Introduction to Simulation

System : Process or facility is called system.
A system is defined to be a collection of entities, e.g., people or machines, that act and interact together toward the accomplishment of some logical end.

"A group of objects that are joined together in some regular interaction or interdependence toward the accomplishment of some purpose" (Banks et al)

State : Collection of variables necessary to describe a system at a particular time

The state of a system is the variables (and their values) at one instance in time

System Components

- Entities
  - Objects of interest within a system
    - Typically "active" in some way
    - Ex: Customers, Employees, Devices, Machines, etc
  - Contain attributes to store information about them
    - Ex: For Customer: items purchased, total bill
  - May perform activities while in the system
    - Ex: For Customer: shopping, paying bill

Category of System : Discrete and Continuous

A *discrete system is one for which the state variables change instantaneously* at separated points in time

Example1: A bank is an example of a discrete system , since state variables--e.g., the number of customers in the bank change only when a customer arrives or when a customer finishes being served and departs.

Example2: Number of students in MSIT When a registration or add is completed, number of students increases, and when a drop is completed, number of students decreases

A *continuous system is one for which the state variables change continuously with respect to time.*

Example1: An airplane moving through the air is an example of a continuous system, since state variables such as position and velocity can change continuously with respect to time

Example2: Volume of CO_2 in the atmosphere CO_2 is being generated via people (breathing), industries and natural events and is being consumed by plants

How to Study the System

Experiment with the Actual System vs . Experiment with a Model of the System:
If it is possible (and cost-effective) to alter the system physically and then let it operate under the new conditions, it is probably desirable to do so, for in this case there is no question about whether what we study is

System Model: A representation of the system to be used / studied in place of the actual system Allows us to study a system without actually building it. The assumptions that is used to study a system scientifically about how it works. These assumptions are called Models and can be in the form of mathematical or logical relationships

Physical Model vs. Mathematical Model :

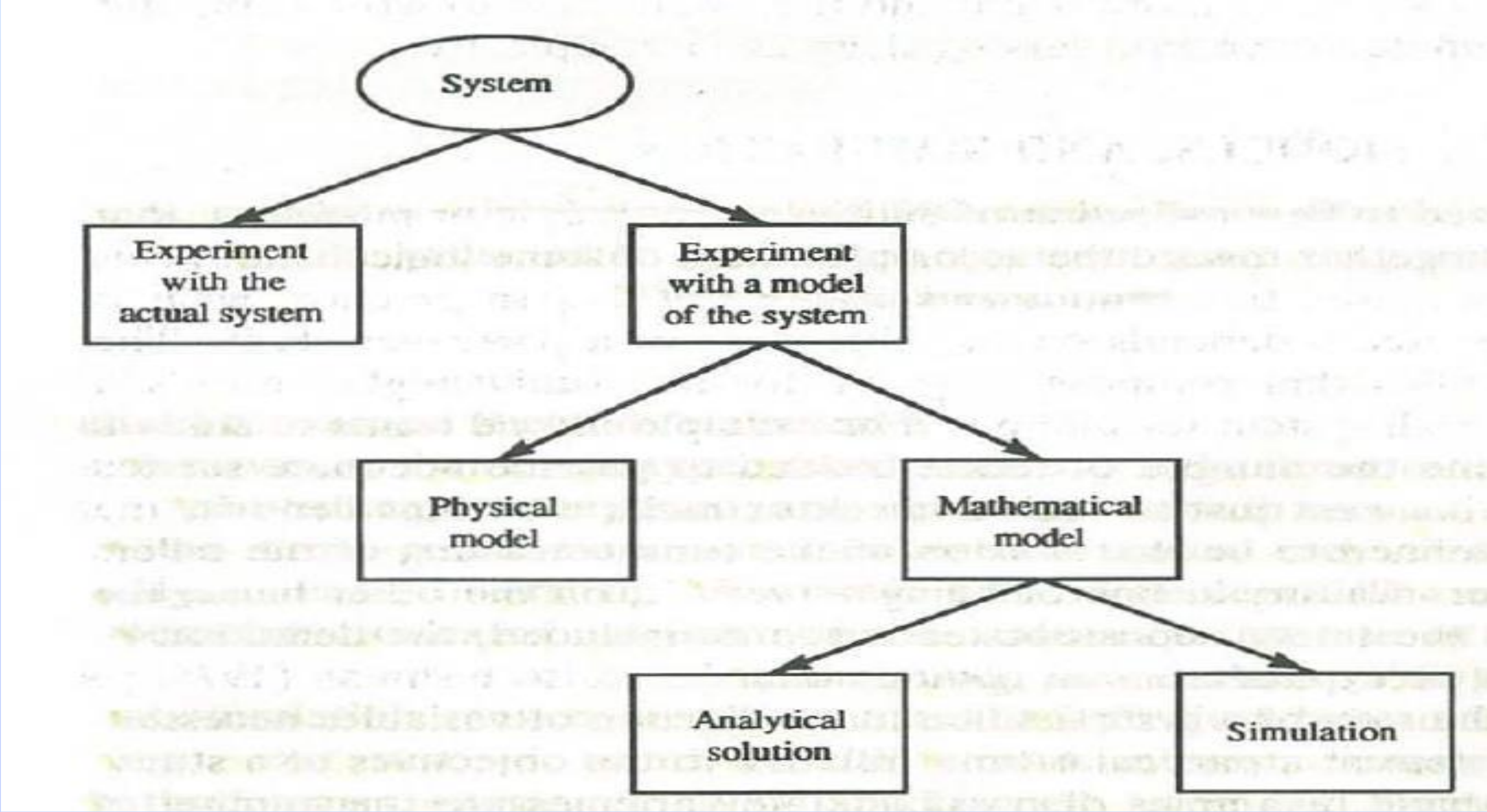
Physical Model: A physical representation of the system (often scaled down) that is actually constructed

