CO-205

DISCRETE STRUCTURES

ASSIGNMENT-1

CHESHTA KAPOOR

(2K21/CO/139)

Q1 (a) Donot cross the road when the signal is red.
It is not a proposition.

(b) What is the time now?
It is not a proposition

(c) There are no windmills in the university. (Truth value: True)
It is a proposition. It can be either true or false.

(d) $3+x=12$
It is not a proposition.

(e) The moon is made of green cheese.
It is a proposition. Its truth value is false.

(f) $5x \geq 67$
It is not a proposition.

Q2. (a) Ram and Vinod are not friends.

(b) There are not 13 items in a baker's dozen.

(c) It is not the case that Bobby sent more than 50 text messages yesterday on his mobile phone.

(d) 144 is not a perfect square.

Q3 (a) $\sim p$
I didn't buy a lottery ticket this week

(b) $p \vee q$
I bought a lottery ticket this week or won the million dollar jackpot.

(c) $p \to q$
If I bought a lottery ticket this week then I won million dollar jackpot.

(d) $p \wedge q$
I bought a lottery ticket this week and won the million dollar jackpot.

(e) $p \leftrightarrow q$ I bought a lottery ticket this week if & only if I won a million dollar jackpot.

(f) $\sim p \to \sim q$

If I didn't buy a lottery ticket this week then I didn't win the million dollar jackpot.

(g) $\sim p \wedge \sim q$

I didn't buy a lottery ticket this week and didn't win the million dollar jackpot.

(h) $\sim p \vee (p \wedge q)$

I didn't buy a lottery ticket this week or I bought the lottery ticket this week and won the million dollar jackpot.

Q4 p: You drive over 80 Km/h

q: You get a speeding ticket

(a) You don't drive over 80 km/h. ($\sim p$)

(b) You drive over 80 km/h but you dont get a speeding ticket. ($p \wedge \sim q$).

(c) You will get a speeding ticket if you drive over 80 km/h. ($p \to q$)

(d) If you donot drive over 80 km/h then you will not get a speeding ticket. ($\sim p \to \sim q$)

(e) Driving over 80 km/h is sufficient for getting a speeding ticket.
($p \to q$)

(f) You get a speeding ticket, but you donot drive over 80 km/h.
($\sim p \wedge q$)

(g) Whenever you get a speeding ticket, you are driving over 80 km/h.
($q \to p$)

Q5. (a) $\underbrace{2+2=4}_{T}$ if and only if $\underbrace{1+1=2}_{T}$.

$T \leftrightarrow T \equiv T$ $\therefore$ True

(b) $\underbrace{1+1=2}_{T}$ if and only if $\underbrace{2+3=4}_{F}$,

$T \leftrightarrow F \equiv F$ (False)

(c) $\underbrace{1+1=3}_{F}$ if and only if $\underbrace{\text{monkeys can fly}}_{F}$.

$F \leftrightarrow F \equiv T$ (true)

(d) $\underbrace{0>1}_{F}$ if & only if $\underbrace{2>1}_{T}$, $F \leftrightarrow T \equiv F$ (false)

Q6. (a) If it snows tonight then I will stay at home. $(p \to q)$

p: it snows tonight q: I will stay at home

Converse: If I will stay at home then it snows tonight.

Contrapositive: If I will not stay at home then it doesn't snow tonight.

Inverse: If it doesn't snow tonight then I will not stay at home.

(b) I go to the beach whenever it is a sunny summer day $(p \to q)$

p: It is a sunny summer day q: I go to the beach

Converse: If I go to the beach then it is a sunny summer day.

Contrapositive: If I do not go to the beach then it is not a sunny summer day.

Inverse: If it is not a sunny summer day then I donot go to the beach.

(c) When I stay up late, it is necessary that I sleep until noon.

~~converse~~ p: I sleep until noon q: I stay up late

Converse: If I stay up late then I sleep until noon.

