Recursion tree method

Key Points

1) Step by step draw the recursion tree.
2) find the time complexity using recurrence tree.
3) Second term in our recurrence rel^n become our root node.
4) Keep drawing till we find a pattern among levels.
5) The pattern is typically a A.P or G.P series.

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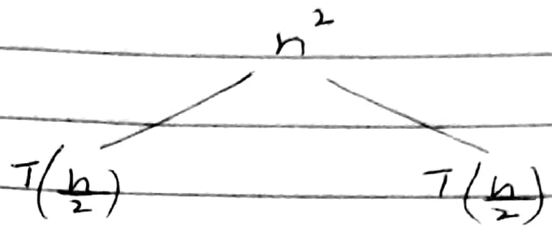
Q $T(n) = 2T\left(\frac{n}{2}\right) + n^2$ — ①

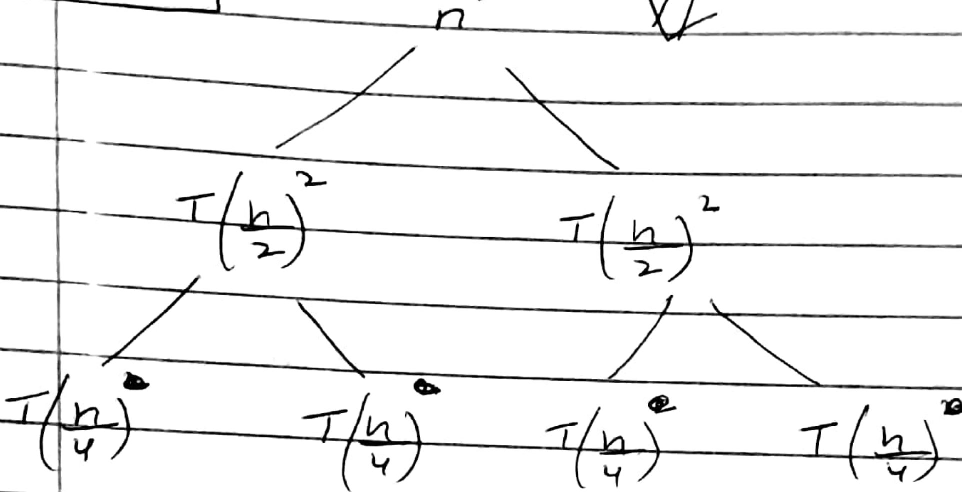
$T(n) \longrightarrow$



Substitute $T(n)$ by $T(n/2)$ in eq ①

$T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{4}\right) + \left(\frac{n}{2}\right)^2$

$\Downarrow$



level

No of nodes

$0 \left(\frac{n}{2^0}\right)^2$ — $2^0 = 1$

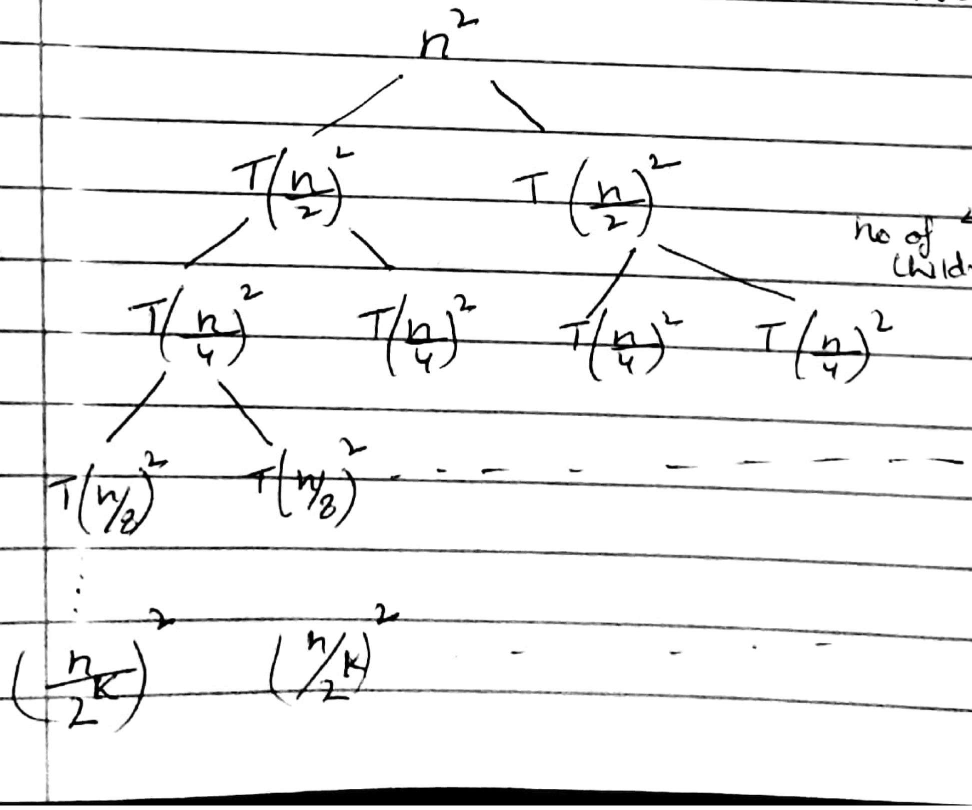
$1 \left(\frac{n}{2^1}\right)^2$ — $\boxed{2}^1 = 2$ ← no of children

$2 \left(\frac{n}{2^2}\right)^2$ — $2^2 = 4$

$3 \left(\frac{n}{2^3}\right)^2$ — $2^3 = 8$

⋮

2^K



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$n^2 + 2\left(\frac{n}{2}\right)^2 + 4\left(\frac{n}{4}\right)^2 + 8\left(\frac{n}{8}\right)^2 + \cdots$

to find upper bound lets assume this series will go infinite.

