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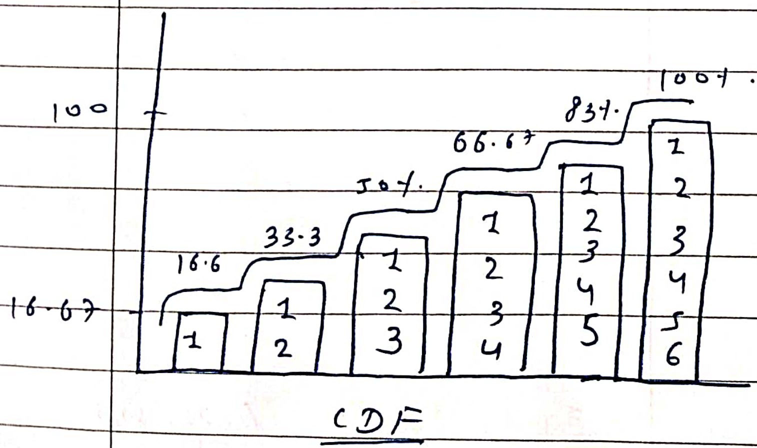
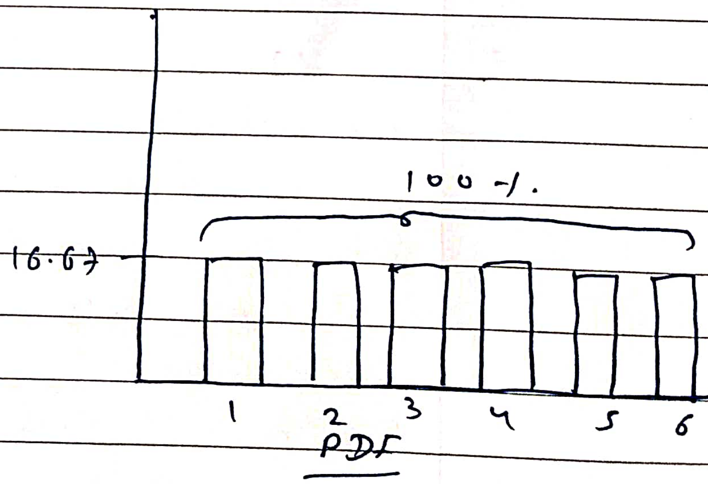
Topic: assignment 2 MDS Date: 10 Nov 2022

Q1) Difference between probability density functions (PDFs) and cumulative distribution functions (CDFs).

→ Probability density functions (PDFs) is a probability that a random variable, will take a value exactly equal to it.
for example : if you roll a dice, the probability of getting 1, 2, 3, 4, 5, 6 is (1/6) each. The PDF that we will get exactly 2 is (1/6).

→ The cumulative distribution function (CDF) is the probability that a random variable say X will take a value equal to or less than x.
for example: if you roll a dice, the probability of obtaining a 1 or 2 or 3 or 4 or 5 or 6 is (1/6) individually. The CDF of 1 is the probability that the next roll will take a value less than or equal to 1 is (16.667)%. The CDF of 2 is 33.33%. as there are 2 possible ways (1 is less than 2 and 2 is equal to 2).





→ For continuous random variable, we cannot use a PDF directly, since the probability that x takes on any exact value is zero.

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CDF	PDF
→ cumulative distribution function	→ probability density function
→ CDF is the probability that random variable values less than or equal to x ~~whereas~~. ~~PD~~	→ PDF is the probability that a random variable say X, will take a value exactly equal to x.
→ Slope of CDF must alway be equal to or greater than zero.	→ PDF is simply a derivative of CDF and as it is slope of CDF so it must always be positive i.e. PDF ≥ 0.
→ CDF values describe probability of a value being less than or equal to given number.	→ PDF values describe the probability of a value falling within a given range.
→ CDF values are often used to describe discrete random variables.	→ PDF values are often used to describe continuous random variables.
→ It is more accurate representation of random value since it takes in account all of the possible outcomes.	→ It can be misleading because it only shows a small snapshot of data.
→ $F_x(x) = P(X \le x)$	→ $f_x(x) = \frac{d}{dx} F_x(x)$

