 Roll No. DSE/2K9/036

FIRST SEMESTER **B.E.(SOFTWARE ENGINEERING)**

END SEMESTER EXAMINATION ***NOVEMBER-2010***

SW-206 DISCRETE MATHEMATICS

Time: 3 Hours ***Max. Marks : 70***

Note :	Answer ***ALL*** questions by selecting any ***TWO*** parts from each. Assume suitable missing data, if any.

1[a] (i) Show that the relation (x,y) R(a,b)⇔ $x^2 + y^2 = a^2 + b^2$ is an equivalence relation on the plane and describe the equivalence classes.

(ii) Prove that the inverse of an invertible function is unique.

[b] (i) Show by induction that $2n < 3^n$ for all $n \in N$.

(ii) Define extended pigeonhole principle. Seven members of a family have total Rs.2886/- in their pocket. Show that at least one of them have at least Rs.416/- in his pocket.

[c] Define primitive recursive function. Show that the function $f(x,y) = x + y$ is primitive recursive.

2[a] Prove that a non-empty set H of a group G is a subgroup of $G\ iff\ a,b \in H \Rightarrow ab^{-1} \in H$.

[b] Show that the set S_n of all the n1 permutations of n elements is a finite non-abelian group when $n \geq 3$ w.r.t. product of permutations.

[c] Let X be a non-empty set and (A,+, ·) be a ring. Define $B =.\{f / f{:}X \to A\}$. Then show that the set B with addition and multiplication defined by

$(f + g)(x) = f(x) + g(x) and\ (f.g)(x) = f(x).g(x,)\forall f,g \in B$ forms a ring.

3[a] Define " Lattice as a poset" and as "an algebraic structure", Let $(L, \leq)$be a lattice such that $a \leq b\ and\ c \leq d, a, b, c, d \in L$. Then prove that $a.c \leq b.d$.

[b] Let S={2,3,4,6,12,18,36}. Define $a \leq b\ iff$ a is multiple of b. Is this a partial order on S? If so, draw the Hasse diagram?

[c] Write the principle of a duality w.r.t Boolean algebra. Convert the Boolean expression $(xy' + xz)' + x'$ into its disjunctive normal form and conjunctive normal form.

4[a] What are normal forms? Find PCNF and PDNF of the following:

$(p \Rightarrow (q \wedge r)) \wedge (\sim p \Rightarrow (\sim q \wedge \sim r))$

[b] What do you mean by " Logical equivalence". By using algebra of propositions show that

(i) $(\sim p \wedge (\sim q \wedge r) \vee (q \wedge r) \vee (p \wedge r) \cong r$

(ii) $\sim (p \Leftrightarrow q) \cong (\sim p \Leftrightarrow q) \equiv (p \Leftrightarrow \sim q)$

[c] Let p denotes the statement " the material is interesting", q denotes " the exercises are challenging" and r denotes " the course is enjoyable". Write the following in symbolic form:

(i) the material is interesting and the exercises are challenging.

(ii) if the material is uninteresting then the exercises are not challenging and the course is not enjoyable.

(iii) If the material is not interesting and the exercise are not challenging then the course is not enjoyable.

(iv) The material is interesting means the excercises are challenging and conversely.

(v) Either the material is interesting or the exercises are not challenging but not both.

5[a]. Solve the recurrence relation

$$a_{n+2} - 6a_{n+1} + 8a_n = n.4^n \; where \;\; a_0 = 8 \; and \; a_1 = 22$$

[b] By using generating function, solve the recurrence relation

$$a_{n+2} - 2a_{n+1} + a_n = 2^n, a_0 = 2 \; and \; a_1 = 1$$

[c] A disconnected graph on n vertices having 5 components is given. Construct a graph on n vertices having the same number of components but having maximum number of edges by giving detailed arguments.