Unit - 2

DATE: / /
PAGE:

~~Sol^n~~- Direct Method of Proofs:-

(Direct Method of proof): A direct Proof of $P \Rightarrow q$ is logical valid argument in which are start with the assumption that 'P' is true, and then using 'p' as well as other axioms show directly that 'q' is true.

$P \rightarrow q$

Q- Give Direct proof of the Statement :
"the product of two add integers is odd".

Soln- let us say 2 odd integer is

$3 \times 5 = 15, \quad 5 \times 7 = 35$

All is odd According to Mathematics.

DATE: __/__/____
PAGE: ____

→ if two odd integer p and q.

$p \Rightarrow q$ (product of two integer)

x and y be two integer rather say odd integer.

$x = 2n+1$ (odd representation) → True.
$y = 2m+1$

xy Ka Product

$xy = (2n+1)(2m+1) \Rightarrow 4m + 2n + 2m + 1$
L.H.S

$\Rightarrow 2\underbrace{(mn + n + m)}+1$

This is integer 'a'.

$\Rightarrow 2a+1$

I want to prof that x, y product is also add.

That shows R.H.S is also an odd integer.

Q- Show that Sequence of an even No. is an even No.

Solⁿ. According to Mathematics -

$2^2 = 4, \quad 4^2 = 16$.

let x be an even NO.

DATE: __/__/__
PAGE:

$x = 2n$ $(n = 1, 2, 3, 4 \ldots)$

$\downarrow$

Means ye true hai assume ker liya to iska Squre Bhi true hoga.

$x^2 = (2n)^2$

$x^2 = 4n^2$

$x^2 = 2\underline{(2n^2)}$

$\downarrow$

This is an integer 'a'

$x^2 = 2a$

Hence x^2 is also even.

Hence the result follows.

<u>Q-3</u> Show that Sum of two odd No. is an even No.

<u>Sol^n</u> :- $3+3 = 6$, $5+5 = 10$

$3+7 = 10$,

P: x is odd and y is odd

Proof $\Rightarrow$ Q: $x+y$ is even.

$x = 2n+1$, $y = 2m+1$ (In odd case)

then sum,

$x+y = (2n+1) + (2m+1)$

$= 2(n+m) + 2$

$= 2\underline{(n+m+1)}$

$\downarrow$

integer (a)

$x+y = 2a$] $\rightarrow$ Hence it is also even No.

DATE : ___/___/___
PAGE : ______

Hence the Result follows.

Indirect Method of Proof –

1) Proof by Contrapositive –

It says that $p \Rightarrow q$ is logically equivalent to its contrapositive.

$$\sim q \Rightarrow \sim p$$

Show by truth table.

p	q	$p \to q$	$\sim p$	$\sim q$	$\sim q \to \sim p$
T	T	T	F	F	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	T	T

They are same means logically Equivalence.

Q- Prove that if $xy \in Z$ (set of integers) such that xy is odd then both x and y are odd.

Sol^n- P: xy is odd $\qquad P \to Q$

Q: x and y are odd.

By Method of Contraposition,

DATE: __/__/____
PAGE: ______

$\sim Q \rightarrow \sim p$

$\sim Q$ = x and y are even
$\sim p$ = xy is even.

$x = 2n$, $n \in$ any Z
$y = 2m$, $m \in$ any Z

$xy = 2n \times 2m$

$xy = 2\underbrace{(2mn)}_{\text{any integer } a}$

$xy = 2a$] it is even
xy is even.

By logically equivalence,

$\sim Q \Rightarrow \sim P \implies P \Rightarrow Q$

DATE: __/__/____
PAGE: ________

Proof of Contradiction -

$P \Rightarrow q$

In this, we assume that q is false ($\sim q$) is true) then by logical argument we arrive at situation where $\sim q$ ~~is false~~ ~~which implies that q~~

implies a contradiction. This can Happen only where $\sim q$ is false, which implies that q must be true.

Q. Proof that if n is an integer and 3n+2 is even then n is even using proof by Contradiction

Soln.

p = 3n+2 is even , q = n is even.

$\neg q$ = n is odd.

