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SIMPLEX METHOD-2

- min $c^T x$ n variables (original)

  s.t. $Ax \leq b$ m constraints

  $x \geq 0$

- constraints are modified with additional slack variables to obtain:

  $Ax = b$ $A: m \times (m+n)$

  $x \geq 0$

- basic feasible solution (BFS) by solving

  $Bx = b$ $B: m \times m$ BFS is vertex / corner

- iterate j enters and I exists from the basic variable set B.

- let j be the entering variable and I be the existing variable.

  $d_j = 1,$

  $d_i = 0$ for i not in basic variable set

$$AX_{new} = A(x + \theta d) = Ax = b \Rightarrow Ad = 0$$

$$\sum_{i=1}^{m} d_{B(i)} A_{B(i)} + A_j = 0,$$

$\Rightarrow \cancel{\sum_{i=1}^{m} d_{B(i)} A_{B(i)} + A_j}$

$$\Rightarrow \bar{B} d_B + A_j = 0, \quad \Rightarrow d_B = -\bar{B}^{-1} A_j$$

SIMPLEX ALGORITHM

- $x_{new} = x_0 I_d + \theta^* d$

- $d_j = 1; \quad d_B = -B^{-1}A_j$

- $\theta = \min_{\substack{i \in B \\ d_i < 0}} \left\{ -\frac{x_i}{d_i} \right\}$

- which j to pick? for a given variable j, difference in cost due to j^{th} variable being basic is:

$$c_B^T d_B + c_j = c_j - c_B^T B^{-1} A_j$$

we do for each j and select:

$$\bar{c} = | c_1, c_2, \ldots c_n |$$

where $\bar{c}_j = c_j - c_B^T B^{-1} A$

- Compute the reduced costs

$$\bar{c_j} = c_j - c_B^T B^{-1} A$$

for all nonbasic indices j. if they are all non-negative, the current basic feasible soln. is optimal, and the algorithm terminates; else choose some j for which $\bar{c_j} < 0$.

- $B^{-1}B = I = [e_1\ e_2 \dots e_m]$

$$= [e_1 \dots e_{l-1}\ u\ e_{l+1} \dots e_m]$$

apply row operations to product matrix, changing the column u to e_i.

SIMPLEX TABLEU

$-C_B^T B^{-1} b$	$C^T - C_B B^{-1} A$
$B^{-1} b$	$B^{-1} A$

often start with a simple case of $B = I$

1. we start with a basis

$$\bar{B} = [A_{B(1)}, \dots, A_{B(m)}]$$

and associated solution x.

2. compute the reduced cost $\bar{c}_j = c_j - C_B^T \bar{B}^{-1} A_j$

for each non-basic variable j if they are all positive, current solution is optimal, so exit else choose j such that $\bar{c}_j < 0$.

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3. compute $u = B^{-1}A_j$ if no component of u is positive, we have $\theta^* = \infty$ and optimal cost $= -\infty$. exit.

4. if for some computation, u_i is positive then,

$$\theta^* = \min_{\{i / u_i \geq 0\}} (x_{B(i)} / u_i)$$

5. if I is the variable which minimizes then I exits and j enters for a new basis by replacing $A_{B(i)}$ by A_j.

EXAMPLE 1

min $-x_1 - x_2$

st. $-x_1 + x_2 + x_3 = 1$

$x_1 + x_4 = 3$

$x_2 + x_5 = 2$

$x_i \geq 0$