07-08-2019 **Discrete Structures**

negate

* Proposition logic :- Either T or F   p∨q , p∧q , ¬p , ¬q , p⊕q (XOR)

* Predicate logic :- Represents in form of functions.

input → $f(x) = x + 3$ → quantifier.

* Mathematical logic : It is a tool for working with complicated compound statements which includes :-

(i) language for expressing.
(ii) concise notation for writting.
(iii) Methadalogy for objectively reasoning about where truth or falcity (T/F).
(iv) It is the foundation for expressing formal proof in all branches of maths.

* There are two types of logic → Proposition, → Predicate.

→ Proposition :- * logic of compound statements built from simpler statements using so called → ~~boolean~~ boolean connectives.

→ Applications :- ① Design of digital electronics.
② Expressing condition in programs.
③ queries to search engine and data bases.

* A proposition (p, q, r) is simply a statement i.e. a declarative sentence with a definite meaning, having a truth value that is either true or false.

operator :- binary, unary,

operands :- 2 + 3

Boolean operators :- work on proposition

(i) Negate , ¬ , NOT , UNARY .

* eg

p = you study hard

q = you will get a good grade

p → q = If you study hard you will get good grade

(2) conjuction , ∧ , AND, Binary.

(3) Disjunction , ∨ , OR , Binary

(4) Exclusive , ⊕ , XOR, Binary.

(5) Implication , → Implies, Binary .

(6) Biconditional ↔ iff , Binary .

* (Negation)

Y	F
0F	1T
1T	0F

(AND)

X	Y	F
1	0	0
0	1	0
0	0	0
1	1	1

(OR)

X	Y	F
1	0	1
1	1	1
0	0	0
0	1	1

(XOR).

X	Y	F
0	0	0
1	1	0
1	0	1
0	1	1

Precedence

↑ NOT
AND
OR

Q. p = It rained last night

q = The sprinkles came on last night

r = The lawn was wet this morning

Solve

(1) $\neg p$ = It did not rain last night

(2) $r \wedge \neg p$ = The lawn was wet this morning and It did not rain last night

(3) $\neg r \vee p \vee q$ → Either the lawn was not wet ... or ... or ...

* where, two V then use either or :

* Either this or ___ or ___ but not both (XOR) → only one is true

* $P \to q$ means if P is true then q is true if either true or false. $p = !T$ then q would be either true or false.

x	y	$F = P \to q$	
1	1	1	F
1	0	0	F
0	1	1	T
0	0	1	T

x	y	$F = P \leftrightarrow q$	
1	0	F	0
1	1	T	1
0	1	F	0
0	0	T	1

$P \leftrightarrow q$ means P and q have same truth value

Date 09/08/2019:

some english phrases which implies $p \to q$.

① If p, then q
② p implies q
③ If p, q
④ when p, q
⑤ whenever p, q
⑥ q when p
⑦ q if p
⑧ p only if q
⑨ q is necessary for p
⑩ p is sufficient for q
⑪ q is implied by p
⑫ q follows from p.

① converse :- $q \to p$.
② Inverse :- $\neg p \to \neg q$
③ contrapopsition :- $\neg q \to \neg p$.

P	q	!p	!q	$P \to q$	$!P \to !q$
1	1	0	0	1	1
1	0	0	1	0	1
0	1	1	0	1	0
0	0	1	1	1	1

* Propocitional equivalence :- Two syntactically different compound propositions may be symantically identical i.e. have same meaning called equivalent

e.g :- $P \to q$; $\neg q \to \neg p$.

Tautology :- compound proposition i.e. true no matter what the truth values of its atomic propositions are :-

eg.. $P \vee \neg P$

contradiction :- compound proposition which is alway false.

contingency :- the other proposition except tautology and contradiction.

Logical equivalence :- compound proposition p is logically equivalent to P q if and only if $P \Leftrightarrow q$ is tautology.

- If p and q are exactly same.
- If P and q have same truth value.

Prove that $p \vee q \Leftrightarrow \neg(\neg p \wedge \neg q)$.

p	q	$p \vee q$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$	$\neg(\neg p \wedge \neg q)$
1	1	1	0	0	0	1
0	1	1	1	0	0	1
1	0	1	0	1	0	1
0	0	0	1	1	1	0

<u>Equivalence laws</u>

1. Identity :- $p \wedge T \Leftrightarrow p$, $p \vee F \Leftrightarrow p$.
2. Domination :- $p \vee T \Leftrightarrow T$, $p \wedge F \Leftrightarrow F$.
3. Idempotent :- $p \vee p \Leftrightarrow p$, $p \wedge p \Leftrightarrow p$.
4. Double negation :- $\neg\neg p \Leftrightarrow p$
5. commutative :- $p \vee q \Leftrightarrow q \vee p$, $p \wedge q \Leftrightarrow q \wedge p$.
6. Associative :- $p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r$.
   $p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$.
7. Distributive $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$.
   $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

⑧ De Morgan's law :-

$$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$$

$$\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$$

$$\boxed{p \rightarrow q \Leftrightarrow \neg p \vee q} \rightarrow (I)$$

$$\boxed{p \leftrightarrow q \Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p)} \text{ — (II)}$$

$$\boxed{p \leftrightarrow q \Leftrightarrow \neg(p \oplus q)} \text{ — (III)}.$$

Q. $p \wedge \neg q \rightarrow (p \oplus r) \Leftrightarrow \neg p \vee q \vee \neg r$.

? $\neg(p \wedge \neg q) \vee (p \oplus r) \rightarrow$ using (I)

p	r	$p \oplus r$	$\neg p$
T	T	F	F
T	F	T	F
F	T	T	T
F	F	F	T

Solution on next page

Ex ① $(x, y, z) \rightarrow x$ gave y the grade z.
$x =$ "Mike", $y =$ "Mary", grade $=$ "A".

ex ②

$P(x) =$ "$x+1 > x$", $x \in I$

"universe of discourse" – collection of values x can take in predicate.

Laws.

① Modes Ponens,
if $(p \rightarrow q)$ is T
$(p) \wedge$ is T.

② Modes Tollens.

$$\begin{array}{c} p \rightarrow q \\ \neg q \\ \hline \neg p \end{array}$$

③ Disjunctive syllogism

$$\begin{array}{c} p \vee q \text{ is T} \\ \neg p \wedge \text{ is T} \\ \hline q \end{array}$$

④ Hypothetical syllogism

$$\begin{array}{c} p \rightarrow q \\ q \rightarrow r \\ \hline p \rightarrow r \end{array}$$

Predicate logic :-

- Generalizes the grammatical notion of predicate to also include propositional functions of any no. of arguments, each of which may take any grammatical role.

universe of discourse : The collection of values that a variable can take.

Quantifiers :- Provide a notation that allows us to quantify or count how many objects in the universe of discourse satisfy a given predicate.

① universal quantifier : $(\forall)$ for all

eg. $\forall x \; P(x)$
for all x this predicate is T.

② Existential quantifiers : ($\exists$) there exists $\exists x\, P(x)$

there exists at least one x for which the predicate is true.

Variable :- $\forall x\, P(x,y)$ = x gives money to y.

Free

Bound

$\forall x\, \exists y\; P(x,y)$

Homework.

Q. $R(x,y)$ = x relies upon y domain : humans

$\forall x\, (\exists y\; R(x,y))$ = everyone has some to rely upon

$\exists y\, (\forall x\; R(x,y))$ = there is someone for everyone to rely upon.

$\forall y\, (\exists x\; R(x,y))$ = everyone is relied upon by someone.

$\forall x\, (\forall y\; R(x,y))$ = everyone relies upon everyone.

previous page one

$(\neg p \vee q) \vee \{(p \vee r) \wedge (\neg p \vee \neg r)\}$.

$(\neg p \vee q) \vee \{(p \vee r) \wedge (\neg p)) \vee ((p \vee r) \wedge (\neg r))\}$.

$(\neg p \vee q) \vee \{[(\neg p \wedge p) \vee (\neg p \wedge r)] \vee [(\neg r \wedge p) \vee (\neg r \wedge r)]\}$.

$(\neg p \vee q) \vee (\neg p \wedge r) \vee (\neg r \wedge p)$.

$[[(\neg p \wedge r) \vee (\neg r \wedge p)] \wedge [(\neg p \wedge r) \vee (p)]]$.

$[[(\neg r \vee \neg p) \wedge (\neg \cancel{q} \vee r)] \wedge [(p \vee \neg p) \wedge (p \vee r)]$

$[(\neg r \vee \neg p) \wedge (p \vee r)]$.

$\neg(p \wedge \neg q) \vee [(p \vee r) \wedge \neg(p \wedge r)]$

$(\neg p \vee q) \vee ((p \vee r) \wedge \neg(p \wedge r))$.