Contrapositive: If I donot stay up late then I donot sleep until noon

Inverse: If I donot sleep until noon then I donot stay up late

Q7 (a) $p \oplus p$

p	p	$p \oplus p$
T	T	T
F	F	T

(b) $p \oplus \sim p$

p	$\sim p$	$p \oplus \sim p$
T	F	T
F	T	T

(c) $p \oplus \sim q$

p	q	$\sim q$	$p \oplus \sim q$
T	F	T	F
F	F	T	T
T	T	F	T
F	T	F	F

(d) $\sim p \oplus \sim q$

p	q	$\sim p$	$\sim q$	$\sim p \oplus \sim q$
T	F	F	T	T
F	F	T	T	F
T	T	F	F	F
F	T	T	F	T

(e) $(p \oplus q) \vee (p \oplus \sim q)$

p	q	$p \oplus q$	$\sim q$	$p \oplus \sim q$	$(p \oplus q) \vee (p \oplus \sim q)$
T	F	T	T	F	T
F	F	F	T	T	T
T	T	F	F	T	T
F	T	T	F	F	T

(f) $(p \oplus q) \wedge (p \oplus \sim q)$

p	q	$p \oplus q$	$p \oplus \sim q$	$(p \oplus q) \wedge (p \oplus \sim q)$
T	F	T	F	F
F	F	F	T	F
T	T	F	T	F
F	T	T	F	F

Q8. $(p \vee \sim q) \wedge (q \vee \sim r) \wedge (r \vee \sim p)$

Since the expressions $(p \vee \sim q)$, $(q \vee \sim r)$, $(r \vee \sim p)$ are joined by disjunction, ∴ to make entire compound statement true, these expression should be true and hence p, q, r should have same truth value for these expressions to be true.

If one or more of these expression is false, if p, q, r are different.

Hence compound statement becomes false for different p, q, r values.

Q9 (a) $x+2=3$

$1+2=3 \rightarrow$ True

then $x = 1+1 = \underline{2}$ ∴ $\underline{x=2}$

(b) $x+1=3$ $1+1=3 \rightarrow$ false

$2x+2=4$ $2+2=3 \rightarrow$ false

Since both are false, so 'then' statement won't be executed.

∴ $\underline{x=1}$ only

(c) $2x+3=5$ $2+3=5$ True

$3x+4=7$ $3+4=7$ True

Since both are true, $x = x+1 = 1+1$

$\underline{x=2}$

(d) $x+1=2$ $1+1=2 \rightarrow$ true

$x+2=3$ $1+2=3 \rightarrow$ True

Xor give false ∴ 'then' statement won't be executed. ∴ $\underline{x=1}$ (donot change

(e) $x < 2$ $1 < 2 \rightarrow$ True

then $x = x + 1 = 1 + 1 = \underline{2}$

$\underline{x = 2}$

Q10 $m \rightarrow a \vee p$

Q11 $\sim s \rightarrow (w \rightarrow d)$

Q12 (a) $r \wedge \sim p$ (b) $(p \wedge r) \rightarrow q$,   (c) $\sim r \rightarrow \sim q$ (d) $(\sim p \wedge r) \rightarrow q$

Q13. (1) Either Kelvin, or Heather, or both, are chatting $(K \vee H)$

(2) Either Randy or Vijay, but not both, are chatting $(R \oplus V)$

(3) If Abby is chatting, so is Randy. $(A \rightarrow R)$

(4) Either both Vijay and Kevin are chatting or neither is chatting. $(V \leftrightarrow K)$

(5) If Heather is chatting, then Abby & Kevin are also chatting. $(H \rightarrow A \wedge K)$

To get answer, all propositions must be true.

