

Total No. of Pages: 02

Roll No.....

THIRD SEMESTER

B.Tech.(COE)

MID SEMESTER EXAMINATION

SEP-2019

**CO 205 Discrete Structures**

Time: 1:30 Hours

Max. Marks : 25

**Note :** Answer all questions.

Assume suitable missing data, if any.

Q.1 Let  $P(x)$ ,  $Q(x)$ ,  $R(x)$  and  $S(x)$  be the statements " $x$  is a baby", " $x$  is logical", " $x$  is able to manage a crocodile" and " $x$  is despised" respectively. Suppose that the domain consist of all people. Express each of these statements using quantifiers, logical connectives and  $P(x)$ ,  $Q(x)$ ,  $R(x)$  and  $S(x)$ . (5)

- a) Babies are illogical.
- b) Nobody is despised who can manage a crocodile.
- c) Illogical persons are despised.
- d) Babies cannot manage crocodile.
- e) Does d) follow from a), b) and c)? If not, is there a correct conclusion?

Q.2 [a] In a class of 100 students, 39 play Tennis, 58 play Cricket, 32 play Hockey, 10 play Cricket and Hockey, 11 play Hockey and Tennis, 13 play Tennis and Cricket. How many students play

- i. All 3 games
- ii. Just one game
- iii. Tennis and cricket and not Hockey?

[b] Find the conjunctive normal form of the function

$$f = [x \wedge (y' \vee z)] \vee z' \quad (3+2)$$

Q.3 Show that  $2^n > n^3$ ,  $n \geq 10$  using mathematical induction. (5)

Q.4 [a] In how many ways can a team of 11 cricketers be chosen from 6 bowlers, 4 wicket keepers and 11 batsmen to give a majority of batsmen if at least 4 bowlers are to be included and there is one wicket keeper.

[b] Give a recursive algorithm for finding reversal of a bit string.

(3+2)

Q.5 Find the explicit formula for the given recurrence relation with initial conditions  $a_0 = 0, a_1 = 1$ . (5)

$$a_r - 7a_{r-1} + 10a_{r-2} = 2r^2 + 2$$

Total No. of Pages: 03

Roll No. \_\_\_\_\_

THIRD SEMESTER

B.Tech.

## SUPPLEMENTARY EXAMINATION

FEB-2020

### CO 205 DISCRETE STRUCTURES

Time: 3:00 Hours

Max. Marks : 50

#### Instructions :

- 1) Attempt any five questions.
- 2) Calculator is allowed.
- 3) Assume suitable missing data, if any.

Q.1 [a] i) Suppose that there are 1807 freshmen at your school. Of these, 453 are taking a course in computer science, 567 are taking a course in mathematics, and 299 are taking courses in both computer science and mathematics. How many are not taking a course either in computer science or in mathematics?

ii) How many elements are in  $A_1 \cup A_2$  if there are 12 elements in  $A_1$ , 18 elements in  $A_2$ , and

a)  $A_1 \cap A_2 = \emptyset$

b)  $|A_1 \cap A_2| = 1$

c)  $A_1 \subseteq A_2$

(2+3)

[b] i) How many cards must be selected from a standard deck of 52 cards to guarantee that at least three cards of the same suit are chosen?  
ii) How many must be selected to guarantee that at least three hearts are selected? (3+2)

Q.2 [a] Show by induction that for any positive integer  $n$ ,  $6^n - 1$  is divisible by 5. (5)

[b] Show that  $\neg(p \vee (\neg p \wedge q))$  and  $\neg p \wedge \neg q$  are logically equivalent by developing a series of logical equivalences. (5)

Q.3 [a] Let  $p$ ,  $q$  and  $r$  be the propositions (5)

$p$ : You get an A on the final exam.

$q$ : You do every exercise in this book.

$r$ : You get an A in this class.

Write these propositions using  $p$ ,  $q$ , and  $r$  and logical connectives (including negations).