$(q \vee \neg p) \vee ((p \vee r) \wedge \neg(p \wedge r))$

$q \vee (\neg p \vee ((p \vee r) \wedge (\neg p \vee \neg r)))$

$q \vee (\underbrace{(\neg p \vee p \vee r)}_{T} \wedge (\neg p \vee \neg p \vee \neg r))$

$q \vee (\underbrace{\neg p \vee \neg p}_{\text{Idempotent}} \vee \neg r)$

$q \vee (\neg p \vee \neg r)$ Ans proved

Ans R : Everybody likes somebody.

$\forall x \, (\exists y \, R(x, y))$.

$\forall x \, P(x) \wedge Q(x)$

$\forall x \, (P(x) \wedge Q(x))$

$\forall x \, P(x) \wedge \exists x \, Q(x)$.

$\forall x \, \forall y \, P(x, y) \equiv \forall y \, \forall x \, P(x, y)$.

$\forall x \, (P(x) \wedge Q(x)) \equiv \forall x \, P(x) \wedge \forall x \, Q(x)$.

$\exists x \, (P(x) \vee Q(x)) \equiv \exists x \, P(x) \vee \exists x \, Q(x)$.

$H(x) = x$ is human

$M(x) = x$ is mortal

$G(x) = x$ is god.

Premises → $\forall x \, H(x) \rightarrow M(x)$ // humans are mortal

$\forall x \, G(x) \rightarrow \neg M(x)$ // Gods are immortal

Deduce → $\neg \exists x \, (H(x) \wedge G(x))$ // there exists no humans who is god.

Translate the following into english.

① $\forall x \forall y ((x>0) \land (y<0) \rightarrow (xy<0))$ Domain: real no.

→ The product of a positive real no. and negative real no. is always negative.

② The sum of two +ve integer is always positive

$\forall x \forall y ((x>0) \land (y>0) \rightarrow (x+y>0))$ Domain: integers.

$\forall x \forall y \; (x+y)>0$ Domain: positive integers

③ For every real no. x if $x!=0$, then there is a real no. y such that $xy=1$.

$\forall x [(x \neq 0) \rightarrow \exists y (xy=1)]$

④ $\forall x (C(x) \lor \exists y (C(y) \land F(x,y)))$.

$C(x)$: x has a computer

$F(x,y)$: x and y are friend.

Everyone has computer or friend with computer.

⑤ If a person is a student & is c.s major, then this person takes a course in mathematics.

$S(x)$: x is a student

$C(x)$: x is a c.s major.

$P(x,y)$: x takes course in y.

x: all people, y: course in maths

$\forall x (S(x) \land C(x) \rightarrow \exists y (P(x,y)))$.

⑥ Everyone has exactly one best friend

$B(x,y)$: y is BFF of x.

✗ $\forall x (\exists y (y=1) \rightarrow B(x,y))$

$\forall x \exists y (B(x,y) \; z (y \neq z) \rightarrow \neg B(x,y))$

Disjunctive normal form.

$P(x) \vee Q(x) \vee R(x)$

can be combination of $(p \vee q \wedge r)$.

Conjuctive normal form.

$P(x) \wedge Q(x) \wedge R(x)$

Disjunctive normal form :-

A disjunctive if fundamental conjuctions.

$(p \wedge q \wedge r) \vee (q \wedge r) \vee (p \wedge q)$.

$(p \wedge q) \vee \sim q$.

$(\sim p \wedge q) \vee (q \wedge r)$.

Que obtain disjunctive normal form for $(p \to q) \wedge (\sim p \wedge q)$

$(\sim p \vee q) \wedge (\sim p \wedge q)$

$\sim p \wedge (\sim p \wedge q) \vee q \wedge (\sim p \wedge q)$

$(\sim p \wedge q) \vee (q \wedge \sim p)$.

Que obtain the DNF for $\sim(p \to (q \wedge r))$

$\sim (\sim p) \vee (q \wedge r)$

$\sim (\sim p \vee (q \wedge r))$

$p \wedge \sim(q \wedge r)$

$\cancel{p \wedge \sim(q \wedge r)}$

$p \wedge \neg(q \wedge r)$

$p \wedge (\neg q \vee \neg r)$.

$(p \wedge \neg q) \vee (p \vee \neg r)$

conjuctive normal form.

→ A statement which consists of conjunction of fundamental disjunctions.

$p \wedge k$; $\sim p \wedge (p \vee q)$;

Ques obtain the conjuctive normal form.

$(p \wedge q) \vee (\sim p \wedge q \wedge r)$.

Solve

$p \vee (\sim p \wedge q \wedge r) \wedge (q \vee (\sim p \wedge q \wedge r))$

$((p \vee \sim p) \wedge (p \vee q) \wedge (p \vee r)) \wedge ((q \vee \sim p) \wedge (q \vee q) \wedge (q \vee r))$

$((p \vee q) \wedge (p \vee r)) \wedge ((q \vee \sim p) \wedge (q \vee r) \wedge q)$.

Ques $(\sim p \rightarrow r) \wedge (p \leftrightarrow q)$

$(p \vee r) \wedge ((p \rightarrow q) \wedge (q \rightarrow p))$.

$(p \vee r) \wedge ((\neg p \vee q) \wedge (\neg q \vee p))$

$(p \vee r) \wedge (\neg p \vee q) \wedge (\neg q \vee p)$.

Que find the DNF for :-

p	q	r	f (p, q, r)
F	F	F	T ✓
F	F	T	F
F	T	F	T ✓
F	T	T	F
T	F	F	T ✓
T	F	T	F
T	T	F	F
T	T	T	T ✓

$(\sim p \wedge \sim q \wedge \sim r) \vee (\sim p \wedge q \wedge \sim r) \vee (p \wedge \sim q \wedge \sim r) \vee (p \wedge q \wedge r)$.

Que p: Food is good.
q:- Service is good.
r:- the food restaurent is 5*.

① write the following in symbolic notations :-
ⓐ* Either food is good or service is good or both.
ⓑ* either food is good or service is good not both.
ⓒ food is good while service is poor
ⓓ It is not the case that both food is good and restaurent is 5*.
ⓔ If both the food and service are good then the rating is 5*.
ⓕ It is not true that a 5* rating always mean good food and good service.

Ans ⓐ $(p \vee q) \vee (p \wedge q)$
ⓑ $(p \vee q) \wedge \sim (p \wedge q)$
ⓒ $p \wedge \sim q$
ⓓ $(p \vee \sim r) \vee (\sim p \vee r)$ Ans :- $\sim (p \wedge r)$
ⓔ $(p \wedge q) \rightarrow r$
ⓕ $\sim (r) \rightarrow (p \wedge q))$.

find the truth value of :-
$[p \rightarrow ((q \wedge (\sim r)) \vee s] \wedge [(\sim t \leftrightarrow (s \wedge r)]$
where t is false & p, q, r, s are true.

A
B

$A = p \rightarrow A'$
$A' = (q \wedge (\sim r)) \vee s$,
$(T \wedge F \vee T)$
$F \vee T$

$A = t \rightarrow T = T$., $B = (\sim t) \leftrightarrow (s \wedge r)$
$= T \leftrightarrow T$
$= T$

* show that $(p \wedge (p \to q)) \to q$ is tautology.

$(p \wedge (\neg p \vee q)) \to q$

$\neg(p \wedge (\neg p \vee q)) \vee q$

$\neg((p \wedge \neg p) \vee (p \wedge q)) \vee q$

$\neg((p \wedge q)) \vee q$

$\neg p \vee \neg q \vee q = T$.

CNF, DNF for $(\sim p \vee q) \Leftrightarrow (p \wedge q)$.

p	q	$(\sim p \vee q)$	$p \wedge q$	F
1	1	1	1	1
1	0	0	0	1
0	1	1	0	0
0	0	1	0	0

$\to (p \wedge q) \vee (p \wedge \sim q) \to$ DNF

$\to (\sim p \vee q) \wedge (\sim p \ \sim q) \to$ CNF

$(\sim p \vee q) \to (p \wedge q) \wedge (p \wedge q) \to (\sim p \vee q)$.

* If I am not in good mood or I am not busy then I will go for movies.

solve $(\sim p \vee \sim q) \to r$.

$(\sim(p \wedge q)) \to r$.

* If you know C++ & crack them you will get a job.

$(p \wedge q) \to r$

* I will score good marks iff I study hard.

$p \Leftrightarrow q$

* Program is readable only it is well structured.

$p \Leftrightarrow q$

* unless ~~she~~ he studies he will fail in examination.

p :- he studies.

q : he passes.

$\sim p \rightarrow \sim q$.

* There will be no test exam. tom if prof. is out of town or transportation is out of service.

p :- test exam

q :- prof out

r : transportat is working.

Ans. $(q \vee \sim r) \rightarrow \sim p$.

Date. 27/08/2019.

Methods of proof.