① $K \vee H$

K	H	$K \vee H$
T	F	T
F	T	T
T	T	T

② $R \oplus V$

R	V	$R \oplus V$
T	F	T
F	T	T

③ $A \rightarrow R$

A	R	$A \rightarrow R$
T	T	T
F	F	T
F	T	T

④ $V \leftrightarrow K$

V	K	$V \leftrightarrow K$
F	F	T
T	T	T

⑤ $H \rightarrow A \wedge K$

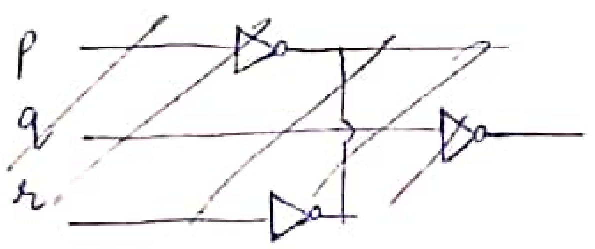
H	A	K	$A \wedge K$	$H \rightarrow (A \wedge K)$
T	T	T	T	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

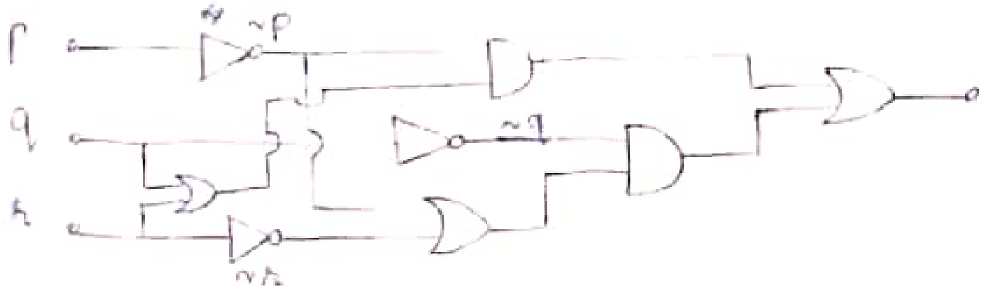
$\therefore$ for all propositions to be true,

$R \rightarrow F$ $V \rightarrow T$ $K \rightarrow T$ $A \rightarrow F$ $H \rightarrow F$

$\therefore$ only Vijay & Kevin are chatting.

Q14 $((\sim p \vee \sim r) \wedge \sim q) \vee (\sim p \wedge (q \vee r))$





5.5 (a)

p	q	$\sim p$	$p \vee q$	$\sim p \wedge (p \vee q)$	$(\sim p \wedge (p \vee q)) \to \sim q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

(b) $[(p \to q) \wedge (q \to r)] \to (p \to r)$

p	q	r	$p \to q$	$q \to r$	$(p \to q) \wedge (q \to r)$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	T
T	F	T	F	T	F	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	T	F	T	F	F	T
F	F	T	T	T	F	T
F	F	F	T	T	T	T

(c) $[p \wedge (p \to q)] \to q$

p	q	$p \to q$	$p \wedge (p \to q)$	$[p \wedge (p \to q)] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

(d) $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$

p	q	r	$p \vee q$	$p \to r$	$q \to r$	$[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$
T	T	T	T	T	T	T
T	T	F	T	F	F	T
T	F	T	T	T	T	T
T	F	F	T	F	T	T
F	T	T	T	T	T	T
F	T	F	T	T	F	T
F	F	T	F	T	T	T
F	F	F	F	T	T	T

Q16 $(\sim p \wedge (p \to q)) \to \sim p\, q$

p	q	$\sim p$	$p \to q$	$\sim p \wedge (p \to q)$	$(\sim p \wedge (p \to q)) \to \sim q$
T	T	F	T	F	T
T	F	F	F	F	T
F	T	T	T	T	F
F	F	T	T	T	T

Hence, it is not a tautology. as $P \to F$ $Q \to T$ expression becomes false.