- i) You get an A in this class, but you do not do every exercise in this book.
- ii) You get an A on the final, you do every exercise in this book, and you get an A in this class.
- iii) To get an A in this class, it is necessary for you to get an A on the final.
- iv) You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.
- v) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.

[b] Draw the Hasse diagram for the partial ordering  $\{(a, b) | a \text{ divides } b\}$  on  $\{1, 2, 3, 4, 6, 8, 12\}$ .

Q.4 [a] Use K-map to find a minimal sum of products form for

$$x\bar{y} + xyz + \bar{x}\bar{y}\bar{z} + \bar{x}y\bar{z}\bar{t} \quad (5)$$

[b] Find the general solution of following recurrence relation : (5)

$$a_r - 6a_{r-1} + 9a_{r-2} = n 3^n$$

**THIRD SEMESTER****B.Tech.(SE)****MID SEMESTER EXAMINATION****SEPTEMBER-2010****SW- 206 DISCRETE MATHEMATICS****Time: 1 Hour 30 Minutes****Max. Marks : 20**

**Note :** Answer **ALL** questions by selecting any **TWO** parts from each.  
Assume suitable missing data, if any.

1[a] Let the proposition p be "Mark is rich" and q be "Mark is happy". Write each of the following in symbolic form :

- (i) ✓ Mark is poor but happy.
- (ii) ✓ Mark is neither poor nor happy
- (iii) ✓ Mark is either rich or happy
- (iv) ✓ Mark is either poor or else he is both rich and happy
- (v) ✓ Mark is either poor or happy.
- (vi) ✓ If mark is happy then he is not rich.

[b] By using algebra of proposition, show that

- (i)  $[\sim p \wedge (\sim q \wedge r) \vee (q \wedge r) \vee (p \wedge r)] \Leftrightarrow r$
- (ii)  $[(\sim p \vee q) \wedge (p \wedge (p \wedge q))] \Leftrightarrow p \wedge q$

[c] (i) ✓ Prove that the following argument is valid : If a baby is hungry then the baby cries. If the baby is not mad then he does not cry. If a baby is mad, then he has a red face. Therefore if a baby is hungry then he has a red face.

- (ii) ✓ Without using truth table, prove that  $\sim p$  is a valid conclusion from  $p \Rightarrow \sim q, r \Rightarrow q, r$

2[a] Define the following terms;

Well formed formula, predicate, compound proposition, tautology, contradiction, contingency and valid argument.

- [b] A relation  $R$  on a set  $X$  is called circular if  $(a, b) \in R$  and  $(b, c) \in R$  implies  $(c, a) \in R$ .

Show that a relation  $R$  is reflexive and circular iff it is an equivalence relation.

- [c] Let  $R$  be an equivalence relation on a set  $X$ , show that the equivalence classes of  $R$  are either disjoint or identical.

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- [d] Consider an algebraic structure  $(G, *)$  where  $G$  is the set of all non-zero real numbers and  $*$  is a binary operation on  $G$  defined by

$$a * b = \frac{ab}{4}$$

show that  $(G, *)$  is an abelian group.

- [e] Prove that the inverse of an element in the group is unique.

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- [f] Define partial order relation and show that the relation "divides" defined on the set of natural numbers is a partial order relation.

**FIRST SEMESTER****B.E.(SOFTWARE ENGINEERING)****END SEMESTER EXAMINATION****NOVEMBER-2010****SW-206 DISCRETE MATHEMATICS****Time: 3 Hours****Max. Marks : 70**