$= n^2 + 2\frac{n^2}{4} + 4\frac{n^2}{16} + 8\frac{n^2}{64} + \cdots$

$= n^2 + \frac{n^2}{2} + \frac{n^2}{4} + \frac{n^2}{8} + \cdots$

$= n^2\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots\right)$

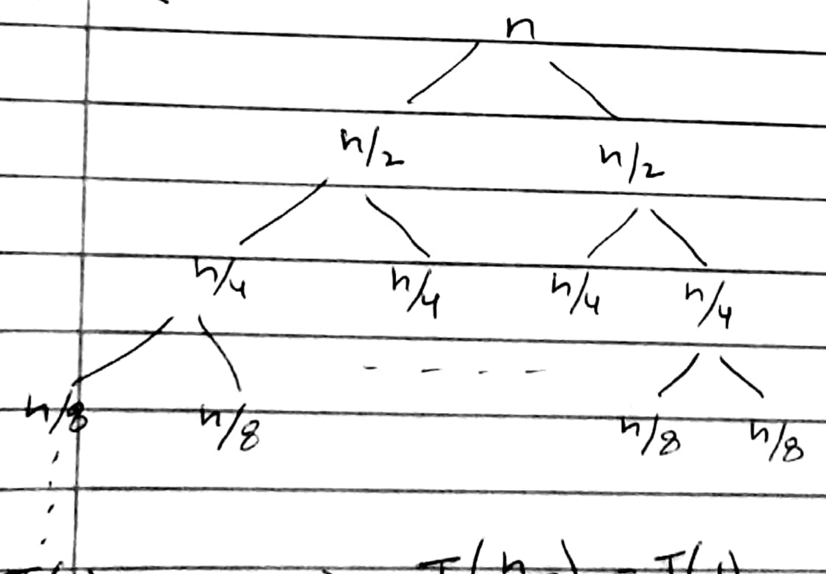
G.P $= \frac{a}{1-r}$

$= n^2\left(\frac{1}{1-\frac{1}{2}}\right) = 2n^2$

$a = 1,\ r = \frac{1}{2}$

$\therefore$ Time Complexity $= O(n^2)$

2) $T(n) = 2T(n/2) + n$



No of nodes	Cost
$2^0 = 1$	$1.n = n$
$2^1 = 2$	$2(n/2) = n$
$2^2 = 4$	$4(n/4) = n$
$2^3 = 8$	$8(n/8) = n$
$\vdots$	$\vdots$
2^k	kn

$T(1) \longrightarrow T\left(\frac{n}{2^k}\right) = T(1)$

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$T\left(\frac{n}{2^k}\right) = T(1)$

$\frac{n}{2^k} = 1$

take log on both sides

$\log_2 n = k\log_2 2$

$\because \log_2 2 = 1$

Put the value in last $\quad k = \log_2 n$

i.e kn

$\Rightarrow n\log_2 n$

$\therefore$ time complexity $= O(n\log n)$

<u>Master Method</u> (in divident form) used recurrence relation

$T(n) = aT(n/b) + \theta(n^k \log^p n)$

$a \geq 1$, $b > 1$, $k \geq 0$ and P is real no

1) If $a > b^k$, then $T(n) = \theta(n^{\log_b a})$

2) If $a = b^k$

a) if $P > -1$ then $T(n) = \theta(n^{\log_b a} \log^{P+1} n)$

b) if $P = -1$, then $T(n) = \theta(n^{\log_b a} \log\log n)$

c) if $P < -1$, then $T(n) = \theta(n^{\log_b a})$

3) If $a < b^k$

a) if $p \geq 0$ then $T(n) = \Theta(n^k \log^p n)$

b) if $p < 0$, then $T(n) = O(n^k)$

Q $T(n) = 2T(n/2) + 1$ $k=0$ $p=0$

$a=2$, $b=2$ $\log_2 2 = 1 > k = 0$

$f(n) = \Theta(1)$

$= \cancel{\Theta(n \log_2 2)} = \Theta(n)$

Q $T(n) = 4T(n/2) + n$ $p=0$, $k=1$

$a=4$, $b=2$ $\log_2 4 = 2 > k=1$

$f(n) = \Theta(n^k \log^p n)$ $\Rightarrow \Theta(n^2)$

$k=1$, $p=0$

Q $T(n) = 2T(n/2) + n$ $p > -1$; $\Theta(n^{\log_b a} \log^{p+1} n)$

$a=2$, $b=2$

$\log_2 2 = 1 \,\Big|\, = \,\Big|\, k=1$, $p=0$

$\Rightarrow \Theta(n \log n)$

Q $T(n) = 2T(n/2) + \frac{n}{\log n}$

$\log_2 2 = 1 \Big| = \Big| K=1$, $P = -1$

$= \Theta(n^K \log \log n)$
$= \Theta(n \log \log n)$

Q $T(n) = 2T(n/2) + \frac{n}{\log^2 n}$

$\log_2 2 = 1 \Big| = \Big| K=1$ $P = -2$

$= \Theta(n^K)$
$= \Theta(n)$

Q $T(n) = T(n/2) + n^2$

$\log_2 1 = 0$ $<$ $K=2$

$= \Theta(n^2)$

Q $T(n) = 2T(n/2) + n^2 \log n$

Q $T(n) = 2T(n/2) + n^2 \log^2 n$

Q $T(n) = 4T(n/2) + \frac{n^3}{\log n}$

Assignment

Q $T(n) = 2T(n/2) + 1$

Q $T(n) = 8T(n/2) + n^2$

Q $T(n) = 2T(n/2) + \frac{n^2}{\log n}$

Q $T(n) = 4T(n/2) + (n\log n)^2 \log n$