images of clay cars in wind tunnels, cockpits disconnected from their airplanes to be used in pilot training, miniature supertankers.
Most real-world systems are too complex to allow realistic models to be evaluated analytically

Mathematical Model: Representing the system using logical and mathematical relationships

Algebra, Calculus, or probability theory

For Example: relation $d = rt$, *where r is the rate of travel*, *t is the time spent traveling, and d is the distance traveled.*



Analytical Solution

After Building a mathematical model examined the model to answer the question of interest about the system it is supposed represent *analytical solution.*

In the d = rt example, if we know the distance to be traveled and the velocity, then we can work with the model to get $t = dlr$

Many systems are highly complex, so that valid mathematical models of them are themselves complex and possibility of an analytical solution is difficult.

In this case, the model must be studied by means of *simulation, i.e.* , *numerically exercising the model for the inputs* in question to see how *they affect the output measures of performance*

Simulation: using computers to imitate, or simulate, the system.

- What is simulation?
  - Banks, et al:
    - "A simulation is the imitation of the operation of a real-world process or system over time". It "involves the generation of an artificial history of a system, and the observation of that artificial history to draw inferences … "
  - Law & Kelton:
    - "In a simulation we use a computer to evaluate a model (of a system) numerically, and data are gathered in order to estimate the desired true characteristics of the model"
  - More specifically (but still superficially)
    - We develop a **model** of some real-world system that (we hope) represents the essential characteristics of that system
      - Does not need to exactly represent the system – just the relevant parts
    - We use a **program** (usually) to test / analyze that model
      - Carefully choosing input and output
    - We use the **results** of the program to make some deductions about the real-world system

- **Why (or when) do we use simulation?**
  - Consider arbitrary large system X
    - Could be a computer system, a highway, a factory, a space probe, etc.
  - We'd like to evaluate X under different conditions
    - Option 1: Build system X and generate the conditions, then examine the results
      - This is not always feasible for many reasons:
        - X may be difficult to build
        - X may be expensive to build
        - We may not want to build X unless it is "worthwhile"
        - The conditions that we are testing may be difficult or expensive to generate for the real system