**Note :** Answer **ALL** questions by selecting any **TWO** parts from each.  
Assume suitable missing data, if any.

- 1 [a] (i) Show that the relation  $(x,y) R(a,b) \Leftrightarrow x^2 + y^2 = a^2 + b^2$  is an equivalence relation on the plane and describe the equivalence classes.
- (ii) Prove that the inverse of an invertible function is unique.
- [b] (i) Show by induction that  $2n < 3^n$  for all  $n \in N$ .
- (ii) Define extended pigeonhole principle. Seven members of a family have total Rs.2886/- in their pocket. Show that at least one of them have at least Rs.416/- in his pocket.
- [c] Define primitive recursive function. Show that the function  $f(x, y) = x + y$  is primitive recursive.
- 2 [a] Prove that a non-empty set H of a group G is a subgroup of G iff  $a, b \in H \Rightarrow ab^{-1} \in H$ .
- [b] Show that the set  $S_n$  of all the  $n!$  permutations of n elements is a finite non-abelian group when  $n \geq 3$  w.r.t. product of permutations.
- [c] Let X be a non-empty set and  $(A, +, \cdot)$  be a ring. Define  $B = \{f / f: X \rightarrow A\}$ . Then show that the set B with addition and multiplication defined by  
 $(f + g)(x) = f(x) + g(x)$  and  $(f \cdot g)(x) = f(x) \cdot g(x), \forall f, g \in B$  forms a ring.
- 3 [a] Define " Lattice as a poset" and as "an algebraic structure", Let  $(L, \leq)$  be a lattice such that  $a \leq b$  and  $c \leq d, a, b, c, d \in L$ . Then prove that  $a \cdot c \leq b \cdot d$ .
- [b] Let  $S = \{2, 3, 4, 6, 12, 18, 36\}$ . Define  $a \leq b$  iff a is multiple of b. Is this a partial order on S? If so, draw the Hasse diagram?

- [c] Write the principle of a duality w.r.t Boolean algebra. Convert the Boolean expression  $(xy' + xz)' + x'$  into its disjunctive normal form and conjunctive normal form.

A[a] What are normal forms? Find PCNF and PDNF of the following:

$$(p \Rightarrow (q \wedge r)) \wedge (\neg p \Rightarrow (\neg q \wedge \neg r))$$

- [b] What do you mean by "Logical equivalence". By using algebra of propositions show that

(i)  $(\neg p \wedge (\neg q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) \equiv r$

(ii)  $\neg (p \Leftrightarrow q) \equiv (\neg p \Leftrightarrow \neg q) \equiv (p \Leftrightarrow \neg q)$

- [c] Let  $p$  denotes the statement "the material is interesting",  $q$  denotes "the exercises are challenging" and  $r$  denotes "the course is enjoyable". Write the following in symbolic form:

(i) the material is interesting and the exercises are challenging.

(ii) if the material is uninteresting then the exercises are not challenging and the course is not enjoyable.

(iii) If the material is not interesting and the exercise are not challenging then the course is not enjoyable.

(iv) The material is interesting means the excercises are challenging and conversely.

(v) Either the material is interesting or the exercises are not challenging but not both.

5[a]. Solve the recurrence relation

$$a_{n+2} - 6a_{n+1} + 8a_n = n \cdot 4^n \text{ where } a_0 = 8 \text{ and } a_1 = 22$$

[b] By using generating function, solve the recurrence relation

$$a_{n+2} - 2a_{n+1} + a_n = 2^n, a_0 = 2 \text{ and } a_1 = 1$$

- [c] A disconnected graph on  $n$  vertices having 5 components is given. Construct a graph on  $n$  vertices having the same number of components but having maximum number of edges by giving detailed arguments.

Total No. of Pages 2

III SEMESTER

## END SEMESTER EXAMINATION

### IT-205 DISCRETE STRUCTURES

Roll No. ....

B.Tech.

Nov/Dec-2019

Time: 3:00 Hours

Note: Answer all question by Selecting any two parts from each questions.

All questions carry equal marks.

Assume suitable missing data, if any.

Max. Marks: 50

Q.1 [a] i) Over the Universe of animals. Let

$P(x)$ :  $x$  is a whale.

$Q(x)$ :  $x$  is a fish.

