CS2050 Spring 2023 Homework 1

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1. Rewrite each of the following in the form "if ...., then ...". (You may adjust verb tense as you wish to make the sentences sound natural.) (6 points)

   (a) You will do well in discrete math unless you do not study for the exams.

   If you study for the exams, then you will do well in discrete math.

   (b) You can go out to eat only if you remember to bring money.

   If you remember to bring money, then you can go out to eat.

   (c) Doing well on exams is sufficient to pass the course.

   If you do well on exams, then you will pass the course.

2. Evaluate each of the following propositions as True or False. (8 points)

   (a) If three is an even number, then triangles have ten sides.

   True

   (b) If $0 \leq 0$, then $1 > -1$.

   True

   (c) If $3 * 3 = 33$, then $2 + 9 = 11$.

   True

   (d) If 9 is not even, then 10 is odd.

   False

3. Let s be the proposition "You are a student.", let d be the proposition "You are taking Discrete Math.", and let n be the proposition "It is Snowing." Expressing the following as English sentences. (You may adjust tense as you like.) (6 points)

   (a) $(d \wedge n) \rightarrow s$

   If you taking Discrete Math and it is snowing, then you are snowing.

(b) $\neg n \vee (n \to s)$

It is either not snowing or if it is snowing, then you are a student.

(c) $(n \wedge d) \leftrightarrow s$

It is snowing and you are taking Discrete Math if and only if you are a student.

4. Let l be the proposition "You are late for class.", let a be the proposition "You set an alarm", and let h be the proposition "You did your homework." Represent each of the following statements using only l, a, h and logical operators. Then, negate the statements you identify pushing all negations in as far as possible. Then translate it back to English. (6 points)

   (a) You are late for class only when you do not set an alarm or did not do your homework.
      i. $l \to (\neg a \vee \neg h)$
      ii. $l \wedge a \wedge h$
      iii. You are late for class and you set an alarm and you did your homework.

   (b) You are late for class unless you set your alarm.
      i. $a \to \neg l$
      ii. $a \wedge l$
      iii. You set your alarm and you are late to class.

5. Give the converse, contrapositive, and inverse of the statement "I go to bed whenever I finish my school work." (Don't worry about tense, just get the idea correct.) (6 points)

   Converse: I finish my school work whenever I go to bed.

   Contrapositive: I do not finish my school work whenever I don't go to bed.

   Inverse: I do not go to bed whenever I don't finish my school work.

6. Construct truth tables for the following propositions. Include all intermediate columns, in an appropriate order, for full credit. (8 points each)

   (a) $\neg p \to \neg q$

p	q	$\neg p$	$\neg p$	$\neg p \to \neg q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

(b) $(\neg p \wedge q) \rightarrow \neg r$

p	q	r	$\neg p$	$\neg r$	$(\neg p \wedge q)$	$(\neg p \wedge q) \rightarrow \neg r$
T	T	T	F	F	F	T
T	T	F	F	T	F	T
T	F	T	F	F	F	T
T	F	F	F	T	F	T
F	T	T	T	F	T	F
F	T	F	T	T	T	T
F	F	T	T	F	F	T
F	F	F	T	T	F	T

(c) $(p \rightarrow \neg q) \leftrightarrow \neg(p \vee q)$

p	q	$\neg q$	$p \vee q$	$p \rightarrow \neg q$	$\neg(p \vee q)$	$(p \rightarrow \neg q) \leftrightarrow \neg(p \vee q)$
T	T	F	T	F	F	T
T	F	T	T	T	F	F
F	T	F	T	T	F	F
F	F	T	F	T	T	T

7. Simplify each of the following to p, q, $\neg p$, $\neg q$, T or F using logical equivalences. State the equivalence used at each step. Do not skip steps. You can only use one equivalence or definition per step (even if the same one can be applied multiple times). Do not forget about the double negation law. (8 points each)

   (a) $q \rightarrow (p \vee q)$

$$\begin{aligned}
&\equiv \neg q \vee (p \vee q) && \text{(Conditional Identity)}\\
&\equiv \neg q \vee (p \vee q) && \text{(Commutative Law)}\\
&\equiv p \vee (\neg q \vee q) && \text{(Associative Law)}\\
&\equiv p \vee (q \vee \neg q) && \text{(Commutative Law)}\\
&\equiv p \vee T && \text{(Complement Law)}\\
&\equiv T && \text{(Domination Law)}
\end{aligned}$$

   (b) $(p \rightarrow q) \wedge (p \rightarrow \neg q)$.

