LP AND IP

$\max \; f(s,u) + f(s,v)$

subject to

$f(s,u) = f(u,v) + f(u,t)$

$f(s,v) + f(u,v) = f(v,t)$

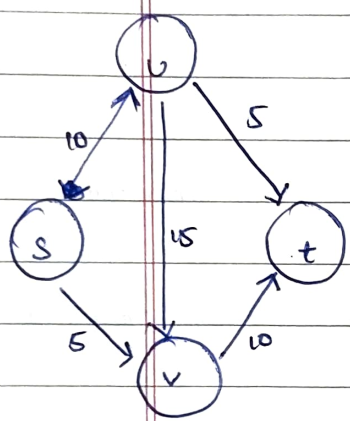
$0 \leq f(s,u) \leq 10$

$0 \leq f(s,v) \leq 5$

$0 \leq f(u,t) \leq 5$

$0 \leq f(u,v) \leq 15$

$0 \leq f(v,t) \leq 10$



$\min \; c'^T x$ | $\max \; c^T x$ | max flow = min cut

$A'x \geq b$ | $Ax \leq b$ | $LP^* = IP^*$

$x \geq 0$ | $x \geq 0$ | duals

edge $\longrightarrow$ $x \in \{0,1\}$ | simplex algorithm

cut or not $\Rightarrow$ 0 or 1

some lucky IP problems can be solved easily.

MINIMIZING NORMS

y

minimize $\|Ax - b\|_1$ L1 norm

subject to $\|x\|_\infty \leq 1$ max. norm

$|y_1| + |y_2| + |y_3| \ldots$

rewrite the above as:

$$\min \sum_{i=1}^{n} y_i$$

$\min 1^T y$

subject to

$$-y_i \leq \sum_{j=1}^{n} (a_{ij} x_j) - b_i \leq y_i \qquad i = 1 \ldots m$$

$$-1 \leq x_j \leq 1 \qquad j = 1 \ldots n$$

what are c, A, b for the LP problem as per the standard norm?

IP WITH BRANCH AND BOUND - EXAMPLE

maximize $x_1 + x_2$

subject to $x_2 - x_1 \leq 2$

$8x_2 + 2x_1 \leq 19$

$x_1, x_2 \geq 0$

$x_1, x_2 \in \mathbb{Z}$

classmate

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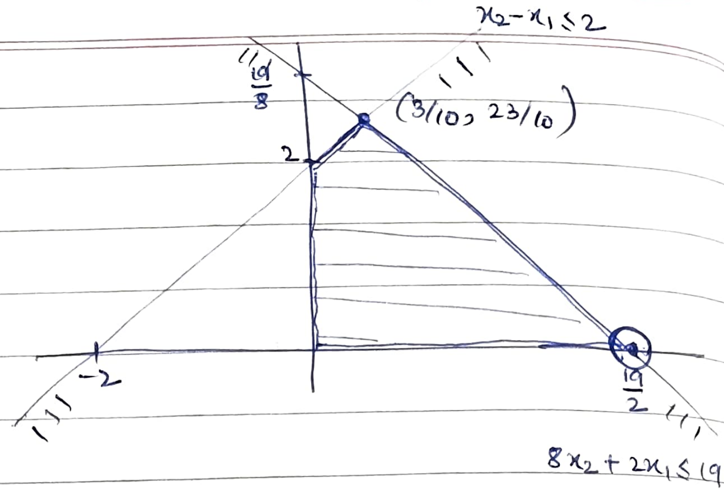