$R(x)$ :  $x$  lives in water

Translate the following into English-

I.  $\exists x (\neg R(x))$

II.  $\exists x (Q(x) \wedge \neg P(x))$

III.  $\forall x (P(x) \wedge R(x)) \rightarrow Q(x)$

ii) Find the disjunctive normal form for the proposition  $p \rightarrow q$ . [3]

[b] i) Write the converse, contrapositive and negation of the following statement: For every integer  $n$ , if  $n$  is

divisible by 3 then  $n^2$  is divisible by 3. [3]

ii) Use the logical equivalences above to show that  $\neg(p \vee \neg(p \wedge q))$  is a contradiction. [2]

[c] Show that the hypotheses "It is not sunny this afternoon and it is colder than yesterday," "We will go swimming only if it is sunny this afternoon," "If we do not go swimming, then we will take a canoe trip," and "If we take a canoe trip, then we will be at home by sunset" lead to the conclusion "We will be home by sunset." [5]

Q.2 [a] Calculate the time complexity of Quick Sort algorithm in terms of recurrence relation. Sort the list  $X = \{64, 25, 12, 22, 11\}$  using quick sort. [5]

[b] i) Prove by Contradiction that  $\sqrt{10}$  is irrational. [2.5]

ii) Let  $A = \{1, 2, 3, 4\}$  and  $R = \{(a, b) : a+b > 4\}$  be a relation on  $A$ . Draw the graph of relation  $R$ . [2.5]

[c] i) Prove for finite sets  $A$  and  $B$ ;  $n(A \cup B) = n(A) + n(B) - n(A \cap B)$  [2.5]

ii) In a class of 50 students, 15 play Tennis, 20 play Cricket, 20 play Hockey, 3 play Tennis and Cricket, 6 play Cricket and Hockey, and 5 play Tennis and Hockey, 7 play no game at all. How many play Cricket, Tennis and Hockey? [2.5]

Q.3 [a] i) What is closure of relations? Let  $A = \{1, 2, 3, 4\}$  and  $R = \{(1, 2), (2, 3), (3, 4)\}$  be a relation in  $A$ . Find its reflexive closure, symmetric closure and transitive closure. [3]

ii) Minimize the following Boolean function using K Map:

$$F(A, B, C) = A'BC + A'BC' + AB'C' + AB'C$$

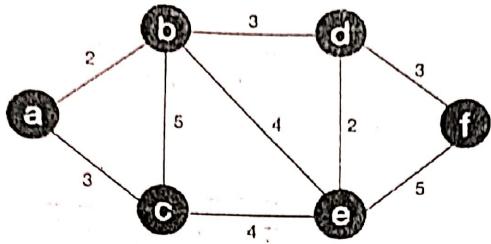
[2]

[b] Define Equivalence Relation. If  $R$  and  $S$  be two equivalence relations in a set  $A$ , then prove that  $R \cup S$  is also an equivalence relation in  $A$ . Also, give suitable example. [5]

[c] Let  $f: R \rightarrow R$  be a function defined as  $f(x) = 2x - 1$  and  $g: R \rightarrow R$  be a function

defined as  $g(x) = \frac{1}{2}x + 4$ . Find  $f^{-1}(x)$ ,  $g^{-1}(x)$ ,  $(f \circ g)^{-1}(x)$  and  $(g^{-1} \circ f^{-1})(x)$ . What can you conclude? [5]

Q.4[a] Define Spanning Tree and Minimal Spanning Tree. Find Two spanning trees of following graph. Also, Find Minimal Spanning Tree of following graph using Prim's algorithm. [5]



[b] Let  $x = \{1, 3, 5, 7, 15, 21, 35, 105\}$  and  $R$  be the relation ' $/$ ' (divides) on the set  $x$  then  $x$  is the Poset.

Draw the Hasse diagram of the given Poset. Determine the following:

- LUB of 3 and 7.
- GLB of 15 and 35.
- Greatest and Least element of  $x$ .
- Is  $x$  a Lattice?

