OM-S20-02: Linear Programming Formulations

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07 Jan 2020

Review LP and Graphical Method

$$\max_{x} \; Z = c^T x$$

subject to

$$Ax \leq b; x \geq 0$$

- Draw the constraints as lines and shade the feasible region.
- If feasible region is bounded, find the coordinates of corner points.
- Find which corner point gives the optimal value of the objective.
- If feasible region is empty, then no solution exists.
- Special cases: LP can be either (1) infeasible, (2) unbounded (3) have unique optimal solution value Z^*
  - Q: Does (3) imply every LP have a unique optimal solution x^*
  - Q: Can (2) and (3) occur simultaneously

Line fitting as LP

Problem: We are given a set of N points (x_i, y_i), and we are asked to fit (or find) a line (say $ax + b = y$) that minimize an "error" (in predicting y). i.e., find a and b by

$$\min_{a,b} \sum_{i=1}^{N} |y_i - (ax_i + b)|$$

Our objective is to find a and b corresponding to the optimal line $y = ax + b$.
We can re-write the problem as

$$\min \sum_{i=1}^{N} e_i$$

- What are the constraints? Are they
- What are c, A and b for this LP problem as per the standard form?
- What about L1, L0, L2, $L\infty$ norms?

Pattern classification as LP

Problem: We are given N_1 positive examples and N_2 negative examples. How do we find a separating line that also maximizes a margin/distance from the line.

Maximize δ subject to

$$y_i^+ \geq ax_i^+ + b + \delta \forall i = 1, 2, \ldots, N_1$$

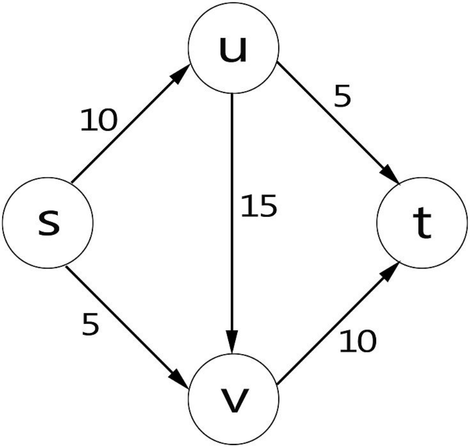
$$y_i^- \leq ax_i^- + b - \delta \forall i = 1, 2, \ldots, N_2$$

- Again find the c, A and b for this LP problem as per the standard form?
- Note that the distances have been measured along Y-axis; not the orthogonal distance from the line $y = a \cdot x + b$
- What if we keep the LHS and RHS as is for both equations but flip the inequality signs. Does anything change?

Max Flow Problem

- Max flow passing from a source node S to a destination node T in a graph G(V,E) is the minimum capacity which when removed from the network results in zero flow from S to T.
- Think of this as the maximum amount of water that can go from A to B in a pipe network in a city.
- You have seen this in you algorithms class as Ford-Fulkerson algorithm.
- This is exactly equal to to the Min S-T cut problem: A cut on minimum sum of edge weights such that S and T are on opposite sides of the cut.
- Review: Maxflow-Mincut theorem

Max Flow Problem



Max Flow Problem

$$\max f(s,u) + f(s,v)$$

subject to

$$f(s,u) = f(u,v) + f(u,t)$$

$$f(s,v) + f(u,v) = f(v,t)$$

$$0 \leq f(s,u) \leq 10$$

$$0 \leq f(s,v) \leq 5$$

$$0 \leq f(u,t) \leq 5$$

$$0 \leq f(u,v) \leq 15$$

$$0 \leq f(v,t) \leq 10$$

Find the c, A, b for the given graph.

Minimizing Norms

Minimize

$$||Ax - b||_1$$

subject to

$$||x||_\infty \leq 1$$

Re-write the above as:

$$\min \sum_{i=1}^{n} y_i$$

subject to

$$-y_i \leq \sum_{j=1}^{n}(a_{ij}x_j) - b_i \leq y_i, i = 1, 2, .., m$$

$$-1 \leq x_j \leq 1, j = 1, 2, .., n$$

What are c, A and b for this LP problem as per the standard form?

Summary

- Two popular problem formulations: LP and IP.
- Many examples of LP
- Once the problem is formulated as LP, “solution is easy”.
- LP:Solving with hand— Graphical Method