① Direct method

→ A direct method of proof $P \Rightarrow Q$ is logical valid argument in which we start with assumption that P is true then using P as well as other axioms, shows directly Q is true.

Que Product of two odd integer is odd.

Solve

P: Product of two odd Integer.

Q: Result is odd

Let two odd integer be, $x = 2m+1$

$y = 2n+1$

where n and m are any positive integers

$$x * y = (2m+1)(2n+1) = 4mn + 2m + 2n + 1$$

$$= 2(2mn + m + n) + 1 = 2z + 1$$

where z is again a positive integers.

$$x * y = 2z + 1$$

Hence, It is proved $x * y$ is of the form $2z+1$,

so, $x * y$ is odd.

Que/ Square of an even no is an even no.

Solve P: square of even no.

Q: Result is even no.

let x be some even no.

$$x = 2n.$$

So $$x^2 = (2n)^2 = 4n^2.$$

Square of an integer is integer.

So $$x^2 = 2(2n^2),$$

$2n^2$ is some integer, let us take $a = 2n^2$

$$x^2 = 2a,$$

Que/ ~~[illegible]~~ Sum of two odd no.s is even no.

Solve. P: sum of two odd no.

Q: Result is even no,

let us take two odd no. x and y.

$$x = 2n+1$$
$$y = 2m+1$$

So sum of two odd no, $x+y$. is

$$x+y = 2n+1+2m+1$$
$$= 2n+2m+2 = 2(n+m+1)$$
$$= 2K \quad \text{where } K = n+m+1$$

which is integer

So. $x+y = 2K$ which is even no. Proved

Indirect method of proof.

① Proof by ~~contrapositia~~ contraposition.

→ It says that $P \Rightarrow Q$ is logically equivalent to its contrapositive $\bar{Q} \Rightarrow \bar{P}$. Assume that Q is false and then show that P is false.

Que. Prove that if n^2 is odd then n is odd.

Solve

P: n^2 is odd

Q: n is odd.

Now we are assuming Q is false, means n is even.

$$n = 2p$$

Then

$$n^2 = (2p)^2 = 4p^2 = 2(2p^2)$$

$n^2 = 2a$, So, n^2 is also even,

Hence, P is also false

Que/ Prove that if $x * y$ belongs to set of integers such that $x * y$ is odd, then both x and y is odd,

Solve.

P: xy is odd.

Q: x and y is odd.

Assume both x and y is even, then

$x = 2n$,

$y = 2m$

So

$xy = 2n \times 2m = 4nm = 2(2nm) = 2a$

$xy = 2a \Rightarrow xy$ is even.

So Q is false, which implies P is also false; so by method of contraposition this eq. holds.

~~negation~~

Que. If $3n+2$ is odd, then n is odd.

P : $3n+2$ is odd

Q : n is odd.

Now Assume, n is even, then:

$n = 2m$.

So

$3n+2 = 3(2m)+2 = 6m+2$

$3n+2$ is also even, hence negation Q is true which implies P is also true. So negation P is also true.

So By method of contraposition this equation holds

② Proof by contradiction.

* In this, we assume the Q is false, i.e. negation Q is true. then by logical argument we arrive at situation where negation Q implies a contradiction. this can happen only when negation Q is false. which implies that Q must be true.

Ex: let $3n+2$ is odd, then n is odd.

P: $3n+2$ is odd

Q: n is odd

Assume that Q is false, means negation Q is true, means n is even.

$n = 2a$.

$3n+2 = 3(2a)+2 = 6a+2$.

$= 2(3a+1) = 2P$.

But P says that $3n+2$ is odd. So we can say this is contradiction which means Q is true means is odd.

Que. Show that $\sqrt{2}$ is an irrational no.

Solve By contradiction, we assume that $\sqrt{2}$ is rational no.

So rational can be written in form of p/q where $q \neq 0$.

$$\sqrt{2} = \frac{x}{y}.$$

$$2 = \frac{x^2}{y^2}.$$

$$2y^2 = x^2,$$

$$x^2 = 2y^2.$$

Here, we can say that x^2 is even, so we can say x is even.

Then we can write $x = 2k$.

Then $x^2 = 4k^2$,

$$2y^2 = 4k^2 \Rightarrow y^2 = 2k^2.$$

Here, we can say that y^2 is even, so y is also even,

$$y = 2p.$$

Now both x and y are even, and have common factor. So our assumption is false, so $\sqrt{2}$ is irrational no.

Ques/ x belongs to real no. if $x^3 + 4x = 0$ then $x = 0$,

Solve. ~~Let us assume~~

p: $x^3 + 4x = 0$

q: $x = 0$.

By contradiction. let $x \neq 0$, then

$$x(x^2+4) = 0 \Rightarrow x^2 + 4 = 0.$$

$$x^2 = -4 \Rightarrow x = \pm 2i.$$

Here, the conclusion $x = \pm 2i$ contradict the fact that ~~$x^2+4x=0$~~. x is real no. as $\pm 2i$ is complex no. which make our assumption false, ~~and~~ thus q is true which means p is also true.

Proof by mathematical induction.

→ A proof by mathematical induction says that $P(n)$ is true for every positive integer n consists of following two steps.

(I) Basic steps - (II) Induction steps -

In basic steps, proposition $P(1)$ is shown to be true, or any basic value of n

In inductive steps we assume that $P(n)$ is true and prove that $P(n+1)$ is true.

For ex:- the sum of n positive integer is $\frac{n(n+1)}{2}$.

Proof:- $P(n) = \frac{n(n+1)}{2}$ $2n-1$

let $n = 1$,

then sum of integers $= 1$

$P(n) = \frac{2 \times 1}{2} = 1$.

Now $n = k$, the statement holds

$1+2+3+4+5 \ldots + k = \frac{k(k+1)}{2}$. —— (1)

Proof for $n = K+1$

$$1+2+3+4+5 \ldots K + K+1 = \frac{(K+1)(K+2)}{2}.$$

$$\frac{K(K+1)}{2} + K+1 = \frac{(K+1)(K+2)}{2}.$$

$$\frac{K(K+1) + 2K+2}{2} = \frac{(K+1)(K+2)}{2}$$

$$\frac{K^2 + 3K + 2}{2} = \frac{(K+1)(K+2)}{2}.$$

$$\frac{(K+1)(K+2)}{2} = \frac{(K+1)(K+2)}{2}.$$

Hence $L.H.S = R.H.S$

Que ① Prove that sum of the first n odd positive integers is n^2.

Que ② $1^2 + 2^2 + 3^2 \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}.$

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Solve ①:

$P(n) = n^2$

let $n = 1$.

The sum of first 1 odd positive integer = 1

$P(1) = 1$

Now:

$n = K$, The statement holds.

$1 + 3 + 5 + 7 \ldots 2K - 1 = K^2$

Then for, $n = K + 1$

$1 + 3 + 5 + 7 \ldots 2K - 1 + 2K + 1 = K^2 + 2K + 1$

$= (K+1)^2$

for $n = K + 1$,

$R.H.S = (K+1)^2$

Solve ②.

$$P(n) = \frac{n(n+1)(2n+1)}{6}$$

for $n = 1$

$$P(1) = \frac{1 \times 2 \times 3}{6} = \frac{6}{6} = 1$$

and sum of square of first n natural no. is 1

Now. for $n = K$, the statement holds

$$1^2 + 2^2 + 3^2 \ldots K^2 = \frac{K(K+1)(2K+1)}{6} \quad -①$$

for $n = K + 1$

$L.H.S = 1^2 + 2^2 + 3^2 \ldots K^2 + (K+1)^2$

using eq ①.

$$LHS = \frac{K(K+1)(2K+1)}{6} + (K+1)^2$$

$$L.H.S = (K+1)\left[\frac{2K^2+K}{6} + K+1\right]$$

$$= (K+1)\left[\frac{2K^2+7K+6}{6}\right]$$

$$= (K+1)\left[\frac{2K(K+2)+3(K+2)}{6}\right]$$

$$= \frac{(K+1)(K+2)(2K+3)}{6} = R.H.S$$

Proved.

write all the steps while writting solution for getting full marks. Like mathematical step, Induction step

Principle of complete induction.

Let $P(n)$ be the statement defined on positive integer such that.

(i) $P(m)$ is true for some $m \in N$,

(ii) whenever $P(m), P(m+1) \ldots P(K)$ are true, then $P(K+1)$ is true where $K \geq m$.

Que (1). Prove that any integer $n \geq 2$. is either a prime or a product of prime.

Solve Let $P(n)$ be the statement that n is prime or product of primes.

Step (i). $P(2) = 2$ is prime no,

$P(3) = 3$ is prime no.

$P(4) = 4$ is prime no.

$P(5) = 5$ is prime no.