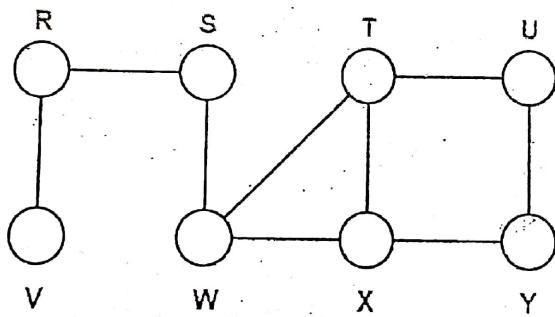
[5]

[c] Define 'Join' and 'Meet' in terms of Boolean Matrices. Compute Join and Meet of following Boolean Matrices:

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

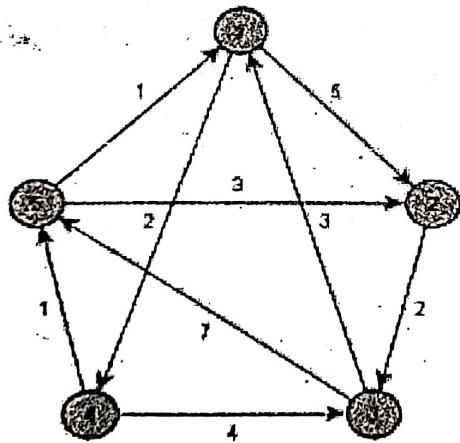
[5]

Q.5[a] What is difference between Breadth First and Depth First Graph Traversals? Apply Breadth First Search to explore all the vertices from the vertex S of the graph given in figure and find the Breadth-first search tree.



[5]

[b] Apply Floyd Warshall algorithm to find all pair shortest path in the following graph.



[5]

[c] Explain Euler's Formula with Proof in Graph Theory. Let  $G$  be a graph that has: 21 edges and 7 vertices of degree 1 each; 3 vertices of degree 2 each; 7 vertices of degree 3 each;  $x$  vertices of degree 4 each. Compute how many vertices are in  $G$ . [2+3]

(b) An intelligence agency forms a code of two distinct digits selected from 0, 1, 2, ..., 9 such that the first digit of the code is nonzero and digit repetition is not allowed. The code, handwritten on a slip, can however potentially create confusion, when read upside down-for example, the code 91 may appear as 16. How many codes are there for which no such confusion can arise? [2]

4. (a) Explain the principle of mathematical induction. Consider the sequence  $\alpha_0, \alpha_1, \alpha_2, \dots$  defined by  $\alpha_0 = 1/4$  and  $\alpha_{n+1} = 2\alpha_n(1 - \alpha_n)$  for  $n \geq 0$ . A formula for the sequence  $\alpha_n$  defined above, is

$$\alpha_n = (1 - 1/2^{2^n}) / 2, \forall n \geq 0.$$

Prove that the recursive formula is true by using proof by mathematical induction. [5]

(b) Write the pseudocode for selection sort. Sort the following list of elements using selection sort.

29, 72, 98, 13, 87, 66, 52, 51, 36

What is the best case and worst case complexity of selection sort?

Justify.

-END-

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Roll No. ....

THIRD SEMESTER

B.Tech. (IT)

MID SEMESTER EXAMINATION

(Sep-2018)

### IT205 DISCRETE STRUCTURES

Time: 1 Hour 30 Minutes

Max. Marks: 25

Note: Answer all questions.

Assume suitable missing data, if any.

1. Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consists of all people.

- (i) Someone in your class can speak Hindi.
- (ii) Everyone in your class is friendly.
- (iii) There is a person in your class who was not born in California.
- (iv) A student in your class has been in a movie.
- (v) No student in your class has taken a course in logic programming.

[5]

2. (a) Using rules of inference, show that the premises

$\sim(r \vee s), \sim p \rightarrow s, p \rightarrow q$  leads to the conclusion  $q$ .

[3]

- (b) Use proof by contradiction to prove that cube root of 2 is irrational.

[3]

3. (a) Explain the principle of inclusion and exclusion. Give a formula for number of elements in the union of four sets.

[2]

P.T.O

P.T.O

Total No. of Pages: 02

Roll No.....