$$\begin{aligned}
&\equiv (\neg p \vee q) \wedge (p \rightarrow \neg q) && \text{(Conditional Identity)}\\
&\equiv (\neg p \vee q) \wedge (\neg p \vee \neg q) && \text{(Conditional Identity)}\\
&\equiv \neg p \vee (q \wedge \neg q) && \text{(Distributive Law)}\\
&\equiv \neg p \vee F && \text{(Complement Law)}\\
&\equiv \neg p && \text{(Domination Law)}
\end{aligned}$$

8. Prove that $(p \wedge q) \rightarrow q \equiv (p \wedge \neg q) \rightarrow \neg q$ in both of the following ways.

(a) truth table (10 points)

p	q	$p \wedge q$	$\neg q$	$(p \wedge \neg q)$	$(p \wedge q) \rightarrow q$	$(p \wedge \neg q) \rightarrow \neg q$
T	T	F	F	F	T	T
T	F	F	T	T	T	T
F	T	F	F	F	T	T
F	F	F	T	F	T	T

(b) logical equivalences (10 points)

$$
\begin{aligned}
&(p \wedge q) \rightarrow q \\
&\equiv \neg(p \wedge q) \vee q && \text{(Conditional Identity)} \\
&\equiv (\neg p \vee \neg q) \vee q && \text{(De Morgan's Law)} \\
&\equiv \neg p \vee (\neg q \vee q) && \text{(Associative Law)} \\
&\equiv \neg p \vee (q \vee \neg q) && \text{(Commutative Law)} \\
&\equiv \neg p \vee T && \text{(Complement Law)} \\
&\equiv T && \text{(Domination Law)} \\
&\equiv \neg p \vee T && \text{(Domination Law)} \\
&\equiv \neg p \vee q \vee \neg q && \text{(Complement Law)} \\
&\equiv \neg p \wedge \neg\neg q \vee \neg q && \text{(Double Negation Law)} \\
&\equiv \neg(p \wedge \neg q) \vee \neg q && \text{(De Morgan's Law)} \\
&\equiv (p \wedge \neg q) \rightarrow \neg q && \text{(Conditional Identity)}
\end{aligned}
$$

$$\square \quad (p \wedge q) \rightarrow q \equiv (p \wedge \neg q) \rightarrow \neg q$$

9. Vikings always tell the truth and Saxons always lie. Given the following information, use a truth table to determine what type each person is or if their status cannot be determined. Be sure to provide a conclusion based on your work. (8 points)

Person A says: “I am a Viking or B is a Saxon”
Person B says: “A is a Saxon if C is a Viking”
Person C says “I am Saxon or A is a Saxon only if B is a Viking”

Let p be ”Person A is a Viking”, let q be ”Person B is a Viking”, and let r be ”Person C is a Viking”

Given

$$p \vee \neg q \tag{1}$$
$$r \rightarrow \neg p \tag{2}$$
$$q \rightarrow (\neg r \vee \neg p) \tag{3}$$

p	q	r	$\neg p$	$\neg q$	$\neg r$	$\neg r \vee \neg p$	$(p \vee \neg q)$	$(r \to \neg p)$	$(q \to (\neg r \vee \neg p)$
T	T	T	F	F	F	F	T	F	F
T	T	F	F	F	T	T	T	T	T
$\boldsymbol{T}$	$\boldsymbol{F}$	$\boldsymbol{T}$	$\boldsymbol{F}$	$\boldsymbol{T}$	$\boldsymbol{F}$	$\boldsymbol{F}$	$\boldsymbol{T}$	$\boldsymbol{F}$	$\boldsymbol{T}$
T	F	F	F	T	T	T	T	T	T
$\boldsymbol{F}$	$\boldsymbol{T}$	$\boldsymbol{T}$	$\boldsymbol{T}$	$\boldsymbol{F}$	$\boldsymbol{F}$	$\boldsymbol{T}$	$\boldsymbol{F}$	$\boldsymbol{T}$	$\boldsymbol{T}$
F	T	F	T	F	T	T	F	T	T
F	F	T	T	T	F	T	T	T	T
F	F	F	T	T	T	T	T	T	T

There are two row where p, q, r are true or false only when their statements are the same. The values of p, q, r for the rows are T, F, T and F, T, T in respective values of p, q, r

Because there has to be one Viking and one Saxon among Person A and Person B, we can not be sure which is which. However in both situations, Person C is a Viking.

Person A is undetermined.
Person B is not the same type of person as Person A.
Person C is a Viking.