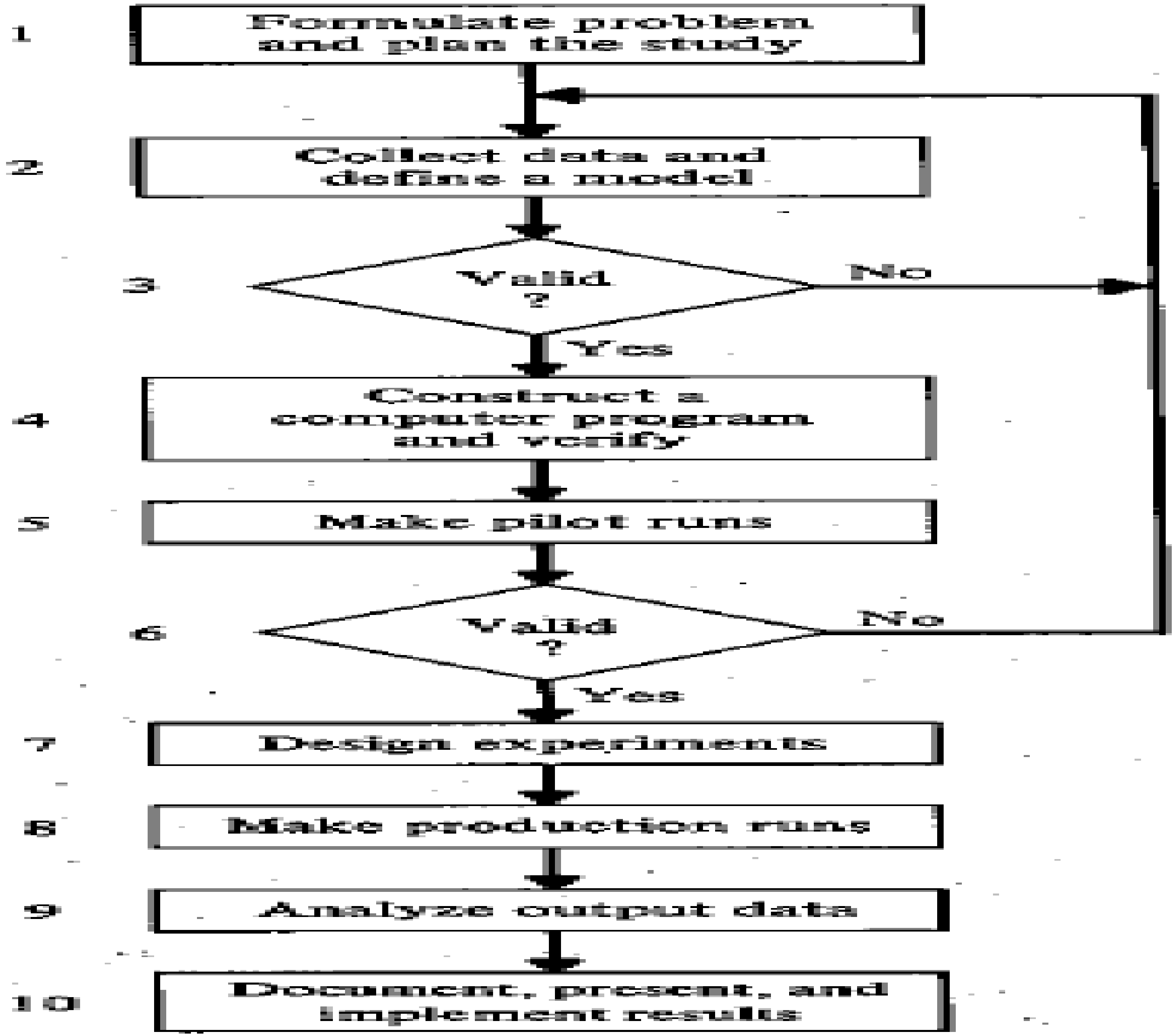
- **Why (or when) do we use simulation?**
  - For example:
    - A company needs to increase its production and needs to decide whether it should build a new plant or it should try to increase production in the plants it already has
      - > Which option is more cost-effective for the company?
    - Clearly, building the new plant would be very expensive and would not be desirable to do unless it is the more cost-effective solution
    - But how can we know this unless we have built the new plant?
  - Another (ongoing) example:
    - NASA wants to know if damage on the Space Shuttle will threaten it upon re-entry
    - If they wait until re-entry to make a judgment, it is already too late
    - In this case it is not feasible to do the real-world test