IIIrd SEMESTER

B.Tech. (Information Technology)

MID SEMESTER EXAMINATION

September-2019

**IT205 Discrete Structures**

*Time: 1:30 Hours*

*Max. Marks: 25*

Note : Answer all question.

Assume suitable missing data, if any.

Q.1 [A] Write the converse, inverse, contrapositive, and negation of the following statement.

"If Sandra finishes her work, she will go to the basketball game."

(2.5 Marks)

[B] Determine whether the conclusion follows logically from the premises.

Premises:  $(\neg p \vee q) \rightarrow r$

$r \rightarrow (s \vee t)$

$\neg s \wedge \neg u$

$\neg u \rightarrow \neg t$

Conclusion:  $p$

(2.5 Marks)

Q.2 [A] Check the following argument logically correct?

Premises: There are men who are soldiers.

All soldiers are strong.

All soldiers are brave.

Conclusion: Therefore, some strong men are brave.

(2.5 Marks)

[B] Let A, B be sets. Prove

$$A - (A - B) = A \cap B.$$

(2.5 Marks)

Q.3 [A] Prove "A positive integer n is odd if and only if  $n^2$  is odd.", using Direct and Contraposition proof technique.

(2.5 Marks)

[B] Find the recurrence relation for the "number of binary sequences of length n that have no consecutive 0's". Solve it using characteristic root method.

(2.5 Marks)

Q.4 [A] Let's consider a propositional language where

A = "Angelo comes to the party",  
B = "Bruno comes to the party",  
C = "Carlo comes to the party",  
D = "Davide comes to the party".

Formalize the following sentences:

- a) "If Angelo and Bruno come to the party, then Carlo comes provided that Davide doesn't come".  
b) "Either Carlo comes to the party, or Bruno and Davide don't come".

(2.5 Marks)

[B] If  $R(x,y) = "x \text{ relies upon } y"$ , express the following in unambiguous English:

- a)  $\forall x (\exists y R(x,y))$   
b)  $\exists y (\forall x R(x,y))$

(2.5 Marks)

Q.5 [A] Let  $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ . Then prove that  $A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ , for all  $n \geq 1$ ; Using mathematical Induction.

(2.5 Marks)

[B] What is the worst case time complexity of following implementation of subset sum problem.

```
bool isSubsetSum(int set[], int n, int sum)
{
    // Base Cases
    if (sum == 0)
        return true;
    if (n == 0 && sum != 0)
        return false;
```

```
// If last element is greater than sum, then ignore it
if (set[n-1] > sum)
    return isSubsetSum(set, n-1, sum);
```

```
/* else, check if sum can be obtained by any of the following
(a) including the last element
(b) excluding the last element */
return isSubsetSum(set, n-1, sum) ||
```

```
isSubsetSum(set, n-1, sum-set[n-1]);
}
```

(2.5 Marks)

Total No. of Pages 2

Roll No. ....

THIRD SEMESTER

B.Tech. (IT)

SUPPLEMENTARY EXAMINATION

(Feb-2019)

IT-205 DISCRETE STRUCTURES

Time: 3 Hours

Max. Marks: 50

Note: Answer all questions.

Assume suitable missing data, if any.

1. Answer the following questions. Each question carries 2 marks.

(a) Construct a truth table for  $(p \leftrightarrow q) \leftrightarrow (r \leftrightarrow s)$ .

(b) What is Selection Sort? Explain with an example.

(c) Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consists of all people.

(i) Everyone in your class has a cellular phone.

(ii) Everyone in your class is friendly.

(d) Explain the principle of inclusion and exclusion. Give a formula for number of elements in the union of four sets.

(e) Obtain Disjunctive normal form of  $\neg(p \wedge q) \leftrightarrow (p \vee q)$

2. Answer the following questions. Each question carries 3 marks.

(a) Explain Isomorphism of graphs with suitable examples.

(b) What is depth first search. Write its algorithm.

(c) Use proof by cases to show that  $|xy| = |x| |y|$ , where x and y are real numbers.

