

- 1) In single server queuing system, if $\lambda = 20$ and $\mu = 25$, what is utilization factor?

soln,

The probability that the server is busy (i.e. probability that customer has to wait) is called utilization factor i.e.

$$U = \frac{\lambda}{\mu} \quad \begin{array}{l} \text{(arrival rate)} \\ \text{(service rate)} \end{array}$$

$$= \frac{20}{25}$$

$$\therefore U = 0.8 \text{ which is required utilization factor}$$

- 2) Customers enter the waiting line at a cafeteria on a first come, first serve basis. The arrival rate follows a poisson distribution, while service time follows an exponential distribution. If average number of arrivals is 6/min and average rate of server is 10/min, what is average time a customer spends waiting in line for service?

soln

$$\text{given, } \lambda = 6, \mu = 10$$

$$\text{average time waiting for service } (W_q) = \frac{\lambda}{\mu(\mu - \lambda)}$$

$$= \frac{6}{10(10 - 6)} \\ = 0.15$$

Teacher's Sign

- 3) customers enter the waiting line at a cafeteria on first-come first-serve basis, the arrival rate follows a poisson distribution while service follows exponential distribution. If average arrivals per is 6/min and average service rate of single server is 10/min. what is average no. of customers in system?

solⁿ,

given: $\lambda = 6$, $\mu = 10$,

average customer no. in system $(L) = \frac{\lambda}{\mu - \lambda}$

$$= \frac{6}{10 - 6} = \frac{6}{4} = 1.5 \#$$

average customer in waiting line is

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{6^2}{10(10 - 6)} = 0.9$$

- 4) Which model/system applies deductive reasoning of mathematical theory to solve a model? Solve support your answer.

λ = arrival rate (av. no. of customers arriving / time period)

μ = service rate (av. no. of customers being served / time period by teller)

1) Probability that no customers are in the queueing system (idle) is $P_0 = \left(1 - \frac{\lambda}{\mu}\right)$ (prob. that customer can be served is $1 - \frac{\lambda}{\mu}$)

2) Probability that n customers are in the system (busy):

$$P_n = \left(\frac{\lambda}{\mu}\right)^n \times P_0 = \left(\frac{\lambda}{\mu}\right)^n \times \left(1 - \frac{\lambda}{\mu}\right)$$

3) average number of customers in the queueing system (customers waiting and being served)

$$L = \frac{\lambda}{\mu - \lambda}$$

4) average customers in waiting line is

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

5) average time customer spends in the total queueing system (waiting and being served)

$$W = \frac{1}{\mu - \lambda} = \frac{L}{\lambda}$$

6) average time a customer spends waiting in the queue to be served is

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)}$$

7) utilisation factor $U = \frac{\lambda}{\mu}$

8) Prob that server is idle (customer can be served) = $1 - U$

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