OM-M20-05: IP Formulations

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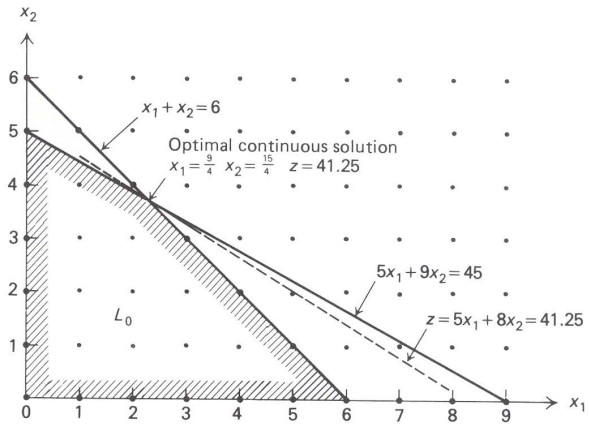
IP formulations

Maximize $5x_1 + 8x_2$ s.t $x_1 + x_2 \leq 6$ and $5x_1 + 9x_2 \leq 45$, $x_1, x_2 \leq 0$ and integer



	Continuous optimum	*Round off*	*Nearest feasible point*	*Integer optimum*
x_1	$\frac{9}{4} = 2.25$	2	2	0
x_2	$\frac{15}{4} = 3.75$	4	3	5
z	41.25	Infeasible	34	40

- **Knapsack Style** Max $c^T x$ s.t $Ax \leq b$.
  - x real number
  - x integers
  - x 0 or 1 (Binary)
- **Terminologies:** LP, IP, MIP, BIP

MIP formulation - Function of K discrete variables

- Say you have to go from A to B; You can go by bus, bike or car.
- You are allowed to spend Rs 5 if you choose bus, Rs 10 if you choose bike and Rs 20 if you choose car.
- We introdice a set of new variables y_i. Let y_i, denote whether you choose $vehicle_i$ and x_i denote the amount spent on $vehicle_i$.
- The constraints can be written as

$$y_1 + y_2 + y_3 = 1$$

$$x_1 + x_2 + x_3 = 5y_1 + 10y_2 + 20y_3$$

$$y_i \in \{0, 1\}; i = 1, 2, .., 5$$

- Notice that y_i's need to be 0 or 1 but no such constraint x_i's.

Setting up a Warehouse

- Let f_i is the fixed operating cost of warehouse i. c_{ij} is the per unit operating cost of warehouse i plus transportation cost for shipping from warehouse i to customer j.
- $y_i = 1$, if warehouse is opened; Goods can be shipped only if it is opened.
- d_j is the demand of customer j

$$Minimize \sum_i \sum_j c_{ij} x_{ij} + \sum_i f_i y_i$$

- Complete the problem statement and write the entire constraints.

Constraints

- Multiple choice problems:

$$\sum_{i=1}^{n} y_i \leq 1 \text{ or } = 1$$

- A specific constraint is satisfied:

$$f(x_1, x_2, \ldots, x_n) < b$$

How do we make a constraint to be trivially True/Satisfied?

- Either of the constraints to be satisfied?

$$f_1(x_1, x_2, \ldots, x_n) - By < b_1$$

$$f_2(x_1, x_2, \ldots, x_n) - B(1 - y) < b_2$$

- What does it mean when y is 0 and 1 respectively?
- When only m out of the n constraints to be true?
- When only one set of the multiple sets of constraints to be true?

IP formulations: Cost of Production

- **Fixed Cost + Variable Cost**

$$K + Cx$$

- **Piece-wise Linear Cost**
  - If the production is below 4000 units, unit price is c_1
  - If the production is between 4000 and 9000 units, unit price is c_2
  - If it is above 8000, and below 15000then cost is c_3

$$0 \leq x_1 \leq 4000; 0 \leq x_2 \leq 5000; 0 \leq x_3 \leq 6000$$

$$cost = c_1 x_1 + c_2 x_2 + c_3 x_3$$

$$4000 w_1 \leq x_1 \leq 4000; 9000 w_2 \leq x_2 \leq 9000 w_1; 0 \leq x_3 \leq 6000 w_2$$