Q17 $(\sim q \wedge (p \to q)) \to \sim p$

p	q	$p \to q$	$\sim q$	$\sim q \wedge (p \to q)$	$(\sim q \wedge (p \to q)) \to \sim p$
T	T	T	F	F	T
T	F	F	T	F	T
F	T	T	F	F	T
F	F	T	T	T	T

Hence, it is a tautology

Q18 $(p \to q) \wedge (q \to r) \to (p \to r)$

p	q	r	$p \to q$	$q \to r$	$p \to r$	$(p \to q) \wedge (q \to r) \to (p \to r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	T
T	F	T	F	T	T	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	T	F	T	F	T	T
F	F	T	T	T	T	T
F	F	F	T	T	T	T

∴ yes, it is a tautology

Q19 $(p \vee q) \wedge (\sim p \vee r) \to (q \vee r)$

p	q	r	$p \vee q$	$\sim p \vee r$	$q \vee r$	$(p \vee q) \wedge (\sim p \vee r) \to (q \vee r)$
T	T	T	T	T	T	T
T	T	F	T	F	T	T
T	F	T	T	T	T	T
T	F	F	T	F	F	T
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	F	T	T	T
F	F	F	F	T	F	T

∴ yes, it is a tautology

Q20 (a) $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

$$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$$
$$\equiv (\sim p \vee q) \wedge (\sim q \vee p)$$
$$\equiv (p \wedge q) \vee (\sim p \wedge \sim q)$$

p	q	$p \leftrightarrow q$	$p \wedge q$	$\sim p \wedge \sim q$	$(p \wedge q) \vee (\sim p \wedge \sim q)$
T	T	T	T	F	T
T	F	F	F	F	F
F	T	F	F	F	F
F	F	T	F	T	T

$\therefore p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

(b) $\sim(p \leftrightarrow q) \equiv p \leftrightarrow \sim q$

$$\sim(p \leftrightarrow q) \equiv \sim[(p \to q) \wedge (q \to p)]$$
$$\sim[(\sim p \vee q) \wedge (\sim q \vee p)]$$
$$(p \wedge \sim q) \vee (q \wedge \sim p)$$
$$\equiv p \leftrightarrow \sim q$$

p	q	$p \leftrightarrow q$	$\sim(p \leftrightarrow q)$	$\sim q$	$p \leftrightarrow \sim q$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	F	T	F	T
F	F	T	F	T	F

$\therefore \sim(p \leftrightarrow q) \equiv p \leftrightarrow \sim q$

(c) $\cancel{\sim p \leftrightarrow q \equiv p \leftrightarrow \sim q}$

$p \to q \equiv \sim q \to \sim p$

LHS: $p \to q$ RHS: $\sim q \to \sim p$

$\sim p \vee q$ $q \vee \sim p$

LHS = RHS

p	q	$p \to q$	$\sim q$	$\sim p$	$\sim q \to \sim p$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

$\therefore p \to q \equiv \sim q \to \sim p$

(d) $\sim p \leftrightarrow q \equiv p \leftrightarrow \sim q$

p	q	$\sim p$	$\sim p \leftrightarrow q$	$\sim q$	$p \leftrightarrow \sim q$
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	T	F	T
F	F	T	F	T	F

$\therefore \sim p \leftrightarrow q \equiv p \leftrightarrow \sim q$

(e) $\cancel{\sim p \leftrightarrow q}$

$\sim (p \oplus q) \equiv p \leftrightarrow q$

p	q	$p \oplus q$	$\sim (p \oplus q)$	$p \leftrightarrow q$
T	T	F	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	T	T

$\therefore \sim (p \oplus q) \equiv p \leftrightarrow q$

(f) $\sim (p \leftrightarrow q) \equiv \sim p \leftrightarrow q$

p	q	$p \leftrightarrow q$	$\sim (p \leftrightarrow q)$	$\sim p$	$\sim p \leftrightarrow q$
T	T	T	F	F	F
T	F	F	T	F	T
F	T	F	T	T	T
F	F	T	F	T	F