Step (ii) Assume that $P(n)$ is true for $2 \leq n \leq K$. If $(K+1)$th

case (i) is itself a prime.

then $P(K+1)$ is true.

case ② If $(K+1)$ is not prime, then we can write $K+1 = UV$
then $2 \le U \le K$, $2 \le V \le K$.
called the above as $P(U)$ and $P(V)$
$P(K+1) = P(U)\,P(V)$

Que. prove that $n^3 - n$ is divisible by 3. whenever n is positive integer.

solve Basic steps :- $P(n) = n^3 - n$
let us take for $n = 1$,
$P(1) = 1 - 1 = 0$.
And ∴ 0 is divisible by 3, so this eq. holds. for $P(1)$.

$$\frac{P(1)}{3} = \frac{0}{3} = 0,$$

Induction step :- let us assume $P(n) = n^3 - n$ is divisible by 3 for $n = K$, where K is positive integer.

i.e. $$\frac{K^3 - K}{3} = \text{some constant integer},$$
$$\Rightarrow K^3 - K = 3t.$$

for $n = K+1$,

$$P(K+1) = \frac{(K+1)^3 - (K+1)}{3} = \frac{K^3 + \cancel{1} + 3K^2 + 3K - K - \cancel{1}}{3}$$

$$= \frac{K^3 - K}{3} + \frac{\cancel{3}K(K+1)}{\cancel{3}}$$

$$= \frac{K^3 - K}{3} + K^2 + K = 3t + 3r = 3(t + r),$$

which is divisible by 3.

DST

Recurrance Relation

→ If n^{th} term of sequence depend on previous terms then the relation is called ~~pre~~ recurrance relation.

→ Recurrance relation is a sequence which can be defined by giving a general formula for its n^{th} term. An alternative approach is to write its sequence by finding a relationship among its terms such relationship is called recurrance relations.

Que find recurrance relation $S = \{5, 8, 11, 14, 17 ...\}$

$$a_n = a_{n-1} + 3 . \quad \text{where or} \quad \left.\begin{matrix} \\ a_0 = 5 \end{matrix}\right\} \text{Initial condition}$$

If n^{th} term is depend on previous term then one initial condition is enough but if more than one previous term then more than one initial condition

Que find first four terms of following $a_1 = 1$, $k \geq 2$.

$a_k = 2a_{k-1} + k$

$a_2 = 2a_1 + 2 = 4$

$a_3 = 2 \times 4 + 3 = 11$

$a_4 = 2 \times 11 + 4 = 26$

$a_5 = 2 \times a_4 + 5$

$= 2 \times 26 + 5$

$= 52 + 5 = 57$

An explicit formula which satisfy the recurrance relation with initial condition is called solution to recurrance relation can be found using three method :-

1. substitution method Iteration
2. characteristic equation
3. Generating function

First order Linear recurrance relation.

$$a_{n+1} = 2a_n \quad , \quad a_0 = 1$$

Procedures

convert in Linear equation

↓

find root (r)

↓

$a_n = A(r^n)$ — find A using initial condition.

---

step ① $r^{n+1} = 2 \cdot r^n$

step ② Divide this eq. by lowest power of r.

$$\frac{r^{n+1}}{r^n} = 2\,\frac{r^n}{r^n}$$

$$r = 2$$

step ③ solution is represent by

$$a_n = A\,2^n$$

Initially, for $n=0$, $a_0 = A\,2^0$,

$$1 = A \times 1$$

$$A = 1.$$

So, the final answer is $a_n = 2^n$

Ques. $a_n = 7a_{n-1}$, where $a_2 = 98$.

Solve. first of all convert it into linear equation.

$$r^n = 7r^{n-1}$$

now divide eq. above by by dividing r^{n-1}

$$\frac{r^n}{r^{n-1}} = 7\frac{r^{n-1}}{r^{n-1}}$$

$$r = 7.$$

Now the root of eq. is $r = 7$.

Then. the answer is given as $a_n = A r^n$

$$a_n = A 7^n.$$

using initial condition as $a_2 = 98$.

for $n = 2$,

$$a_2 = A 7^2 = A \cdot 49.$$

$$a_2 = 98.$$

$$\cancel{98}^{2} = A \cdot \cancel{49} \Rightarrow A = 2$$

$$\boxed{a_n = 2 \cdot 7^n} \text{ Ans}$$

Second order linear homogeneous recurrence relation.

$B a_{n+2} + C a_{n+1} + D a_n = 0$ → this is homogeneous because it is equate to zero.

$B a_{n+2} + C a_{n+1} + D a_n = f n$ → non-homogeneous.

Now.

$B \cdot r^{n+2} + C r^{n+1} + D r^n = 0$.

Divide by lowest power of r,

$$B \frac{r^{n+2}}{r^n} + \frac{C r^{n+1}}{r^n} + \frac{D r^n}{r^n} = 0$$

$$\boxed{B r^2 + C r + D = 0}$$

Three types of roots :-

① Distinct real roots.

$$a_n = A (r_1)^n + B (r_2)^n$$

Que. $a_n = 5 a_{n-1} + 6 a_{n-2}$, $a_0 = 1$, $a_1 = 1$

$a_n - 5a_{n-1} + 6a_{n-2} = 0$

$r^2 - 5r + 6 = 0$ → this is known as characterist equation of this.

$(r-3)(r-2) = 0$ $r = 3$, $r = 2$

$$a_n = A\,3^n + B\,2^n$$

for $n=0$,

$$1 = A + B.$$

for $n=1$,

$$1 = 3A + 2B$$

$$2 = 2A + 2B$$

$$-1 = A \Rightarrow \underline{A=-1},\ \boxed{B=2}$$

$$a_n = -1\times 3^n + 2\cdot 2^n.$$

Que $\quad a_n - a_{n-1} - 6a_{n-2} = 0. \qquad a_0 = 1, \ a_1 = 8.$

$$r^2 - r - 6 = 0$$

$$(r-3)(r+2) = 0$$

$$r = 3, -2,$$

$$a_n = A\,3^n + B(-2)^n$$

$$1 = A + B. \quad , \quad 8 = 3A - 2B$$

$$2 = 2A + 2B.$$

$$5A = 10. \quad A = 2$$

$$B = -1$$

$$\boxed{a_n = 2\,3^n - 1(-2)^n}$$ Ans

Ans $a_r = 6a_{r-1} - 8a_{r-2}$ $a_0 = 0$

$a_1 = 4$

$r^2 - 6r + 8 = 0$

$(r-4)(r-2) = 0$

$r = 4, 2,$

$a_n = A \cdot 4^n + B\,2^n$

$0 = A + B,$ $4 = 4A + 2B$

$A = -B.$ $4 = -4B + 2B$

$\boxed{B = -2}$

$A = 2$

$\boxed{a_n = 2 \cdot 4^n - 2 \cdot 2^n}$. Ans.

Exam step $a_r - 6a_{r-1} + 8a_{r-2}$

$r^n - 6r^{n-1} + 8r^{n-2} = 0$ Imp

Characteris eq. $\rightarrow r^2 - 6r + 8 = 0$

Ans $t^2 - 7t + 10 = 0$

$(t-5)(t-2) = 0$ $t = 5, 2$

$a_n = A\,5^n + B\,2^n$

$5 = A + B,$ $16 = 5A + 2B$.

$10 = 2A + 2B$

$3A = 6$

$A = 2$ $\boxed{B = 3}$

$\boxed{a_n = 2 \cdot 5^n + 3 \cdot 2^n}$

Ans

(II) Repeated real roots..

$$r_1 = r_2$$

$$a_n = A(r_1)^n + B \times n (r_1)^n$$

Que

$$a_n = 6a_{n-1} - 9a_{n-2}, \quad a_0 = 1, \quad a_1 = 1$$

$$a^2 - 6a + 9 = 0$$

$$(a-3)^2 = 0 \quad a = 3.$$

$$a_n = A(3)^n + B \times n (3)^n.$$

$$1 = A$$

$$1 = 3A + 3B.$$

$$3B = -2 \Rightarrow B = -2/3$$

$$\boxed{a_n = (3)^n - \frac{2}{3} n (3)^n}.$$ ans

Que

$$a_n - 4a_{n-1} + 4a_{n-2} = 0. \quad a_0 = 1, \quad a_1 = 3$$

$$(a-2)^2 = 0 \quad a = 2$$

$$a_n = A(2)^n + B \times n (2)^n$$

$$1 = A, \quad 3 = 2A + 2B.$$

$$\frac{3-2}{2} = B \Rightarrow B = \frac{1}{2}$$

$$\boxed{a_n = 2^n + \frac{1}{2} n 2^n}$$ ans

Fibonacci sequence

0, 1, 1, 2, 3, 5, 8 ----

$a_n = a_{n-1} + a_{n-2}$ $a_0 = 0$

$a_1 = 1$

$a^2 - a - 1 = 0$.

