TUTORIAL-1

BRANCH AND BOUND TECHNIQUE

① divide a problem into subproblems.

② for each subproblem:

(a) if it has no feasible soln; done

(b) if it has an integer optimal soln, done. compare the optimal soln. with the best soln. we know till now.

(c) if it has an optimal soln., that is coorse than the previous best soln., done.

(d) if it has an optimal soln. that are not all integer, better than the previous solution, then we would have to divide this subproblem further and repeat.

P:
$$\max \ z = -x_1 + 4x_2$$

Such that
$$-10x_1 + 20x_2 \leq 22$$

$$5x_1 + 10x_2 \leq 49$$

$$x_1 \leq 5$$

$$x_i \geq 0, \quad x_i \in \mathbb{Z}$$

for LP relaxation:

Optimal solution of the relaxation is $(3.8, 3)$ with $z = 8.2$

then we consider 2 cases $x_1 \geq 4$, $x_1 \leq 3$
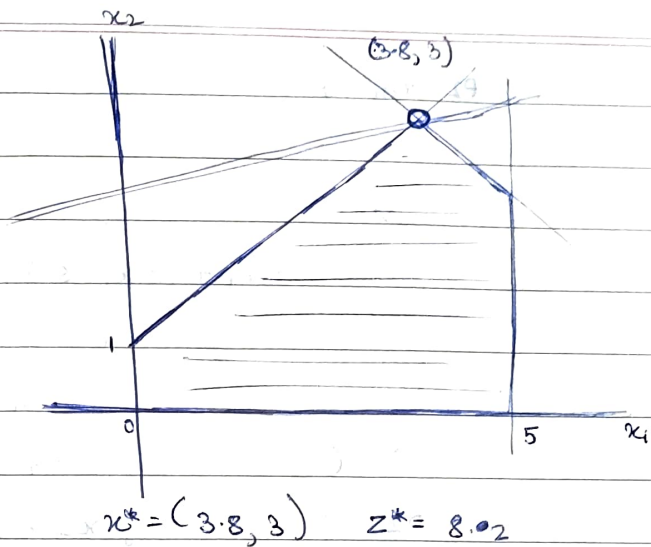
P_1 P_2

for lP relaxation

$$\max \quad z = -x_1 + 4x_2$$

$$\text{st.} \quad -10x_1 + 20x_2 \leq 22$$
$$5x_1 + 10x_2 \leq 49$$
$$x_1 \leq 5$$
$$x_i \geq 0$$

(3.8, 3)

$$x^* = (3.8, 3) \qquad z^* = 8.02$$

the lP relaxation

$$\max. \quad z = -x_1 + 4x_2$$

$$\text{st.} \quad -10x_1 + 20x_2 \leq 22$$
$$5x_1 + 10x_2 \leq 49$$
$$x_1 \leq 5$$
$$x_1 \geq 4$$
$$x_2 \geq 0$$

z-line

(4, 2.9)

$$x^* = (4, 2.9) \qquad z^* = 7.6$$
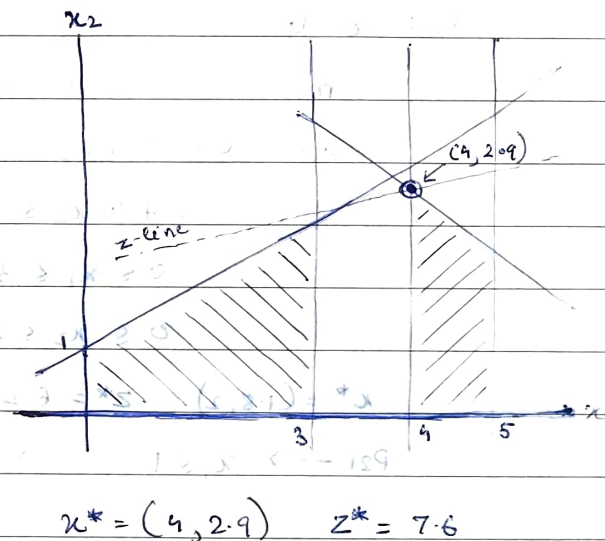
$$P1 \longrightarrow x_2 \leq 2, \quad x_2 \geq 3 \longleftarrow \text{has no feasible soln.}$$
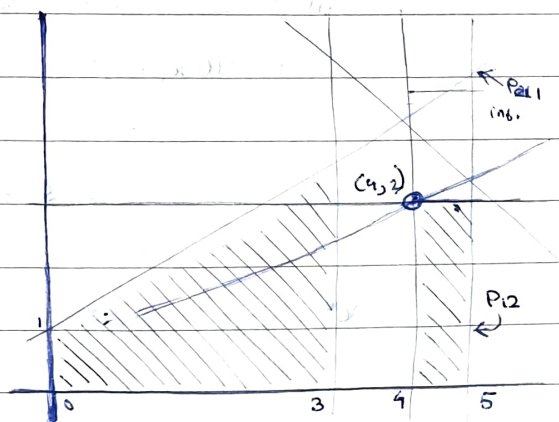
the lp relaxation

$$\max. \quad z = -x_1 + 4x_2$$
$$\text{st.} \quad -10x_1 + 20x_2 \leq 22$$
$$5x_1 + 10x_2 \leq 49$$
$$4 \leq x_1 \leq 5$$
$$0 \leq x_2 \leq 2$$

P₂₁
ing.

