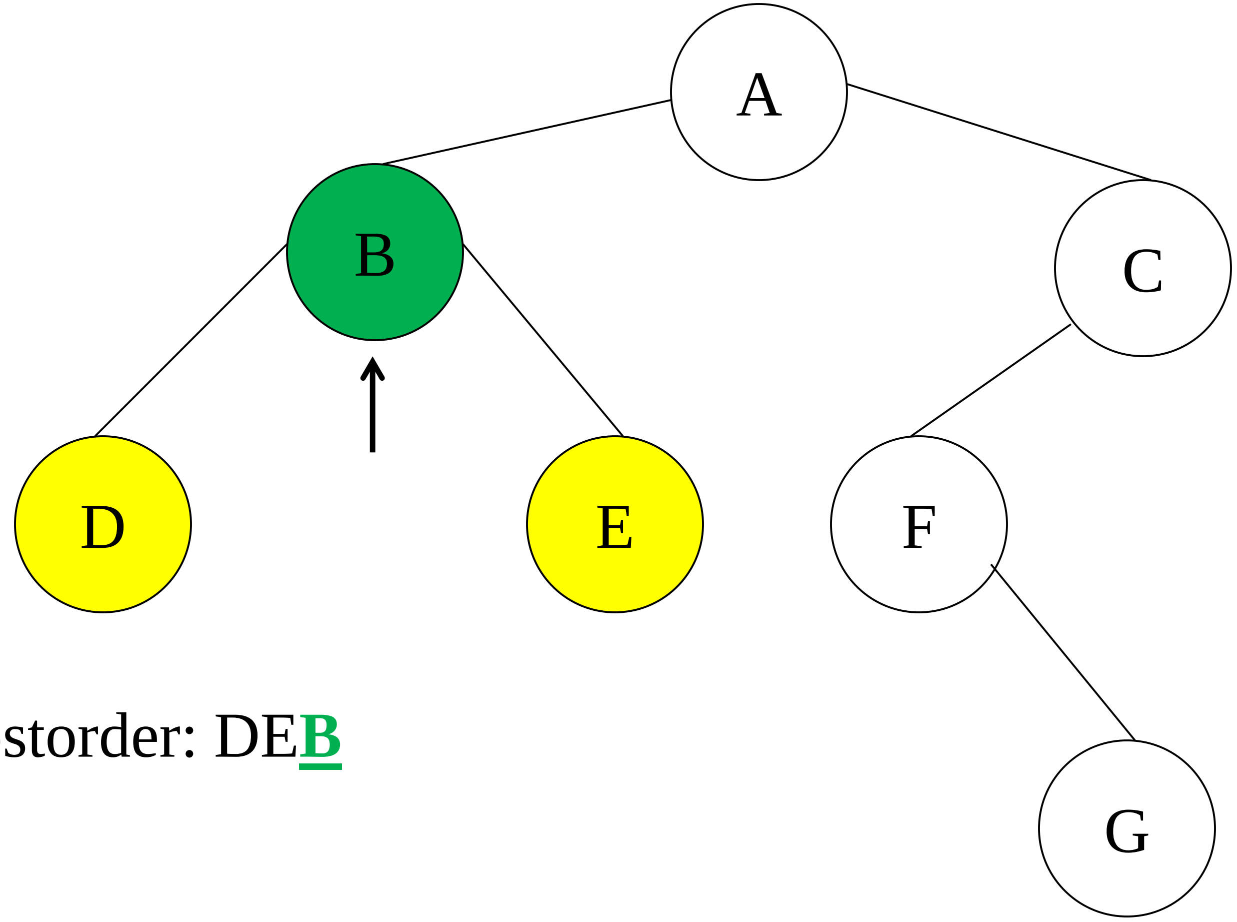


Tree Traversals: An Example



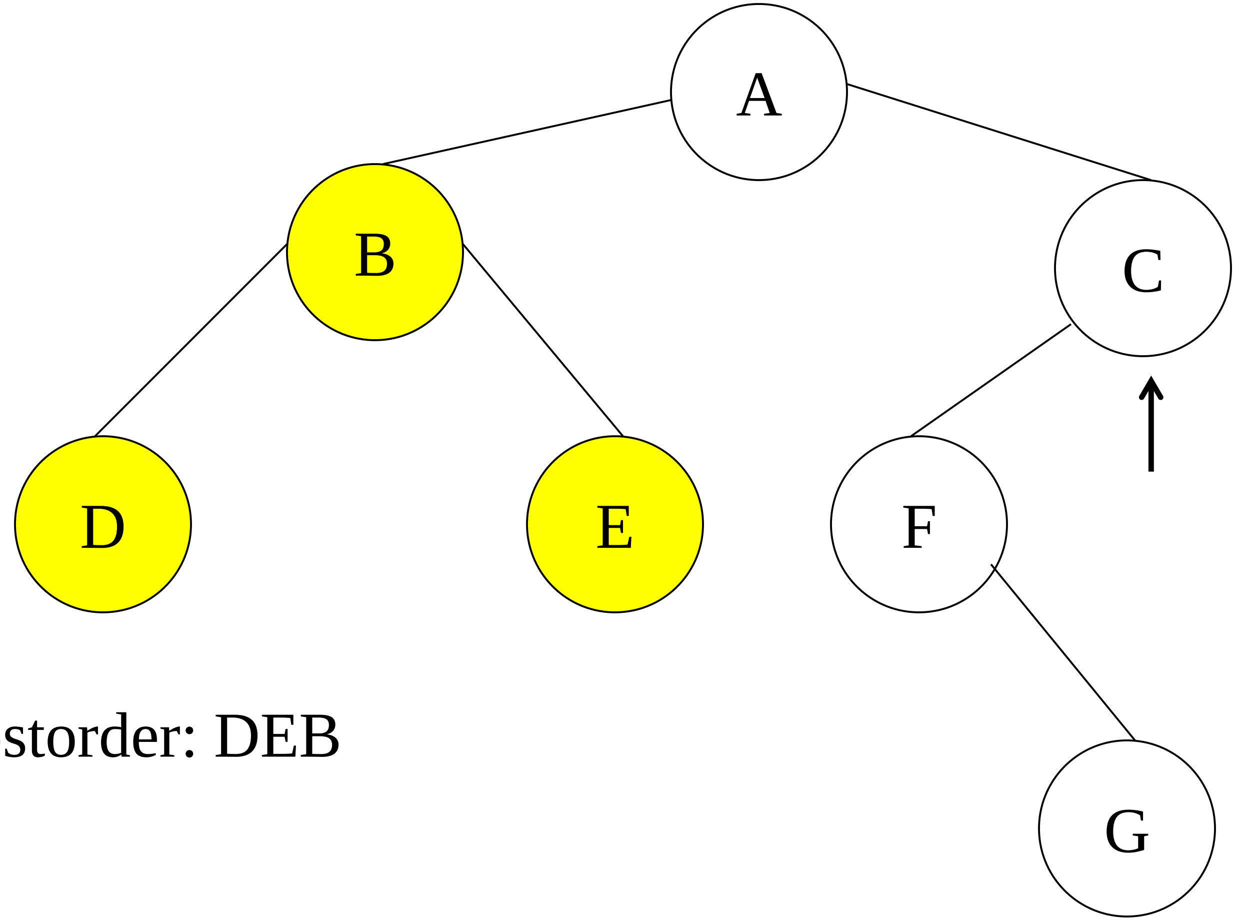
Postorder: D**E**

Tree Traversals: An Example



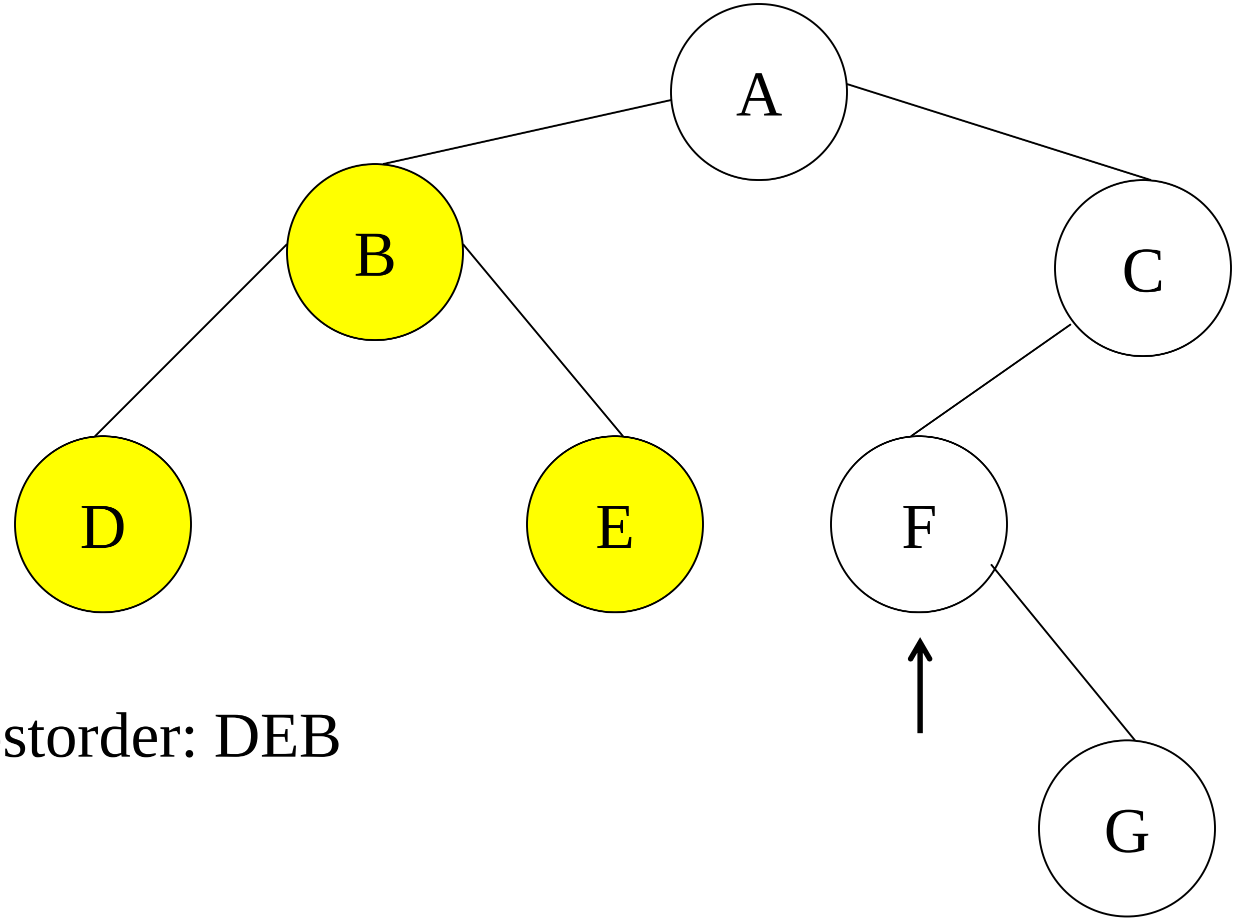
Postorder: DE**B**

Tree Traversals: An Example



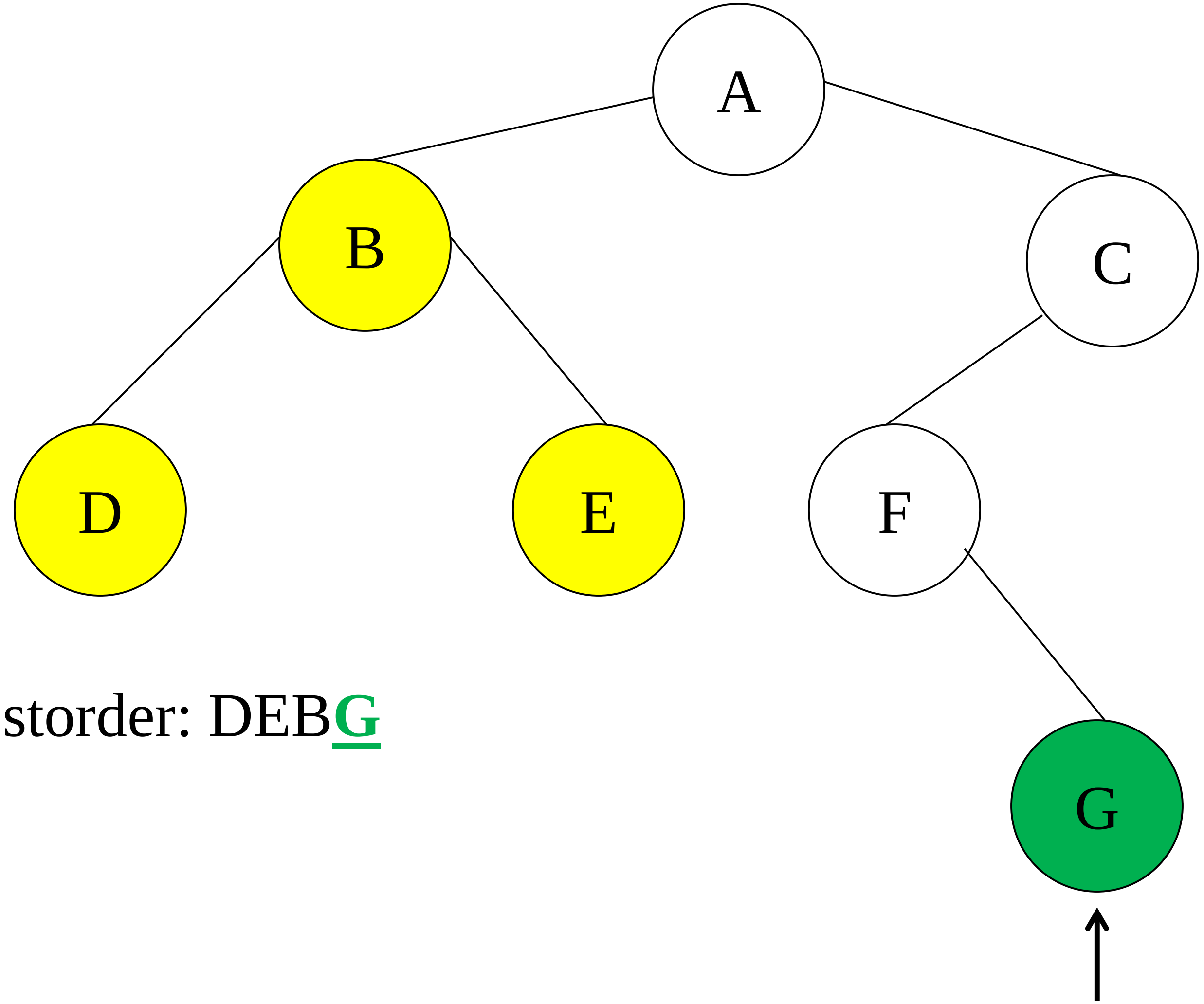
Postorder: DEB

Tree Traversals: An Example



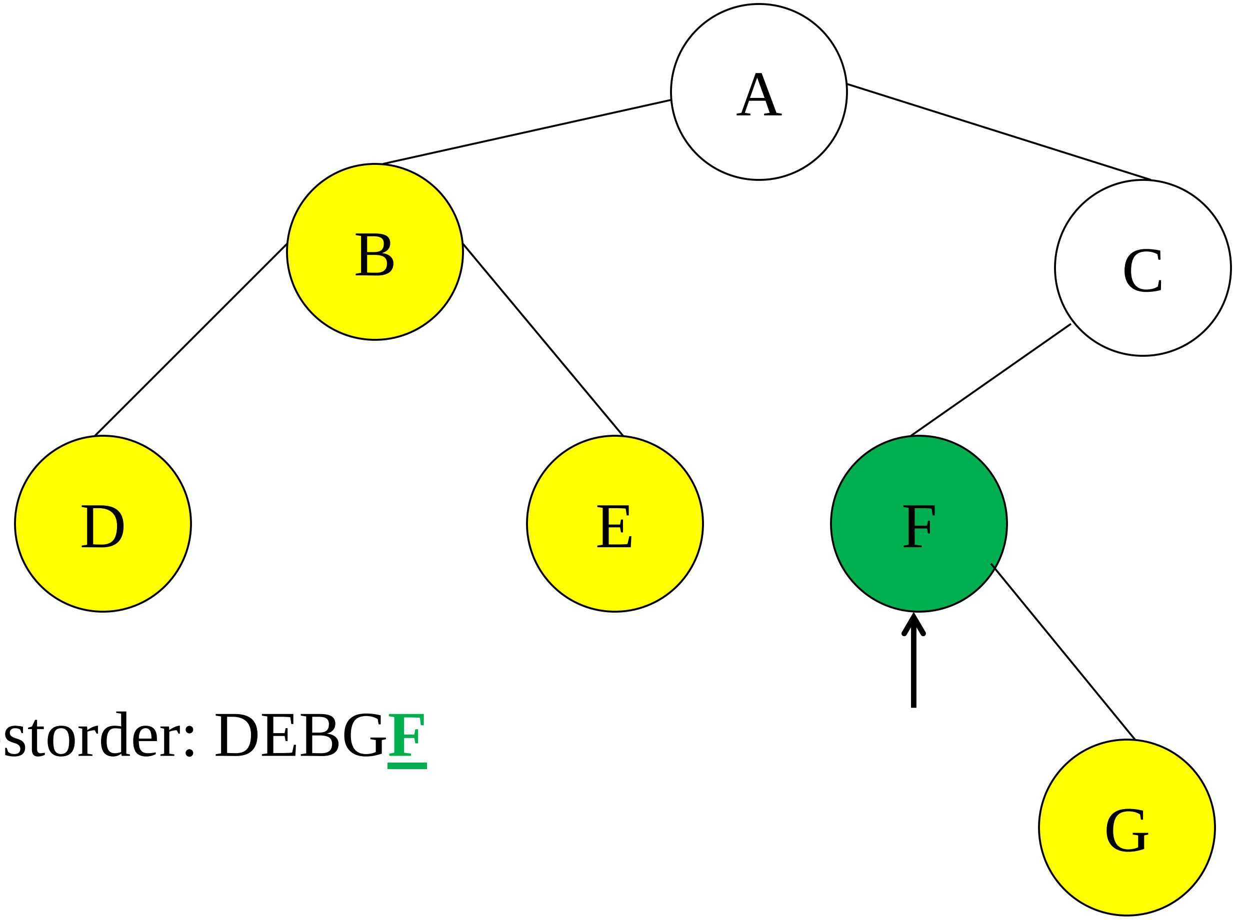
Postorder: DEB

Tree Traversals: An Example



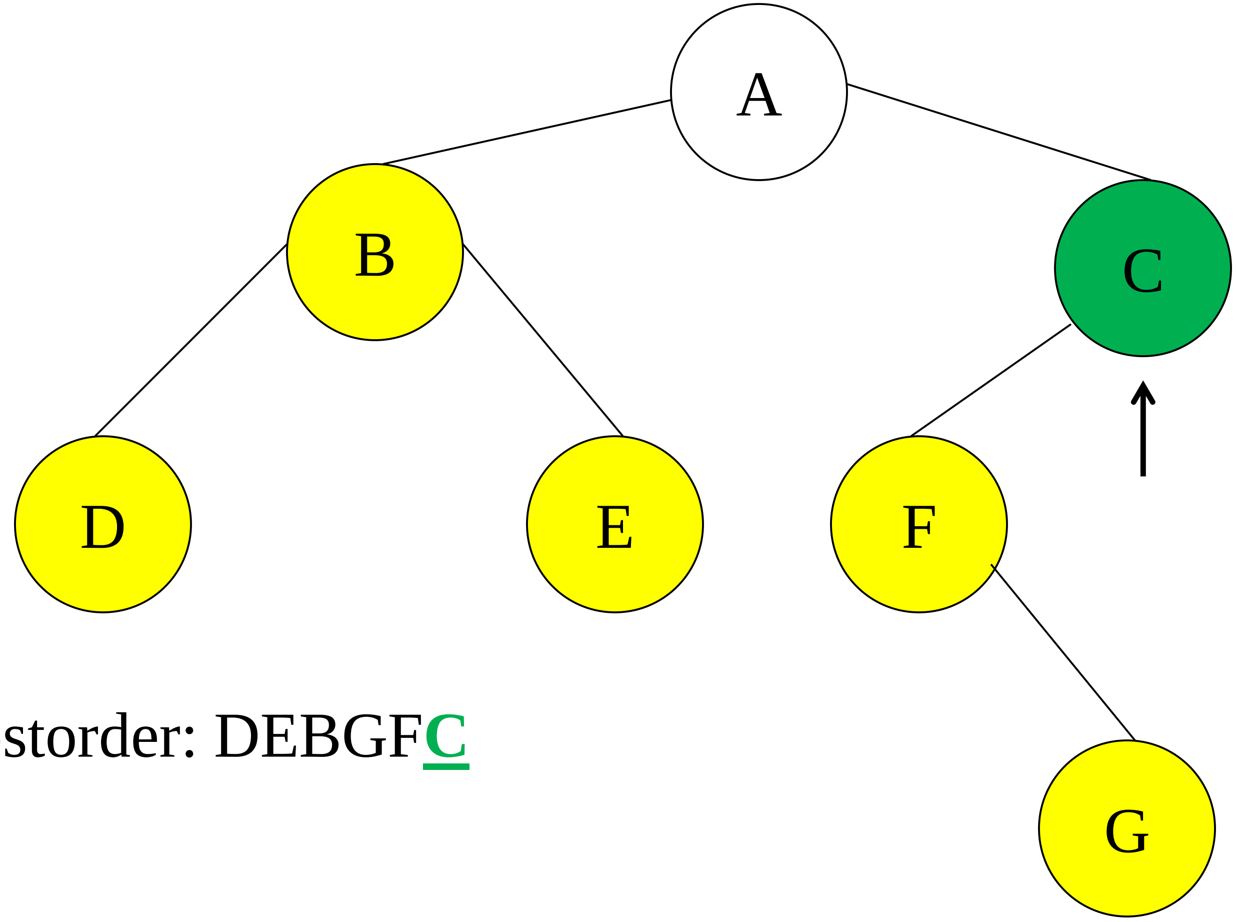