Q2) Linear congruential method

Find the first 5 random numbers where $X_0 = 27$, $a = 17$, $c = 43$ and $m = 100$.

$$X_{i+1} = (a X_i + c) \bmod m \quad , i = 0, 1, 2 \ldots$$

seed value, increment, modulus

$$R_i = \frac{X_i}{m}$$

Soln,

$X_1 = (a X_0 + c) \bmod m$
$= (17 \times 27 + 43) \bmod 100$
$= 502 \bmod 100 = 2$

$R_1 = \frac{X_1}{m} = \frac{2}{100} = 0.02$

$X_2 = (a X_1 + c) \bmod m$
$= (17 \times 2 + 43) \bmod 100 = 77$

$R_2 = \frac{X_2}{m} = 0.77$

$X_3 = (a X_2 + c) \bmod m$
$= (17 \times 77 + 43) \bmod 100$
$= 52$

$R_3 = \frac{X_3}{m} = \frac{52}{100} = 0.52$

$X_4 = (17 \times 52 + 43) \bmod 100$
$= \cancel{0}\ 27$

$R_4 = \frac{X_4}{m} = \frac{27}{100} = 0.27$

$X_5 = (17 \times 27 + 43) \bmod 100$
$= 502 \bmod 100 = 2$

$R_5 = \frac{X_5}{100} = 0.02$

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Q 3) Find the first 5 random numbers where
$X_0 = 27, a = 17, c = 0, m = 100.$

when $c = 0$ in LCG, then we call that multiplicative congruential generator.

Q 2

i	X_i	X_{i+1}	R_i
0	27	2	0.~~0~~27
1	2	77	0.02
2	77	52	0.77
3	52	27	0.52
4	27	2	0.27
5	2		0.02

Q 3)

$X_1 = (aX_0 + c) \bmod m$
$= (17 \times 27 + 0) \bmod 100$
$= 59$

$R_1 = \frac{X_1}{m} = 0.59$

$X_2 = (17 \times 59 + 0) \bmod 100 = 3$
$R_2 = \frac{3}{100} = 0.03$

$X_3 = (17 \times 3 + 0) \bmod 100 = 51$
$R_3 = \frac{51}{100} = 0.51$

$X_4 = (17 \times 51 + 0) \bmod 100 = 67$
$R_4 = \frac{67}{100} = 0.67$

$X_5 = (17 \times 67 + 0) \bmod 100 = 88$
$R_5 = \frac{88}{100} = 0.88$

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(3)

i	X_i	X_{i+1}	R_i
0	27	59	0.27
1	59	03	0.59
2	3	51	0.03
3	51	67	0.51
4	67	88	0.67
5	88		0.88

Q4) Find the mean, median, midrange, mode, variance and standard deviation for the random numbers generated in question 2 and 3.

Question 2 only: random numbers: ~~0.27~~.

3 R_i 0.02, 0.77, 0.52, 0.22, 0.02

~~mean~~ $\Sigma R_i = 1.6$

mean $= \frac{\Sigma R_i}{5} = \frac{1.6}{5} = 0.32$

~~median =~~

R_i	f	cf
0.02	1	1
0.77	1	2
0.52	1	3
0.22	1	4
0.02	1	5
$\Sigma R_i = 1.6$	$N = 6$	

~~median = value of $\left(\frac{N+1}{2}\right)$ th item~~

~~= (3.5) th item~~

=

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R_i	f
0.02	2
0.77	1
0.52	1
0.27	2
~~0.02~~	~~2~~
0.59	1
0.03	1
0.51	1
0.67	1
0.88	1

$\Sigma R_i = 4.28$

R_i	f
0.02	2
0.03	1
0.27	1
0.51	1
0.52	1
0.59	1
0.67	1
0.77	1
0.88	1
$\Sigma R_i = 4.28$	$N = 10$

$$\text{mean} = \frac{\Sigma R_i}{10} = 0.428$$

median = 0.515

mode = 0.02

$\sigma = 0.3061$ (SD)

$\sigma^2 = 0.093756$ (variance)

midrange = 0.45