- **Why (or when) do we use simulation?**
    - Option 2: Model system X, simulate the conditions and use the simulation results to decide
        - Continuing with the same first example:
        - Model both possibilities for increasing production and simulate them both
            - > We then choose the solution that is most economically feasible
        - Continuing with the Space Shuttle example
        - Model the damage and the stress that re-entry imparts on the shuttle
            - > Determine via a simulation if the damage will threaten the shuttle or not

Application Areas of Simulation

- Designing and analyzing manufacturing systems
- Evaluating hardware and software requirements for a computer system
- Evaluating a new military weapons system or tactic
- Determining ordering policies for an inventory system
- Designing communications systems and message protocols for them
- Designing and operating transportation facilities such as freeways, airports, subways, or ports
- Evaluating designs for service organizations such as hospitals, post offices, or fast -food restaurants
- Analyzing financial or economic systems

- STEPS IN A SIMULATION STUDY
- *Formulate problem and plan the study*
- *Collect data and define a model*



1
Formulate problem and plan the study
2
Collect data and define a model
3
Valid ?
No
Yes
4
Construct a computer program and verify
5
Make pilot runs
6
Valid ?
No
Yes
7
Design experiments
8
Make production runs
9
Analyze output data
10
Document, present, and implement results

Issues in Simulation

- Writing computer programs to execute them can be an difficult task.
- simulation of complex systems is that a large amount of computer time is often required.
- Simulation *methodology* is largely independent of the software and hardware required

- Don't use a simulation if the system is too complex to model correctly / accurately
  - This is often not obvious
  - Can depend on cost and alternatives as well
    - > However, a bad model may not be helpful and could actually be harmful
    - > Ex: With the Space Shuttle, lives were at risk – if the model predicts incorrectly the results are catastrophic

- When is simulation NOT a good idea?
  - Don't use a simulation when the problem can be solved in a "simpler" or more exact way
    - Some things that we think may have to be simulated can be solved analytically
    - Ex: Given N rolls of a fair pair of dice, what are the relative expected frequencies of each of the possible values {2, 3, 4, … 12} ?
      - We could certainly simulate this, "rolling" the dice N times and counting
      - However, based on the probability of each possible result, we can derive a more exact answer analytically

- Don't use a simulation if it is easier or cheaper to experiment directly on a real system
  - Ex: A 24 hour supermarket manager wants to know how to best handle the cash register during the "midnight shift":
    - > Have one cashier at all times
    - > Have two cashiers at all times
    - > Have one cashier at all times, and a second cashier available (but only working as cashier if the line gets too long)
  - Each of these can be done during operating hours
    - > An extra employee can be used to keep track of queue data (and would not be too expensive)
    - > Differences are (likely) not that drastic so that customers will be alienated

Type of Simulation Model

- Static Model
  - Models a system at a single point in time, rather than over a period of time
  - Sometimes called Monte Carlo simulations
- **Dynamic Model**
  - Models a system over time
- Deterministic Model
  - Inputs to the simulation are known values
    - No random variables are used
    - Ex: Customer arrivals to a store are monitored over a period of days and the arrival times are used as input to the simulation
- **Stochastic Model**
  - One or more random variables are used in the simulation
    - Results can only be interpreted as estimates (or educated guesses) of the true behavior of the system
    - Quality of the simulation depends heavily on the correctness of the random data distribution