Postorder: DEB**G**

Tree Traversals: An Example



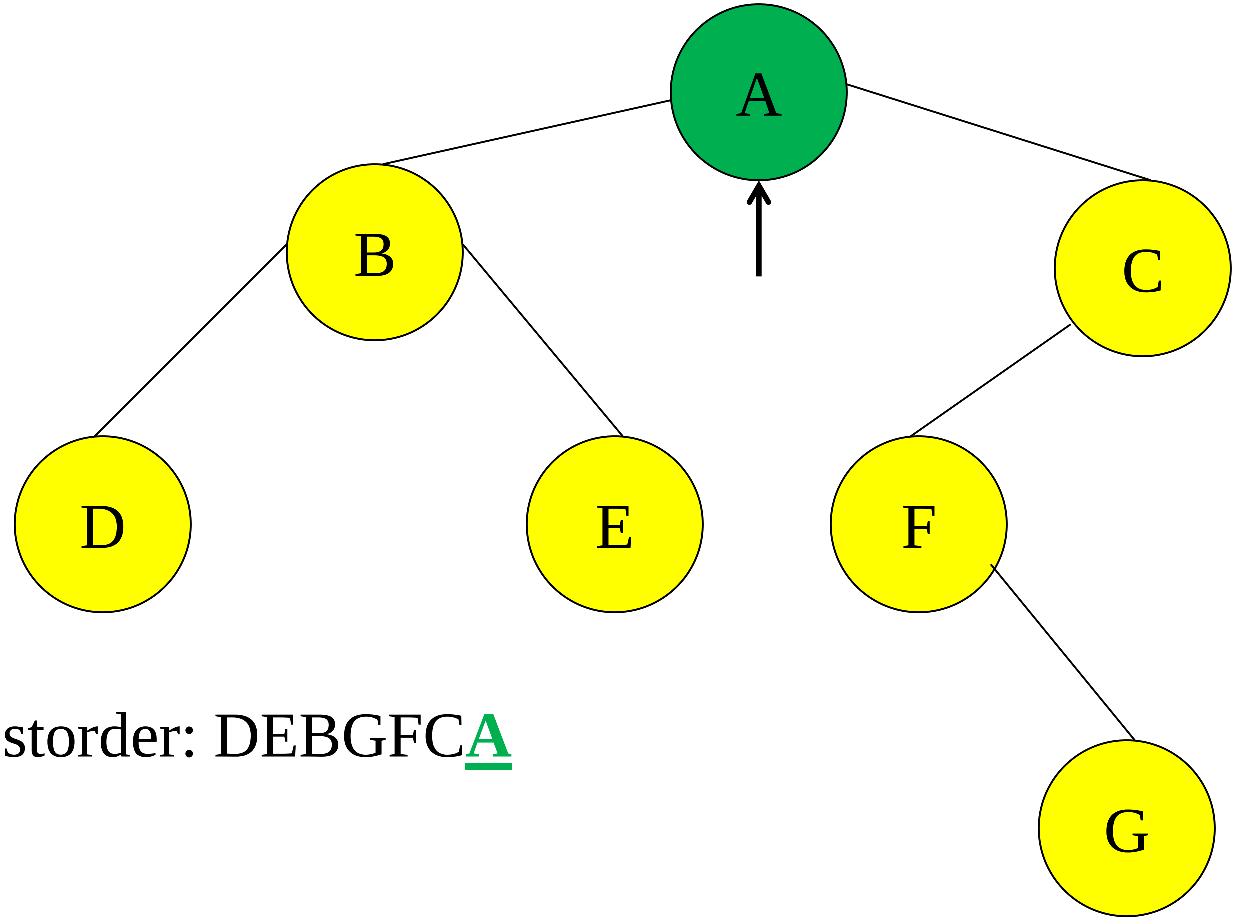
Postorder: DEBG**F**

Tree Traversals: An Example



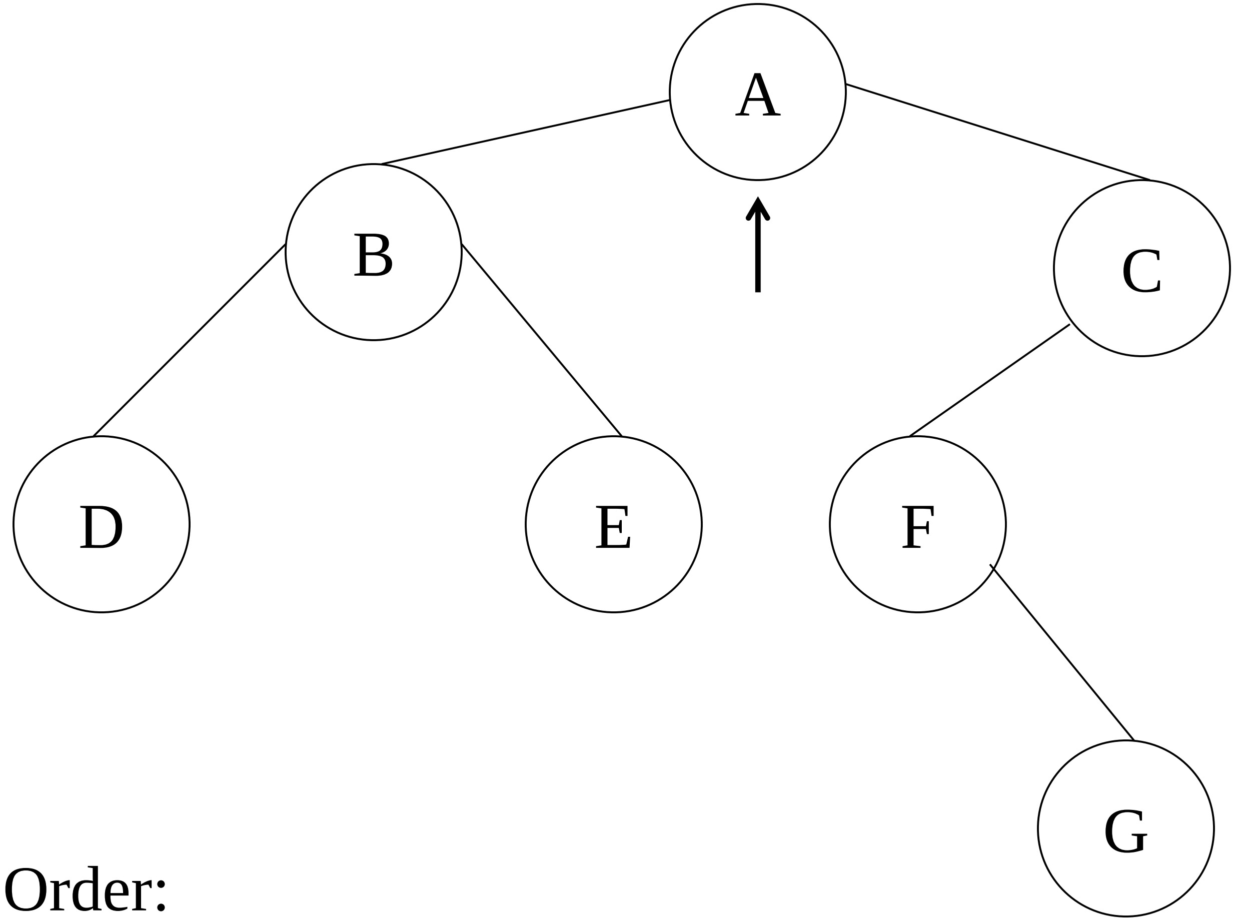
Postorder: DEBGF**C**

Tree Traversals: An Example



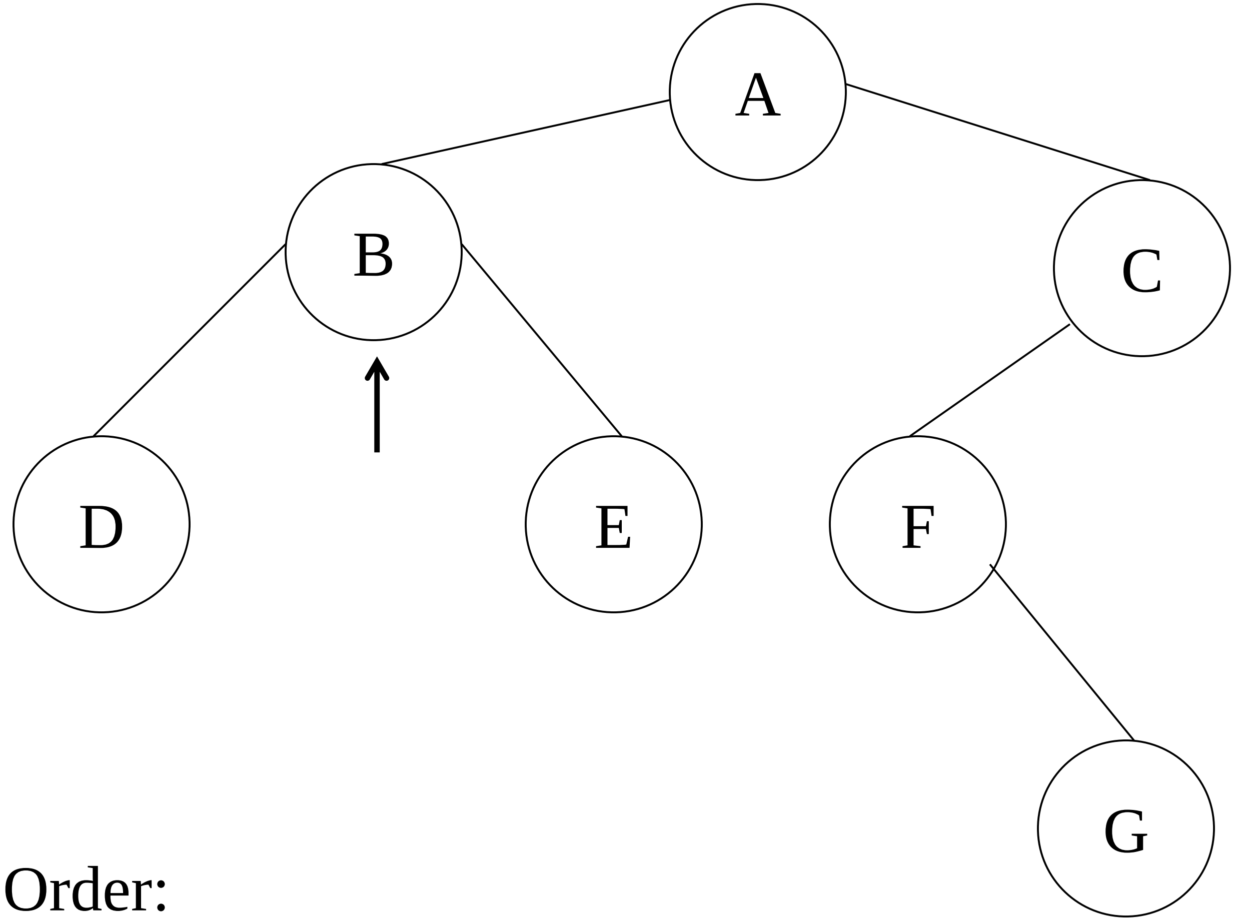
Postorder: DEBGFC**A**

Tree Traversals: An Example



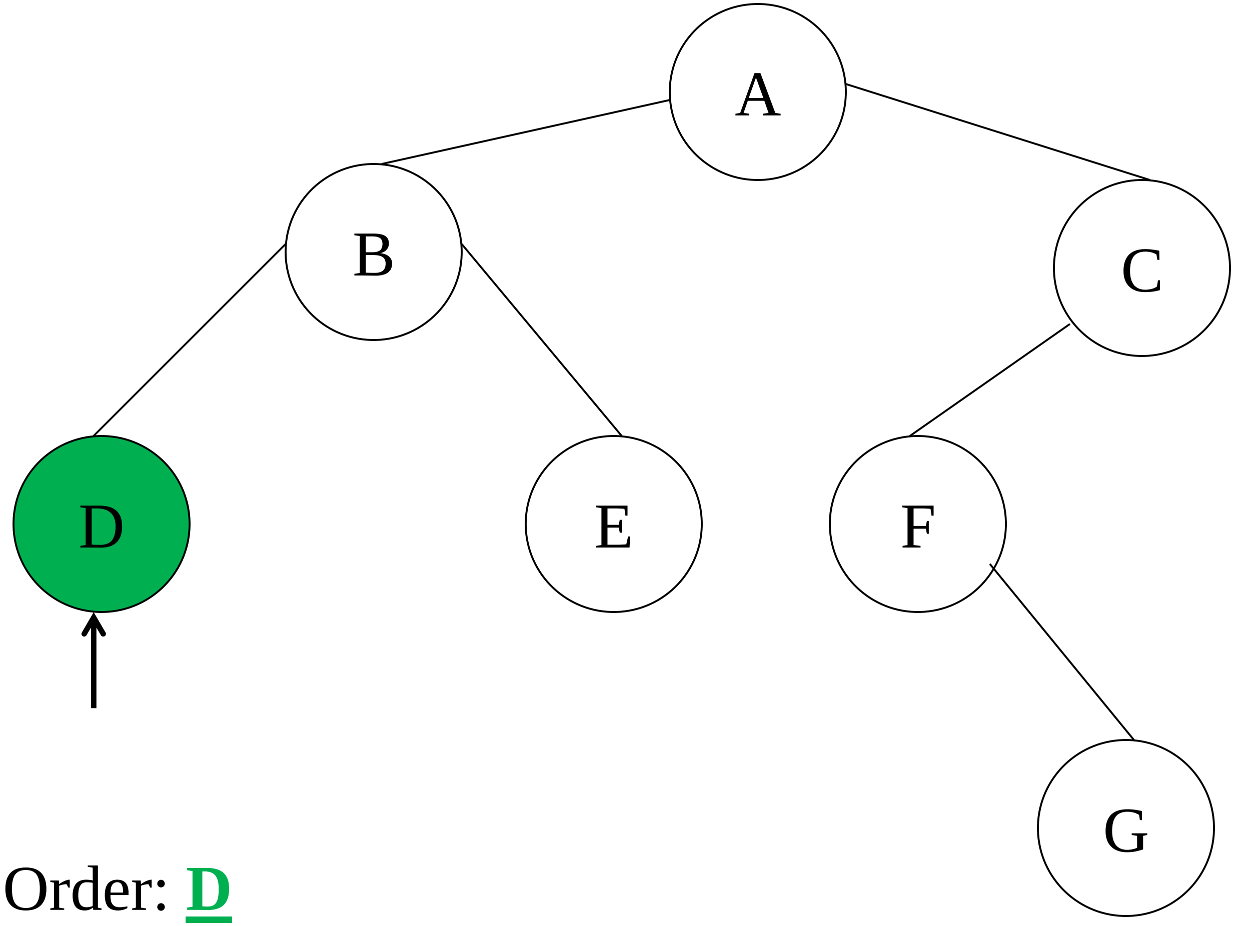
In Order:

Tree Traversals: An Example



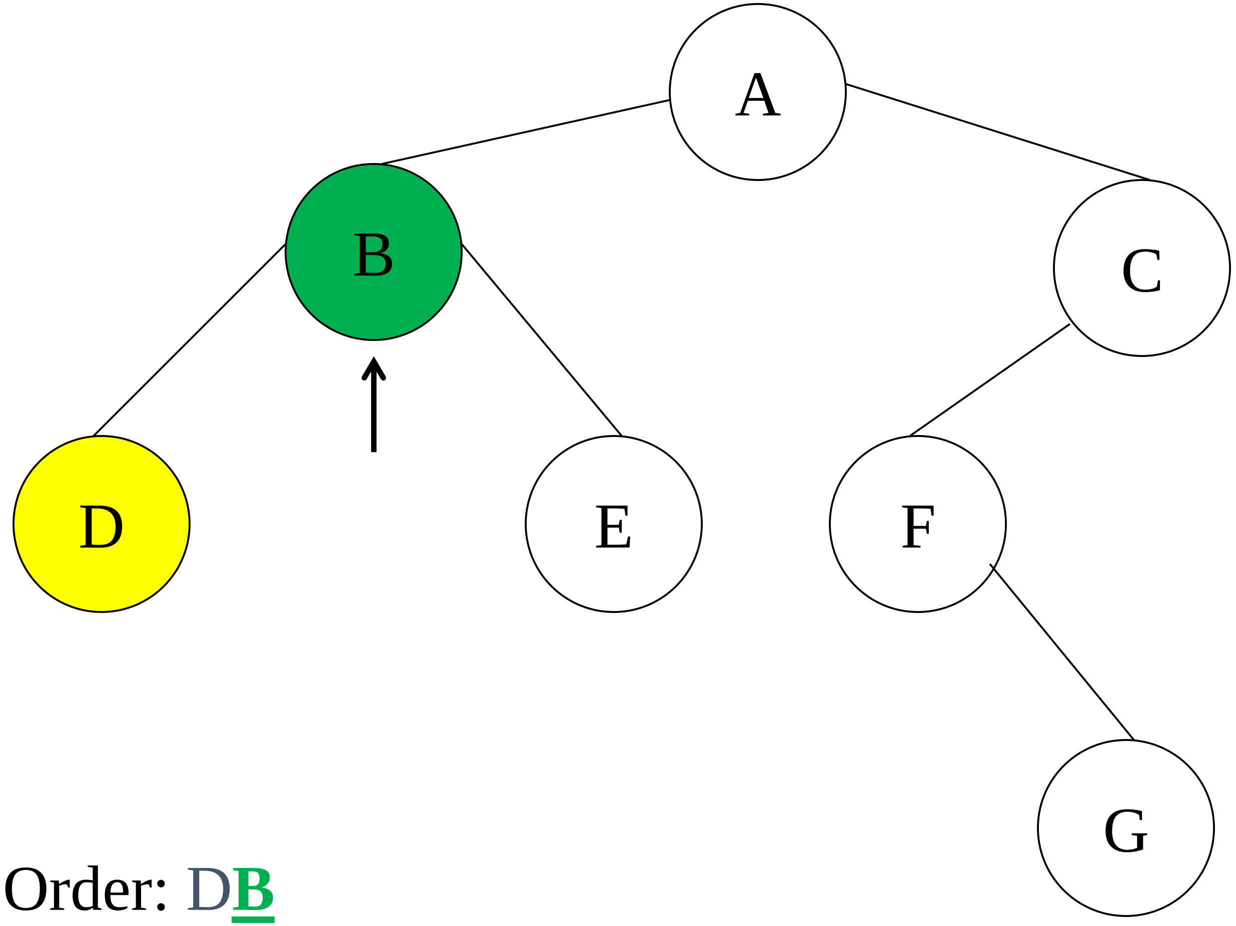
In Order:

Tree Traversals: An Example



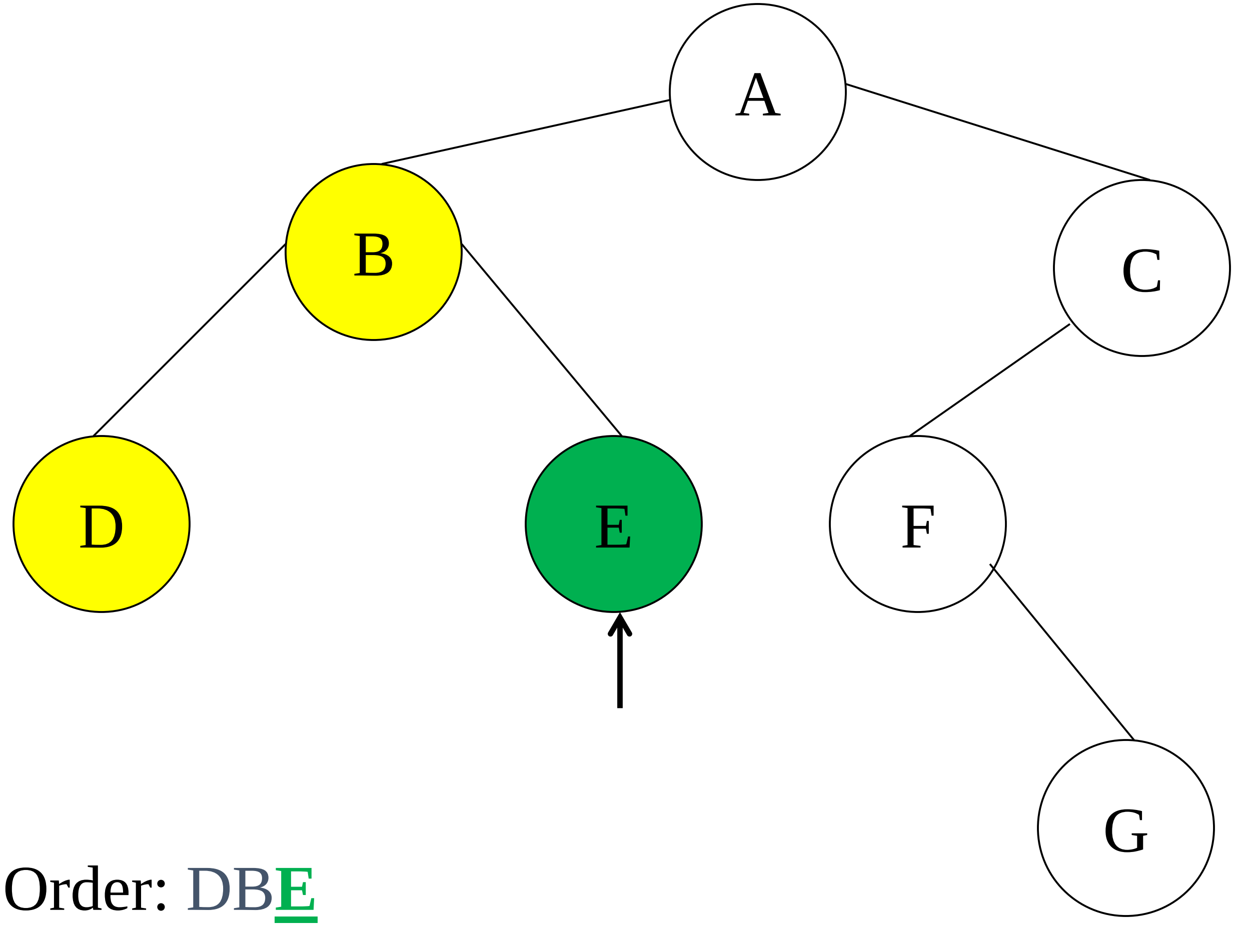
In Order: **D**

Tree Traversals: An Example



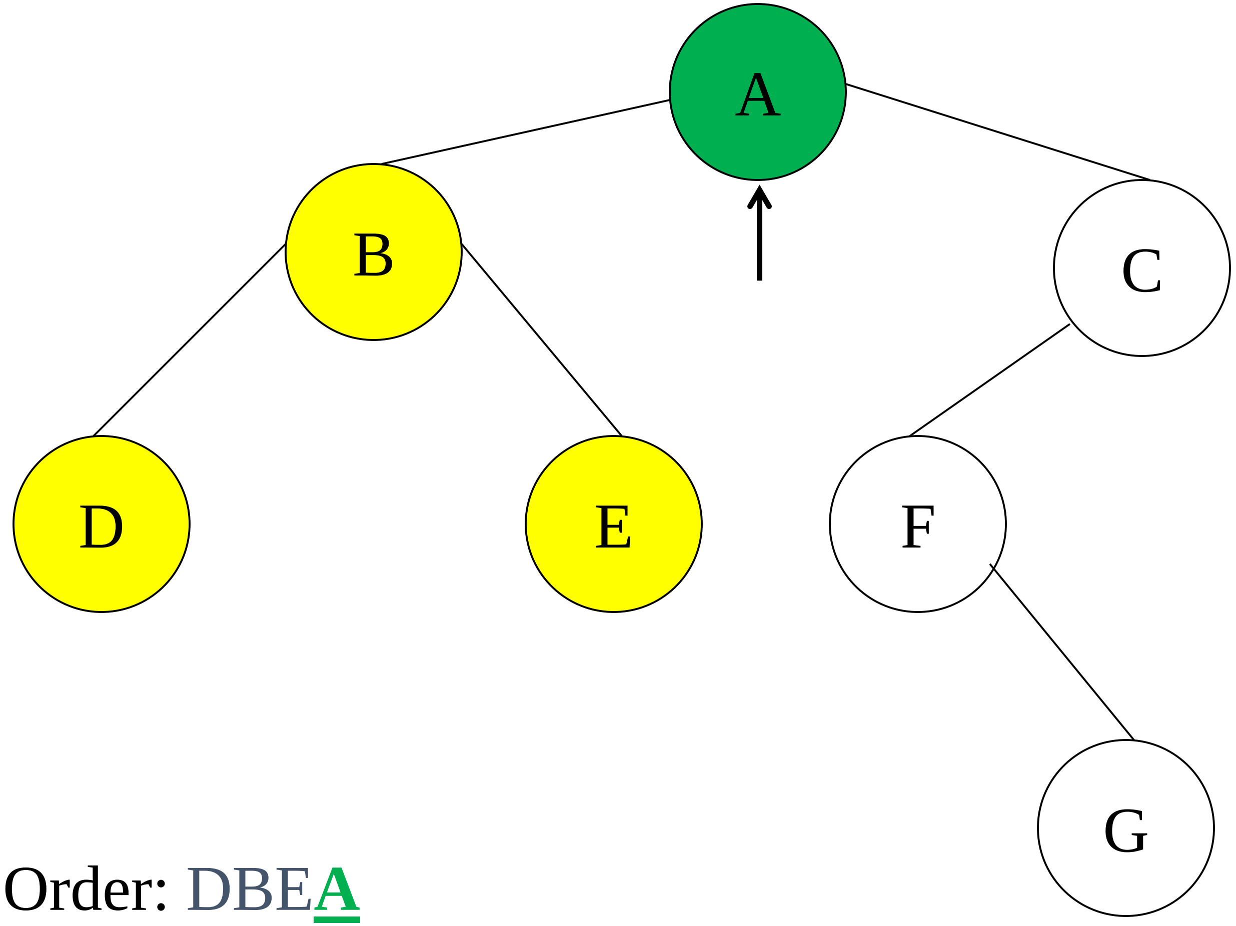
In Order: D**B**

Tree Traversals: An Example



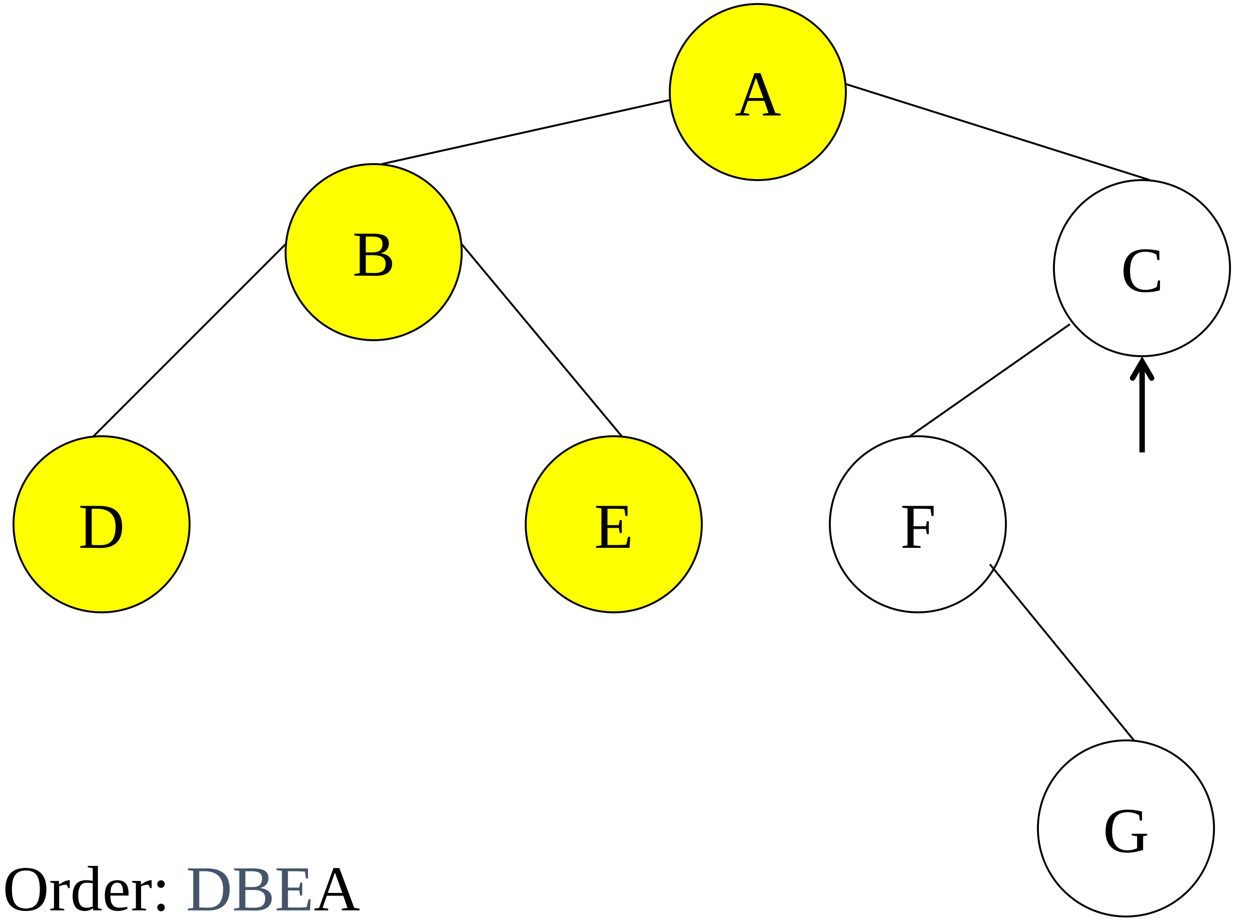
In Order: DB**E**

Tree Traversals: An Example



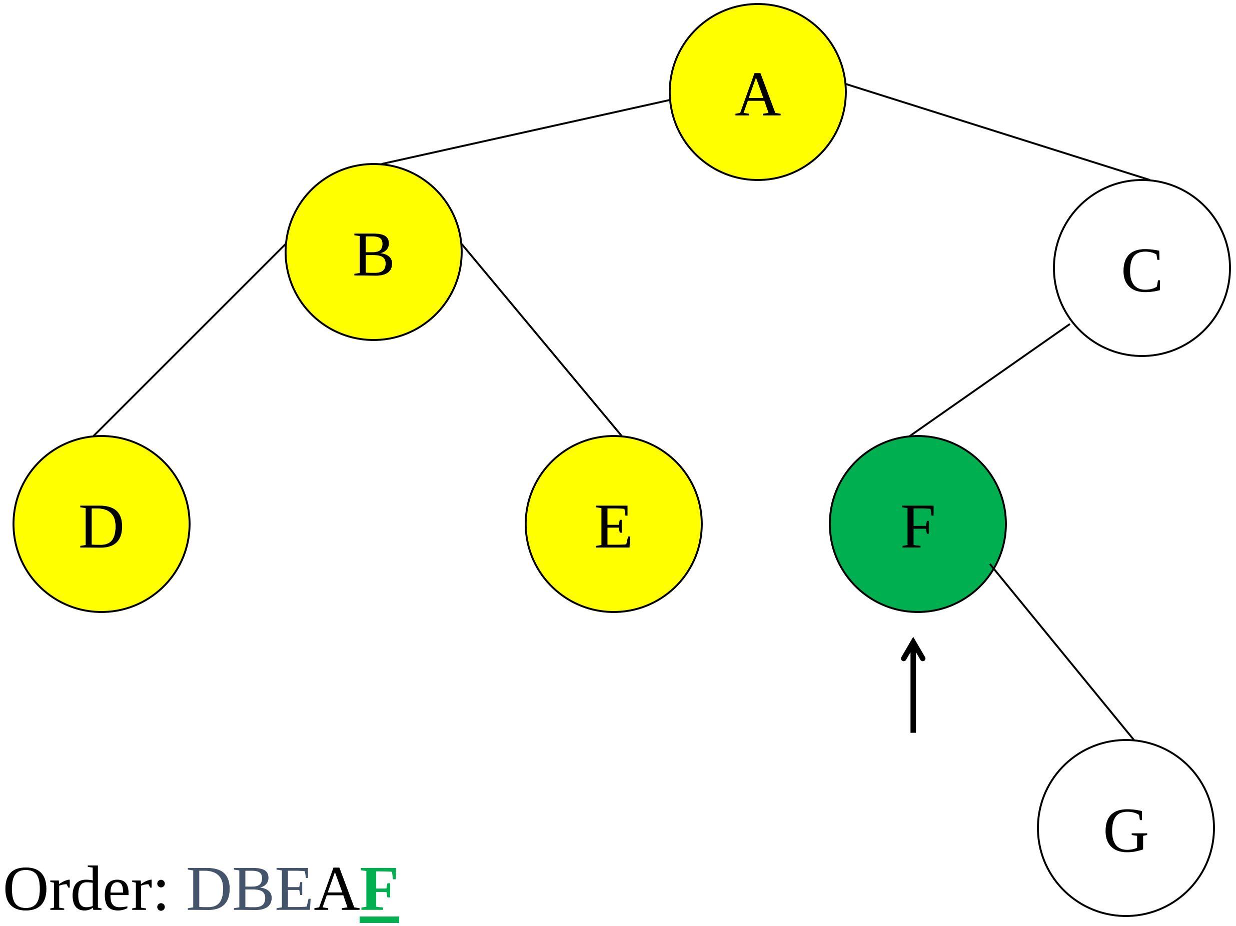
In Order: DBE**A**

Tree Traversals: An Example



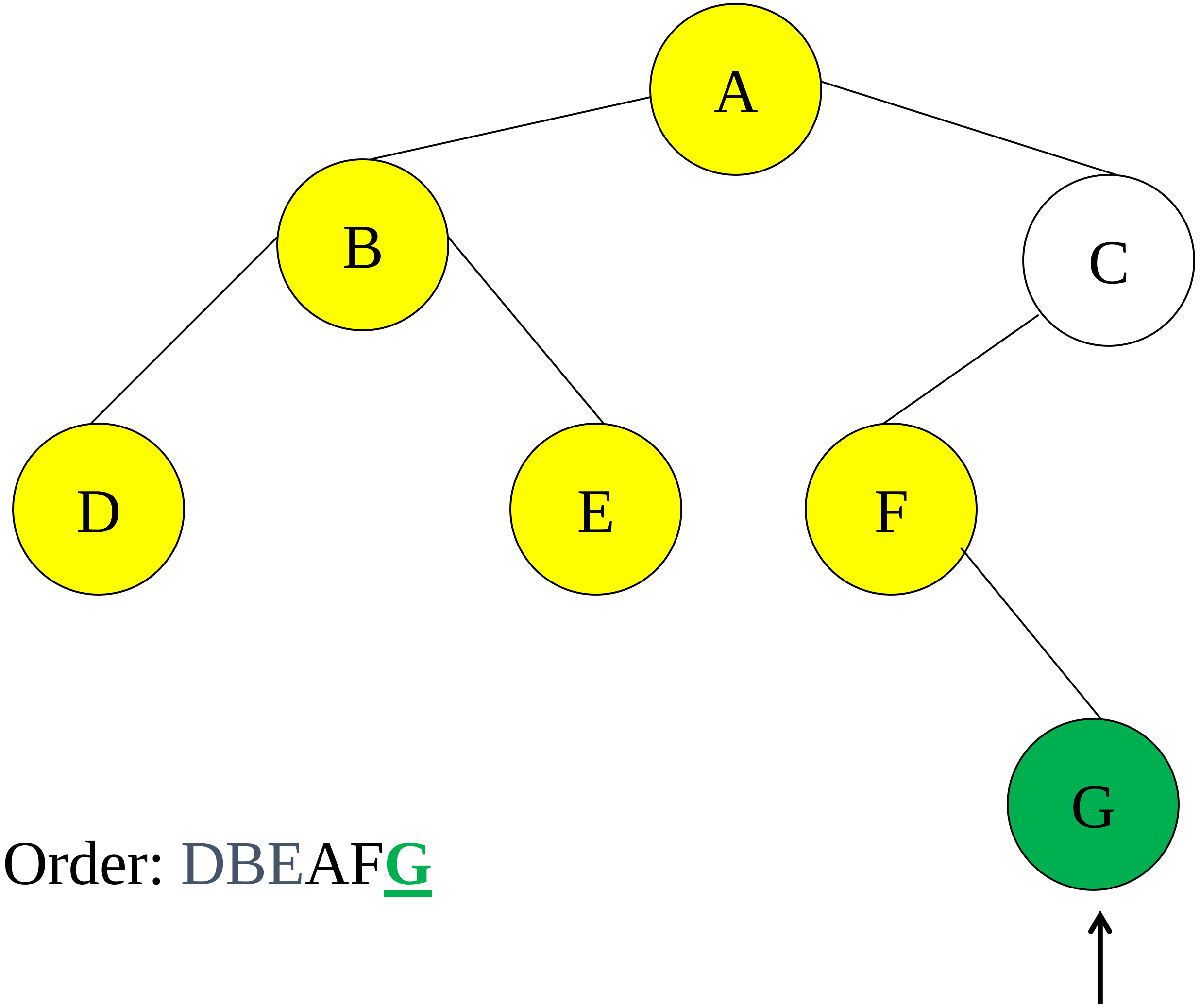
In Order: DBEA

Tree Traversals: An Example



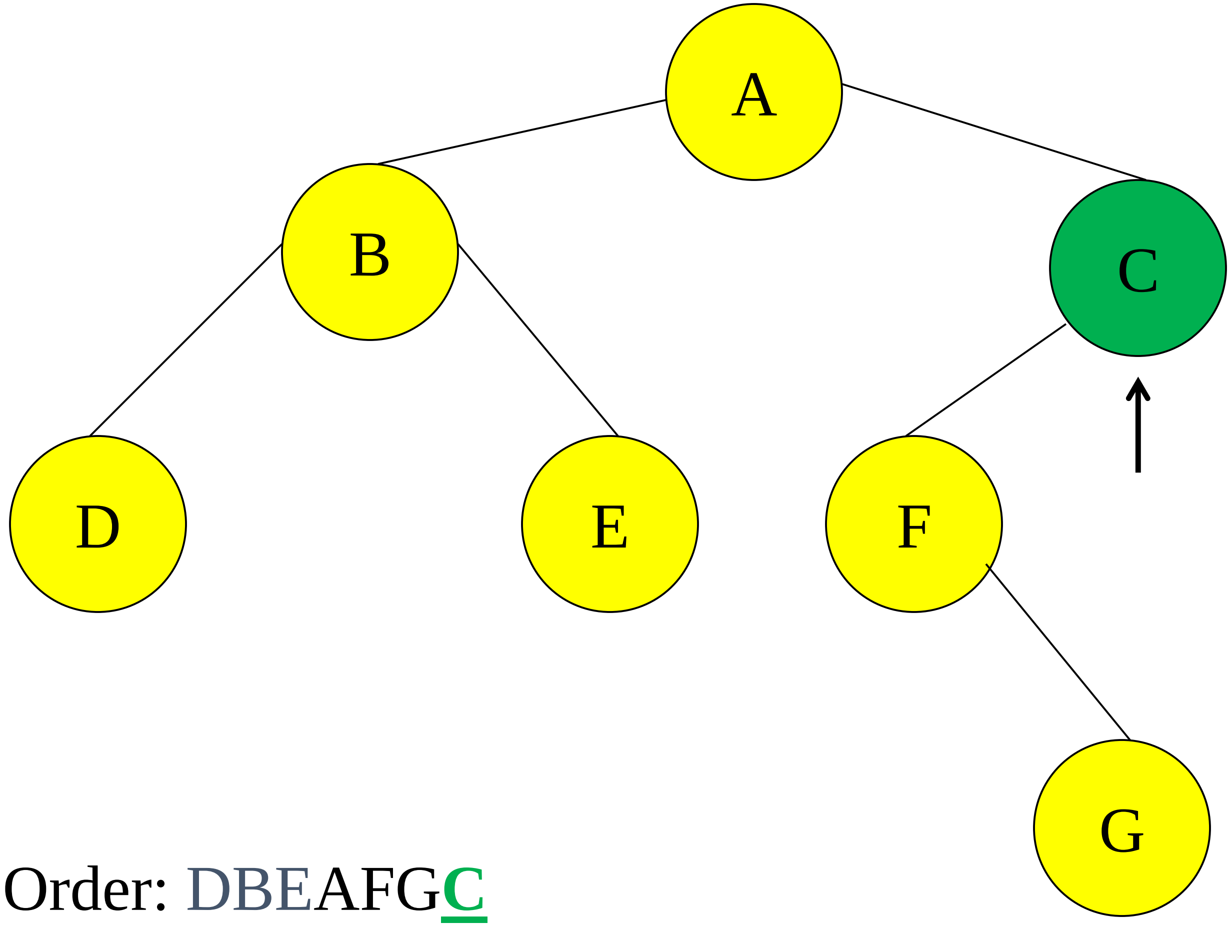
In Order: DBEAF

Tree Traversals: An Example



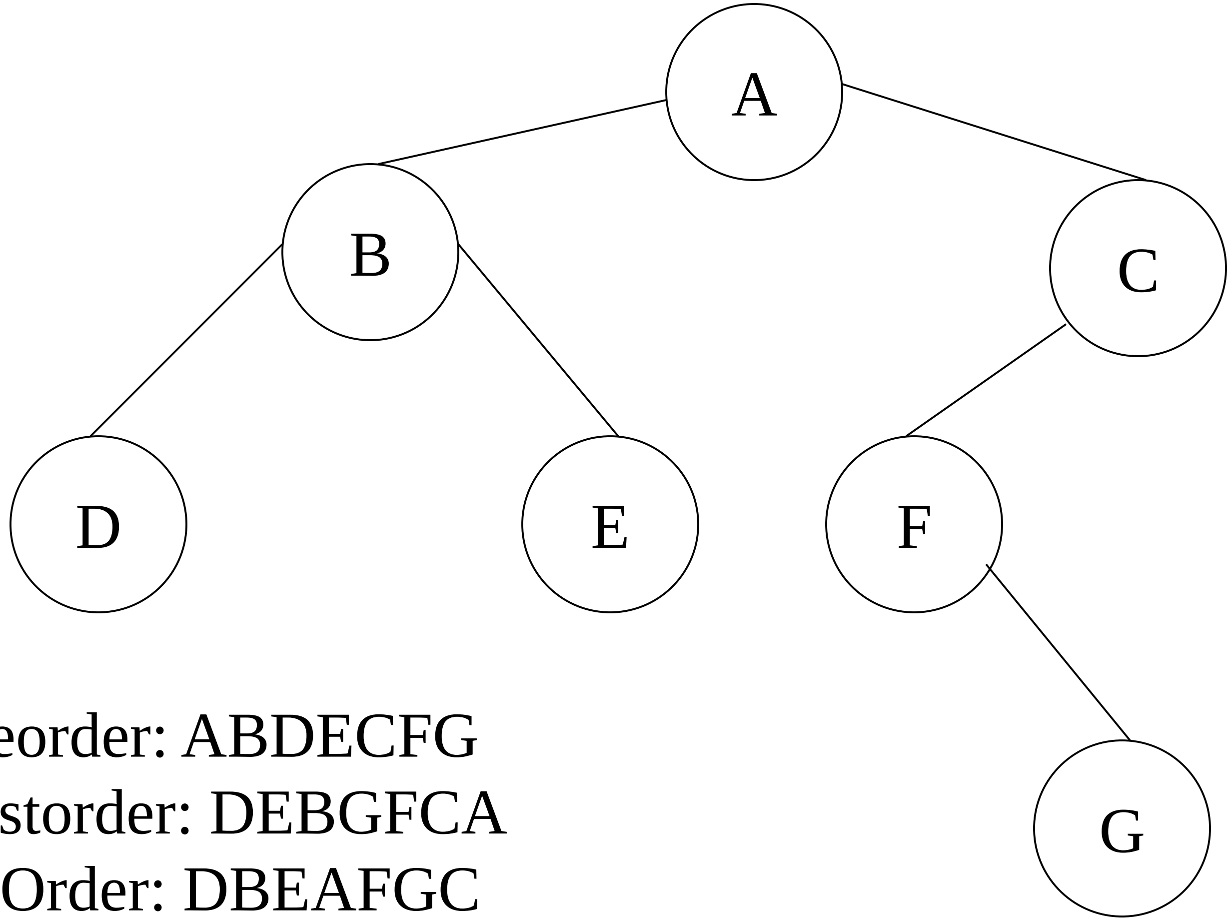
In Order: DBEAF**G**

Tree Traversals: An Example



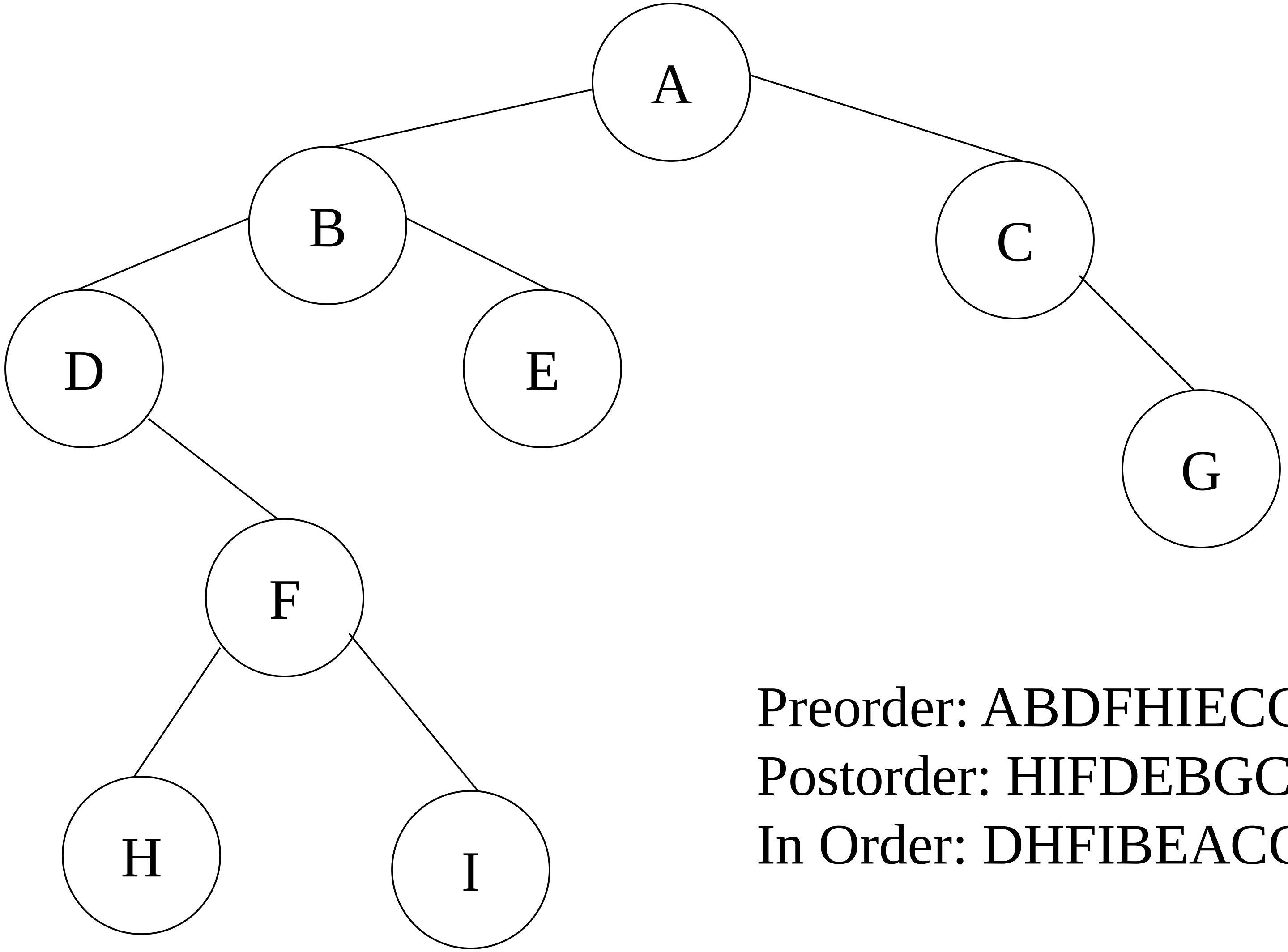
In Order: DBEAFG**C**

Tree Traversals: An Example



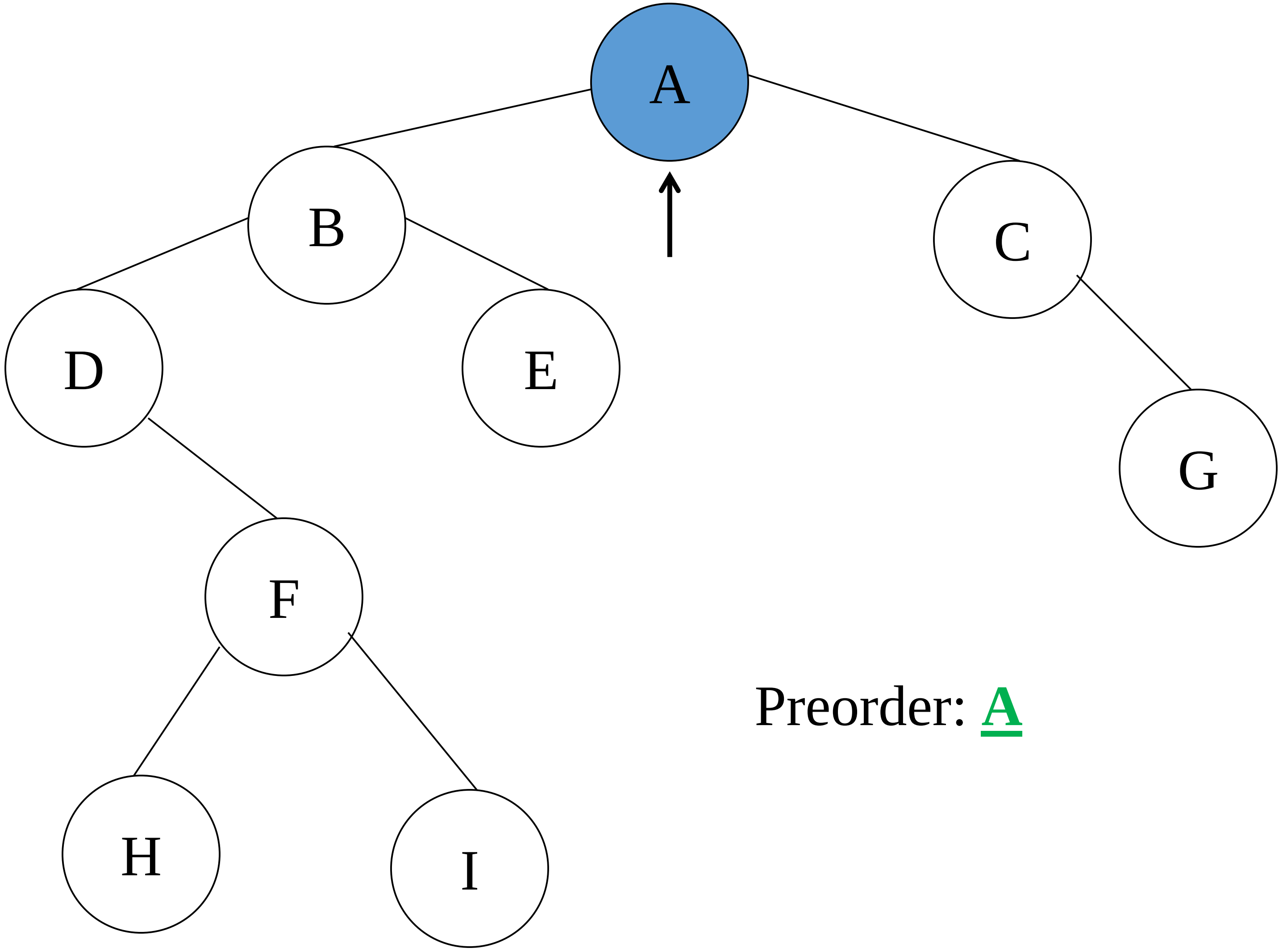
Preorder: ABDECFG
Postorder: DEBGFCA
In Order: DBEAFGC

Tree Traversals: Another Example



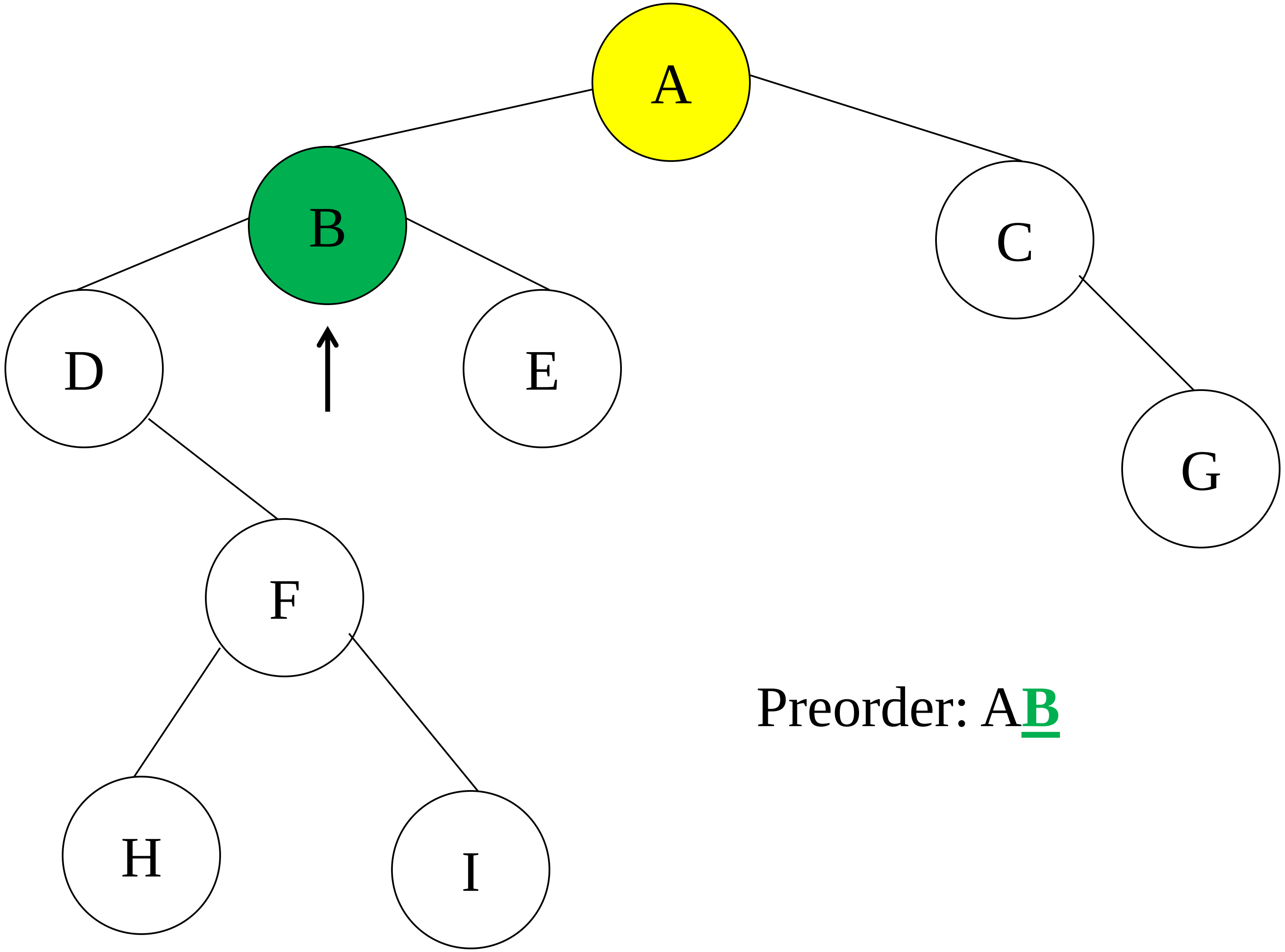
Preorder: ABDFHIECG
Postorder: HIFDEBGCA
In Order: DHFIBEACG

