OM-M20-06: IP formulations (Cont.)

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IP formulations: Cost of Production

- **Fixed Cost + Variable Cost**

$$K + Cx$$

- **Piece-wise Linear Cost**
  - If the production is below 4000 units, unit price is c_1
  - If the production is between 4000 and 9000 units, unit price is c_2
  - If it is above 9000 and below 15000then cost is c_3

$$0 \le x_1 \le 4000; 0 \le x_2 \le 5000; 0 \le x_3 \le 6000$$

$$cost = c_1x_1 + c_2x_2 + c_3x_3$$

$$4000w_1 \le x_1 \le 4000; 5000w_2 \le x_2 \le 5000w_1; 0 \le x_3 \le 6000w_2$$

Bala's Algorithm for BIP (Branch and Bound for BIP?)

$$\min Z = \sum_i c_i x_i ; x_i \in \{0, 1\}$$

subject to

$$\sum a_{ij} x_j \geq b_i \forall i \in 1, 2, .., m$$

$$0 \leq c_1 \leq c_2 \leq .. \leq c_n$$

- Start expanding tree with x_1, then move to x_2 and so on. At each step, evaluate Z, check if any constraint is violated. When you find a feasible point you start expanding down that tree. Also prune a branch if Z_{node} is not good enough w.r.t. best Z you saw till now.

Bala's Algorithm - Example

Minimze

$$Z = 3x_1 + 5x_2 + 6x_3 + 9x_4 + 10x_5 + 10x_6$$

subject to

$$-2x_1 + 6x_2 - 3x_3 + 4x_4 + x_5 - 2x_6 \geq 2$$

$$-5x_1 - 3x_2 + x_3 + 3x_4 - 2x_5 + x_6 \geq -2$$

$$5x_1 - x_2 + 4x_3 - 2x_4 + 2x_5 - x_6 \geq 3$$

$$x_i \in \{0, 1\} \forall i \in 1, 2, .., 6$$

Note: There is an interesting animation demonstrating this algorithm at `http://optlab-server.sce.carleton.ca/POAnimations2007/BalasAddAlg.html`

Maximum weight bipartite matching

Given a bipartite graph G(V,E) with equal number of nodes $|X| = |Y|$ on each side with edges $X \rightarrow Y$ only, a perfect matching $M \subset E$ is such that each vertex in X as well as Y appears only once in M.
We want to find an M with the maximum weight ; $max \sum_{e \in M}$

$$max \sum_{e \in E} w_e x_e$$

subject to

$$\sum_{v \in e} x_e = 1 \forall v \in V$$

$$x_e \in \{0, 1\} \forall e \in E$$

- **Min Vertex Cover** Find $min|V'|, V' \subset V$ such that $\forall (u, v) \in e, e \in E$, either $u \in V'$ or $v \in V'$ or both.
- In English: Smallest set such that at least one end of every edge is a member of the set.
- IP formulation:

$$\min \sum_{v \in V} x_v$$

subject to

$$x_u + x_v \geq 1 \forall (u, v) \in E$$

$$x_u \in \{0, 1\}$$

Maximum Independent Set

- $\max_{v \in V} x_v$ such that $x_u + x_v \leq 1 \forall (u, v) \in E$, $x_u \in \{0, 1\}$
- ie Maximum size set such that no two vertices in it are connected by an edge.

Review Link: `http://tiny.cc/c072iz`