(d) The Indian Cricket team consists of 16 players. It includes 2 wicket keepers and 5 bowlers. In how many ways can a cricket team of eleven players be selected if we have to select 1 wicket keeper and atleast 4 bowlers?

(e) Explain pigeonhole principle. Show that for every integer n there is a multiple of n that has only 0s and 1s in its decimal expansion.

3. Answer the following questions. Each question carries 5 marks.

(a) What is Quicksort? Explain with an example. Apply the Quicksort on the following data

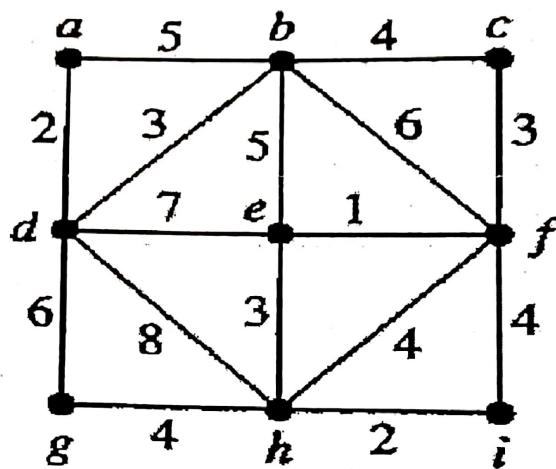
44, 33, 11, 55, 77, 90, 40, 60, 99, 22, 88, 66

(b) Explain the principle of mathematical induction. Consider the sequence  $a_0, a_1, a_2, \dots$  defined by  $a_0 = 1/4$  and  $a_{n+1} = 2 a_n (1 - a_n)$  for  $n \geq 0$ . A formula for the sequence  $a_n$  defined above, is

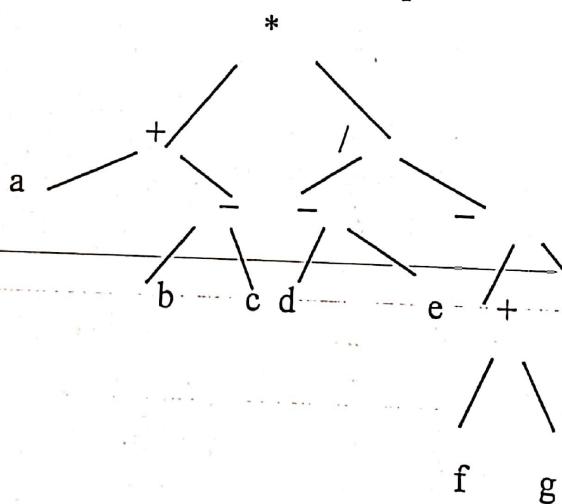
$$a_n = (1 - 1/2^{2n}) / 2, \forall n \geq 0.$$

Prove that the recursive formula is true by using proof by mathematical induction.

(c) Use Kruskal's algorithm to find a minimum spanning tree for the given weighted graph.



(d) Write the inorder, preorder and postorder for the following tree.



(e) Use a K-map to find a minimal expansion as a Boolean sum of products of each of these functions

(i)  $wxyz\bar{z} + wx\bar{y}\bar{z} + w\bar{x}yz + w\bar{x}y\bar{z} + w\bar{x}\bar{y}\bar{z} + \bar{w}xyz + \bar{w}xy\bar{z} + \bar{w}x\bar{y}\bar{z} + \bar{w}x\bar{y}z + \bar{w}\bar{x}y\bar{z} + \bar{w}\bar{x}\bar{y}\bar{z}$

(ii)  $wxyz\bar{z} + w\bar{x}yz + w\bar{x}y\bar{z} + w\bar{x}\bar{y}\bar{z} + w\bar{x}\bar{y}\bar{z} + \bar{w}xyz + \bar{w}xy\bar{z} + \bar{w}\bar{x}y\bar{z} + \bar{w}\bar{x}\bar{y}\bar{z}$

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Roll No.....

III SEMESTER

B.Tech.