$\therefore \sim (p \leftrightarrow q) \equiv \sim p \leftrightarrow q$

(g) $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

p	q	r	$p \to q$	$p \to r$	$(p \to q) \wedge (p \to r)$	$q \wedge r$	$p \to (q \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	F	T	F	F	F
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	F	T
F	F	T	T	T	T	F	T
F	F	F	T	T	T	F	T

$\therefore (p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

(h) $(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$

p	q	r	$p\to r$	$q\to r$	$(p\to r)\wedge(q\to r)$	$p\vee q$	$(p\vee q)\to r$
T	T	T	T	T	T	T	T
T	T	F	F	F	F	T	F
T	F	T	T	T	T	T	T
T	F	F	F	T	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	F
F	F	T	T	T	T	F	T
F	F	F	T	T	T	F	T

$\therefore (p\vee q)\to r \equiv (p\to r)\wedge(q\to r)$

(i) $(p\to q)\vee(p\to r) \equiv \cancel{(p\wedge q)\to r}\ p\to(q\vee r)$

p	q	r	$p\to q$	$p\to r$	$(p\to q)\vee(p\to r)$	$\cancel{p\wedge q}$	$\cancel{(p\wedge q)\to r}$	$p\to(q\vee r)$
T	T	T	T	T	T	T	T	T
T	T	F	T	F	T	T	F	T
T	F	T	F	T	T	F	T	T
T	F	F	F	F	F	F	T	F
F	T	T	T	T	T	F	T	T
F	T	F	T	T	T	F	T	T
F	F	T	T	T	T	F	T	T
F	F	F	T	T	T	F	T	T

$\therefore (p\to q)\vee(p\to r) \equiv p\to(q\vee r)$

(j) $(p\to r)\vee(q\to r) \equiv (p\wedge q)\to r$

p	q	r	$p\to r$	$q\to r$	$(p\to r)\vee(q\to r)$	$p\wedge q$	$(p\wedge q)\to r$
T	T	T	T	T	T	T	T
T	T	F	F	F	F	T	F
T	F	T	T	T	T	F	T
T	F	F	F	T	T	F	T
F	T	T	T	T	T	F	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	F	T
F	F	F	T	T	T	F	T

$\therefore (p\to r)\vee(q\to r) \equiv (p\wedge q)\to r$

(k) $\sim p\to(q\to r) \equiv q\to(p\vee r)$

p	q	r	$q\to r$	$\sim p$	$\sim p\to(q\to r)$	$p\vee r$	$q\to(p\vee r)$
T	T	T	T	F	T	T	T
T	T	F	F	F	T	T	T
T	F	T	T	F	T	T	T
T	F	F	T	F	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	T	F	F	F
F	F	T	T	T	T	T	T
F	F	F	T	T	T	F	T

$\therefore \sim p\to(q\to r) \equiv q\to(p\vee r)$

(l) $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

p	q	$p \leftrightarrow q$	$p \to q$	$q \to p$	$(p\to q)\wedge(q \to p)$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

$\therefore (p \leftrightarrow q) \equiv (p \to q) \wedge (q \to p)$

(m) $p \leftrightarrow q \equiv \sim p \leftrightarrow \sim q$

p	q	$p \leftrightarrow q$	$\sim p$	$\sim q$	$\sim p \leftrightarrow \sim q$
T	T	T	F	F	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

$\therefore (p \leftrightarrow q) \equiv (\sim p \leftrightarrow \sim q)$

Q21 (a) $(p \to q) \to r$ and $p \to (q \to r)$

p	q	r	$p \to q$	$(p \to q) \to r$	$q \to r$	$p \to (q \to r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	T	T	T	T
F	T	F	T	F	F	T
F	F	T	T	T	T	T
F	F	F	T	F	T	T