(4,2)

P₁₂

$$x^* = (4, 2) \qquad z^* = 4 \qquad (\text{lets try } P_2)$$

P2: the lp relaxation

$$\max. \ z = -x_1 + 4x_2$$
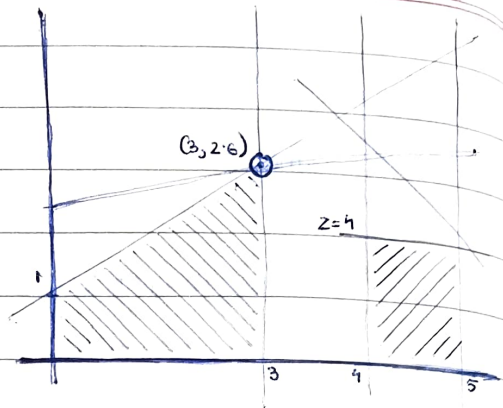
st. $-10x_1 + 20x_2 \le 22$

$5x_1 + 10x_2 \le 49$

$x_1 \le 3$

$0 \le x_i$

$x^* = (3, 2·6) \qquad z^* = 7·4$

P2 $\longrightarrow$ $\underset{P21}{x_2 \le 2}$, $\underset{P22}{x_2 \ge 3}$ $\leftarrow$ has no feasible soln.

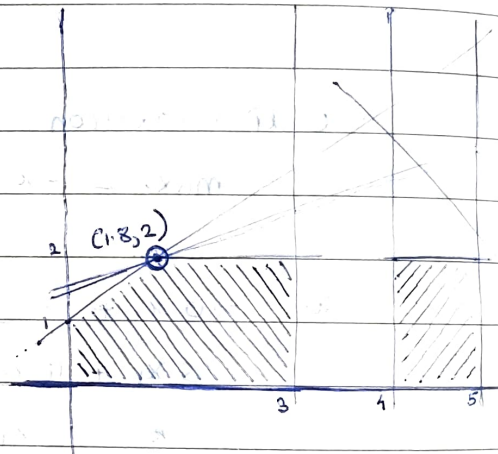(3, 2·6)

z = 4

P21: the lp relaxation

$$\max. \ z = -x_1 + 4x_2$$

st. $-10x_1 + 20x_2 \le 22$

$5x_1 + 10x_2 \le 49$

$0 \le x_1 \le 3$

$0 \le x_2 \le 2$

$x^* = (1·8, 2) \qquad z^* = 6·2$

P21 $\longrightarrow$ $\underset{P211}{x_1 \le 1}$, $\underset{P212}{x_1 \ge 2}$
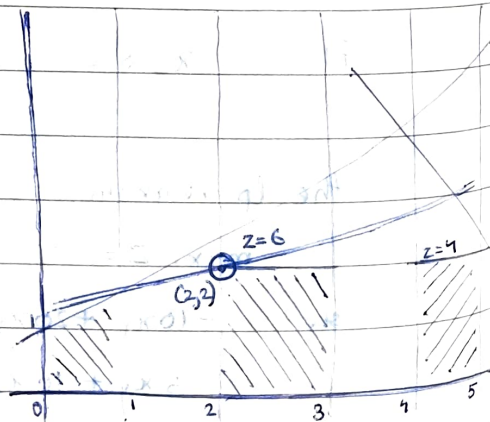
(1·8, 2)

P212: the lp relaxation

$$\max. \ z = -x_1 + 4x_2$$

st. $-10x_1 + 20x_2 \le 22$

$5x_1 + 10x_2 \le 49$

$2 \le x_1 \le 3$

$0 \le x_2 \le 2$

$x^* = (2, 2) \qquad z^* = 6 \qquad \checkmark$
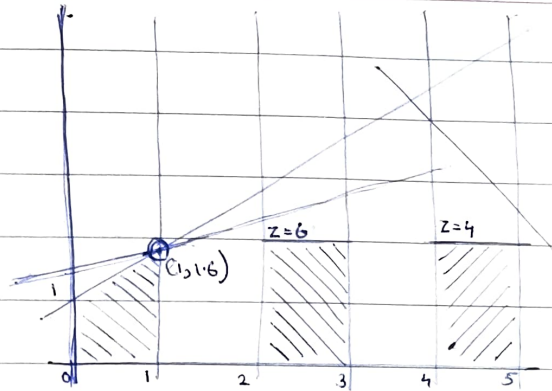
z = 6

(2, 2)

z = 4

P2il. the lp relaxation

max. $Z = -x_1 + 4x_2$

st. $-10x_1 + 20x_2 \leq 22$

$5x_1 + 10x_2 \leq 49$

$0 \leq x_1 \leq 1$

$0 \leq x_2 \leq 2$

$x^* = (1, 1.6)$ $z^* = 5.4$ ✗ too low.

$\rightarrow z^*$ for $P = 6$, $x^* = (2,2)$ ✓