$$a = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}.$$

$$a_n = A\left(\frac{1+\sqrt{5}}{2}\right)^n + B\left(\frac{1-\sqrt{5}}{2}\right)^n$$

$a_0 = 0$ $0 = A + B$

$$\cancel{1 = \frac{A}{2} + \frac{\sqrt{5}A}{2} + \frac{B}{2} - \sqrt{5}}$$

$$1 = A\left(\frac{1+\sqrt{5}}{2}\right) + B\left(\frac{1-\sqrt{5}}{2}\right)$$

$$1 = \cancel{\;} - \cancel{\frac{B}{2}} - \frac{B\sqrt{5}}{2} + \cancel{\frac{B}{2}} - \frac{B\sqrt{5}}{2}$$

$$\boxed{\frac{-1}{\sqrt{5}} = B} \qquad \boxed{A = \frac{1}{\sqrt{5}}}$$

$$a_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n.$$

Substitution method

$a_1 = 2$

$a_n = a_{n-1} + 3$

$a_{n-1} = a_{n-2} + 3$,

$a_n = a_{n-2} + 3 + 3$, $a_{n-2} = a_{n-3} + 3$

$a_n = a_{n-3} + 3 + 3 + 3$

repeat up to k times.

$$a_n = a_{n-k} + 3k$$

$$n-k = 1.$$

$$n = 1+k \Rightarrow k = n-1$$

$$a_n = a_1 + 3k$$

$$a_n = 2 + 3(n-1) \qquad (k = n-1).$$

Que. $H_n = H_{n-1} + (n-1). \qquad n \geq 2$

$$= 0 \,;\, n = 1$$

$$H_{n-1} = H_{n-2} + (n-2).$$

$$H_{n-2} = H_{n-3} + (n-3)$$

$$H_n = H_{n-k} + (n-k) + (n-k+1) + \dots (n-2) + (n-1)$$

Putting $n-k = 1$, $k = n-1$

$$= H_1 + (n-n+1) + (n-n+1+1) \dots (n-2) + (n\dots$$

$$= H_1 + 1 + 2 + \dots (n-2) + (n-1)$$

$$= \frac{n(n-1)}{2}.$$

Que $t_n = t_{n-1} + n. \qquad$ for $n \geq 1$

$$= 1. \qquad n = 0.$$

Sol

$$t_{n-1} = t_{n-2} + (n-1)$$

$$t_n = t_{n-2} + (n-1) + n$$

$$t_n = t_{n-k} + (n-k+1) \dots (n-k+2) \dots n$$

Put $n-k=0$. $\Rightarrow$ $\underline{n=k}$

$t_n = t_0 + 1+2+3 \ldots n$

$t_n = 1 + 1+2+3 \ldots n$

$= 1 + \frac{n(n+1)}{2} = \frac{2+n(n+1)}{2}$

$= \frac{n^2+n+2}{2}$

<u>Que</u> (i) $5, 3, 1, -1, -3$. , $a_n = 5$.

(ii) $16, 8, 4, 2, 1, 1$

(iii) $1, 3, 7, 15, 31, 63$.

<u>Solve</u> (1) $\underline{a_n = a_{n-1} - 2}$ $n \geq 1$

$a_0 = 5$

~~$r^2 - r - 2 = 0$~~

~~$(r-2)(r+1) = 0$~~

$r = 2, 1$

<u>Solve</u> (3) $a_n = 2^n - 1$

$a_n = 2a_{n-1} + 1$ $n \geq 1$

$a_0 = 1$

<u>Solve</u> (2) $a_n = 1$. for $a_n = 1$,

$= \frac{a_n}{2}$, for $\frac{a_n}{2}$ $\underline{even}$

otherwise.

Que (1) $t_n = t_{n-1} + 4n$ $n \geq 1$ using substitution

$n = 0$.

$= 0$

Que (2) $t_n = 7t_{n-1} - 10t_{n-2}$ using characteristic equation

$t_0 = 5$

$t_1 = 16$

Que consider the arithmetic sequence with first term 2 and common difference 3.

(i) Give first six terms

(ii) Define recurrance relation

(iii) compute 123^{rd} term.

Solve

(i) $\boxed{a_n = 2 + (n-1)3}$. $n \geq 1$

~~$a_0 = -1$~~

$a_1 = 2$. $a_2 = 5$ $a_4 = 11$

$a_3 = 8$

(iii) $a_{123} = 2 + 122 \times 3 = 2 + 366 = 368$.

(ii) $\boxed{a_n = a_{n-1} + 3}$ with $a_0 = 2$

$n \geq 0$.

$$t_n = t_{n-1} + 4n$$
$$t_{n-1} = t_{n-2} + 4(n-1)$$
$$t_{n-2} = t_{n-3} + 4(n-2)$$

~~topes~~ ~~t_{n-k}~~

$$t_n = t_{n-3} + 4n + 4(n-1) + 4(n-2)$$

$$t_n = t_{n-k} + 4n + 4(n-1) + 4(n-2) \dots 4(n-k+1)$$

Put, $n-k=0$,

$n=k$,

$$t_n = t_0 + 4n + 4(n-1) + 4(n-2) - 4(n-n+1)$$

$$= t_0 + 4[n + (n-1) + (n-2) \dots 1]$$

$$= t_0 + \cancel{4}^{2}\left(\frac{n(n+1)}{\cancel{2}}\right)$$

Since, $t_0 = 0$ as given in question

$$t_n = 2n(n+1)$$ Ans

Solve (8) $t_n = 7t_{n-1} - 10t_{n-2}$

$$t_n - 7t_{n-1} + 10t_{n-2} = 0$$

$$r^n - 7r^{n-1} + 10r^{n-2} = 0$$

Divide this by r^{n-2}

$r^2 - 7r + 10 = 0$, $\Rightarrow (r-5)(r-2) = 0$

$r = 5, 2$

Since this equation has real and distinct root.

So,

$$t_n = A(5)^n + B(2)^n$$

$n = 0$, $t_0 = 5$ (given).

$$t_0 = A \times 5^0 + B \times 2^0.$$

$$\boxed{5 = A + B} \text{ — (i)}.$$

$n = 1$, $t_1 = 16$. (given)

$$\boxed{16 = 5A + 2B} \text{ — (ii)}$$

Multiply eq. (i) by 2 and subtracting eqn (i) from eq (ii)

$$5A + 2B = 16$$
$$2A + 2B = 10$$
$$3A = 6$$

$$\boxed{A = 2}, \quad \boxed{B = 3}$$

So

$$\boxed{t_n = 2 \cdot (5)^n + 3 \cdot (2)^n}$$

Ans.

void test

{ if (n == 1) $\longrightarrow T(n)$

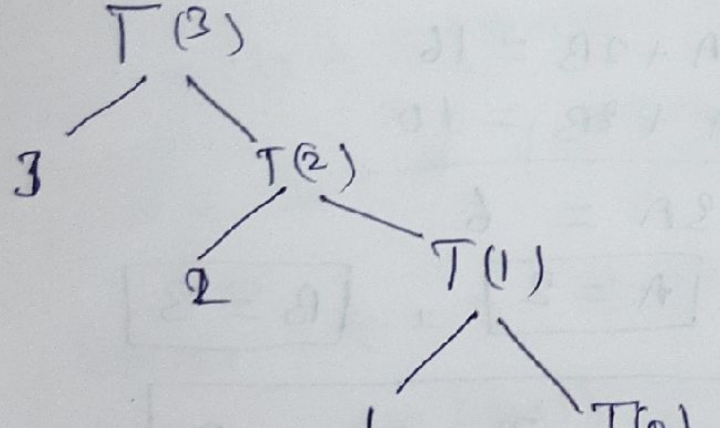
printf ("%d", n);

test (n-1); $\longrightarrow T(n-1)$

}.

$T(n) = T(n-1) + 1 \quad n > 0$

$= 1 \quad n = 0.$



for n=3, it is calling itself 4 times. and it is printing the value three times.

Now for n, it call itself n+1.

$\underline{n \rightarrow n+1}$

Now $T(n) = T(n-1) + 1$. → solve using substitution meth

$T(n) = T(n-2) + 1 + 1$

$T(n) = T(n-3) + 1 + 1 + 1$

$T(n) = T(n-k) + k$.

for, $n-k = 0$, $\Rightarrow n = k$.

$T_n = T(0) + n$

$T_n = 1 + n$ $\qquad T_n = T(n-1) + 1$.

The time complexity of this function is order of n.

Que

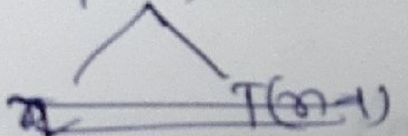
```
Void test (int a)
{ if (a>0).                          — 1
   { for (i=0; i<n; i++)             — n+1
      { printf ("%d", n);            — n
      }.
   test (n-1);
   }.
}.
```

$T(n) = T(n-1) + n + 1 + n + 1$

$= T(n-1) + 2n + 2.$

$= T(n-1) + n \qquad n > 0$.