$\therefore (p \to q) \to r$ is not logically equivalent to $p \to (q \to r)$

(b) $(p \wedge q) \to r$ and $(p \to r) \wedge (q \to r)$

p	q	r	$p \wedge q$	$(p \wedge q) \to r$	$p \to r$	$q \to r$	$(p \to r) \wedge (q \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	F	T	T	T	T
T	F	F	F	T	F	T	F
F	T	T	F	T	T	T	T
F	T	F	F	T	T	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

$\therefore (p \wedge q) \to r$ is not logically equivalent to $(p \to r) \wedge (q \to r)$

(c) $(p \to q) \to (r \to s)$ & $(p \to r) \to (q \to s)$

p	q	r	$p \to q$	$r \to s$
T	T	T	T	
T	T	F	T	
T	F	T	F	
T	F	F	F	
F	T	T	T	
F	T	F	T	
F	F	T	T	
F	F	F	T	

If r is true & p, q, s be false then $(p \to q) \to (r \to s)$ will be false but $(p \to r) \to (q \to s)$ will be True

Q22 (a) $(p \vee q \vee \sim r) \wedge (p \vee \sim q \vee \sim s) \wedge (p \vee \sim r \vee \sim s) \wedge (\sim p \vee \sim q \vee \sim s) \wedge (p \vee q \vee \sim s)$

$p \to$ true, $s \to$ false, $q \to$ false

then compound proposition is true. Hence, satisfiable

(b) $(\sim p \vee \sim q \vee r) \wedge (\sim p \vee q \vee \sim s) \wedge (p \vee \sim q \vee \sim s) \wedge (\sim p \vee \sim r \vee \sim s) \wedge (p \vee q \vee \sim r) \wedge (p \vee \sim r \vee \sim s)$

$p \to$ false $s \to$ false $q \to$ True

compound proposition is true. Hence, satisfiable

(c) $(p \vee q \vee r) \wedge (p \vee \sim q \vee \sim s) \wedge (q \vee \sim r \vee s) \wedge (\sim p \vee r \vee s) \wedge (\sim p \vee q \vee \sim s) \wedge (p \vee \sim q \vee \sim r) \wedge (\sim p \vee \sim q \vee s) \wedge (\sim p \vee \sim r \vee \sim s)$

$p \to$ false $q \to$ false $r \to$ true $s \to$ true

compound proposition is true. Hence, satisfiable.

Q23 (a) $\exists x N(x)$: There exist a student who has visited Nainital.

(b) $\forall x N(x)$: All students of your school have visited Nainital.

(c) $\sim \exists x N(x)$: No student has visited Nainital.

(d) $\exists x \sim N(x)$: Some students of your school have not visited Nainital.

(e) $\sim \forall x N(x) \equiv \exists x \sim N(x)$: Some students of your school have not visited Nainital.

(f) $\forall x \sim N(x)$: No student of your school has visited Nainital

Q24 (a) $\forall x(R(x) \to H(x))$ All Rabbits hop

(b) $\forall x(R(x) \wedge H(x))$ Every animal is a hopping rabbit.

(c) $\exists x\ (R(x) \to H(x))$

There exist an animal such that, if it is a rabbit then it hops

(d) $\exists x(R(x) \wedge H(x))$ Some rabbits hop.

Q25. (a) $Q(0)$ $x+1 > 2x$

$1 > 0$ true

(b) $Q(-1)$ $0 > -2$ true

(c) $Q(1)$ $2 > 2$ false

(d) $\exists x\, Q(x)$

There exists integer in the domain for which $x+1 > 2x$

$\therefore$ True

(e) $\forall x\, Q(x) \to$ false

as for $x=1$ $1+1 > 2 \to$ false

(f) $\exists x \sim Q(x) \to$ true

there exist some integers for which $x+1 \not> 2$

for eg: $x=1$

(g) $\forall x \sim Q(x) \to$ false

$x+1 > 2x$ is not false for all integers.

