OM-S20-16: Simplex method - II

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Simplex: Recap

- Min $c^T x$ subject to $Ax \leq b$ and $x \geq 0$. n variables (original) and m constraints.
- Constraints are modified with additional slack variables to obtain $Ax = b$ with $x \geq 0$. A is $m \times (m + n)$
- Basic Feasible Solution (BFS) by solving $Bx = b$. BFS is a vertex/corner. B is $m \times m$
- Iterate j enters and l exists from the Basic Variable set B
- Let j be the entering variable and l be the exiting variable. $d_j = 1$ and $d_i = 0$ for i not in the basic variabel set.

$$Ax_{new} = A(x + \theta d) = Ax = b \implies Ad = 0$$

$$\sum_{i=1}^{m} d_{B(i)} A_{B(i)} + A_j = 0, \implies \bar{B} d_B + A_j = 0, \implies d_B = -\bar{B}^{-1} A_j$$

Simplex Algorithm

- $x_{new} = x_old + \theta^* d$
  - $d_j = 1$; $d_B = -B^{-1}A_j$
  - $\theta = \min_{\substack{i \in \mathbf{B} \\ d_i < 0}} \left\{ \frac{-x_i}{d_i} \right\}$
- **which j to pick?** For a given variable j, difference in cost due to j^{th} variable being basic is: $c_B^T d_B + c_j = c_j - c_B^T B^{-1} A_j$. We do for each j and select:

$$\bar{C} = [C_1, C_2, .., C_n]$$

$$\text{where } \bar{C_j} = C_j - C_B^T B^{-1} A$$

- Compute the reduced costs $\overline{c}_j = c_j - c_B^T B^{-1} A_j$ for all nonbasic indices j. If they are all nonnegative, the current basic feasible solution is optimal, and the algorithm terminates; else, choose some j for which $c_j < 0$.
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$$\mathbf{B}^{-1}\mathbf{B} = I = [\mathbf{e_1}\ \mathbf{e_2} \cdots \mathbf{e_m}] = [\mathbf{e_1} \cdots \mathbf{e_{l-1}}\ \mathbf{u}\ \mathbf{e_{l+1}} \cdots \mathbf{e_m}]$$

  Apply row operations to product matrix, changing the column u to e_l

Simplex Tableu

-$C_B^T B^{-1} b$	C^T-$C_B B^{-1} A$
$B^{-1}b$	$B^{-1}A$

Often start with a simple case of $B = I$.

1. We start with a basis $\bar{B} = [A_{B(1)}, \ldots, A_{B(m)}]$ and associated solution **x**.
2. Compute the reduced cost $C_j = C_j - C_B^T \bar{B}^{-1} A_j$ for each non basic variable j if they are all positive, current solution is optimal, so exit else choose j such that $C_j < 0$.
3. Compute $u = B^{-1} A_j$ if no component of u is positive, we have $\theta^* = \infty$ and optimal cost $= -\infty$. Exit.
4. If for some computation, u_i is positive then, $\theta^* = \min_{\{i/u_i \geq 0\}} (A_{B(i)}/u_i)$
5. If l is the variable which minimizes then l exits and j enters. For a new basis by replacing $A_{B(i)}$ by A_j.

Example 1 (from last class)

Min $-x_1-x_2$ Subject to: $-x_1+x_2+x_3 = 1$; $x_1+x_4 = 3$; and $x_2+x_5 = 2$; $x_i \geq 0$

1

	0	-1	-1	0	0	0
x_3	1	-1	1	1	0	0
x_4	3	1	0	0	1	0
x_5	2	0	1	0	0	1

3

	3	0	0	-1	0	2
x_2	2	0	1	0	0	1
x_4	2	0	0	1	1	-1
x_1	1	1	0	-1	0	1

2

	1	-2	0	1	0	0
x_2	1	-1	1	1	0	0
x_4	3	1	0	0	1	0
x_5	1	1	0	-1	0	1

4

	5	0	0	0	1	1
x_2	2	0	1	0	0	1
x_3	2	0	0	1	1	-1
x_1	3	1	0	0	1	0

Example 2

Min $-10x_1-12x_2-12x_3$ Subject to: $x_1+2x_2+2x_3 \leq 20$; $2x_1+x_2+2x_3 \leq 20$; $2x_1+2x_2+x_3 \leq 20$; and $x_1,x_2,x_3 \geq 0$

1

	0	-10	-12	-12	0	0	0
x_4	20	1	2	2	1	0	0
x_5	20	2	1	2	0	1	0
x_6	20	2	2	1	0	0	1

2.

	100	0	-7	-2	0	5	0
x_4	10	0	1.5	1	1	-0.5	0
x_1	10	1	0.5	1	0	0.5	0
x_6	0	0	1	-1	0	-1	1

	120	0	-4	0	2	4	0
x_3	10	0	1.5	1	1	-0.5	0
x_1	0	1	-1	0	-1	1	0
x_6	10	0	2.5	0	1	-0.5	1

4

	136	0	0	0	3.6	1.6	1.6
x_3	4	0	0	1	0.4	0.4	-0.6
x_1	4	1	0	0	-0.6	0.4	0.4
x_2	4	0	1	0	0.1	-0.6	0.4