$T(n) = 1 \qquad n = 0$



Solve by substitution method.

$T(n) = T(n-2) + n + (n-1)$

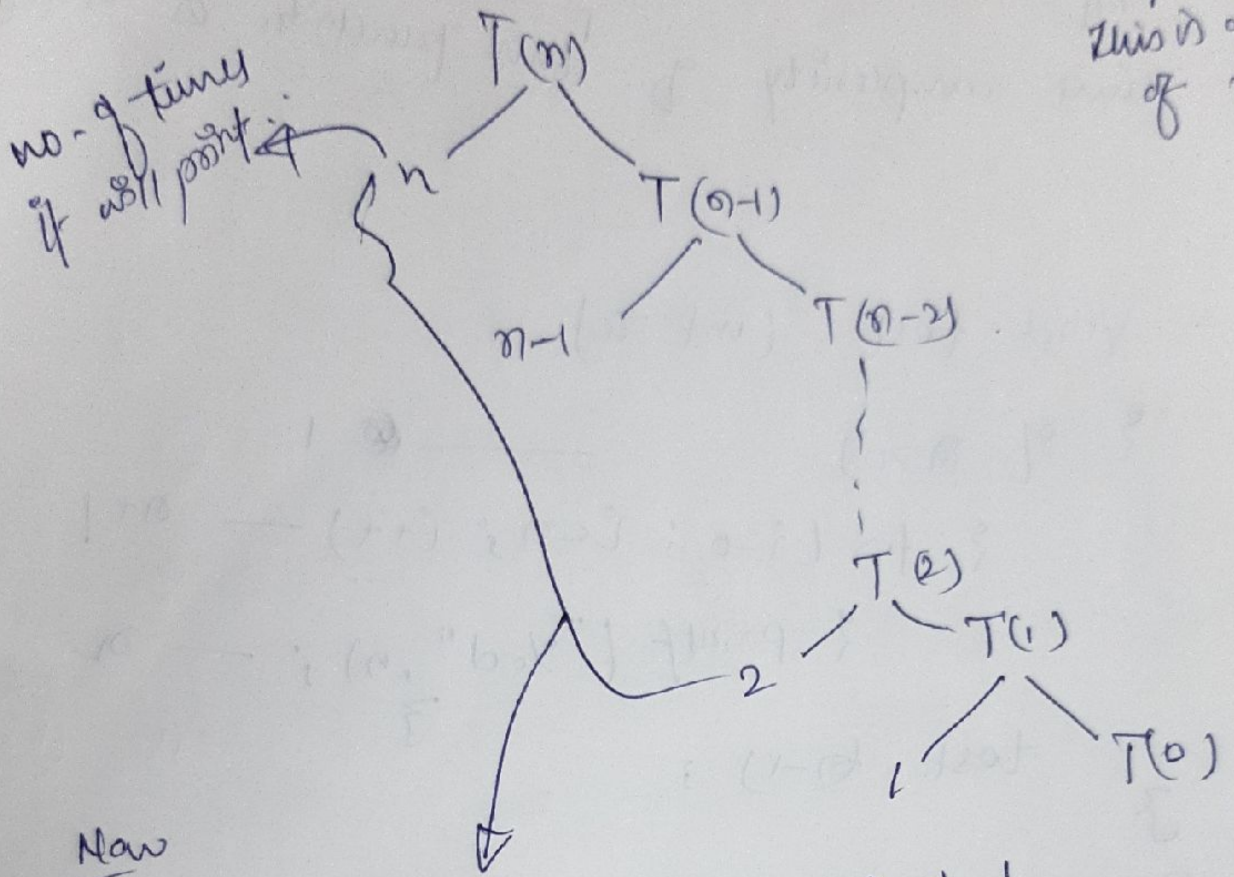
$T(n) = T(n-k) + n + (n-1) \ldots (n-k+1)$.

$n-k = 0$

$T(n) = T(0) + n + (n-1) + (n-2) \ldots 1$

$$T(n) = T(0) + \frac{n(n+1)}{2} = 1 + \frac{n(n+1)}{2}$$

This is of order of n^2



Now

$n + (n-1) + (n-2) \ldots 2 + 1 \ldots$

$= \frac{n(n+1)}{2}$ this is order of n^2 ignoring all the constant term.

* when in exam it is came that to solve recurrence relation of code, then we must solve by algorith tree as well as substitution method.

Que.

```
Void test (int n)
{ if (n>0)
  { for (i=0; i<n; i=i*2)
      { printf ("%d", n); }
    test (n-1)
  }
}
```

This loop will execute for how many times:

$2^0 + 2^1 + 2^2 \ldots 2^k = n$

$2^k = n$

$k = \log_2 n$

Quick sort :- $T(n) = \underbrace{2T(n/2)}_{\text{cost for recursively sorting two half size arrays}} + \underbrace{N+1}$

Divide and conquer sorting relation.

Partitioning cost for n items $= \underbrace{N+1}_{\text{comparison}}$

$T(1) = 1$.

Time complexity is order of $\boxed{n\log_2 n}$.

Merge sort :- $T(n) = \underbrace{2T(n/2)}_{\text{Time taken split list into two halfes}} + \underbrace{n}_{\text{Merging time}}$

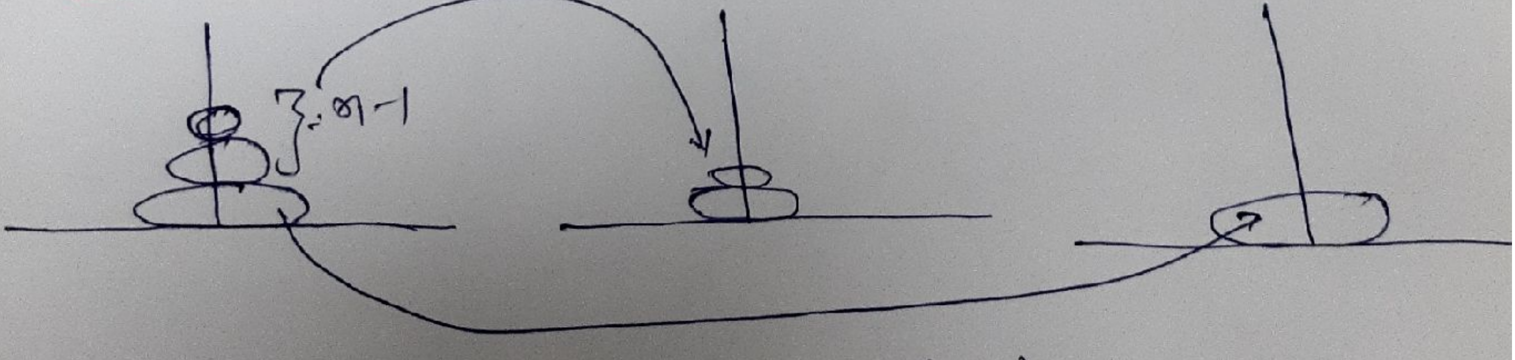
$T(1) = 1$

Time complexity is order of $(n\log_2 n)$

selection sort :- n unsorted element is given. select smallest element, then swap first element with rest element.

