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SIMPLEX METHOD - 2

- min $c^T x$ $\quad$ n variables (original)
  $Ax \leq b$ $\quad$ m constraints
  $x \geq 0$

- Constraints are modified with additional slack variables to obtain
  $Ax = b$ $\quad$ $A: m \times (m+n)$
  $x \geq 0$

- basic feasible solution (BFS) by solving
  $Bx = b$ $\quad$ $B: m \times n$ $\quad$ ↳ vertex / corner

- iterate j and I exists from the Basic Variable set B.

- let j be the entering variable and I be the exiting variable.
  $d_j = 1$ for i not in the basic variable set.
  $\wedge$ $d_i = 0$

$$Ax_{new} = A(x + \theta d)$$
$$= Ax = b \Rightarrow Ad = 0$$

$$\sum_{i=1}^{m} d_{B(i)} A_{B(i)} + A_j = 0$$

$$\Rightarrow \bar{B} d_B + A_j = 0 \quad \Rightarrow d_B = -\bar{B}^{-1} A_j$$

SIMPLEX ALGORITHM

- $x_{new} = x_0 I_d + \theta^* d.$

- $\bullet\ d_j = 1$

  $d_B = -B^{-1}A$

  $\bullet\ \theta = \min_{d_i < 0,\ i \in B} \left\{ \frac{-x_i}{d_i} \right\}$

- which j to pick?

for a given variable j, difference in cost due to j^{th} variable being basic is:

$$c_B^T d_B + c_j = c_j - c_B^T B^{-1} A_j$$

we do for each j and select:

$$\bar{c} = | c_1, c_2, \ldots, c_n |$$

where $\bar{c}_j = c_j - c_B^T B^{-1} A_j$

- compute the reduced costs $\bar{c}_j$

$$\bar{c}_j = c_j - c_B^T B^{-1} A_j$$

for all non-basic indices j.

if they are all non-negative, the current basic feasible solution is optimal, the algorithm terminates; else, choose some j for which $c_j < 0$.

- $B^{-1}B = I = [e_1 \; e_2 \; e_3 \dots e_m]$

$= [e_1 \dots e_{l-1} \; u \; e_{l+1} \dots e_m]$

apply row operations to product matrix, changing the column u to e_l.

SIMPLEX TABLEU

$-c_B^T B^{-1} b$	$c^T - c_B B^{-1} A$
$B^{-1} b$	$B^{-1} A$

often start with a simple case of $B = I$

- we start with a basis $\bar{B} = [A_{B(1)}, \dots, A_{B(m)}]$ and associated solution x.

- compute the reduced cost $\bar{c}_j = c_j - c_B^T \bar{B}^{-1} A$ for each non-basic variable j if they are all positive, current solution is optimal, so exit else choose j such that $\bar{c}_j < 0$.

- compute $u = B^{-1}A_j$ if no component of u is positive, we have $\theta^* = \infty$ and optimal cost $= -\infty$. exit.

- if for some computation, u_i is positive then,
$$\theta^* = \min_{\{i \mid u_i \geq 0\}} (A_{B(i)} / u_i)$$

- if I is the variable which minimizes then I exists and j enters. For a new basis by replacing $A_{B(i)}$ by A_j.

EXAMPLE 1

$$\min \; -x_1 - x_2$$

$$\text{s.t.} \quad -x_1 + x_2 + x_3 = 1$$

$$x_1 + x_4 = 3$$

$$x_2 + x_5 = 2$$

$$x_i \geq 0$$

1

	0	-1	-1	0	0	0
x_3	1	-1	1	1	0	0
x_4	3	1	0	0	1	0
x_5	2	0	1	0	0	1

2

	1	-2	0	1	0	0
x_2	1	-1	1	1	0	0
x_4	3	1	0	0	1	0
x_5	1	1	0	-1	0	1

3

	3	0	0	-1	0	2
x_2	2	0	1	0	0	1
x_4	2	0	0	1	1	-1
x_1	1	1	0	-1	0	1

4

	5	0	0	0	1	1
x_2	2	0	1	0	0	1
x_3	2	0	0	1	1	-1
x_1	3	1	0	0	1	0

EXAMPLE 2

min $-10x_1 - 12x_2 - 12x_3$

s.t. $x_1 + 2x_2 + 2x_3 \leq 20$

$2x_1 + x_2 + 2x_3 \leq 20$

$2x_1 + 2x_2 + x_3 \leq 20$

$x_1, x_2, x_3 \geq 0$

1

	0	-10	-12	-12	0	0	0
x_4	20	1	2	2	1	0	0
x_5	20	2	1	2	0	1	0
x_6	20	2	2	1	0	0	1

2

	100	0	-7	-2	0	5	0
x_4	10	0	1.5	1	1	-0.5	0
x_1	10	1	0.5	1	0	-0.5	0
x_6	0	0	1	-1	0	-1	1

3

	120	0	-4	0	2	4	0
x_3	10	0	1.5	1	1	-0.5	0
x_1	0	1	-1	0	-1	1	0
x_6	10	0	2.5	0	1	-0.5	1

4)

	136	0	0	0	3.6	1.6	1.6
x_3	4	0	0	1	0.4	0.4	-0.6
x_1	4	1	0	0	-0.6	0.4	0.4
x_2	4	0	1	0	-0.1	-0.8	0.4