Tree Traversals: Another Example



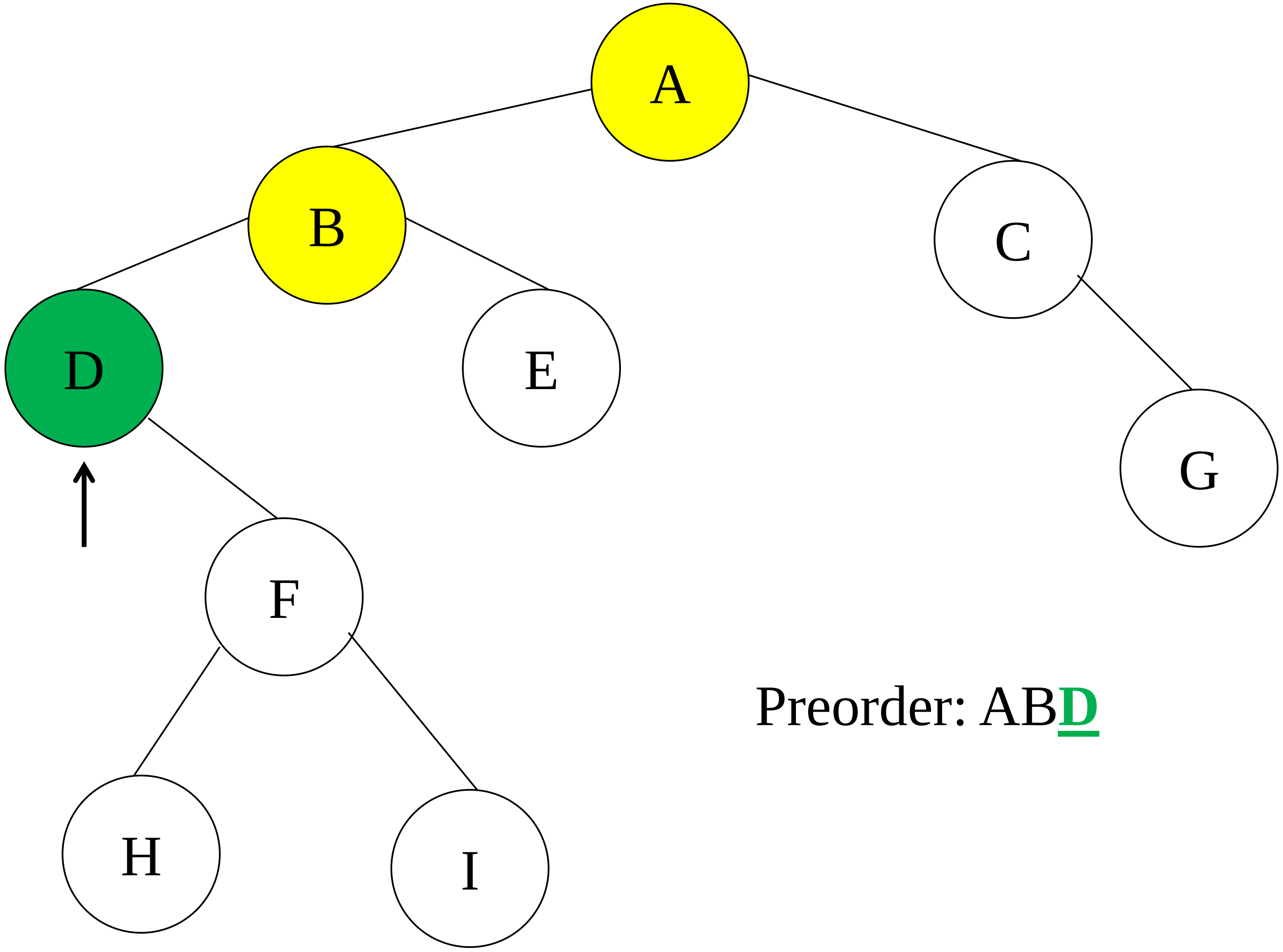
Preorder: **A**

Tree Traversals: Another Example



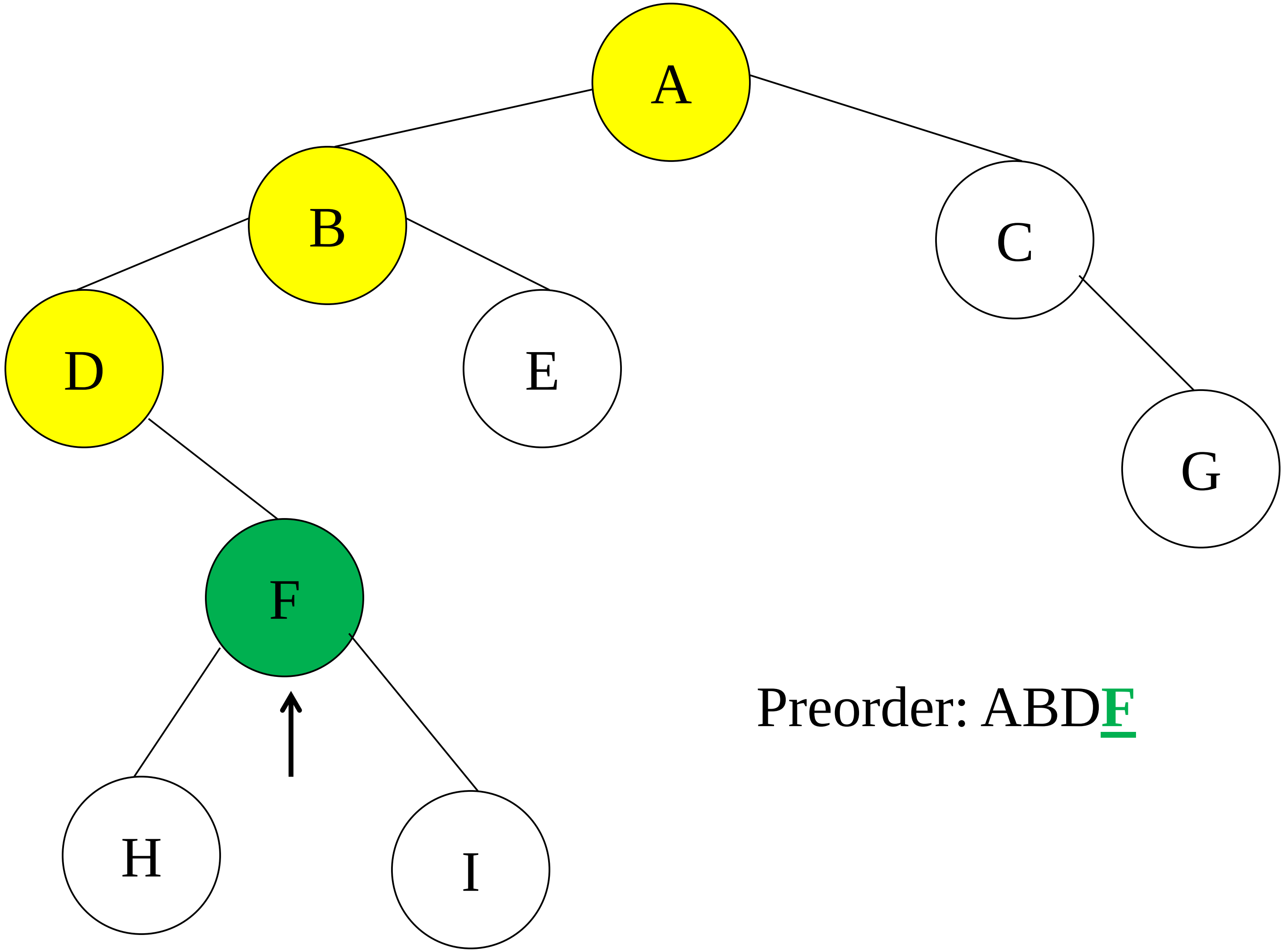
Preorder: A**B**

Tree Traversals: Another Example



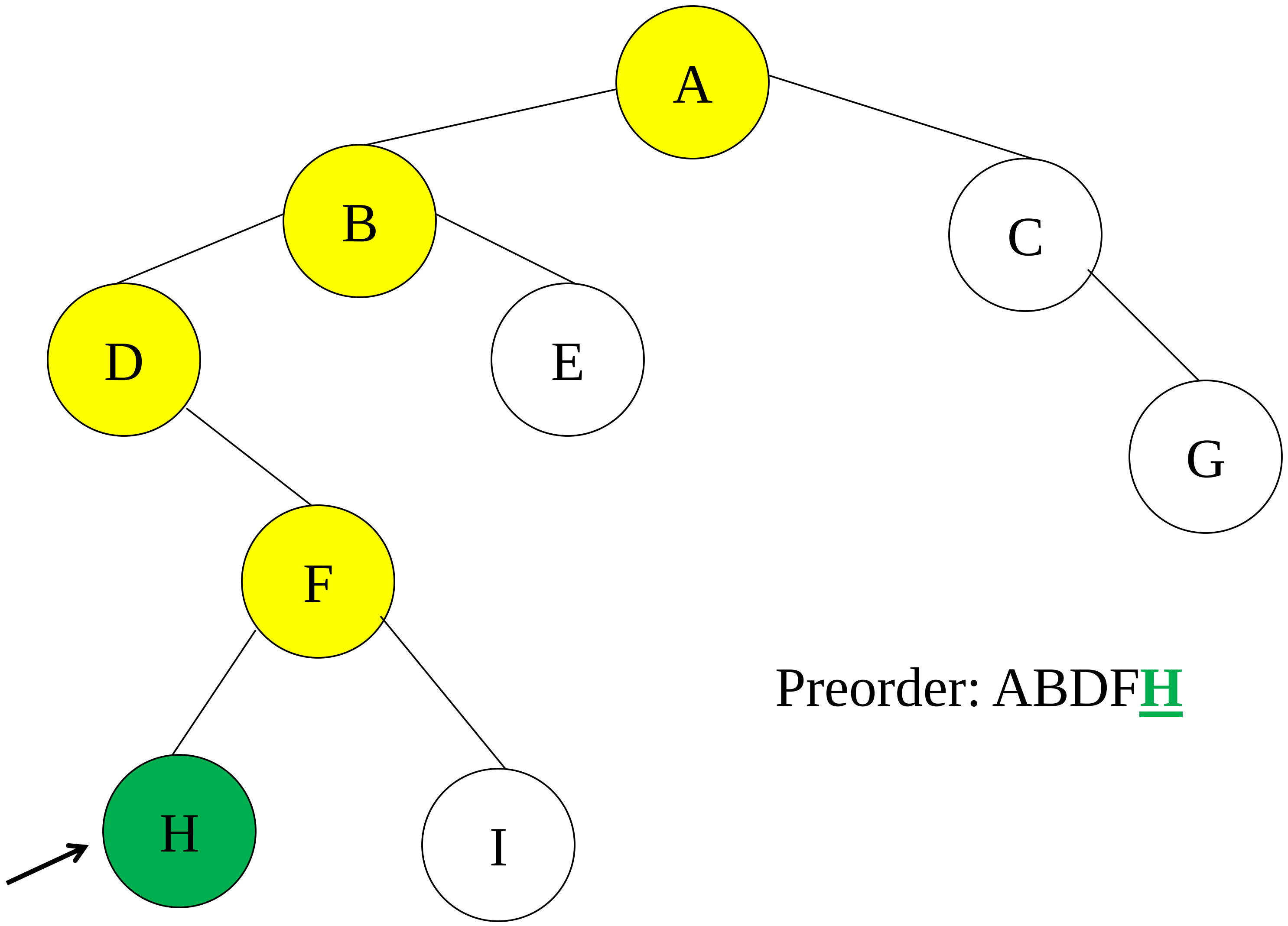
Preorder: AB**D**

Tree Traversals: Another Example



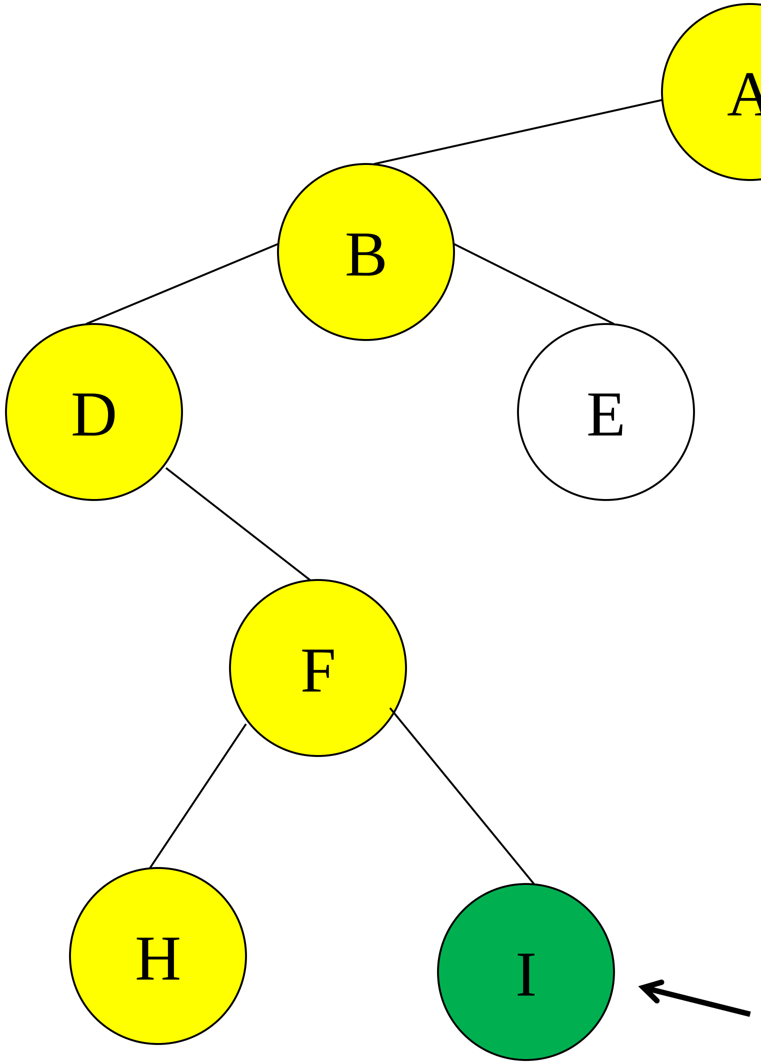
Preorder: ABD**F**

Tree Traversals: Another Example



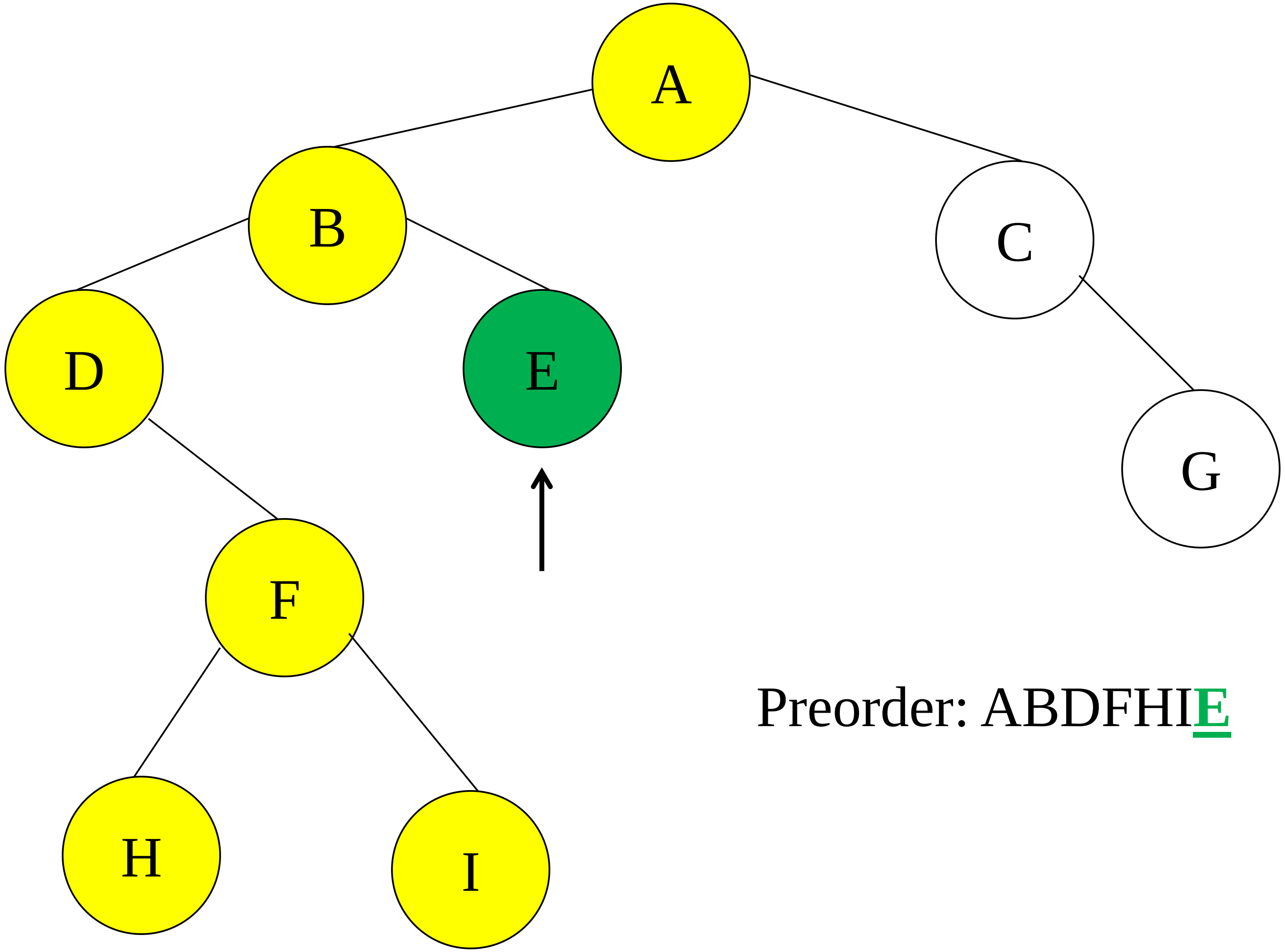
Preorder: ABDF**H**

Tree Traversals: Another Example



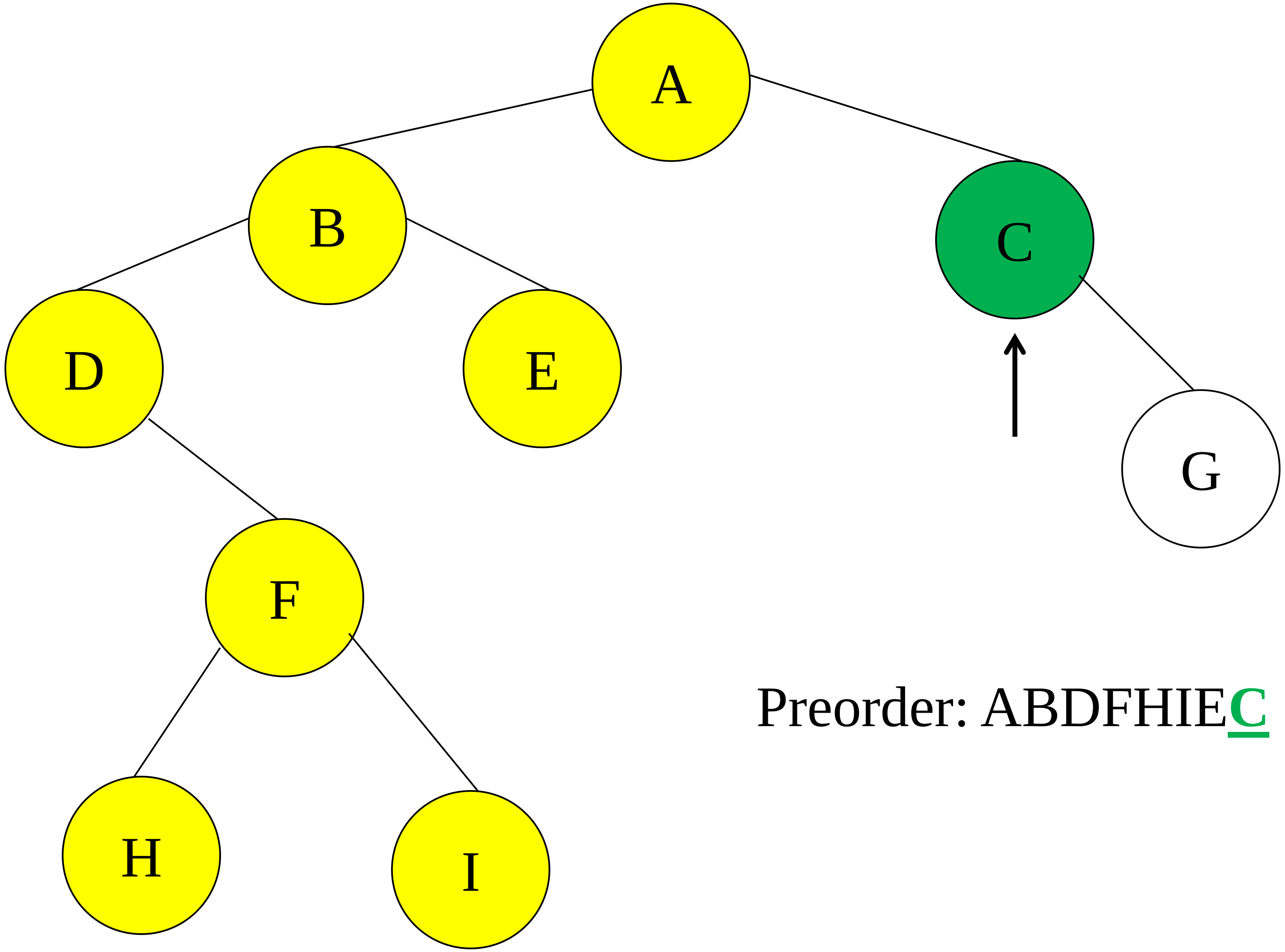
Preorder: ABDFH**I**

Tree Traversals: Another Example