$n = 2K+1$

Put $n = 2K+1$ in $3n+2$

$$3n+2 = 3(2K+1)+2$$
$$= 6K+3+2$$
$$= 6K+5$$

if $K = 1, 2$

$6 \times 1 + 5 = 11$

$6 \times 2 + 5 = 17$ odd value.

This is Contradiction.

DATE: / /
PAGE:

Mathematical Induction -

A proof by Mathematical Induction that P(n) is true for every positive integer 'n' consist of the following two steps :-

1- Basic Step : Proposition P(1) is show given to be true
↳ basic value

Assume $1+2+3 \cdots + n = \frac{n(n+1)}{2}$ Proof Karna hai

$L.H.S = 1$, $R.H.S = \frac{1(1+1)}{2}$

$= 1$, P(1) is true.

(ii) Inductive step : We have to assume that P(n) is true then Also Proof that P(n+1) is true.

$\underline{P(n)} \longrightarrow \underline{P(n+1)}$ is true

$\downarrow$ Assume to true → Proof

let us say,

P(n) is true for n = K, $1+2+3+\cdots K = \frac{K(K+1)}{2}$

→ (i) → this is true.

Proof that P(n+1) is true for n = K+1

$L.H.S = 1+2+3+\cdots+K+K+1 \Rightarrow \frac{K(K+1)}{2} + K+1$

DATE: __/__/__
PAGE: ____

$\Rightarrow (K+1)(K+2)/2$

R.H.S $= \frac{(K+1)(K+2)}{2}$

L.H.S = R.H.S hence proof

<u>Ques</u> → Use M.I to prove that "Sum of the first n odd positive integers is n^2".

<u>Sol^n</u> - P = "Sum of the first n odd positive integers is n^2".

$1+3+5+ - - - - - (2n-1) = n^2$

(i) Basic step :-

P(n) for n=1

L.H.S = 1

R.H.S = $n^2 = 1$

L.H.S = R.H.S

(ii) Assume P(n) is true for n=K

$1+3+5+ - - \cdot (2K-1) = K^2$ —(i) [this is true]

Proof → P(n) is true for n = K+1

L.H.S $= \underbrace{1+3+5+ - - \cdot (2K-1)}_{K^2} + (2K+1) = (K+1)^2$

K^2+2K+1

$= (K+1)^2$ They are equal

∴ P(n) for K+1 is true.

Order - How to find order & degree -

The order of a recurrence relation can be calculated as the difference between the largest and the smallest subscripts of a appearing in the recurrence relation.

Ex- i) $a_r = 2a_{r-1} - a_{r-2}$

Oder = largest Subscript - smallest Subscript

= ~~$a_r - a_r$~~ $r - r + 2$

= 2

Good Write

(ii) $a_r = r\, a_{r-1} + a^2_{r-2}$

order $= r - r + 2 = 2$

$d = 2$

(iii) $a_r = r\, a_{r-1} + a_{r-2} + r^2,$

order $= r - r + 2 = 2$

degree $= 1$

Master Method

$$T(n) = a\,T\left(\frac{n}{b}\right) + f(n)$$

$\boxed{a \geq 1}$, $\boxed{b > 1}$ this type of problem solved by MM.

Ex. $T(n) = (1)\,T\left(\frac{n-1}{1}\right) + 1$

$a = 1,\ b = 1$
but $\rightarrow$ b is not > 1 so this type of problem solved by Substitution Method.

i) $T(n) = T\left(\frac{n}{2}\right) + c$

$\Rightarrow$ solution is :
$$T(n) = n^{\log_b a}\,[U(n)]$$

$\rightarrow$ $U(n)$ depends on $h(n)$
$$h(n) = \frac{f(n)}{n^{\log_b a}}$$

Relation between $h(n)$ and $U(n)$ is $\rightarrow$

if $h(n)$	$U(n)$
$n^r,\ r>0$	$O(n^r)$
$n^r,\ r\leq 0$	$O(1)$
$(\log n)^p,\ p\geq 0$	$\frac{(\log_2 n)^{p+1}}{p+1}$

ii) $T(n) = T\left(\frac{n}{2}\right) + c$

$a = 1, \quad b = 2, \quad f(n) = c$

Sol^n.

