Date 11.2.2000
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## FACTORIZATION OF MATRICES

Cholosky

every PD matrix A can be factorized as.

A = LL<sup>T</sup>

cohere L is a lower A matrix.

[a\_1 A2T] - l\_1 0 | l\_1 L2T |

A21 A22 | L21 L22 | 0 L7Z.

LU

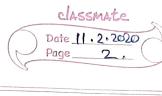
A need not be PD; only needs to be non-singular.

A =  $\kappa = b$ Cotaphitation  $\Rightarrow \kappa = A'b$ 

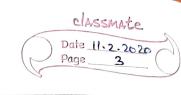
of we have a structure

cold be done efficiently.

I, PDD, Tr. 11 PD matrix, cholesky, W



factorization / decomposition Costly) SVD > cholesky. LUn=b easy solution (20) 1 matrix Ux = w if step () is used extensively, then ok. COMPUTATION Lu cholesky lower D an = lu-lu > lu = vair 0. => Rel21 = 1 A21 (2)



$$A_{22} = L_{21} L_{21} + L_{22} L_{22}$$

$$\Rightarrow$$
  $L_{22}L_{22} = A_{22} - L_{21}L_{21}^{T}$  (3)

another cholesky decomposition (n-1)x(n-1)

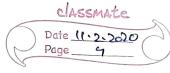
$$\Rightarrow$$
  $o_{ij} = a_{ij}$ 

$$\Rightarrow U_{12} = A_{12} \qquad \boxed{2} \qquad \boxed{1} \qquad \boxed{1}$$

$$A_{21} = L_{21} \cup U_{11}$$

 $\alpha_{ij} = l_{ij} \cup_{ij}$  (6).

$$\Rightarrow L_{21} = \int_{\mathcal{U}_{11}} A_{21} \qquad \boxed{3}$$

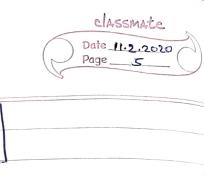


An 
$$2 = L_{21} U_{12} + L_{22} U_{12}$$

$$\Rightarrow L_{22} U_{22} = A_{22} - L_{21} U_{12} \qquad G$$

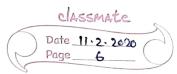
another LU decomposition

$$\frac{1}{3} \frac{1}{3} \frac{$$



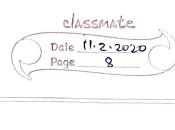
$$L_{21} = \int A_{21}^{1} = \int \times 20 = 5$$

$$\frac{1}{22} \frac{1}{22} = \frac{1}{25} =$$



	L = 1 0 0 0 U = 6 3 1
	Y <sub>3</sub> 1 0 0 3 8/3
	3/2 1/6 1 0 0 1/18
	· ·
	Lw=6 >> w= [18 11 1/6]
	$2 \times 2 \times$
	MD DEALLONGITION
	QR DECOMPOSITION
,	1
1	· a square mortrix Q à souid to be orthogonal.
	when:
	$QQ^{T} = Q^{T}Q = I$
0	QR decomposition: A=QR
	-Q is orthogonal.
	- R is an upper 1 matrix.
	N IS GV) OPSET A MATERIA,
	why is this useful for us?
	- because QRx lets us write
	Rx = QTb which is easy to solve fore
	1
9	many problems can be solved faster (and reliably)
	by QR factorization.
	- least squares
	- least norm.
	Kenst 110 km

Ax = 6 costly step QR x = 6 Rm = QTb upper 1 matrix  $\begin{array}{c|c}
n \times (n-1) & \\
n \times 1 & \\
n \times (n-1) \\
\end{array}$   $Q = \begin{bmatrix} q_1 & Q_2 \\
\end{array}$   $R = \begin{bmatrix} q_1 & Q_2 \\
\end{array}$ (x(n-1). lxl R12 R22 (n-1) x (n-1) QQT = I Given). agueriage leg notice that QTQ = I implies 9,9,=1 2,70 = 0  $Q_2^T Q_2 = I$ conat is this step? placifor a, = 9, n, taking norm gives us "1 = a;  $a_1 = \int_{a_1} a_1$ 



- QIR12=0?

$$A_2 = Q_1 R_{12} + Q_2 R_{22}$$

$$Q_1^T A_2 = R_{12} + 0$$

$$Q_2 R_{22} = A_2 - Q_1 R_{12}$$
 $(n+1)x(n-1)$ 
 $Q_2 R_{23} = A_2 - Q_1 R_{12}$ 
 $Q_3 R_{12} = Q_1 R_{12}$ 

$$2 A^{T}A \pi - 2 A^{T}b = 0$$

$$A^TAx = A^Tb$$