Supplementary EXAMINATION

Feb-2020

IT-205 DISCRETE STRUCTURES

Time: 3:00 Hours

Max. Marks: 50

Note: Answer any FIVE questions.

All questions carry equal marks.

Assume suitable missing data, if any.

Q.1[a] Show that  $(\neg q \wedge (p \Rightarrow q)) \Rightarrow \neg p$  ( $\neg q \wedge (p \Rightarrow q)$ )  $\Rightarrow \neg p$  is a tautology. [5]

[b] Use mathematical induction to show that

$$1 + 2 + 3 + 4 + \dots + n = n(n + 1)/2. \quad [5]$$

Q.2[a] Solve the following recurrence relation:  $a_n - 5a_{n-1} + 6a_{n-2} = 2^n$  with initial conditions  $a_0 = -1$  and  $a_1 = 1$ . [5]

[b] Let the universal set  $U = \{1, 2, 3, \dots, 10\}$ . Let  $A = \{2, 4, 7, 9\}$  B =  $\{1, 4, 6, 7, 10\}$  and  $C = \{3, 5, 7, 9\}$ .  
Find 1)  $A \cup B$ , 2)  $A \cap B$ , 3)  $B \cap C$ , 4)  $(A \cap B) \cup C$ , 5)  $(B \cup C) \cap A$  [5]

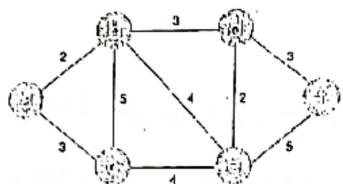
Q.3 [a] Define Equivalence Relation. Let assume that  $F$  be a relation on the set  $R$  real numbers defined by  $xFy$  if and only if  $x-y$  is an integer. Prove that  $F$  is an equivalence relation on  $R$ . [5]

[b] Let  $f: R \rightarrow R$  be a function defined as  $f(x) = 2x+1$  and  $g: R \rightarrow R$  be a function defined as  $g(x) = x/3$ . Find  $f^{-1}(x)$ ,  $g^{-1}(x)$ ,  $(f \circ g)^{-1}(x)$ , and  $(g^{-1} \circ f^{-1})(x)$ . What can you conclude? [5]

Q.4[a] Define Spanning Tree and Minimal Spanning Tree (MST). Discuss the difference between Prim's and Kruskal's algorithm to find MST

Also, Find Minimal Spanning Tree of following graph using Kruskal's algorithm.

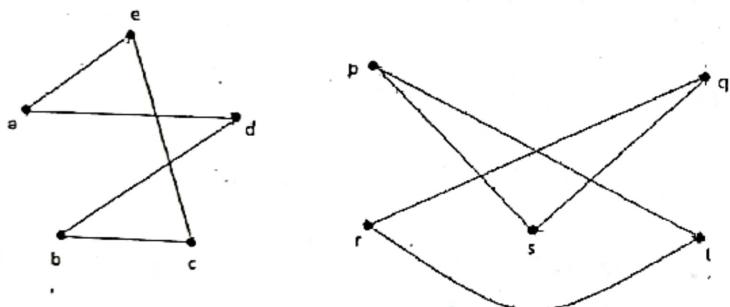
[5]



[b] Draw Hasse diagram for  $(\{3, 4, 12, 24, 48, 72\}, \sqsubset)$  and  $(D_{12}, \sqsubset)$ .

[5]

Q.5[a] What are Isomorphic graphs? Determine whether the following graphs are isomorphic or not?



[5]

[b] Explain Euler's Formula with Proof in Graph Theory. Let G be a graph that has: 21 edges and 7 vertices of degree 1 each; 3 vertices of degree 2 each; 7 vertices of degree 3 each; x vertices of degree 4 each. Compute how many vertices are in G.

[2+3]

Q6 Write short notes on any two:

- a) Pigeonhole principle with example
- b) Partial Order Relation with example
- c) Euler and Hamiltonian Cycle in graph

[END]