Q26 (a) $\exists x P(x) : P(-5) \vee P(-3) \vee P(-1) \vee P(1) \vee P(3) \vee P(5)$

(b) $\forall x\, P(x): P(-5) \wedge P(-3) \wedge P(-1) \wedge P(1) \wedge P(3) \wedge P(5)$

(c) $\forall x((x \neq 1) \to P(x)) : P(-5) \wedge P(-3) \wedge P(-1) \wedge P(3) \wedge P(5)$

(d) $\exists x\ ((x \geq 0) \wedge P(x)) : P(1) \vee P(3) \vee P(5)$

(e) $\exists x\ (\sim P(x)) \wedge \forall x\ ((x < 0) \to P(x))$

$(\sim P(-5) \vee \sim P(-3) \vee \sim P(-1) \vee \sim P(1) \vee \sim P(3) \vee \sim P(5)) \wedge (P(-5) \wedge P(-3) \wedge P(-$

Q27 $P(x)$: x is in correct place

$Q(x)$: x is in excellent condition

a) $\exists x \sim P(x)$

b) $\forall x[P(x) \wedge Q(x)]$

c) $\forall x\ [P(x) \wedge Q(x)]$

d) $\forall x \sim(P(x) \wedge Q(x))$

(e) $\exists x\ [\sim P(x) \wedge Q(x)]$

Q28. (a) $\forall x\,(x^2 \neq x)$ for $x = 1$ $x^2 = x$

(b) $\forall x\,(x^2 \neq 2)$: for $x = \pm\sqrt{2}$ $x^2 = 2$

(c) $\forall x\,(|x| > 0)$. for $x = 0$, $|x| = 0$

Q29 p : there is less than 30 MB free space on hard disk

(a) q warning message is sent to all users

$p \to q$

(b) p. directories in file system can be opened

q: files can be closed

r: system errors have been detected

$r \to (\sim p \wedge \sim q)$

(c) p: file system can be backed up

q: there is user currently logged on

$q \to \sim p$

(d) p: video on demand can be delivered

q: There is atleast 8MB of memory available

r: connection speed is atleast 256 kbps.

$(q \wedge r) \to p$

Q30 Grandfather (X,Y): [father $(X,Z) \wedge$ father(Z,Y)] $\vee$ [father $(X,Z) \wedge$ mother (Z,Y)]

Q31 (a) $\forall x\,[P(x) \to Q(x)]$

(b) $\exists x\,[R(x) \wedge \sim Q(x)]$

(c) $\exists x\,[R(x) \wedge \sim P(x)]$

(d) Yes, c follows a & b

Q32 (a) There exist a real no. x such that for every real no. y, $xy = y$.

(b) for every real no. x and real no y such that x is non-negative & y is negative, then the difference $x - y$ is positive

(c) For every real no. x, y, there exist a real number z such that $x = y + z$

Q33 (a) There exist a student in ~~our~~ your class who has taken a computer science course.

(b) There is a student in your class who has taken every comp sc course.

(c) Every student in your class has taken atleast one computer science course

(d) There exist a course taken by every student in your class

(e) Every course is taken by atleast one student from your class

(f) Every student of your class has taken every computer science course

Q34 (a) $\exists x \exists y\, Q(x,y)$

(b) $\forall x \forall y \sim Q(x,y)$

(c) $\exists x\, (Q(x, KBC) \wedge Q(x, \text{Mastermind India}))$

(d) $\forall y \exists x\, Q(x,y)$

(e) $\exists x_1 \exists x_2\, (Q(x_1, KBC) \wedge Q(x_2, KBC) \wedge x_1 \neq x_2)$

