

LINEAR PROGRAMMING

max  
 $x_1, x_2$

$$3x_1 + 2x_2$$

← objective  
function

subject to

$$2x_1 + x_2 \leq 6$$

$$7x_1 + 8x_2 \leq 28$$

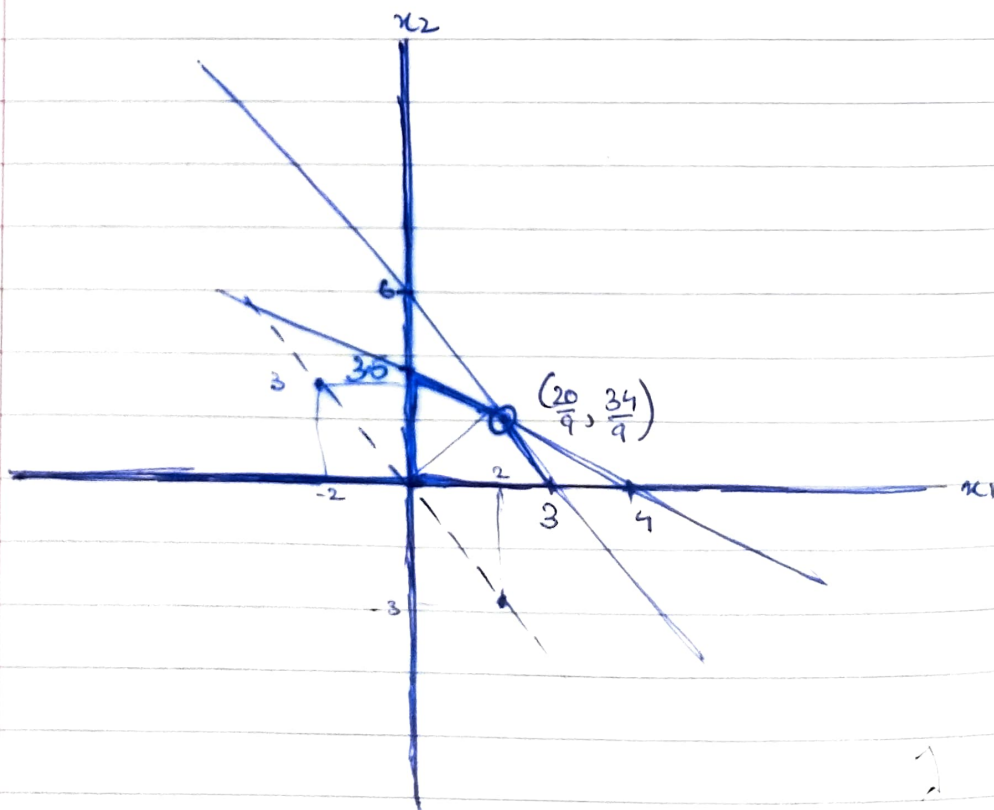
} constraints

$$x_1 \geq 0 \quad x_2 \geq 0 \quad x_1, x_2 \in \mathbb{R}$$

- feasible region  
solution space

- feasible solution  
(any pt. in feasible region)

- optimal solution.



$$2x_1 + x_2 = 6 \quad x_1 \geq 0 \quad \text{and} \quad 16x_1 + 8x_2 = 48$$

$$7x_1 + 8x_2 = 28 \quad x_1 \geq 0$$

$$7x_1 + 8x_2 = 28$$

$$9x_1 = 20$$

$$x_1 \leq \frac{20}{9}$$

$$x_1 \geq 0$$

$$x_1 \geq \frac{20}{9}$$

$$x_2 = 6 - 2x_1 = 6 - \frac{20}{9} = \frac{54 - 20}{9} = \frac{34}{9}$$

53x for minimization problem

n variables

m constraints

max

$$Z = C^T x$$

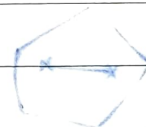
$$C : n \times 1$$

subject to  $Ax \leq b$  and  $x \geq 0$

$$A : m \times n$$



$$x \geq 0$$



$$A : m \times n$$

$$b : m \times 1$$

① for minimization problem:

$$\min_x Z = C^T x = \max_x -C^T x$$

② one of the constraints is  $|x_i| < 3$

$$3 \times \frac{20}{9} + 2 \times \frac{34}{9}$$

$$= \frac{60 + 68}{9}$$

$$x_i < 3$$

$$\text{and } x_i > -3$$

$$3 \pm \epsilon$$

$$x_i > -3 + \epsilon$$

$$= \frac{128}{9}$$

$$9 \times 1 = 9$$

③ what if one of the constraint is  $|x_i| = 3$

$$x_i = +3$$

or

$$x_i = -3$$

$$x_i \geq 3$$

or

$$x_i \leq -3$$

$$x_i \leq 3$$

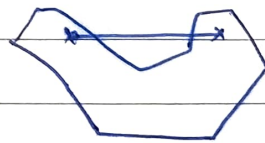
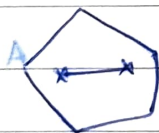
$$x_i \leq -3$$

maybe we have to solve it as 2 separate linear programming problems,

④ problem with added constraint of  $x \in \mathbb{Z}$  (integer programming).

$$x^T c = \sum_j x_j c_j$$

convex polygon & non-convex polygon



minimization problem

$$x^T c = \sum_j x_j c_j$$

$$x^T c = \sum_j x_j c_j$$