- *Static vs. Dynamic Simulation Models: A static simulation model is a* representation of a system at a particular time, or one that may be used to represent a system in which time simply plays no role; examples of static

On the other hand,

A *dynamic simulation model represents a system as it evolves over* time

Deterministic vs. Stochastic Simulation Models: If a simulation model does not contain any probabilistic (i .e., random) components, it is called *deterministic;*

A complicated (and analytically intractable) system of differential equations describing a chemical reaction might be such a model. In deterministic models, the output is "determined" once the set of input quantities and relationships in the model have been specified, even though it might take a lot of computer time to evaluate what it is. Many systems, however, must be modeled as having at least some random input components, and these give rise to *stochastic simulation models.* simulation models produce output that is itself random, and must Therefore be treated as only an estimate of the true characteristics of the model

Simulating a Single-Server Queue

Random Nos. for inter-arrival time are: 913, 727, 015, 948, 309, 922, 753, 235, 302.

Table 4.1 Arrival Time

Arrival Time in Mins (1)	Probability (2)	Cumulative Probability (3)	Range for Random numbers (4)
1	0.125	0.125	0 – 125
2	0.125	0.250	126 – 250
3	0.125	0.375	251 – 375
4	0.125	0.500	376 – 500
5	0.125	0.625	501 – 625
6	0.125	0.750	626 – 750
7	0.125	0.875	751 – 875
8	0.125	1.000	876 - 000

Col: 2 and 3

Table 4.3 Simulation Table

Customer (1)	Inter-arrival time (2)	Arrival time in clock (3)	Service time (4)	Time in clock when service begins (5)	Waiting time in Queue (6)	Time service ends (7)	Time spent by customer in system (8)	Idle time of Server (9)
1	0	0	4	0	0	4	4	-
2	8	8	1	8	0	9	1	4
3	6	14	4	14	0	18	4	5
4	1	15	3	18	3	21	6	0
5	8	23	2	23	0	25	2	2
6	3	26	4	26	0	30	4	1
7	8	34	5	34	0	39	5	4
8	7	41	4	41	0	45	4	2
9	2	43	5	45	2	50	7	0
10	3	46	3	50	4	53	7	0
Total	**46**		**35**		**9**		**44**	**18**

Random Nos. for service time are: 84, 10, 74, 53, 17, 79, 91, 67, 89, 38.

Table 4.2 Service Time

Service Time in Mins (1)	Probability (2)	Cumulative Probability (3)	Range for Random numbers (4)
1	0.10	0.10	0 – 10
2	0.20	0.30	11 – 30
3	0.30	0.60	31 – 60
4	0.25	0.85	61 – 85
5	0.10	0.95	86 – 95
6	0.05	1.00	96 – 00

Col: 4 and 5

Table 4.3 Simulation Table

Customer (1)	Inter-arrival time (2)	Arrival time in clock (3)	Service time (4)	Time in clock when service begins (5)	Waiting time in Queue (6)	Time service ends (7)	Time spent by customer in system (8)	Idle time of Server (9)
1	0	0	4	0	0	4	4	-
2	8	8	1	8	0	9	1	4
3	6	14	4	14	0	18	4	5
4	1	15	3	18	3	21	6	0
5	8	23	2	23	0	25	2	2
6	3	26	4	26	0	30	4	1
7	8	34	5	34	0	39	5	4
8	7	41	4	41	0	45	4	2
9	2	43	5	45	2	50	7	0
10	3	46	3	50	4	53	7	0
Total	46		35		9		44	18