$$T(n) = n + T(n-1) \quad n>1$$
$$= 1 \quad n=1.$$

Tower of Hanoi



Recurrance relation $T(n) = 2T(n-1)+1 \quad n>1$

$T(1) = \quad n=1$

$T(n) = T(n-1) + \log^n \quad n>0$

$\quad = 1 \quad n=0.$

$T(n) = T(n-2) + \log^n + \log^{n-1}$

$T(n) = T(n-k) + \log^n + \log^{n-1} \cdots \log^{(n-k+1)}$

$n-k=0 \Rightarrow n=k$

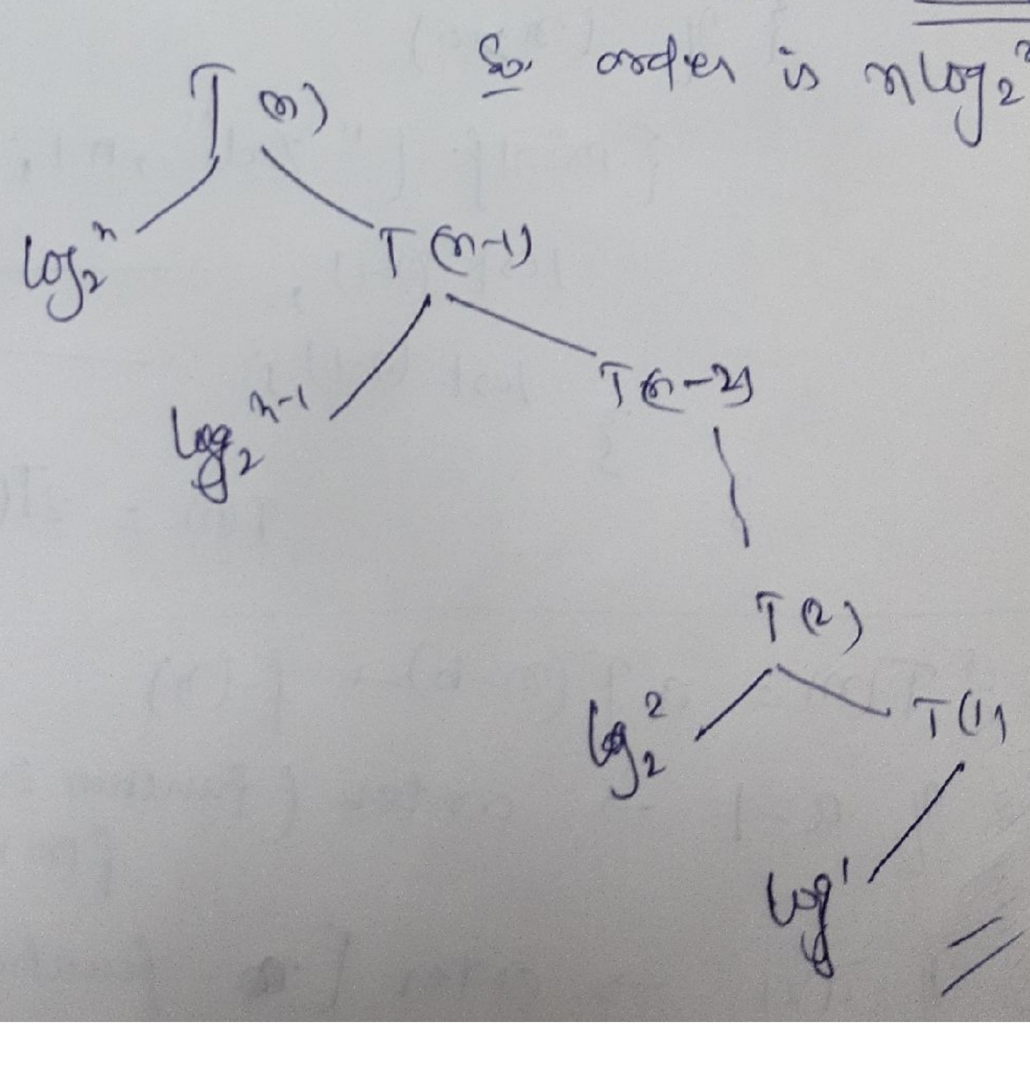
$T(n) = T_0 + \log^n + \log^{n-1} - - - \log^1$

$T(n) = 1 + \log_2^{n(n-1)(n-2)(n-3) - - - 1}$

$T(n) = 1 + \log_2^{n!}$ $\Rightarrow$ the maximum order it take is $\log_2^{n^n} = \underline{n\log_2^n}$

So order is $n\log_2^n$

Tree



① $T(n) = T(n-1) + 1 \rightarrow$ order (n) — n is multiplied with last term.
$\quad\quad\quad\quad\quad\quad\quad\quad \hookrightarrow n(n)$

② $T(n) = T(n-1) + n \rightarrow$ order (n^2)
$\quad\quad\quad\quad\quad\quad\quad\quad \hookrightarrow n(n)$

③ $T(n) = T(n-1) + \log^n \rightarrow$ order $(n \log^n)$
$\quad\quad\quad\quad\quad\quad\quad\quad n(\log^n)$

④ $T(n) = T(n-1) + n^2 \rightarrow$ order (n^3)
$\quad\quad\quad\quad\quad\quad\quad\quad n(n^2)$

⑤ $T(n) = T(n-2) + 1 \rightarrow \frac{n}{2}$ order (n)

⑥ $T(n) = T(n-100) + n \rightarrow \frac{n^2}{100}$ order (n^2).

⑦ $T(n) = 2T(n-1) + 1 \rightarrow$ order (2^n).

Que ② int (int n)

```
{ if (n>0).
    { printf ("%d", n);      — 1
      test (n-1);   ———— n-1
      test (n-1);   ———— n-1
    }
}.
```

$T(n) = 2T(n-1) + 1$

---

⑧ Master's theorem for decreasing function

$T(n) = aT(n-b) + f(n)$

If $a=1 \rightarrow$ order [function is multiplied with n $f(n) * n$]

If $a>1 \rightarrow$ order [function $(f(n)) * a^{n/b}$].

If $a<1 \rightarrow$ order [directly function $f(n)$]

Solve

(a)

$$T(n) = 2T(n-1) + 1 \qquad n > 0$$
$$= 1 \qquad n = 0.$$

$$T(n-1) = 2\,T(n-2) + 1$$
$$T(n-2) = 2\,T(n-3) + 1$$
$$T(n) = 2\,(2T(n-2) + 1) + 1$$
$$T(n) = 2^2\,T(n-2) + 2^1 + 1$$
$$T(n) = 2^2\,(2\,T(n-3) + 1) + 2^1 + 1$$
$$T(n) = 2^3\,T(n-3) + 2^2 + 2^1 + 1$$
$$T(n) = 2^k\,T(n-k) + 2^{k-1} + 2^{k-2} \ldots 2^2 + 2^1 + 2^0.$$

~~T(n) =~~ Put $n - k = 0$

$$n = k,$$

$$T(n) = 2^n\,T(0) + 2^{n-1} + 2^{n-2} \ldots 2^2 + 2^1 + 2^0$$
$$T(n) = 2^n + 2^{n-1} + 2^{n-2} \ldots 2^2 + 2^1 + 2^0.$$

This eq. is of order 2^n

Substitution method:

$$T(n) = n + 2T(n/2)$$

$$T(n/2) = n/2 + 2T(n/2^2)$$

$$T(n/2^2) = n/2^2 + 2T(n/2^3)$$

$$T(n/2) = n/2 + 2\left(2T(n/2^3) + n/2^2\right)$$

$$= \frac{n}{2} + \frac{n}{2} + 2^2 T(n/2^3).$$

$$T(n) = n + 2\left(n + 2^2 T(n/2^3)\right)$$

$$= n + 2n + 2^3 T(n/2^3).$$

$$= 3n + 2^3 T(n/2^3)$$

$n=4$

$$T(n) = kn + 2^k T(n/2^k)$$

$2\log_2 n$

$2(\log_2$

$$\frac{n}{2^k} = 1 \Rightarrow k = \log_2 n$$

$$T(n) = n\log_2 n + 2^{\log_2 n} \times 1.$$

$$T(n) = n\log_2 n + n$$

Ans

if $\frac{n}{2^k} = 1$

then $k = \log_2^n$

$$T(n) = \log_2^n + T(1)$$
$$= \log_2^n + 1 = 1 + \log_2^n.$$

Time complexity is order of $\log_2^n$.

<u>solve</u> by <u>substitution</u>.,

$$T(n) = T(n/2) + 1$$

~~$T(n-1) = T(\frac{n-1}{2}) + 1$~~

~~$T(n-2) = T(n-2/2) + 1$~~

$$T(n/2) = T(n/2^2) + 1$$
$$T(n/3) = T(n/2^3) + 1$$

$$T(n) = T(n/2^3) + 1 + 1 + 1$$

$$T(n) = T(n/2^k) + k$$

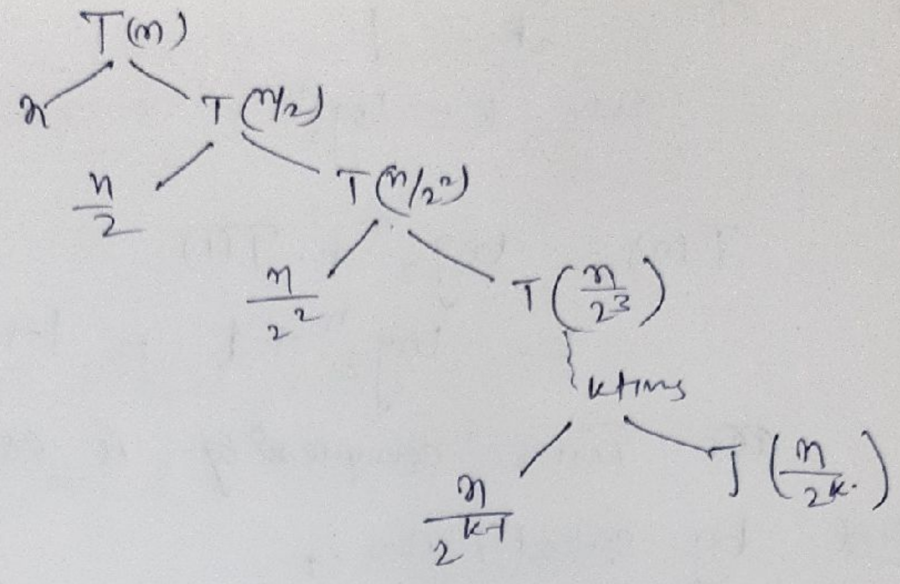
$$\frac{n}{2^k} = 1 \Rightarrow k = \log_2^n$$

$$T(n) = 1 + \log_2^n.$$

Test (int n)

```
{ if (n>1)
   { for (i=0; i<n; i++)            — n
        {printf ("%d", n); }.
     Test (n/2)                    — T(n/2)
   }.
```