$T(n) = n^{\log_b a} \cdot U(n)$

$= n^{\log_2 1} \cdot U(n) = n^0\, U(n)$

$= U(n)$

$h(n) = \frac{f(n)}{n^{\log_b a}} = \frac{c}{n^{\log_2 1}} = \frac{c}{n^0}$

$= c$

Third case apply in this,

$\cancel{h(n) =} \; (\log_2 n)^0 \cdot c = \frac{(\log_2 n)^{0+1}}{0+1}$

$= (\log_2 n) \cdot c$

$= \log_2 n \cdot c$

$O(\log n)$

) $T(n) = 8T\left(\frac{n}{2}\right) + n^2$

$a = 8, \quad b = 2, \quad f(n) = n^2$

Sol^n.,

$T(n) = n^{\log_b a} \cdot U(n)$

$= n^{\log_2 8} \cdot U(n)$

$= n^{\log_2 2^3} \cdot U(n)$

$= n^3 \cdot U(n)$

$h(n) = \frac{f(n)}{n^{\log_b a}} = \frac{n^2}{n^3} = \frac{1}{n} = n^{-1}$

Linear RR with Constant Coefficients -

A linear RR with constant coefficient is a recurrence relation of the form

$$a_r = c_1 a_{r-1} + c_2 a_{r-2} + \dots + c_k a_{r-k} + f(r)$$

where $c_1, c_2 \dots c_k$ are real numbers and $c_k \neq 0$

if $f(r) = 0 \rightarrow$ Homogeneous

otherwise $\rightarrow$ Non-Homogeneous.

Good Write

first order RR -

$a_n = C_K a_{n-K}$ when $K=1$

$\boxed{a_n = c_1 a_{n-1}}$ or $\boxed{a_{n+1} = c_1 a_n}$

Second order $\Rightarrow K = 2$

$$\boxed{a_n = c_1 a_{n-1} + c_2 a_{n-2}}$$

Homogenous Solution -

$a_r = \alpha^r$ where α is a Costant

$a_r = c_1 a_{r-1} + c_2 a_{r-2} + \cdots + c_K a_{r-K}$

$\alpha^r = c_1\alpha^{r-1} + c_2\alpha^{r-2} + \cdots + c_K \alpha^{r-K}$

dividing by α^{r-K}

$\alpha^K = c_1\alpha^{K-1} + c_2\alpha^{K-2} + \cdots + c_K\alpha^{K-K}$

$\alpha^K = c_1\alpha^{K-1} + c_2\alpha^{K-2} + \cdots + c_K\alpha^{0}$

$\alpha^K = c_1\alpha^{K-1} + c_2\alpha^{K-2} + \cdots + c_K$

$\alpha^K - c_1\alpha^{K-1} - c_2\alpha^{K-2} - \cdots - c_K = 0$

$\downarrow$

This equation is known as characteristic equation.

α value is known as characteristic Rules.

DATE: __/__/__
PAGE:

Q- Solve the Recurrance Relation

$$a_r = 6a_{r-1} - 8a_{r-2} \quad (r-k)$$

$$\cancel{k=2}$$

Sol^n. sabKi Power 1 hai.

$d=1, \quad O=2$

$\alpha^k = c_1\alpha^{k-1} - c_2\alpha^{k-2} + \dots - c_k = 0$ chart eqn

$\alpha^2 - 6\alpha + 8 = 0 \quad - c.k$

α Ki highest power 2

$(\alpha-2)(\alpha-4) = 0$

$\alpha = 2, 4.$

Therefore,

$$a_r = c_1 2^r + c_2 4^r$$

$a_0 = 0, a_1 = 4$ (if it is given)

so put this,

$a_0 = c_1 2^0 + c_2 4^0$

$0 = c_1 + c_2 \quad \text{— (i)}$

$a_1 = c_1 2^1 + c_2 4^1$

$4 = 2c_1 + 4c_2 \quad \text{— (ii)}$

$c_1 = -2, \quad c_2 = 2$

$a_r = -2 \cdot 2^r + 2 \cdot 4^r$

$$\boxed{a_r = -2^{r+1} + 2 \cdot 4^r}$$