OM-S20-04: LP and IP

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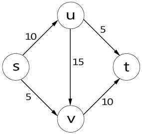
Max Flow Problem (Review)

$$\max f(s,u) + f(s,v)$$

subject to

$$f(s,u) = f(u,v) + f(u,t)$$
$$f(s,v) + f(u,v) = f(v,t)$$

$0 \leq f(s,u) \leq 10; 0 \leq f(s,v) \leq 5; 0 \leq f(u,t) \leq 5; 0 \leq f(u,v) \leq 15; 0 \leq f(v,t) \leq 10$



$$MaxFlow = MinCut$$

$$LP^* = IP^*$$

Possible?

Minimizing Norms

Minimize

$$||Ax - b||_1$$

subject to

$$||x||_\infty \leq 1$$

Re-write the above as:

$$\min \sum_{i=1}^{n} y_i$$

subject to

$$-y_i \leq \sum_{j=1}^{n}(a_{ij}x_j) - b_i \leq y_i, i = 1, 2, .., m$$

$$-1 \leq x_j \leq 1, j = 1, 2, .., n$$

What are c, A and b for this LP problem as per the standard form?

IP with Branch and Bound - Example

Maximize

$$x_1 + x_2$$

subject to

$$x_2 - x_1 \leq 2$$

$$8x_2 + 2x_1 \leq 19$$

$$x_1, x_2 \geq 0$$

$$x_1, x_2 \in Z$$

Note: Nice animation of branch and bound: `http://optlab-server.sce.carleton.ca/POAnimations2007/BranchAndBound.html`

Branch and Bound

Consider the IP problem: Maximize $c^T x$ subject to $Ax \leq b$, $x \in Z$

1. Initialize constraints as $L = \{Ax \leq b\}$
2. Initialize $x^- = \phi, l = -\infty$. Here x^- and l are the optimal solution and value resp.
3. While $L \neq \phi$
   3.1 Pick a subproblem: Maximize $c^T x, A'x \leq b'$ and solve the LP. Also delete subproblem constraint from L.
   3.2 Let x^* be the optimum solution to the LP.
   3.3 If $x^* \in Z$ and $c^T x > l$. Then set $x^- = x^*$ and $l = c^T x^*$
   3.4 If $x_j^* \notin Z$ and $c^T x^* > l$. Then add two subproblems in L for $x_j^* \notin Z$
      - Max $c^T x$ such that $A'x \leq b'$ and $x_j \leq \lfloor x_j^* \rfloor$ and add to L
      - Max $c^T x$ such that $A'x \leq b'$ and $x_j \geq \lceil x_j^* \rceil$ and add to L

IP formulation: Cutting the paper roll

You have paper rolls of width 3m. You have got an order of the form: (i) 97 rolls of width 135 cm (ii) 610 rolls of width 108 cm (iii) 395 rolls of width 93 cm (iv) 211 rolls of width 42 cm
What is the smallest number of rolls you need?

- Can you write this as the standard LP problem. [Hint: List down the possible ways to cut the paper rolls such as 2x135, 1x135+1x108+1x42 etc. Note that the sum total of each possibility has to be less than 300. Your unknowns are the number of rolls for which you will use the i^{th} way of cutting the roll. Your objective: simply the sum of the unknowns]
- Can the number of rolls be non-integer?
- Will the rounded-up/rounded-down optimum LP solution necessarily give us the optimum IP solution?