try to find problems with this pattern

$$\max \; c^T x$$
$$Ax \le b$$
$$x \ge 0$$

$$x_2 - x_1 = 2 \quad \times 2$$

$$8x_2 + 2x_1 = 19$$

$$x_1 = x_2 - 2$$

$$10x_2 = 23 \Rightarrow x_2 = \frac{23}{10}$$

$$\Rightarrow x_1 = \frac{3}{10}$$

let us understand the issues

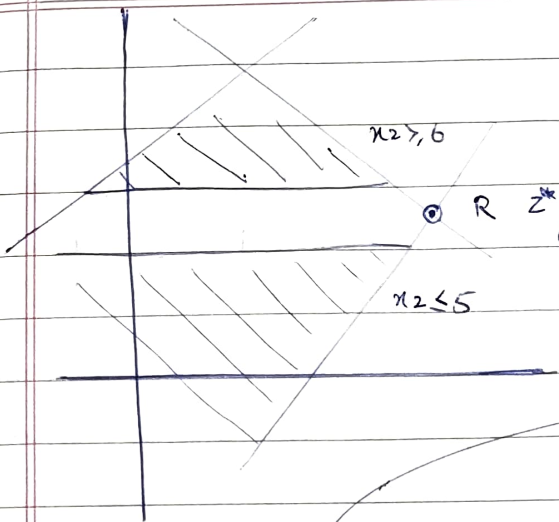
$\boxed{x \in \mathbb{R}^n}$ $\Rightarrow$ LP^* easy

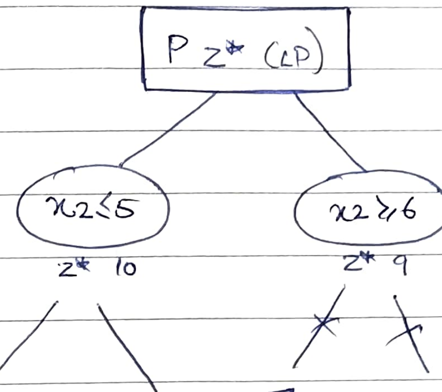
$\boxed{x \in \mathbb{Z}^n}$ $\Rightarrow$ IP^* assume somehow we could solve

① try solving without integer constraint first.

LP relaxation

$\boxed{LP^* \ge IP^*}$ LP more optimal than IP.





until we find an integer
"branch & bound"

$LP^* \geq IP^*$

BRANCH AND BOUND

consider the IP problem.

maximize $c^T x$ subject to $Ax \leq b$, $x \in \mathbb{Z}$

1. initialize constraints as $L = \{ Ax \leq b \}$

2. initialize $\bar{x} = \phi$, $\bar{I} = \infty$. here $\bar{x}$ and I are the optimal soln. and value resp.

3. while $L \neq \phi$

 3.1 pick a sub problem. maximize $c^T x$, $A'x \leq b'$ and solve the LP. also delete subproblem constraint from L.

 3.2 let x^* be the optimum solution to the LP.

 3.3 if $x^* \in \mathbb{Z}$ and $c^T x > I$. then set $\bar{x} = x^*$ and $I = c^T x$.

 3.4 if

(3.2) if $x_j^* \notin \mathbb{Z}$ and $c^T x^* > I$, then add 2 subproblems in L for $x_j^* \notin \mathbb{Z}$.

- max $c^T x$ st. $A'x \leq b'$ & $x_j \leq \lfloor x_j^* \rfloor$ and add to L.
- max $c^T x$ st. $A'x \leq b'$ & $x_j \geq \lceil x_j^* \rceil$ and add to L.

branch & bound is used to solve hard problems.
LP and IP fastest way to get a solution to a problem.

IP FORMULATION: CUTTING THE PAPER ROLL

you have paper rolls of width 3m. you have got an order of the form:

(i) 97 rolls of width 135 cm
(ii) 610 rolls of width 108 cm
(iii) 395 rolls of width 93 cm
(iv) 211 rolls of width 42 cm

- can you write this as the standard LP problem. (List down the possible ways to cut the paper rolls, such as 2×135, $1\times135 + 1\times108 + 1\times42$, etc. note that the sum total of each possibility has to be less than 300. your unknowns are the no. of rolls for which you will use the i^{th} way of cutting the roll. your objective simply the sum of the unknowns).

- can the no. of rolls be non-integer?

- will the rounded-up / rounded-down optimum LP soln. necessarily give us the optimum IP soln.?

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① 2×135

② $1 \times 135 + 1 \times 108 + 1 \times 42$

x_1 rolls in 1st way

x_2 rolls in 2nd way

$\min \sum x_i$

$2x_1 + x_2 \geq 97$

$\vdots$

4 constraints

$x_i \in \mathbb{Z}$