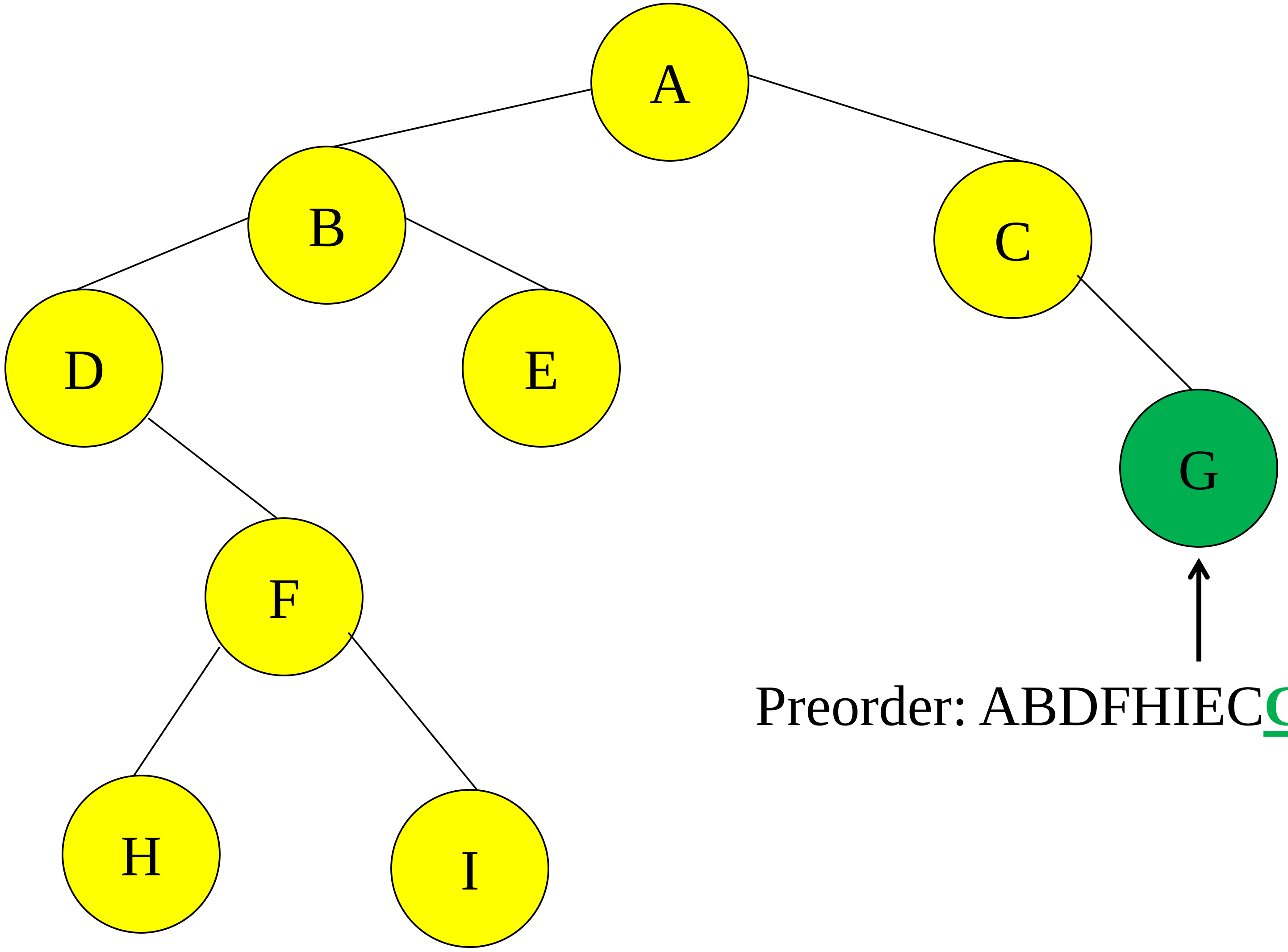
Preorder: ABDFHI**E**

Tree Traversals: Another Example



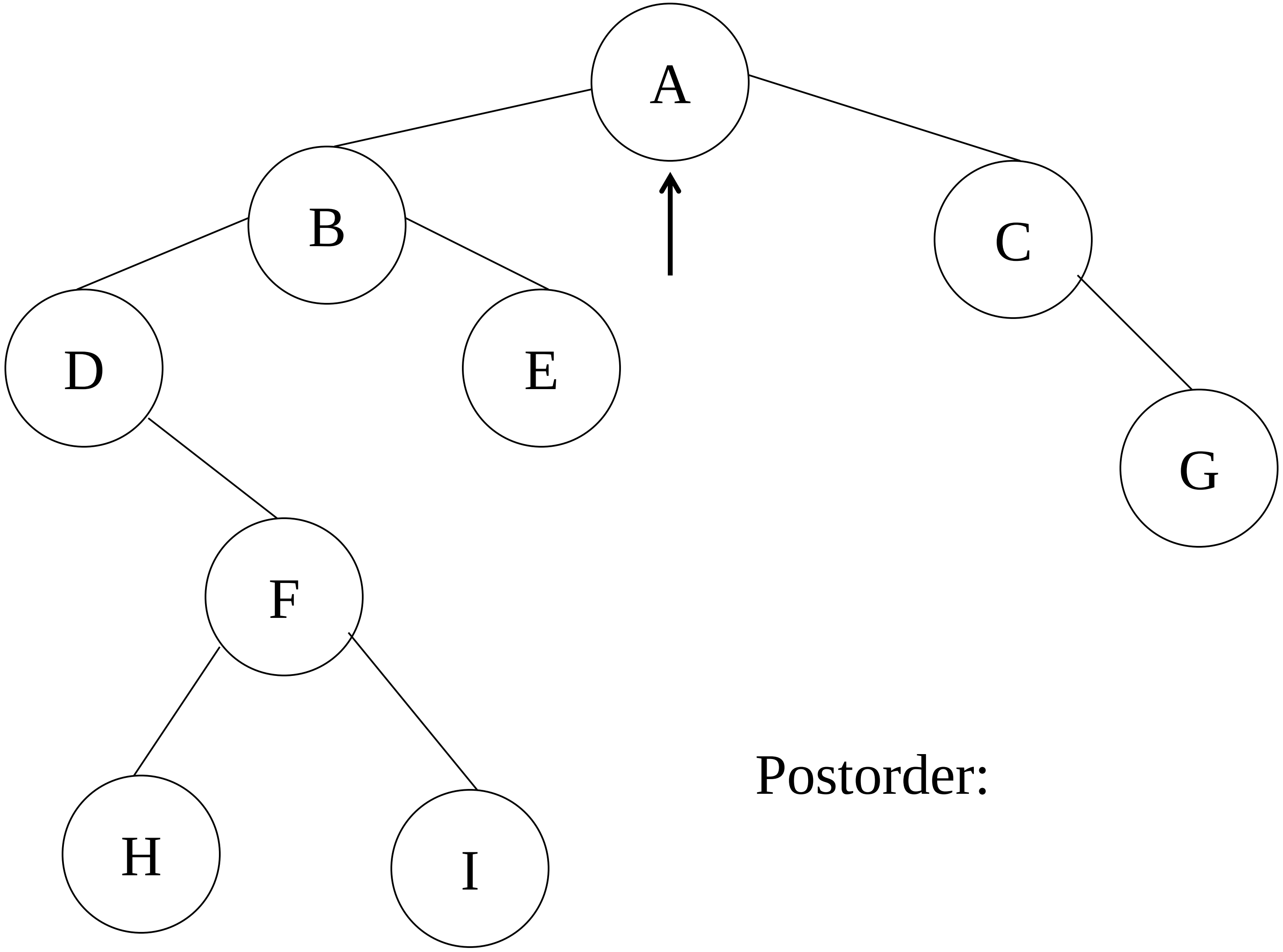
Preorder: ABDFHIE**C**

Tree Traversals: Another Example



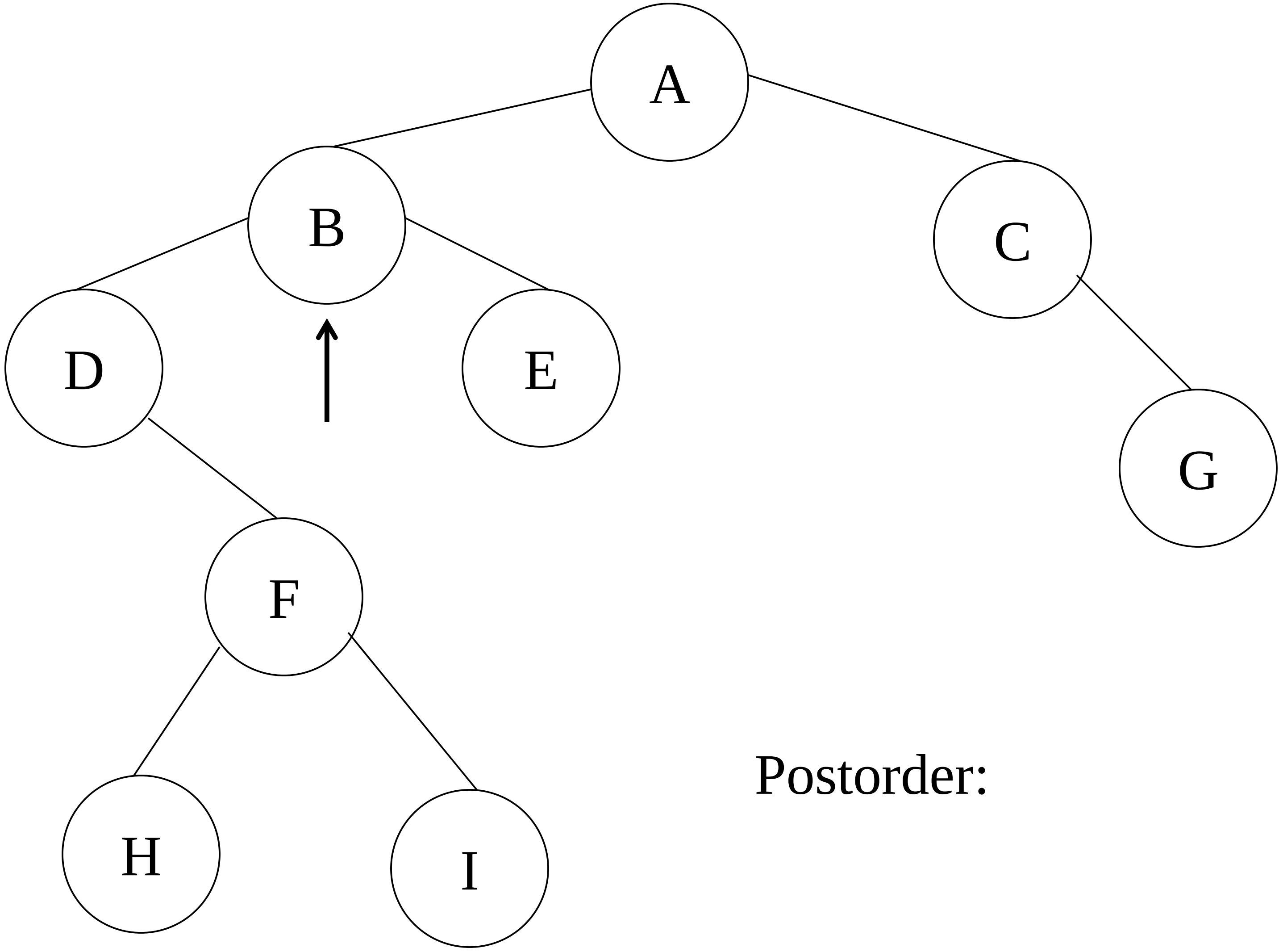
Preorder: ABDFHIEC**G**

Tree Traversals: Another Example



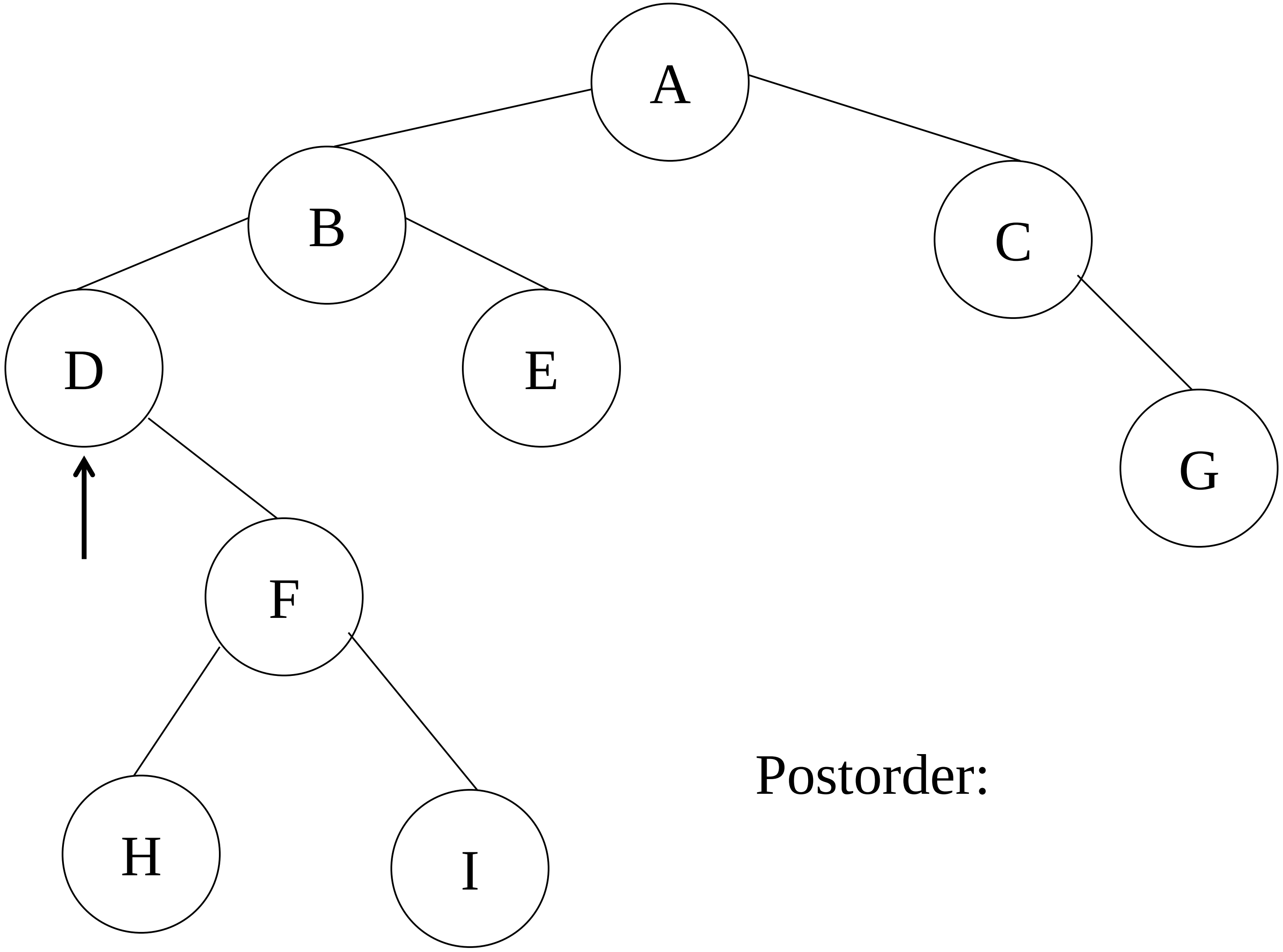
Postorder:

Tree Traversals: Another Example



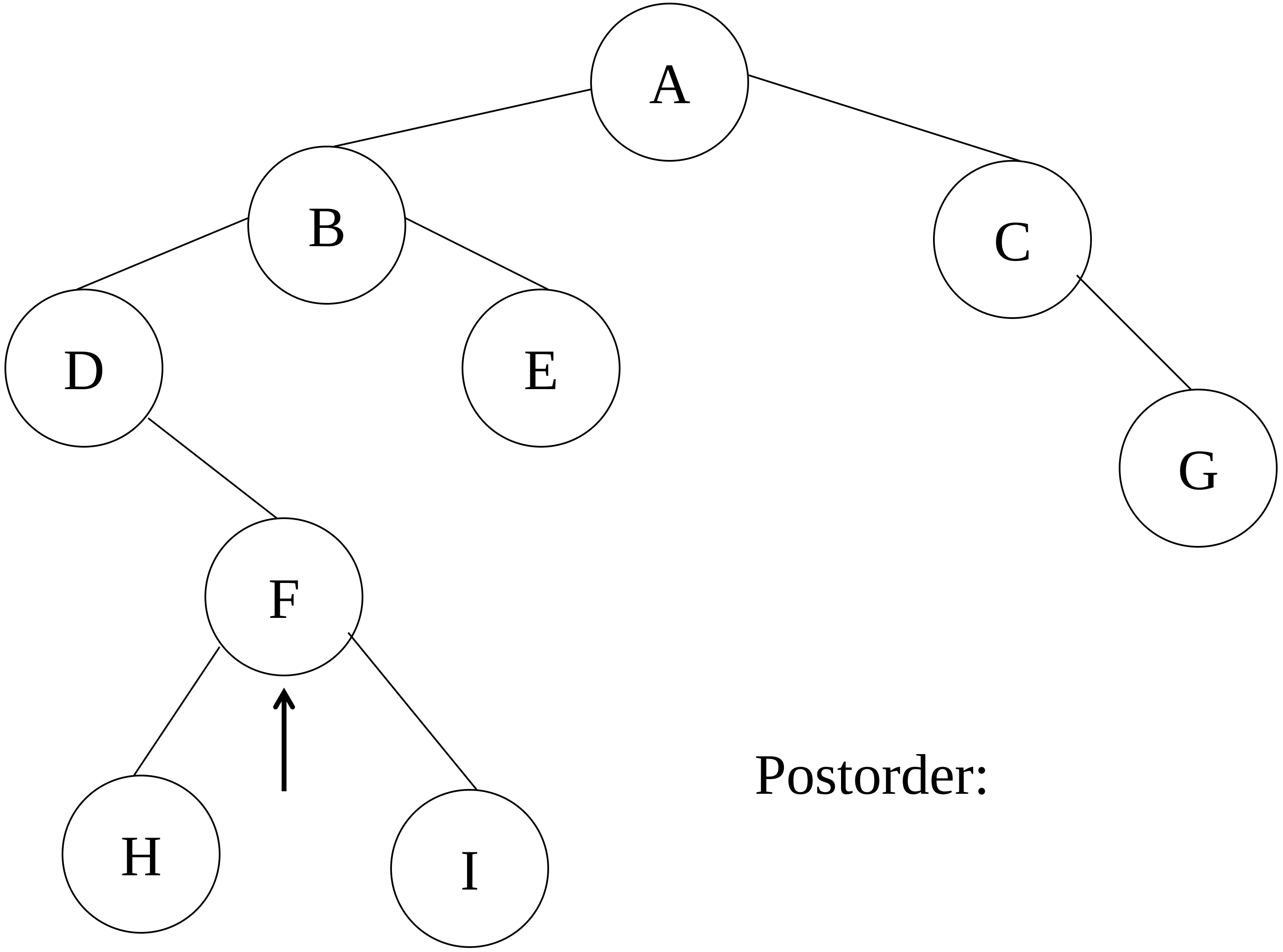
Postorder:

Tree Traversals: Another Example



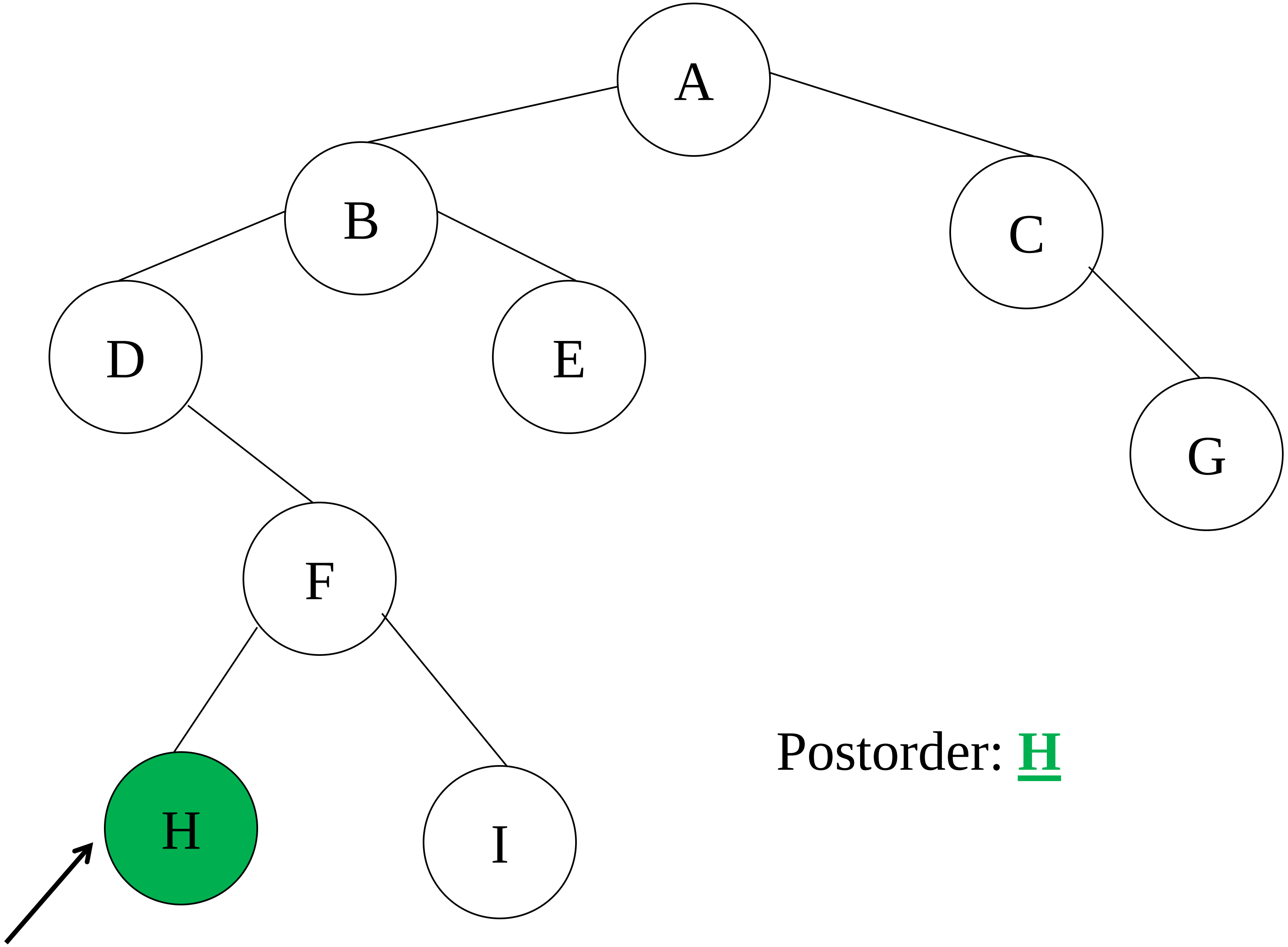
Postorder:

Tree Traversals: Another Example



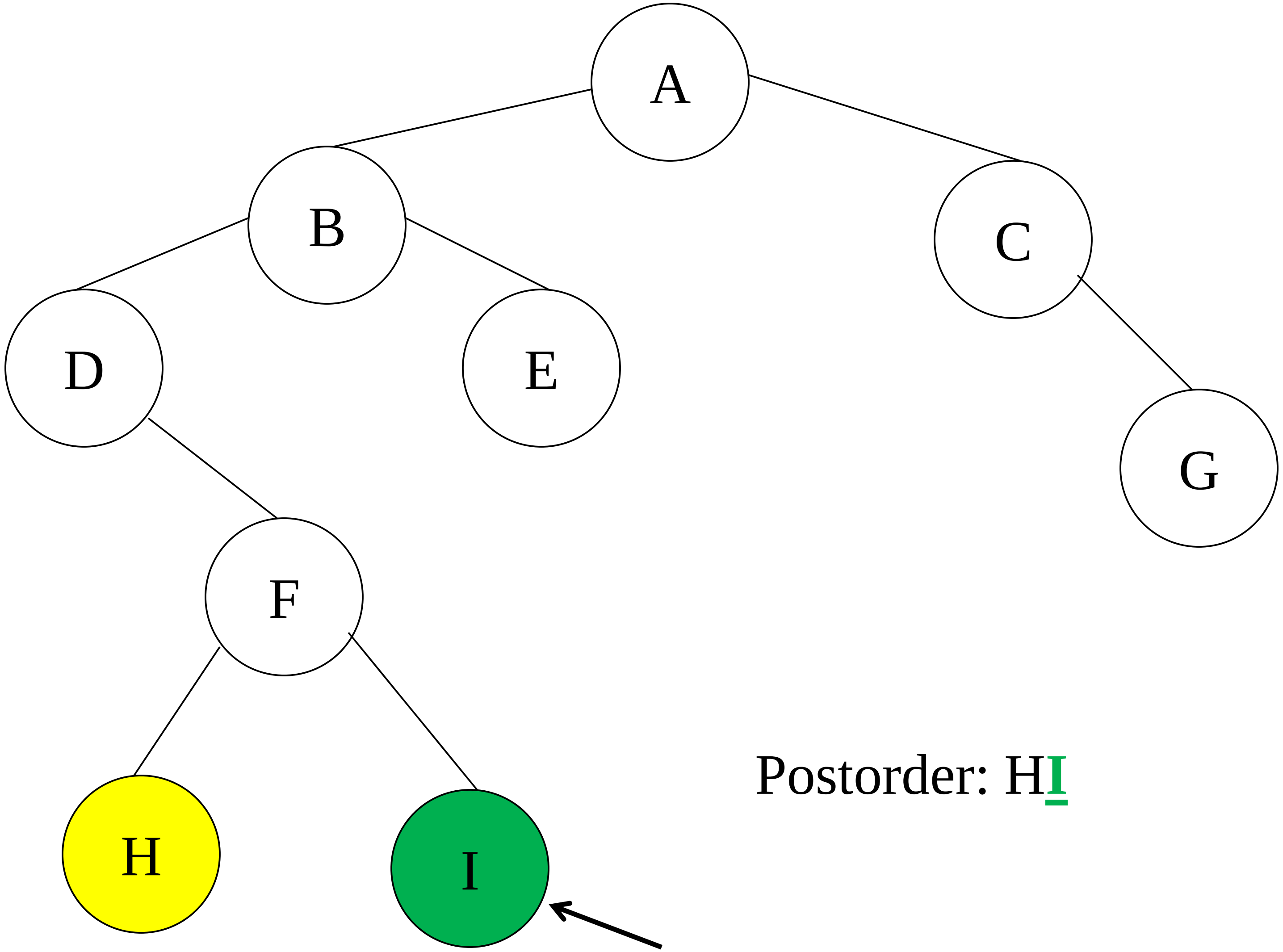
Postorder:

Tree Traversals: Another Example



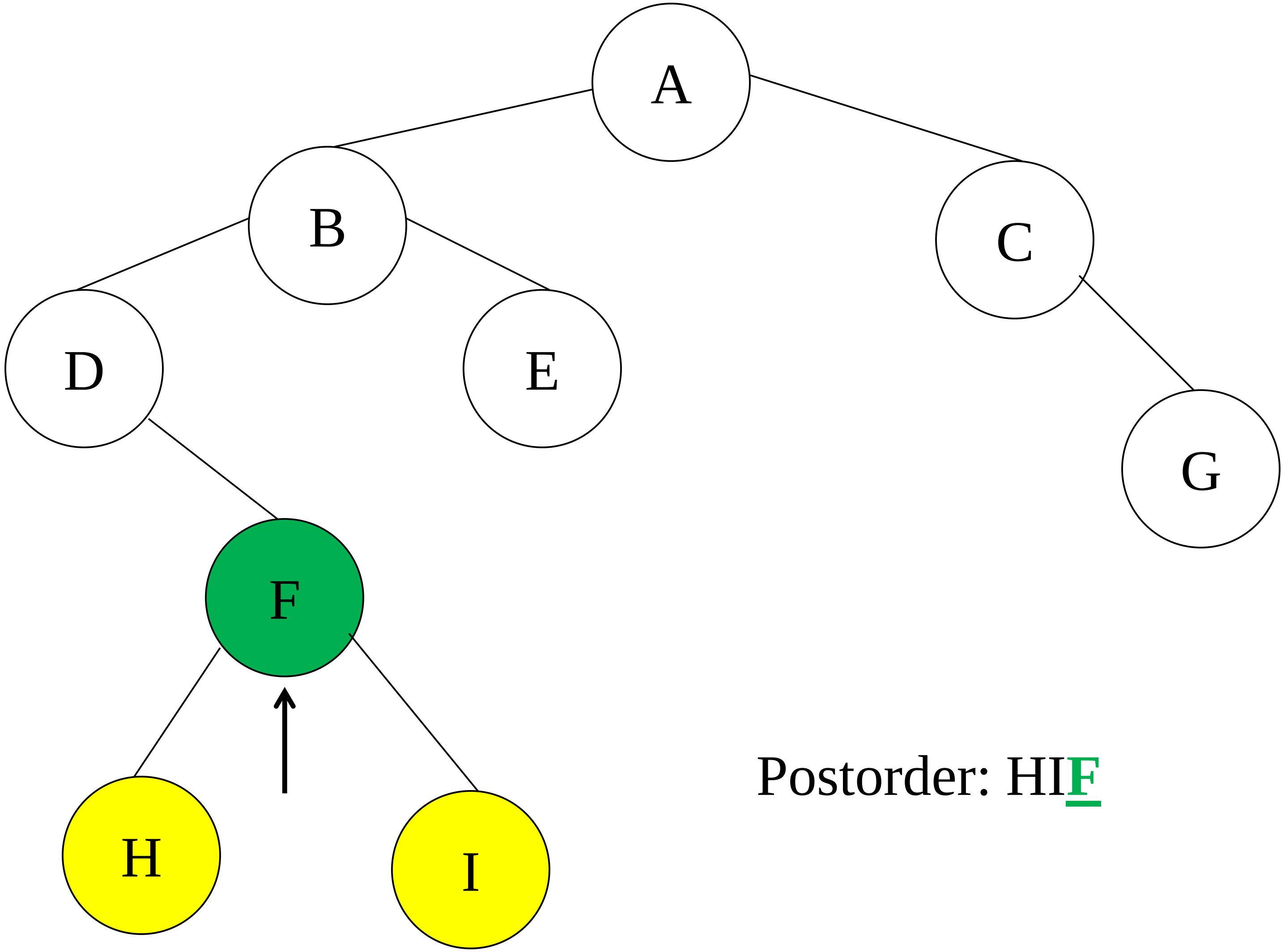
Postorder: **H**

Tree Traversals: Another Example



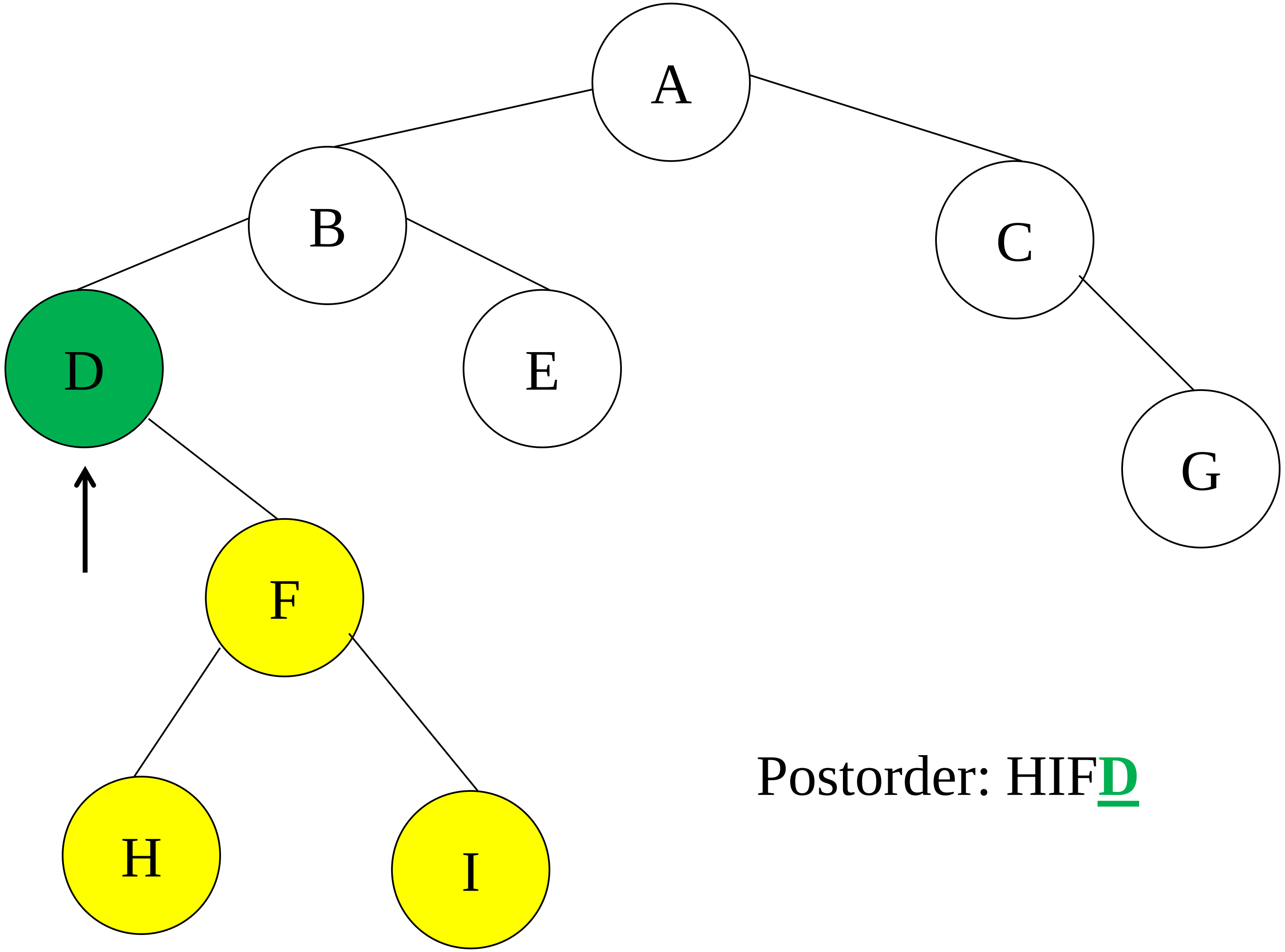
Postorder: H**I**

Tree Traversals: Another Example



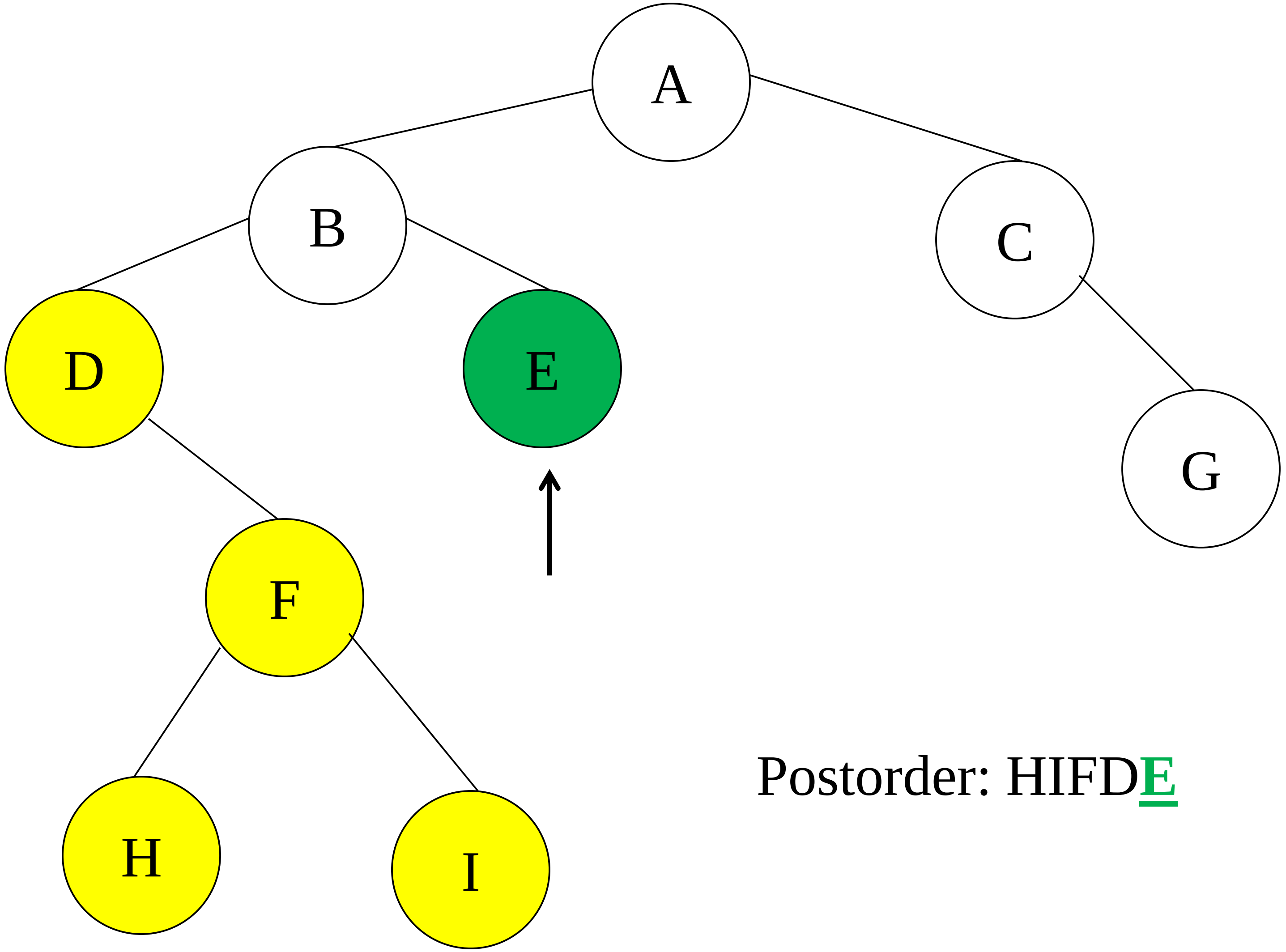
Postorder: HI**F**

Tree Traversals: Another Example



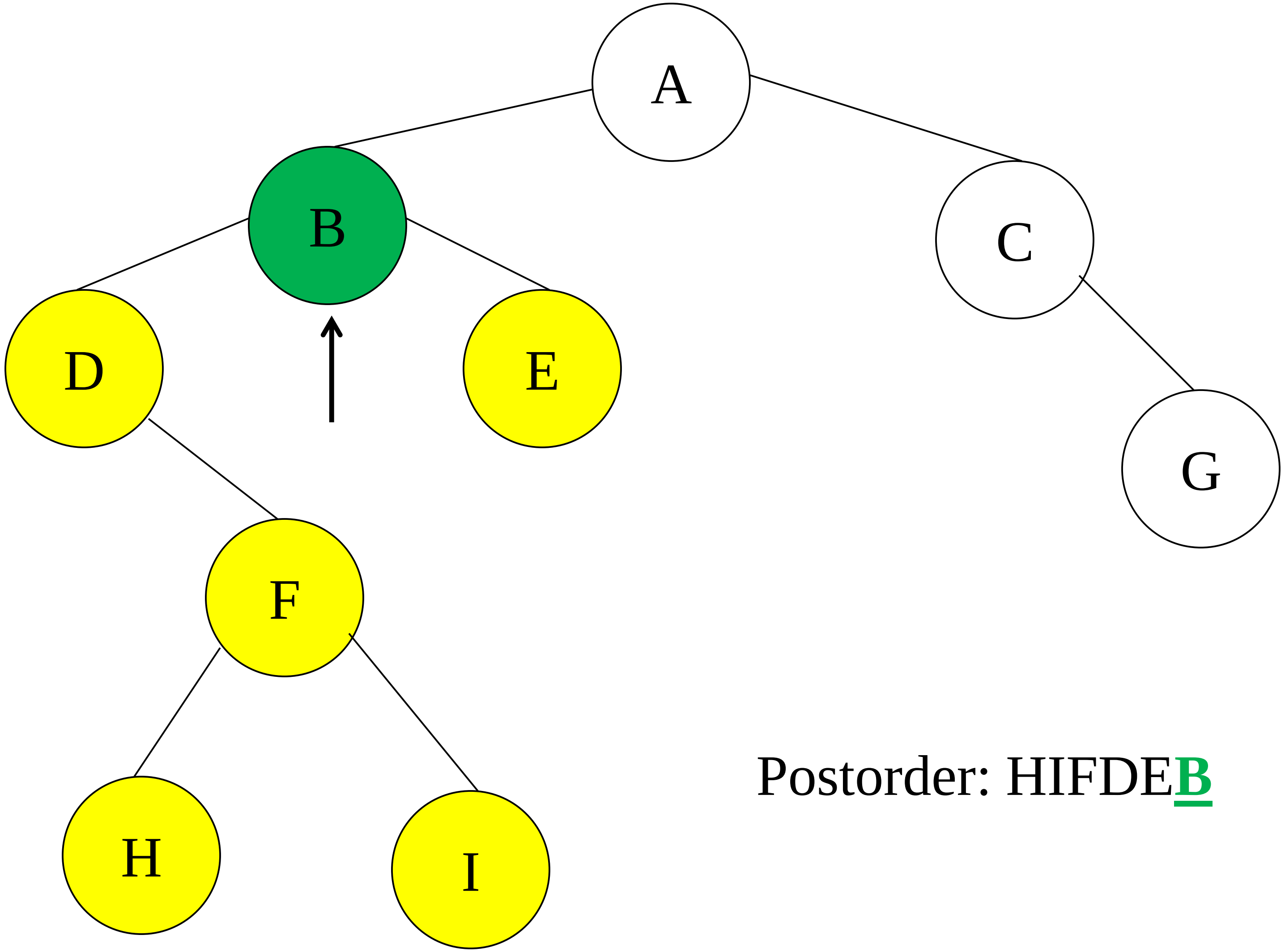
Postorder: HIF**D**

Tree Traversals: Another Example



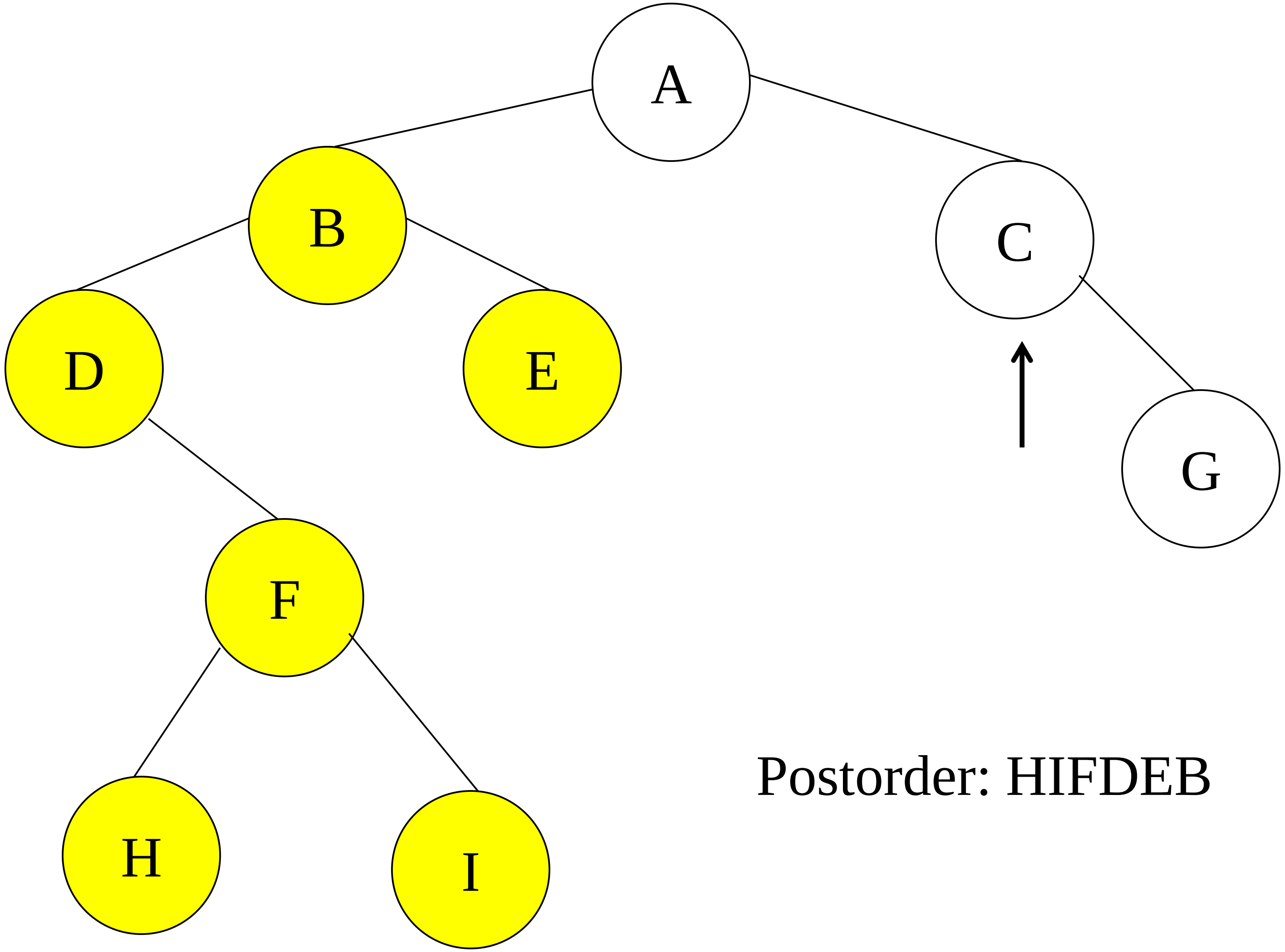
Postorder: HIFD**E**

Tree Traversals: Another Example



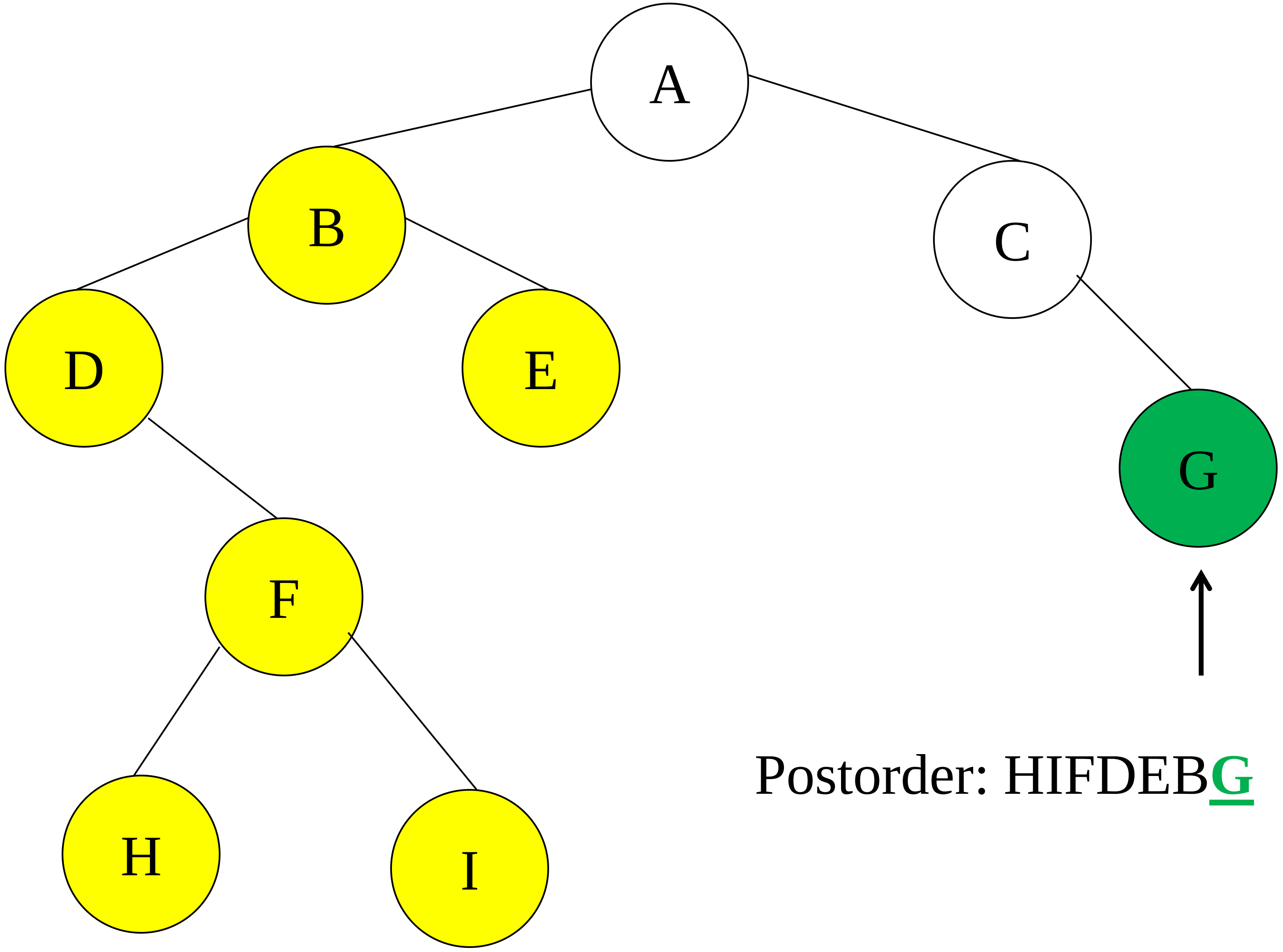
Postorder: HIFDE**B**