Recurrance relation :- $T(n) = n + T(n/2)$



$$\frac{n}{2^k} = 1 \Rightarrow k = \log_2 n.$$

Now $T(n) = n + \frac{n}{2} + \frac{n}{2^2} + \cdots \frac{n}{2^{k-1}} + \overbrace{T\left(\frac{n}{2^k}\right)}^{1}$

$$= 1 + n\left(1 + \frac{1}{2} + \frac{1}{2^2} + \cdots \frac{1}{2^{k-1}}\right].$$

$$= 1 + n\left(\frac{1}{1-\frac{1}{2}}\right) = 1 + n(2)$$

$$= 1 + 2n$$

Now

By substitution method,

$$T(n) = n + T(n/2)$$

$$T(n/2) = n/2 + T(n/2^2)$$

$$T(n) = T(n/2^k) + n + n/2 \cdots \frac{n}{2^{k-1}}$$

put $\frac{n}{2^k} = 1 \Rightarrow k = \log_2 n$

$$T(n) = T(1) + n\left(1 + \frac{1}{2} + \cdots \frac{1}{2^{k-1}}\right]$$

$$= 1 + n\left(\frac{1}{1-\frac{1}{2}}\right) = 1 + n\left(\frac{1}{1/2}\right)$$

$$= 1 + 2n$$

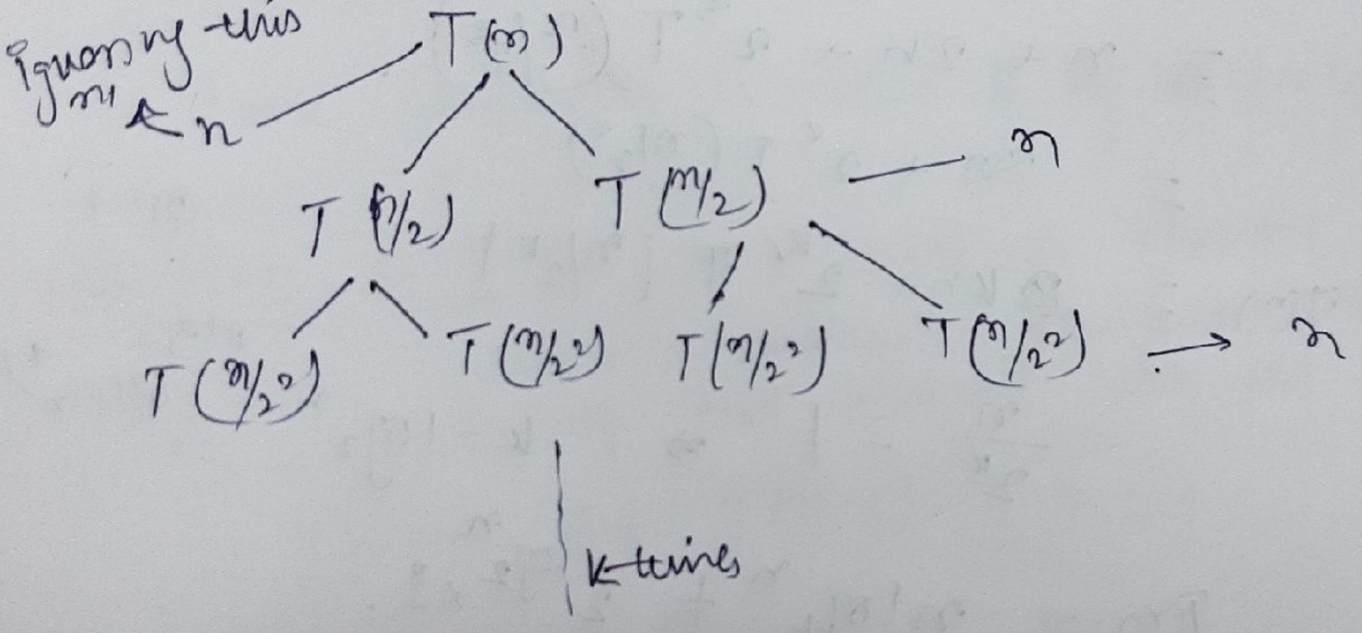
so, time complexity is of order n,

Que: Test(int n)

```
{ if (n>1)
   { for (i=0; i<n; i++)
        printf("%d", n); }
     Test(n/2);
     Test(n/2);
  }.
```

Solve

$$T(n) = n + 2T(n/2) \quad n>1$$
$$= 1 \quad n=0,$$



$$T(n/2^k) \rightarrow kn.$$

$$\frac{n}{2^k} = 1 \Rightarrow k = \log n$$

$$T(n) = kn + T(n/2^k)$$
$$= n \log_2 n + 1.$$

Date: 06-09-2018

Recurrance relation for divide and conquor relation

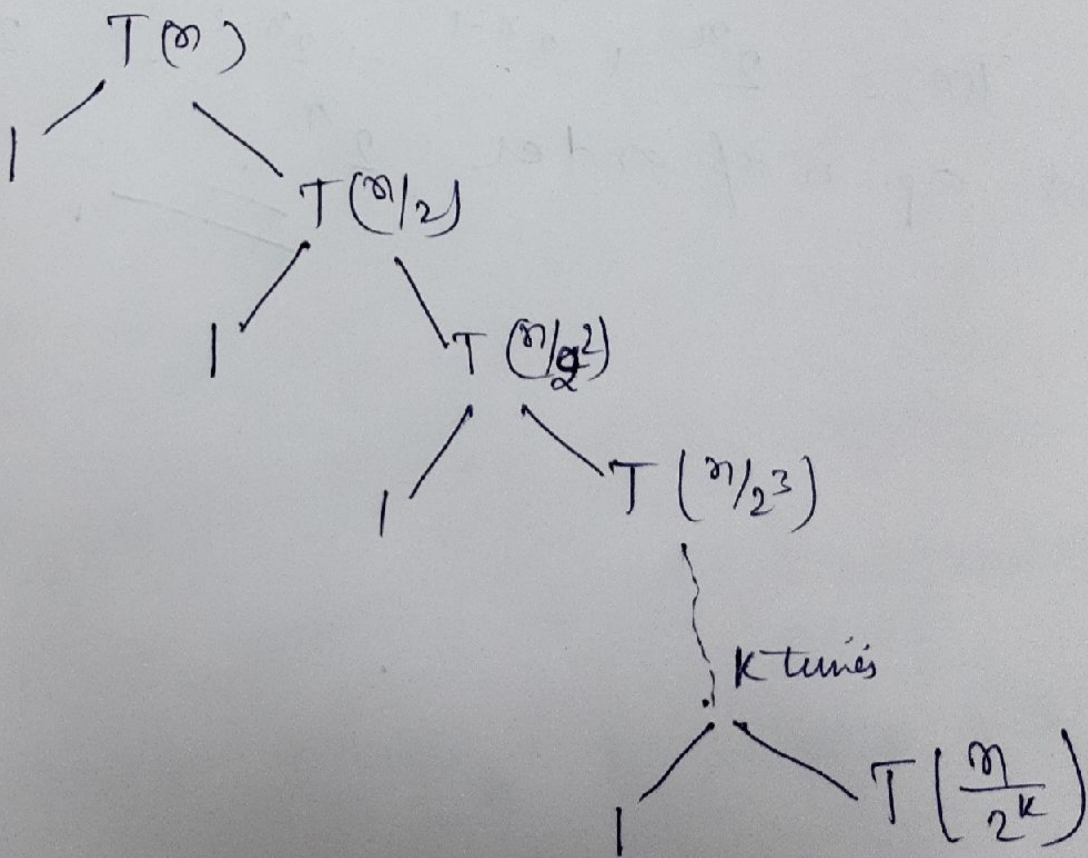
This is also same for binary search

```
Test (int n)                  —— T(n)
{ if (n>1)
    { printf ("%d", n);       —— 1
      Test (n/2);             —— T(n/2)
    }
}
```

Recurrance relation :-

$$T(n) = 1 + T(n/2) \quad n > 1$$
$$= 1 \quad n = 1$$

In exam, In substitution writting each and every step.



After k times,

$$T(n) \; T(n/2^k) + k ,$$

- The time complexity is order of 2^n in tower of Hanoi.