Stressen's Matrix Multiplication

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \times B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$

$m \times n$

$2 \times \boxed{2 \quad\quad 2} \times 2 \qquad\qquad 2 \times 2$

$$C_{ij} = \sum_{k=1}^{n} A_{ik} * B_{kj}$$

Algorithm

```
for (i=0; i<n; i++)
{   for (j=0; j<n; j++)
    {   C[i,j] = 0
        for (k=0; k<n; k++)
        {   C[i,j] = A[i,k] * B[k,j];
        }
    }
}
```

$O(n^3)$

Divide & Conquer Strategy

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \times B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = C \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$

$C_{11} = a_{11} * b_{11} + a_{12} * b_{21}$

$C_{12} = a_{11} * b_{12} + a_{12} * b_{22}$

$C_{21} = a_{21} * b_{11} + a_{22} * b_{21}$

$C_{22} = a_{22} * b_{21} + a_{22} * b_{22}$

If the matrices are in form 2x2 then we can multifdy directly by using above formula.

Each Statement taking one unit time.

=> total time is four unit of time.

i.e Constant.

=> This defined a small problem

Eg: $A = [a_{11}]$ $B = [b_{11}]$

$C = [a_{11} * b_{11}]$

=> If the dimension is ≤ 2 then T.C is Constant.

Note: We assuming that the matrixes are having dimensions in power of two only (Divide & Conquer). If not in power of two dimension then fill matrix with zero & make it Power of two matrix.

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$$
\begin{array}{ccc}
 & A_{11} & A_{12} \\
A = & \left[\begin{array}{cc|cc}
a_{11} & a_{12} & a_{13} & a_{14} \\
a_{21} & a_{22} & a_{23} & a_{24} \\
\hline
a_{31} & a_{32} & a_{33} & a_{34} \\
a_{41} & a_{42} & a_{43} & a_{44}
\end{array}\right] & \\
 & A_{21} & A_{22} \quad \frac{4}{2} \times \frac{4}{2}
\end{array}
\qquad
\begin{array}{ccc}
 & B_{11} & B_{12} \\
B = & \left[\begin{array}{cc|cc}
b_{11} & b_{12} & b_{13} & b_{14} \\
b_{21} & b_{22} & b_{23} & b_{24} \\
\hline
b_{31} & b_{32} & b_{33} & b_{34} \\
b_{41} & b_{42} & b_{43} & b_{44}
\end{array}\right] & \\
 & B_{21} & B_{22} \quad \frac{4}{2} \times \frac{4}{2}
\end{array}
$$

```
Algorithm MM (A, B, n)
{   if (n ≤ 2)
    {    C = 4 formulas
    }
    else
    {    mid = n/2
         MM(A11, B11, n/2) + MM(A12, B21, n/2)
         MM(A11, B12, n/2) + MM(A12, B22, n/2)
         MM(A21, B11, n/2) + MM(A22, B21, n/2)
         MM(A21, B12, n/2) + MM(A22, B22, n/2)
    }
}
```

$C_{11} = A_{11} * B_{11} + A_{12} * B_{21}$

$C_{12} = A_{11} * B_{12} + A_{12} * B_{22}$

$C_{21} = A_{21} * B_{11} + A_{22} * B_{21}$

$C_{22} = A_{21} * B_{12} + A_{22} * B_{22}$

3)

$$T(n) = \begin{cases} 1 & n \leq 2 \\ 8T(n/2) + n^2 & n > 2 \end{cases}$$

Strassen's Matrix multiplication

$P = (A_{11} + A_{22})(B_{11} + B_{22})$

$Q = (A_{21} + A_{22}) \cdot B_{11}$

$R = A_{11}(B_{12} - B_{22})$

$S = A_{22}(B_{21} - B_{11})$

$T = (A_{11} + A_{12}) \cdot B_{22}$

$U = (A_{21} - A_{11}) \cdot (B_{11} + B_{12})$

$V = A(A_{12} - A_{22})(B_{21} + B_{22})$

$C_{11} = P + S - T + V$ $\qquad C_{21} = Q + S$

$C_{12} = R + T$ $\qquad C_{22} = P + R - Q + U$

⇒ Addition & Subtraction takes less time as compared to multiplication of two matrices.

$$T(n) = \begin{cases} 1 & n \leq 2 \\ 7T(n/2) + n^2 & n > 2 \end{cases}$$

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