Q35 (a) $\sim I(\text{Jatin})$

(b) $\sim C\, C(\text{Ruchi}, \text{Chitra})$

(c) $\sim C\, C(\text{Jerry}, \text{Shiva})$

(d) $\sim \exists x\, C(x, \text{Bobby})$

(e) $\forall y\, [y \neq \text{Vijay} \rightarrow C(\text{Sanjay}, y)]$

(f) $\exists x \sim I(x)$

(g) $\sim \forall x\, I(x)$

(h) $\exists x\, (I(x) \wedge \forall y\, (I(y) \rightarrow y = x))$

(i) $\exists x\, (\sim I(x) \wedge \forall y\, (I(y) \rightarrow y = x))$

(j) $\forall x\, (I(x) \rightarrow \exists y\, (C(x,y) \wedge x \neq y))$

(k) $\exists x\, (I(x) \wedge \forall y \sim C(x,y))$

(l) $\exists x \exists y\, (x \neq y \wedge \sim C(x,y))$

(m) $\exists x\, \cancel{\exists z} \forall y\, C(x,y)$

(n) $\exists x \exists y\quad x \neq y \wedge \cancel{\exists}$

(o)

Q37 (a) $\forall x \exists y\, (x^2 = y)$ if $x < 0$, then $x^2 = +ve$, $\exists$ some y

True

(b) $\forall x \exists y\, (x = y^2)$ false

for $x < 0$, y^2 will make y complex.

(c) $\exists x \forall y\, (xy = 0)$

True, when $x = 0$.

(d) $\cancel{\exists x \forall y\ xy =}$ $\exists x \exists y\, (x + y \neq y + x)$

False.

(e) True If $x \neq 0$ then $xy = 1 \Rightarrow y = \frac{1}{x}$

(f) False when $x = 3$ $y = \frac{1}{3}$ $xy = 1$
but when $x = 3$ $y = 2$ $xy \neq 1$

(g) True

(h) False (No solution for given system of eq^n)

(i) False only one unique sol^n $x = 1, y = 1$

(j) True

Q38 (a) $P(1,1) \wedge P(1,2) \wedge P(1,3) \wedge P(2,1) \wedge P(2,2) \wedge P(2,3) \wedge P(3,1) \wedge P(3,2) \wedge P(3,3)$

(b) $P(1,1) \vee P(1,2) \vee P(1,3) \vee P(2,1) \vee P(2,2) \vee P(2,3) \vee P(3,1) \vee P(3,2) \vee P(3,3)$

(c) $P(1,1) \wedge P(1,2) \wedge P(1,3) \cancel{\wedge} \vee P(2,1) \wedge P(2,2) \wedge P(2,3) \vee P(3,1) \wedge P(3,2) \wedge P(3,3)$

(d) $(P(1,1) \wedge P(1,2) \wedge P(1,3)) \vee (P(2,1) \wedge P(2,2) \wedge P(2,3)) \vee (P(3,1) \wedge P(3,2) \wedge P(3,3))$

Q39 (a) $\sim \forall x \, \exists y \, \forall z \, T(x,y,z)$
$\exists x \, \forall y \, \exists z \sim T(x,y,z)$

(b) $\exists x \, \forall y \sim P(x,y) \wedge \exists x \, \forall y \sim Q(x,y)$

(c) $\exists x \, \forall y (\sim P(x,y) \wedge \forall z \sim R(x,y,z))$

(d) $\exists x \, \forall y \, [P(x,y) \wedge \sim Q(x,y)]$

Q40 (a) $\forall x \, \forall y \, (x^2 = y^2 \rightarrow x = y)$
$x = +ve$ $y = -ve$ $x = 3, y = -3$
$x = -ve$ $y = +ve$ $x^2 = 9$ $y^2 = 9$ $x \neq y$

(b) $\forall x \, \exists y \, (y^2 = x)$ x is negative eg $x = -3$

(c) $\forall x \, \forall y \, (xy \geq x)$
when $y < 0$ & $x > 0$, statement $\rightarrow$ false
eg. $y = -1$ $x = 2$ $xy = -2$ $x = 2$ $xy \not\geq x$