

Paper Code: COF-208

Title of the subject: Algorithm Design and Analysis

Time: 3:00 Hours

Max. Marks: 50

Note: Answer 5 questions. Write pseudo code for all algorithms asked.  
Assume suitable missing data, if any.

Q1. (a) Suppose that a divide and conquer algorithm solves a problem of size  $n$  by first solving three instances of size  $n/2$  and then taking  $O(n)$  additional basic steps. If the time to create the three instances is  $O(n^2)$ , what is the running time,  $T(n)$ , of the algorithm.

(b) solve :  $T(n) = 8T(n/2) + n^3$  and  $T(n) = 7T(n/2) + n^3$

(c) Solve following 0/1 Knapsack problem using Dynamic Programming algorithm for  $m=25$ :

Item	1	2	3	4
Weight	9	8	12	14
Value	10	12	14	16

(2+4+4=10)

Q2 (a) We want to merge  $n$  sorted lists  $L_1, L_2, \dots, L_n$  of sizes  $l_1, l_2, \dots, l_n$ , respectively. Suppose that at any point of time, we are allowed to merge only two sorted lists, that is, simultaneously merging  $t$  sorted lists for  $t > 3$  is not allowed. (For example, the sorted lists may be residing in files in a palmtop computer which has very little memory and allows only three opened file pointers at any time.) The effort associated with merging two sorted lists of sizes  $u$  and  $v$  is taken as  $u + v$ . Your task is to select the merging sequence in such a way that the total effort of merging the given  $n$  lists is minimized. Design one efficient algorithm for doing this task.

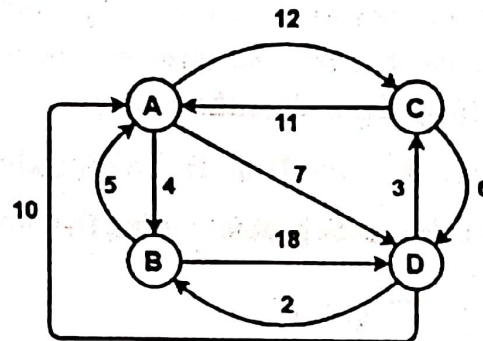
Suppose that  $n = 4$  lists are given with respective sizes 10, 20, 30, 40. Find the efforts of the following two ways of merging  $L_1, L_2, L_3, L_4$ :

$\text{merge}(\text{merge}(L_1, L_2), \text{merge}(L_3, L_4))$  and  $\text{merge}(\text{merge}(\text{merge}(L_1, L_2), L_3), L_4)$ .

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- (b). Compute LCS for following set of sequences:  $X = \text{"PMJYAUZ"}$  and  $Y = \text{"MZJAWPU"}$  using dynamic programming algorithm. Show mathematical formulation of the given problem using dynamic programming. (5+5=10)

- Q3. A traveler needs to visit all the cities from a list, where distances between all the cities are known and each city should be visited just once. What is the shortest possible route that he visits each city exactly once and returns to the origin city? Apply branch and bound paradigm to solve TSP for graph given below.



(10)

- Q4. (a) Solve following 0/1 Knapsack problem using branch and bound technique:

Item	1	2	3	4	5
Weight	2	3	1	5	3
Value	40	50	100	95	30

$W=10$

- (b) Using divide and conquer strategy count the number of inversions (out of order pairs) in an array. Let the elements be  $x_1, \dots, x_n$ . There is an inversion if  $x_i > x_j$  and  $i < j$ .

(5+5=10)

- Q5. (a) Probability of occurrence of various characters in a text segment is as follows:

Character	A	B	C	D	E	F	G	H
Probability	0.10	0.18	0.40	0.05	0.06	0.10	0.07	0.04

Find Huffman encoding for this text and compute average code length. What will be saving compared to fixed length encoding for a text segment of size 1000 characters?

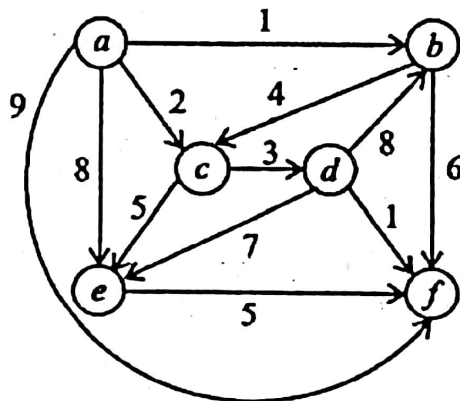
(b) You are given a sequence of  $n$  songs where the  $i^{\text{th}}$  song is  $t_i$  minutes long. You want to place all of the songs on an ordered series of CDs (e.g.  $CD_1, CD_2, CD_3, \dots, CD_k$ ) where each CD can hold  $m$  minutes. Furthermore,

(1) The songs must be recorded in the given order,  $song_1, song_2, \dots, song_n$ .

(2) All songs must be included.

(3) No song may be split across CDs. Your goal is to determine how to place them on the CDs as to minimize the number of CDs needed. Give the most efficient algorithm you can to find an optimal solution for this problem and analyze the time complexity (5+5=10)

Q6. (a) Apply Dijkstra's algorithm to compute Single Source Shortest Path for vertex 'a' as source:



(b) Differentiate between "backtracking" and "branch and bound" strategies.

(5+5=10)

Q7. (a) Prove that vertex cover problem is NP-complete.

(b) Prove that CNF- satisfiability reduces to clique decision problem.

(5+5=10)



Q.8. (a) Solve the following subset problem using backtracking.  $S = (3, 5, 6, 7)$  and  $d = 15$ .

(b) Let  $x = x_1x_2 \dots x_n$  and  $y = y_1y_2 \dots y_m$  and  $z = z_1z_2 \dots z_{n+m}$  be three strings of length  $n$ ,  $m$ , and  $n + m$ , respectively. We say that  $z$  is a merge of  $x$  and  $y$  if  $x$  and  $y$  can be found as two disjoint subsequences in  $z$ . Example: "algotdatastrucrituthresms" is a merge of "algorithms" and "datastructures". For  $0 \leq i \leq n$  and  $0 \leq j \leq m$ ,  $\text{Merge}(i, j)$  is true if  $z = z_1z_2 \dots z_{i+j}$  is a merge of  $x = x_1x_2 \dots x_i$  and  $y = y_1y_2 \dots y_j$  ( $x = x_1x_2 \dots x_i$  is the empty string if  $i = 0$ . Similarly for  $y$  and  $z$ ). Define function  $\text{Merge}(i, j)$  and write code for this.

(5+5=10)