Table 4.3 Simulation Table

Customer (1)	Inter-arrival time (2)	Arrival time in clock (3)	Service time (4)	Time in clock when service begins (5)	Waiting time in Queue (6)	Time service ends (7)	Time spent by customer in system (8)	Idle time of Server (9)
1	0	0	4	0	0	4	4	-
2	8	8	1	8	0	9	1	4
3	6	14	4	14	0	18	4	5
4	1	15	3	18	3	21	6	0
5	8	23	2	23	0	25	2	2
6	3	26	4	26	0	30	4	1
7	8	34	5	34	0	39	5	4
8	7	41	4	41	0	45	4	2
9	2	43	5	45	2	50	7	0
10	3	46	3	50	4	53	7	0
Total	**46**		**35**		**9**		**44**	**18**

Step(5) The time in the clock when the service begins is treated as 0 for the first customer and for all other (Col 5 in Table 4.3) customers (i.e. from Row2 onwards), it is calculated as–

Max{ sum of Col 4 and Col 5 values in previous row, value in Col 3 of current row}

That is, for second row, it would be –

Max{0+4,8} = 8

Col: 5 to 9

Step(6) Waiting time in a queue is : Col 6 = Col 5 – Col 3

Time service ends is: Col 7 = Col 4 + Col 5

Time spent by customer in system is: Col 8 = Col 7 – Col 3

Idle time of Server (Col 9) for 1st customer is not considered. From second customer onwards, it is –

Col 9 = value of Col 5 in current row – value of Col 7 in previous row

- **Average waiting time =**
  Total time spent by customers in queue / total number of customers
  = 9/10 = 0.9 minutes
- **The probability that a customer has to wait in the queue =**
  Number of customers who wait / total number customers
  = 3/10 = 0.3
- **Probability of idle server = total idle time of server / total run time of simulation**
  = 18/53 = 0.3396 (34% aprox)

  That is, server is busy for about 66% of the time.
- **The average service time = total service time / total number of customers**
  = 35/10 = 3.5 minutes

  This result is out of simulation. The expected service time can be computed by multiplying Col 1 and Col 2 in each row of Table 4.2 and then adding all those values. In this example, it would be –

  1*(0.1) + 2*(0.2) + 3*(0.3) + 4*(0.25) + 5*(0.1) + 6*(0.05) = 3.5 minutes
- **The average inter-arrival time =**
  sum of all inter-arrival times / number of arrivals – 1
  = 46 / 9 = 5.11 minutes
- **The average waiting time of those who wait =**
  Total time spent by customers in queue / total number of customers who wait
  = 9/3 = 3
- **The average time spent by customer in a system =**
  total time spent by customers in the system / total number of customers
  = 44/10 = 4.4 minutes

  This can be also calculated as –

  Average waiting time + Average service time = 0.9 + 3.5 = 4.4 minutes

Random Sequence Generator using Linear congruential Method

This Method is defined by given Recurrence Relation

$$X_{n+1} = (aX_n + c) \bmod m$$

where X is the sequence of pseudo-random values, and

$m,\ 0 < m$ — the "modulus"

$a,\ 0 < a < m$ — the "multiplier"

$c,\ 0 \le c < m$ — the "increment"

$X_0,\ 0 \le X_0 < m$ — the "seed" or "start value"

Random Sequence Generator using Linear congruential Method

$$X_{n+1} = (aX_n + c) \bmod m$$

X_0=27, a=17, c=43 and m=100

$X_0 = 27$

$X_1 = (a. X_0 + c) \bmod m$

$= (17 . 27 + 43) \bmod 100$

$= 2$

Now, $R_1 = \frac{2}{100} = 0.02$

$X_2 = (a. X_1 + c) \bmod m$

$= (17 . 2 + 43) \bmod 100$

$= 77$

$R_2 = \frac{77}{100} = 0.77$

$X_3 = (17 . 77 + 43) \bmod 100$

$= 52$

$R_3 = 0.52$

Continuing in the above manner, we can generate as many random numbers as required.

*****