Tree Traversals: Another Example



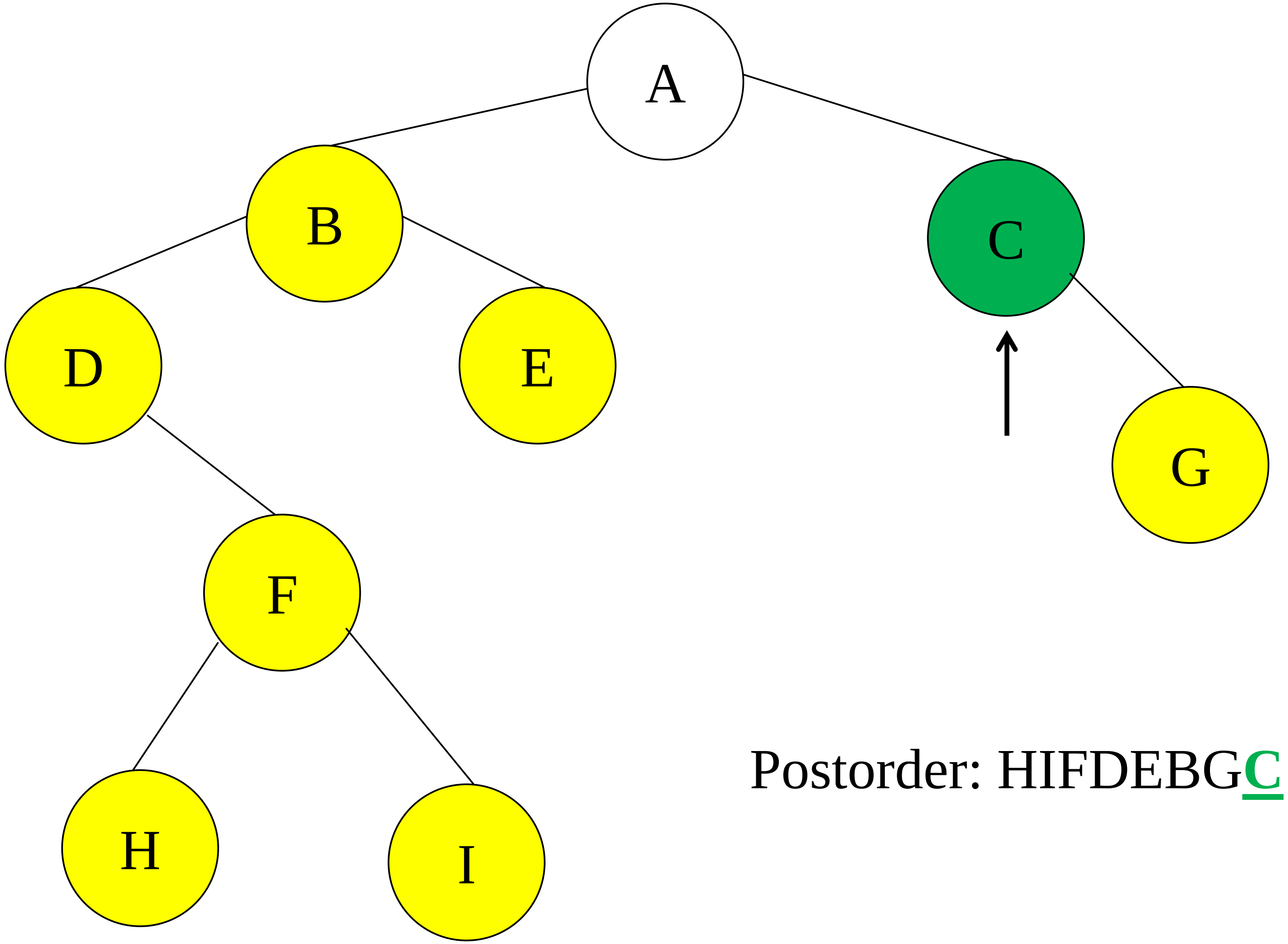
Postorder: HIFDEB

Tree Traversals: Another Example



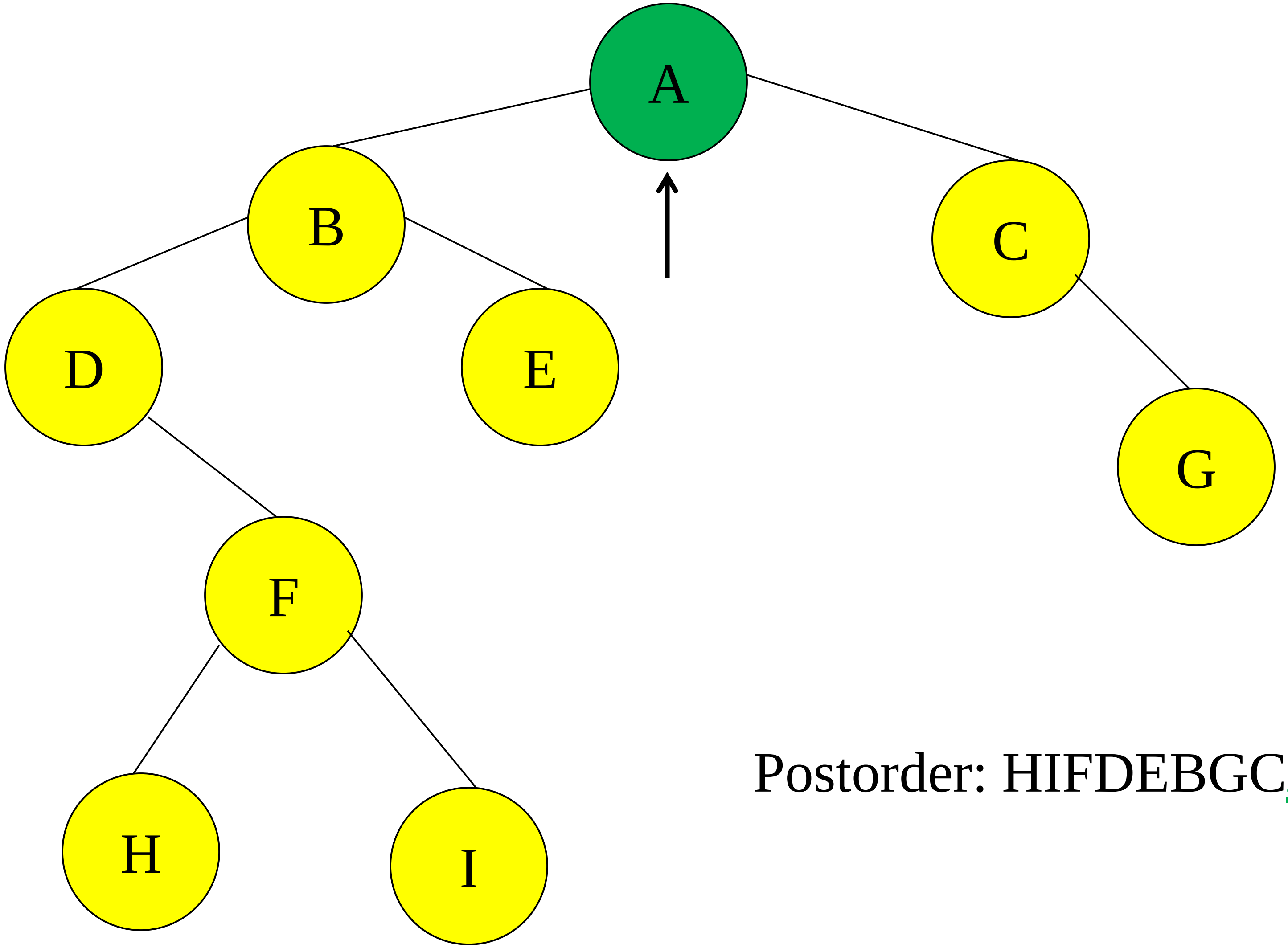
Postorder: HIFDEB**G**

Tree Traversals: Another Example



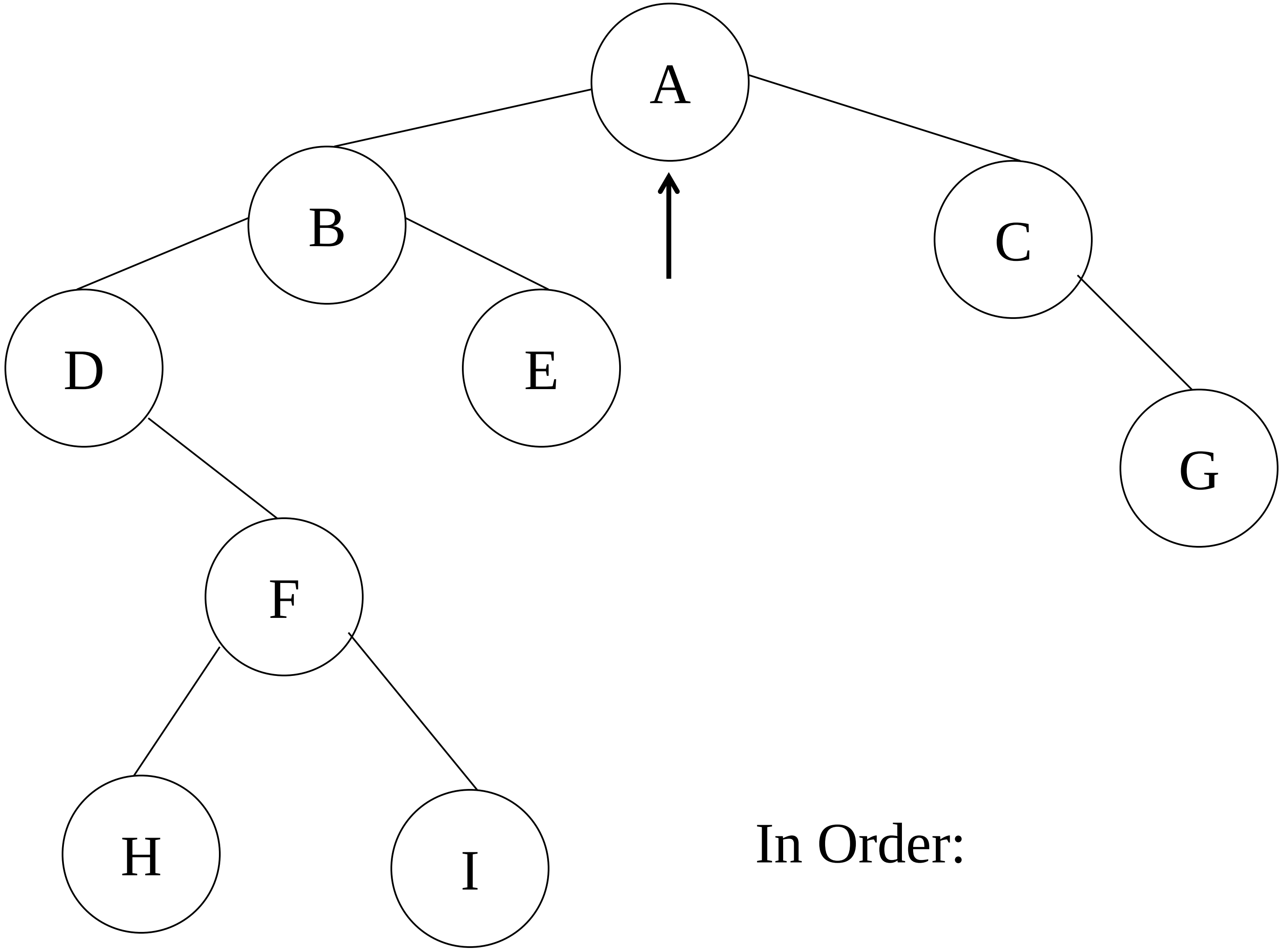
Postorder: HIFDEBG**C**

Tree Traversals: Another Example



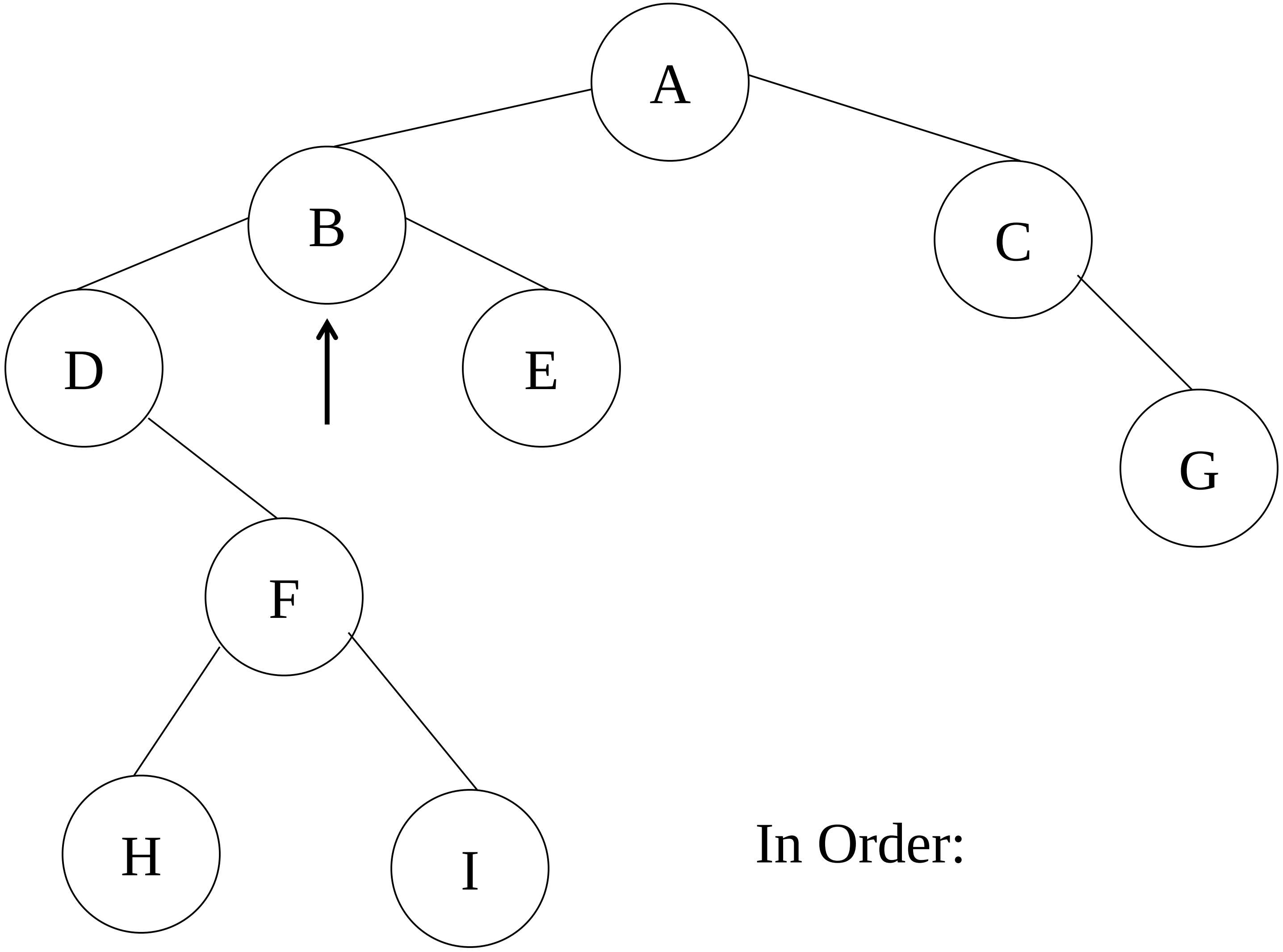
Postorder: HIFDEBGC**A**

Tree Traversals: Another Example



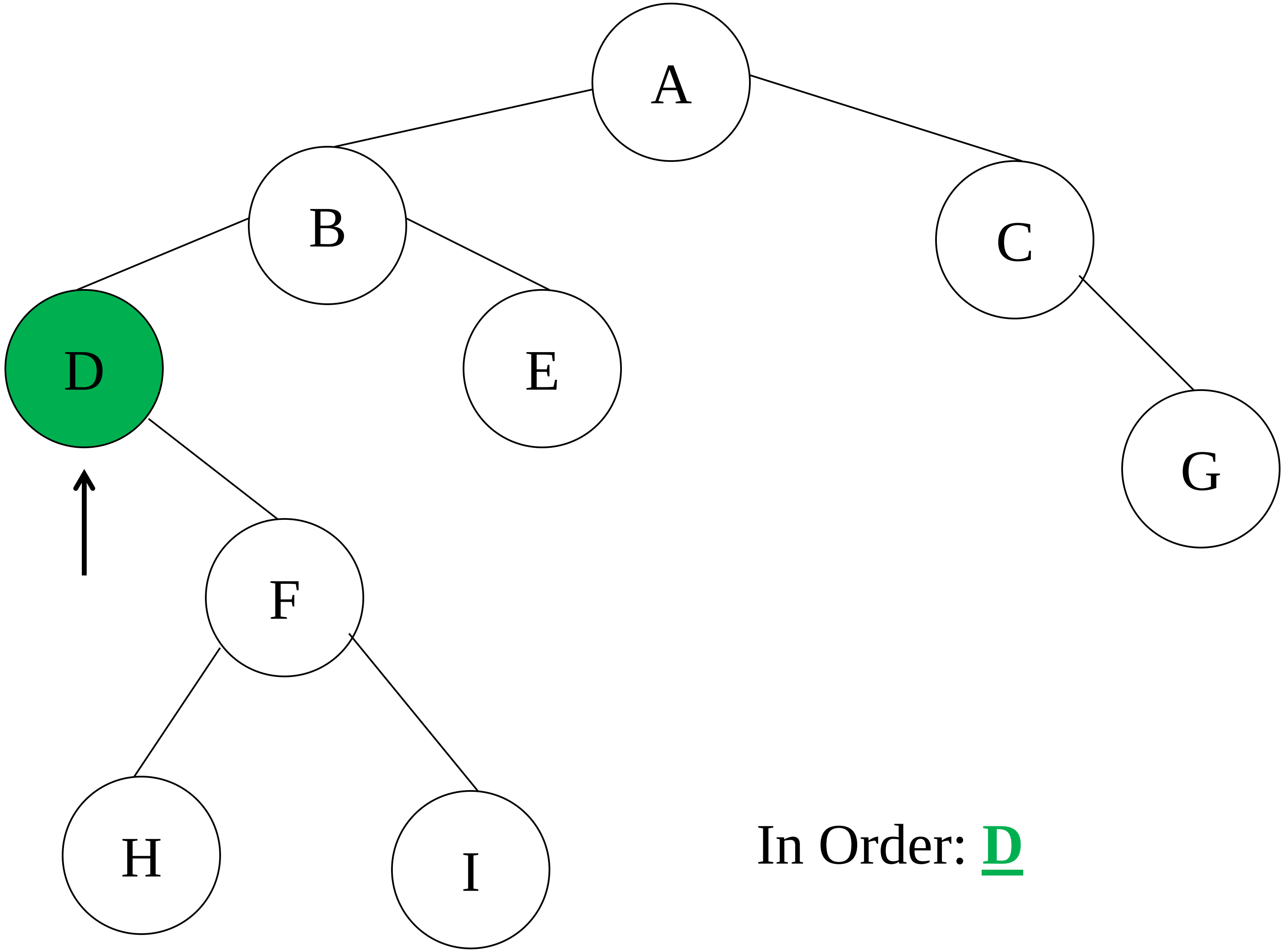
In Order:

Tree Traversals: Another Example



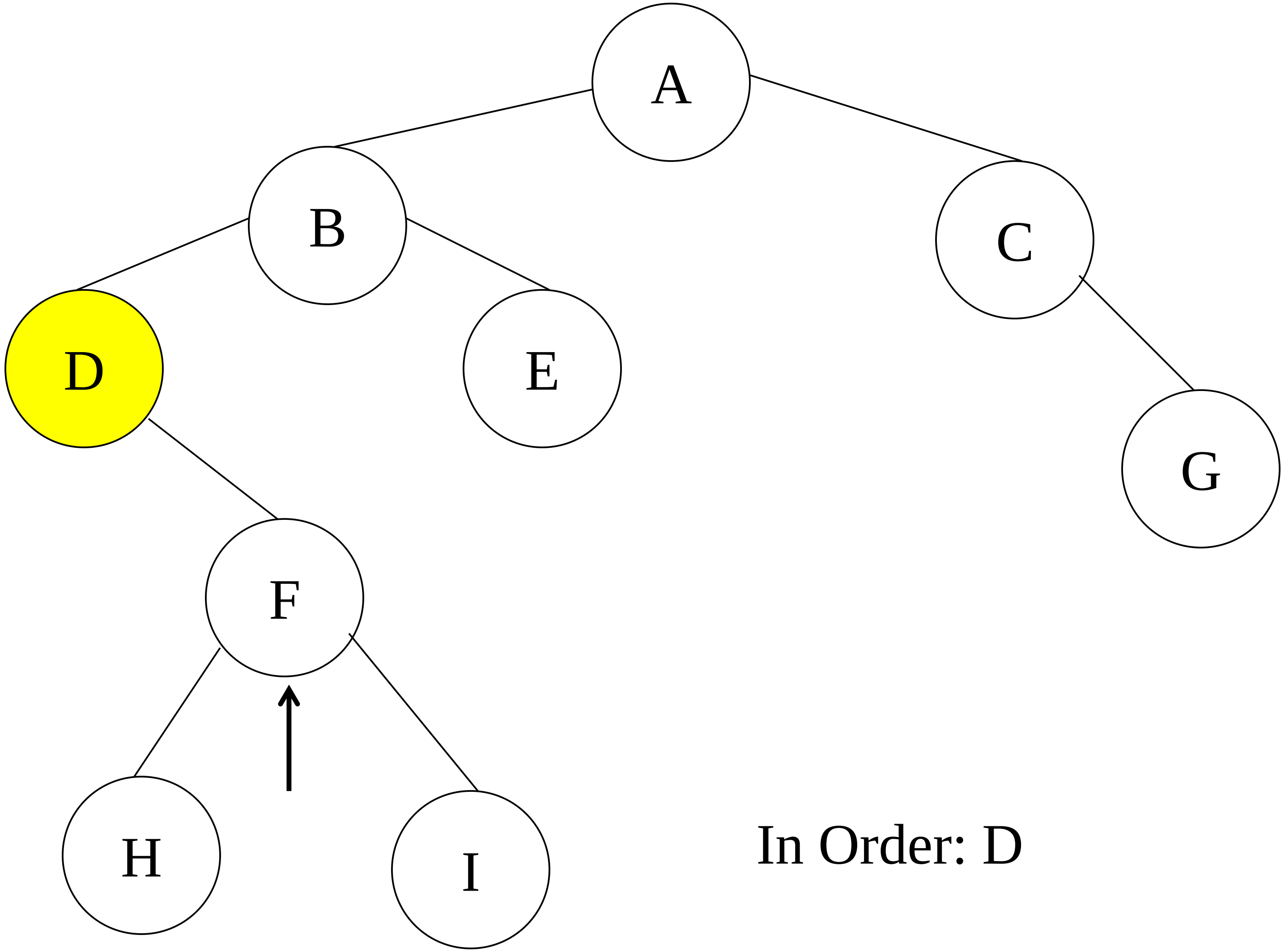
In Order:

Tree Traversals: Another Example



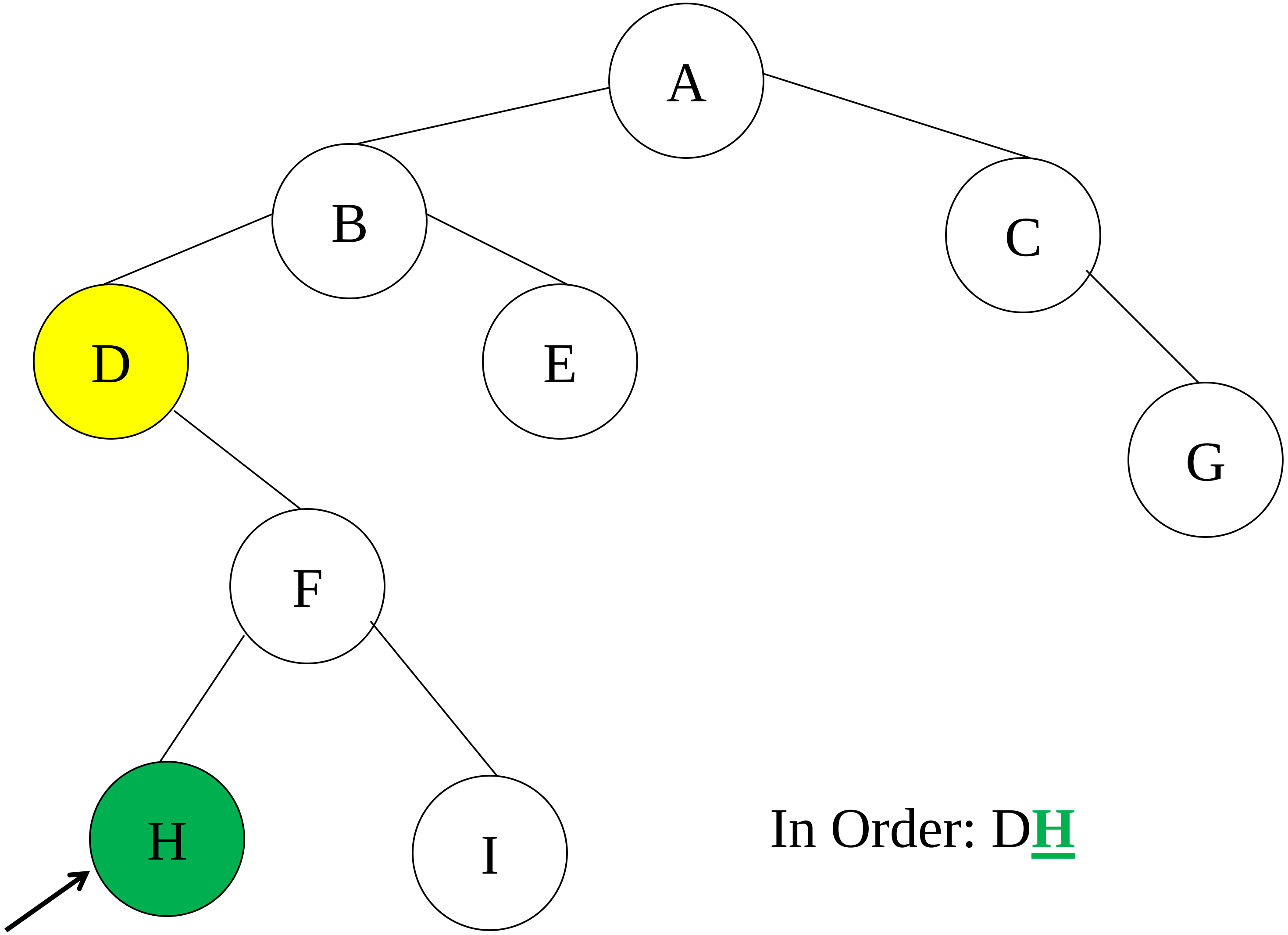
In Order: **D**

Tree Traversals: Another Example



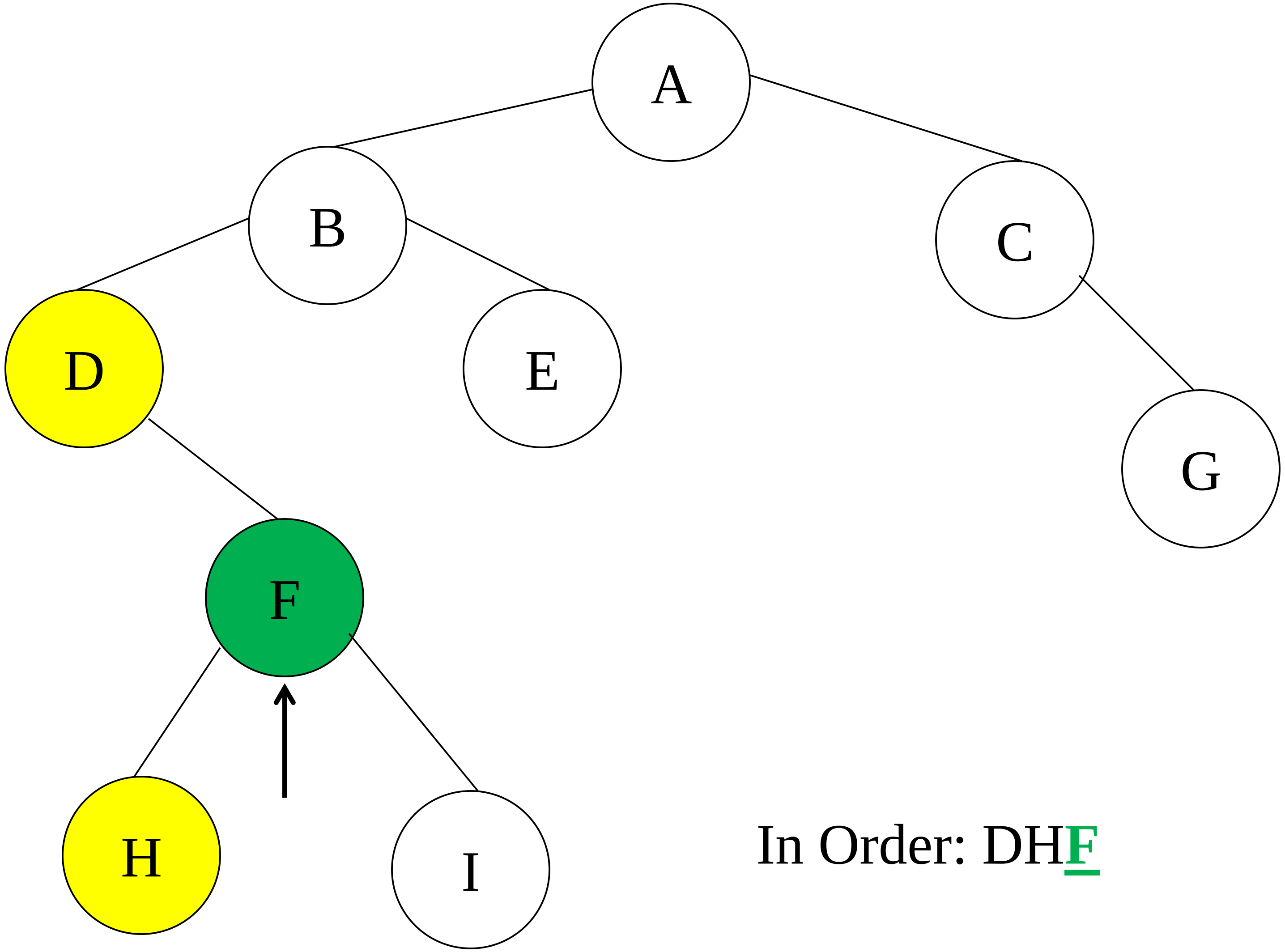
In Order: D

Tree Traversals: Another Example



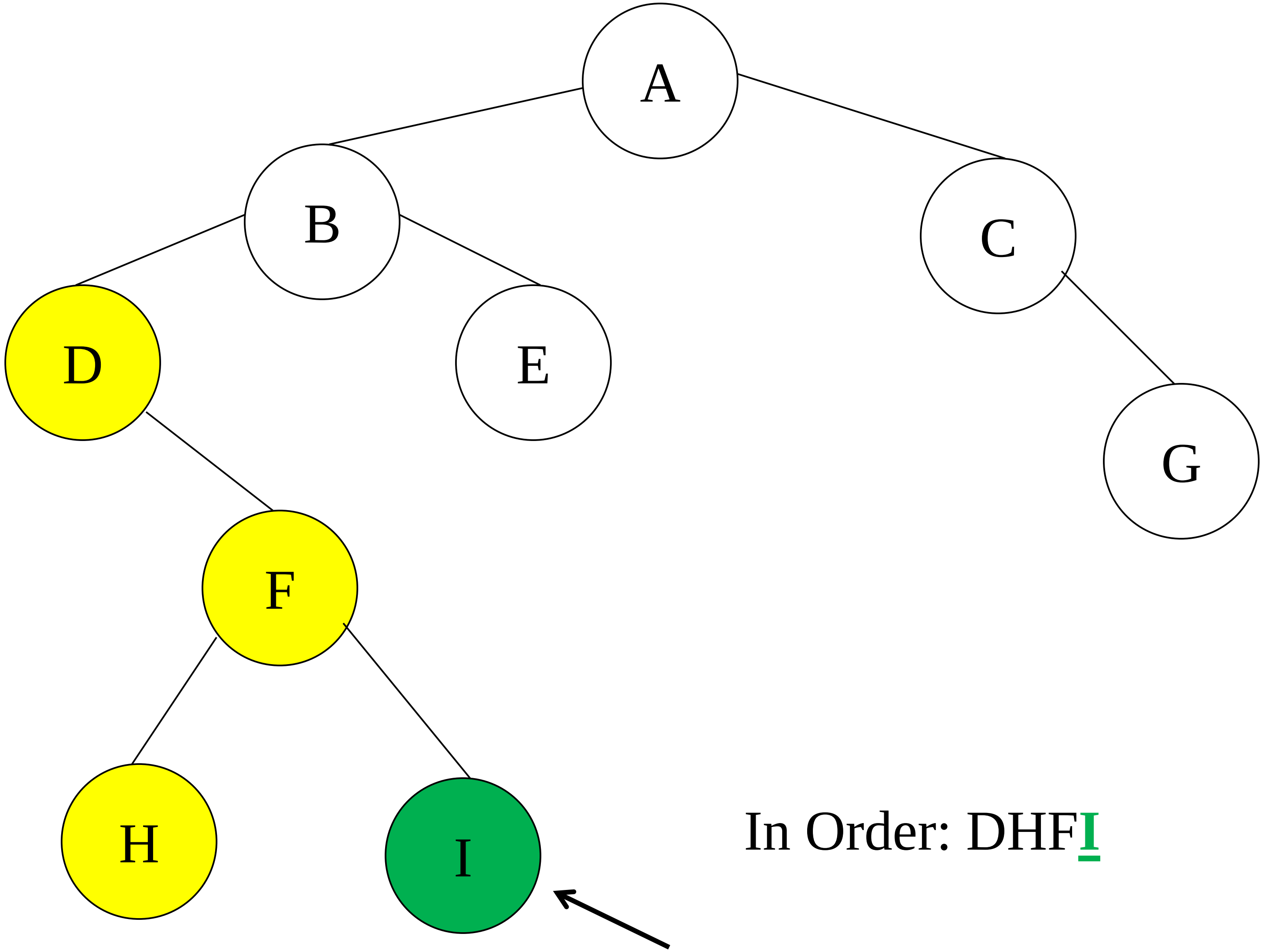
In Order: D**H**

Tree Traversals: Another Example



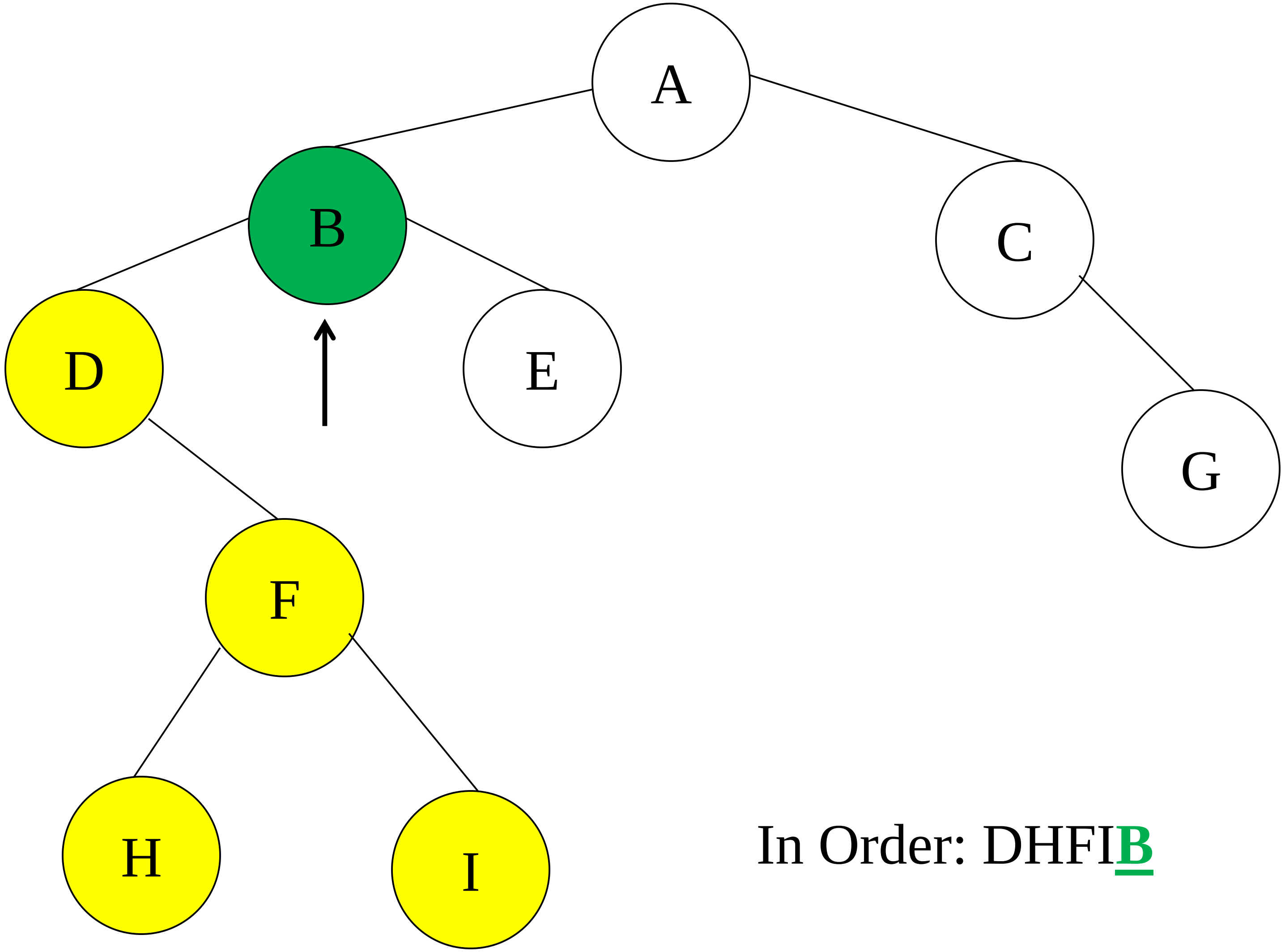
In Order: DH**F**

Tree Traversals: Another Example



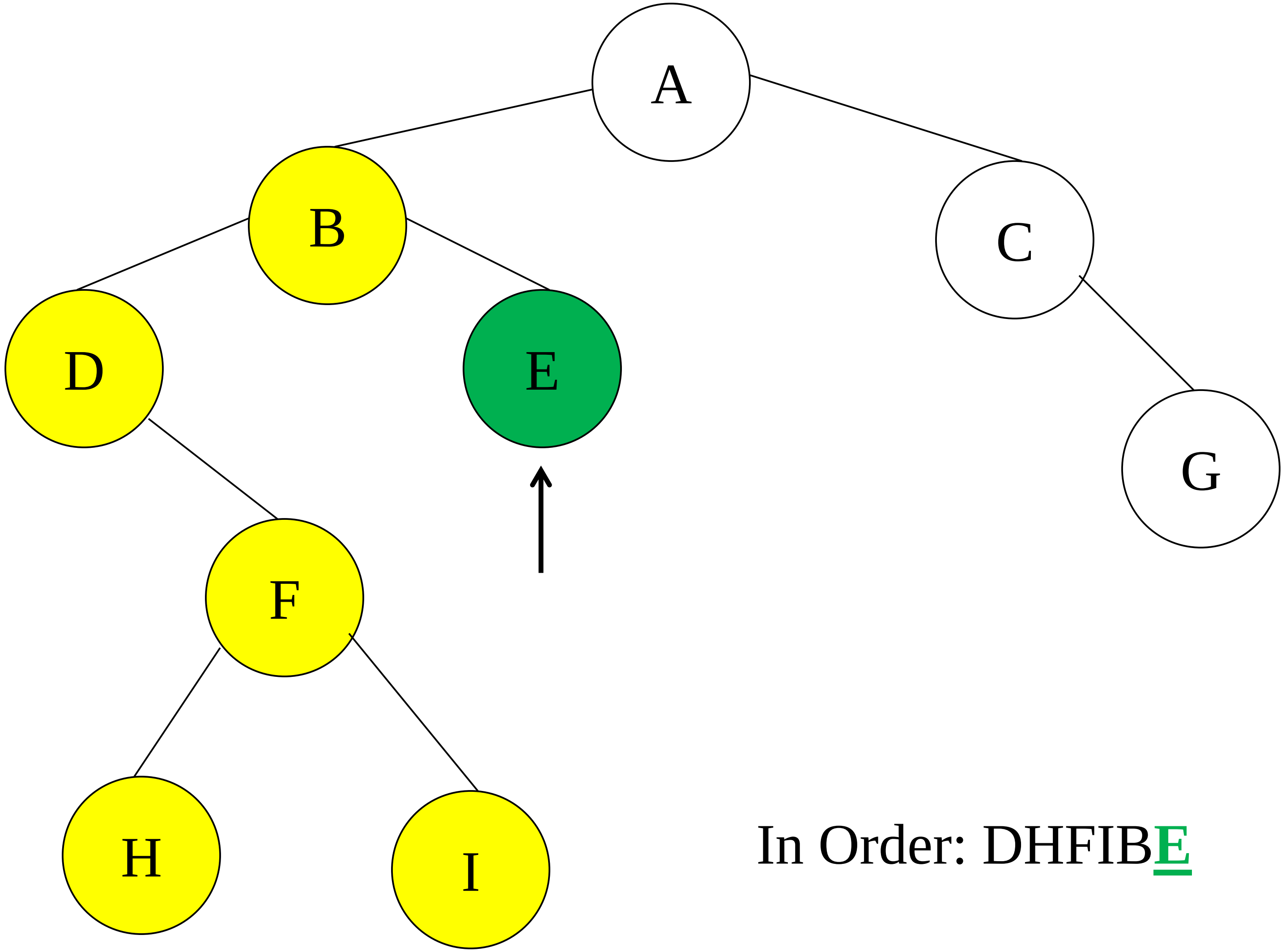
In Order: DHF**I**

Tree Traversals: Another Example



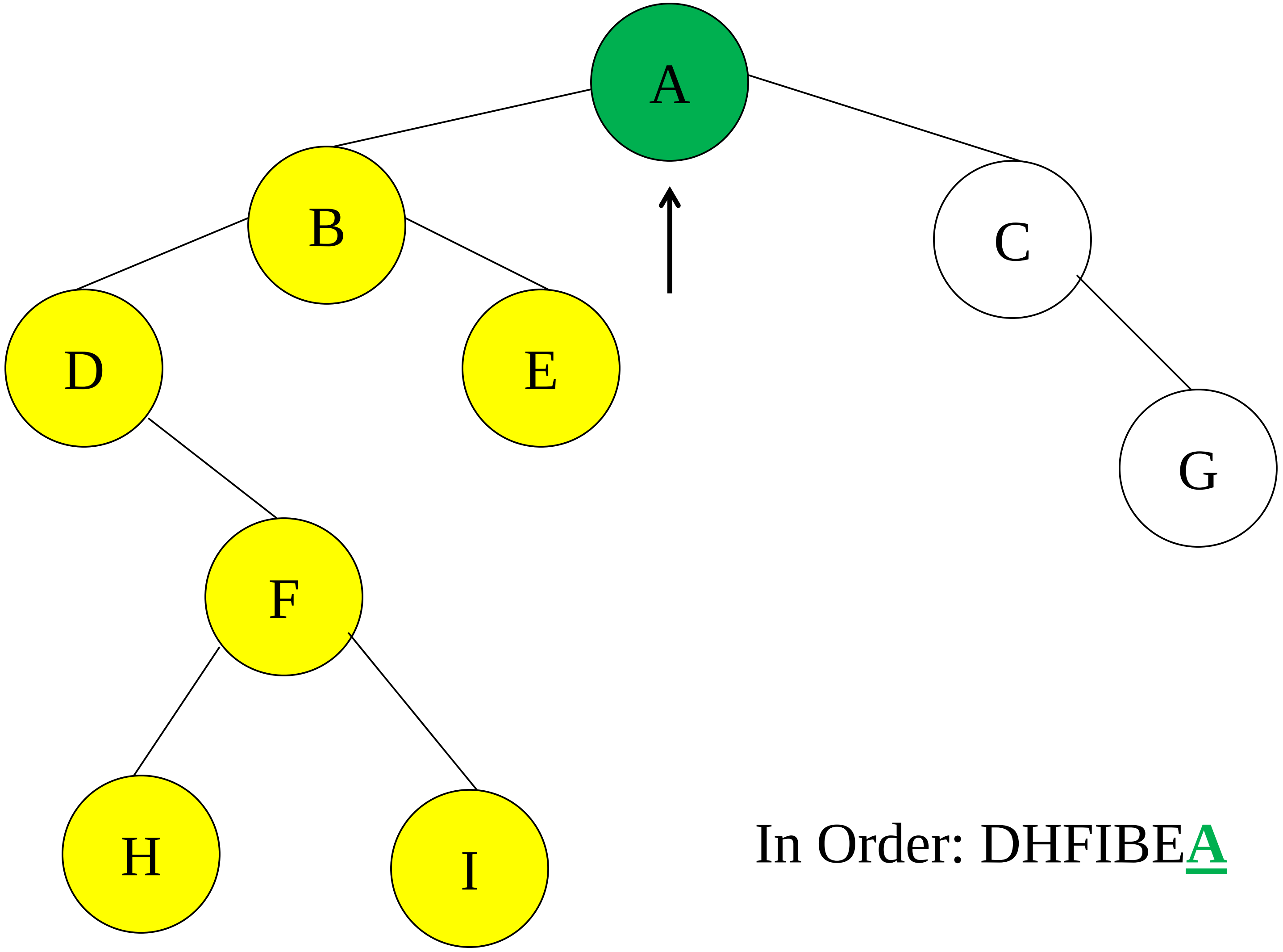
In Order: DHFI**B**

Tree Traversals: Another Example



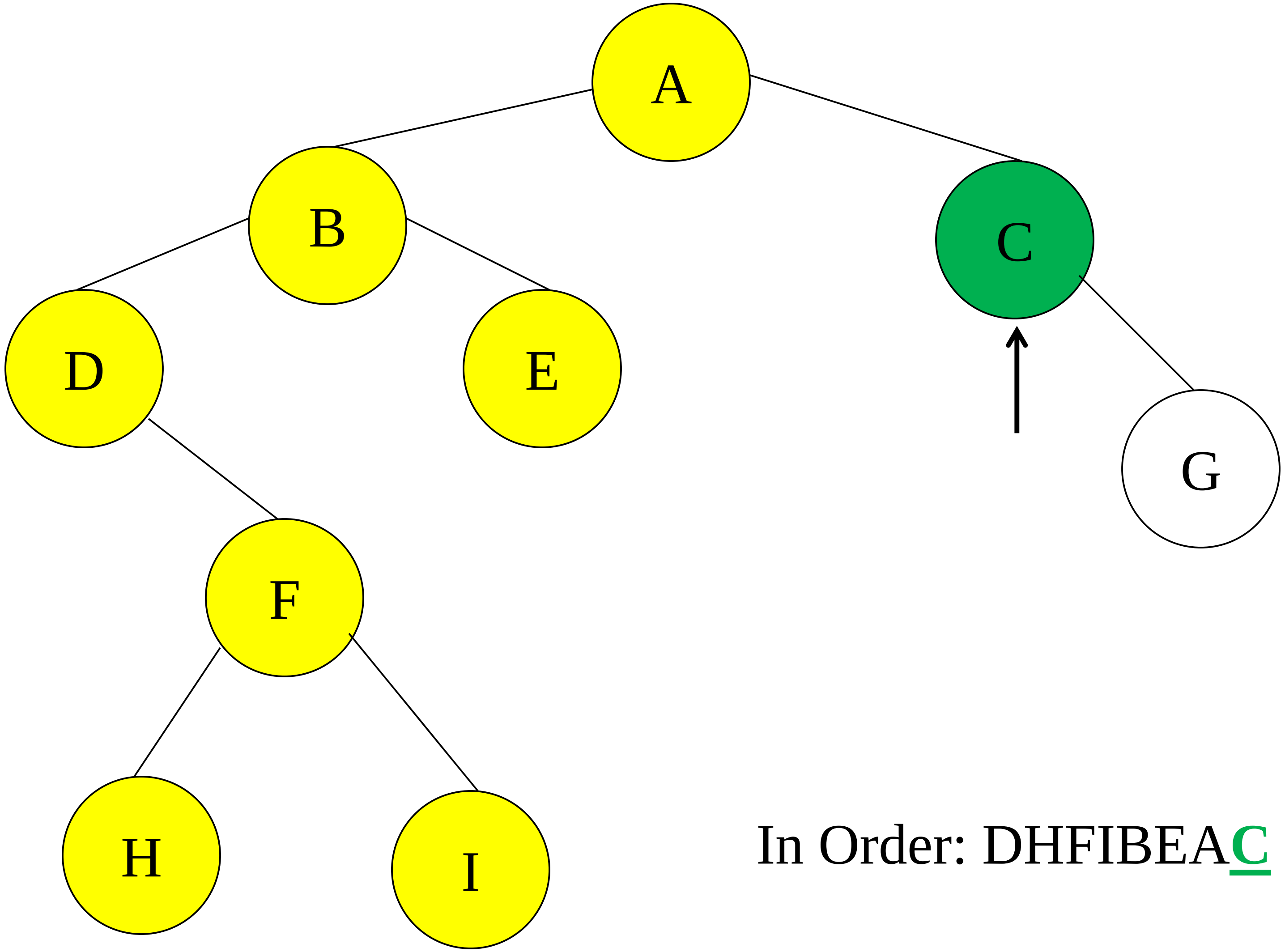
In Order: DHFIB**E**

Tree Traversals: Another Example



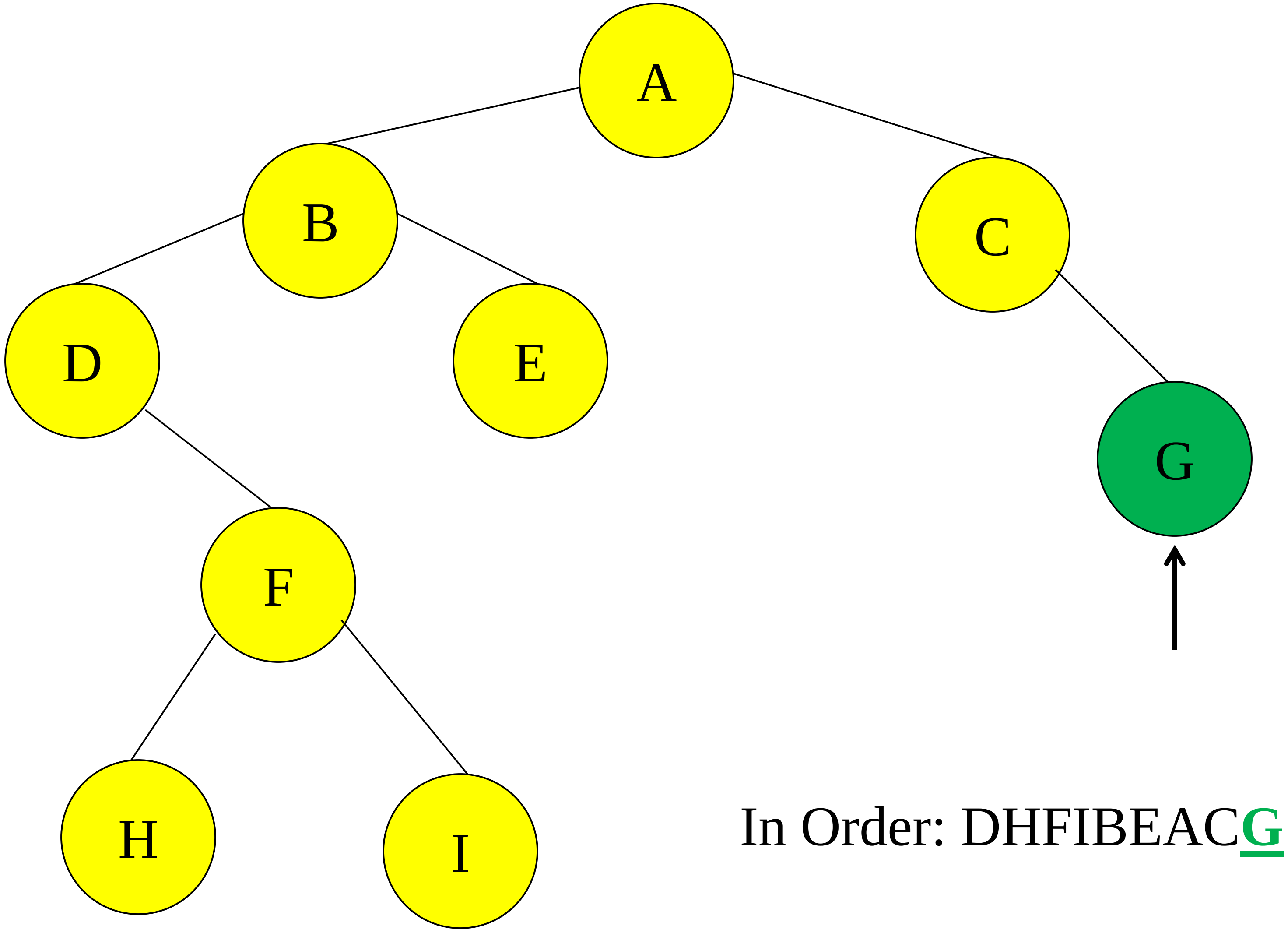
In Order: DHFIBE**A**

Tree Traversals: Another Example



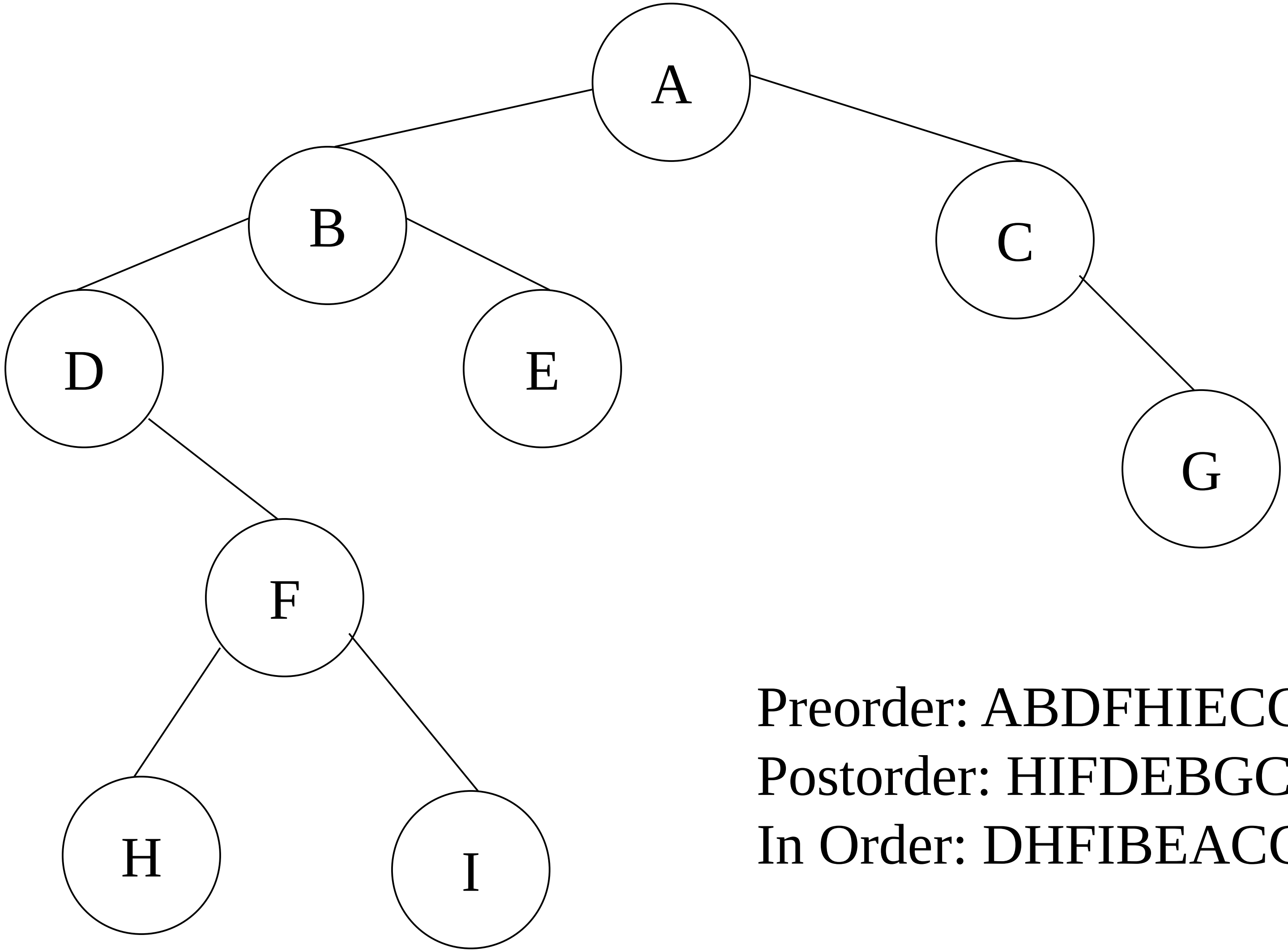
In Order: DHFIBEA**C**

Tree Traversals: Another Example



In Order: DHFIBEAC**G**

Tree Traversals: Another Example



Preorder: ABDFHIECG
Postorder: HIFDEBGCA
In Order: DHFIBEACG