

Paper Code: CDE-208

Title of the subject: Algorithm Design and Analysis

Time: 3:00 Hours

Max. Marks: 50

**Note:** Answer 5 questions. Write pseudo code for all algorithms asked.  
Assume suitable missing data, if any.

Q1. (a) Solve following recurrences:

i.  $T(n) = 3T(n/2) + n$

ii.  $T(n) = 3T(n/3) + n/2$

iii.  $T(n) = 6T(n/3) + n^2 \log n$

(b) Apply Huffman coding and assign binary codes to following characters.

Character	Frequency
A	10
B	4
C	3
D	5
E	15
F	3
G	2
H	1

(6+4=10)

Q2 (a) The Longest Increasing Subsequence (LIS) problem is to find the length of the longest subsequence of a given sequence such that all elements of the subsequence are sorted in increasing order. For example, the length of LIS for {10, 22, 9, 33, 21, 50, 41, 60, 80} is 6 and LIS is {10, 22, 33, 50, 60, 80}.

arr[]	10	22	9	33	21	50	41	60	80
LIS	1	2		3		4		5	6

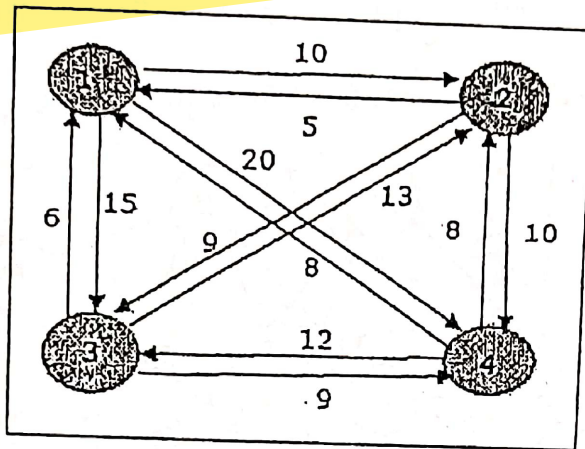
Apply Dynamic programming to solve such problem. Mention the pseudo-code, complexity, Overlapping sub problems and optimal sub structure with respect to the problem.

(b). Compute LCS for following set of sequences:  $X = \langle A, B, C, B, D, A, B, C, D \rangle$  and  $Y = \langle B, A, C, A, D, B, C, A, A, A \rangle$  using dynamic programming algorithm. Show mathematical formulation of the given problem using dynamic programming.

Q3

(5+5=10)

A traveler needs to visit all the cities from a list, where distances between all the cities are known and each city should be visited just once. What is the shortest possible route that he visits each city exactly once and returns to the origin city? Apply branch and bound paradigm to solve TSP for graph given below.



(10)

- Q4 (a) Give a divide and conquer pseudocode to find the minimum and maximum element in an array. Also form the recurrence relation and solve the recurrence relation to obtain the complexity of the pseudo code. Also apply the pseudo code to the given below array to find the minimum and maximum of the elements?

28	60	72	56	18	69	21	95	31	17	23	90	70	44	52	24
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- (b) An array of  $n$  elements contains all but one of the integers from 1 to  $n+1$ .

(i) Give the best algorithm you can for determining which number is missing if the array is sorted, and analyze its asymptotic worst-case running time.

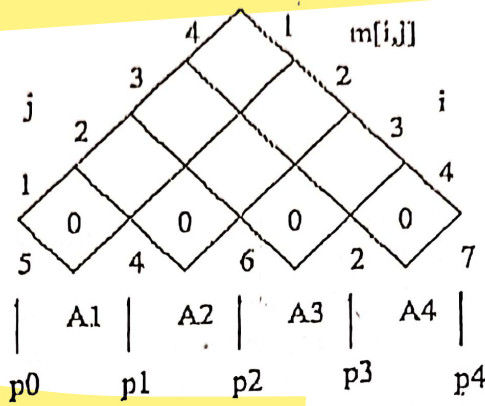
(ii). Give the best algorithm you can for determining which number is missing if the array is not sorted, and analyze its asymptotic worst-case running time.

(5+5=10)

Q5

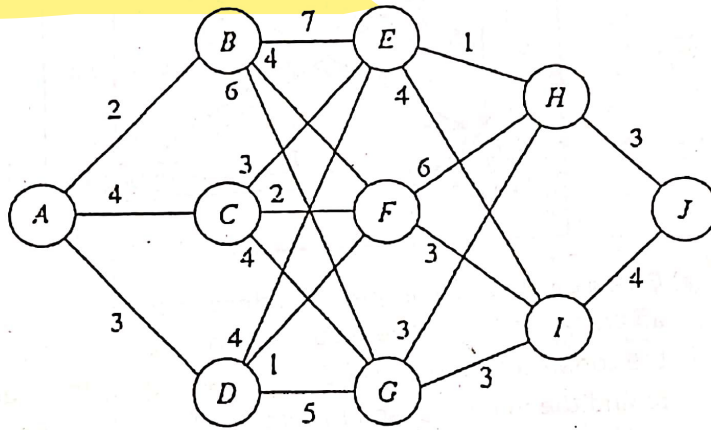
(a) For bit strings  $X = x_1 \dots x_m$ ,  $Y = y_1 \dots y_n$  and  $Z = z_1 \dots z_{m+n}$ , we say that  $Z$  is an interleaving of  $X$  and  $Y$  if it can be obtained by interleaving the bits in  $X$  and  $Y$  in a way that maintains the left-to-right order of the bits in  $X$  and  $Y$ . For example if  $X = 101$  and  $Y = 01$  then  $X_1X_2Y_1X_3Y_2 = 10011$  is an interleaving of  $X$  and  $Y$ , whereas  $11010$  is not. Give the most efficient algorithm you can to determine if  $Z$  is an interleaving of  $X$  and  $Y$ . Prove your algorithm is correct and analyze its time complexity as a function  $m$  and  $n$ .

- (b) Given a chain of four matrices  $A_1$ ,  $A_2$ ,  $A_3$ , and  $A_4$ , with dimensions  $(5 \times 4)$ ,  $(4 \times 6)$ ,  $(6 \times 2)$ , and  $(2 \times 7)$  respectively. Fill the following table in bottom-up fashion and give solution to the problem:



(5+5=10)

- Q6 (a) Apply Kruskal's algorithm to find MST for given graph:



- (b) What are NP-Complete problems. Give example of one NP-Complete problem.

(5+5=10)

- Q7 (a) Write algorithm for solving 8-queen's problem.

- (b) Apply Floyd Warshall all pair shortest path algorithm on following graph: