# LINEAR PROGRAMMING FORMULATIONS

$$\max_{x} \quad Z = c^T x$$

Subject to $\quad Ax \le b$

$$x \ge 0$$

① draw the constraints as lines and shade the feasible region

② if feasible region is bounded, find the coordinates of corner points.

③. find which corner point gives the optimal value for the objective.

④. if feasible region is empty, then no solution exists.

⑤. Special cases :

LP can be either :

(a) infeasible

(b) unbounded

(c) have unique optimal solution value $Z^*$

- Q: does (c) imply every LP have a unique optimal solution $x^*$

A: no, there can be $\infty$ - many solutions as well.

- Q: can (b) and (c) occur simultaneously.

A: if LP is unbounded, it will not have unique optimal solution value $z^*$.

~~PATTERN CLASSIFICATION~~

LINE FITTING AS LP

we are given a set of N points $(x_i, y_i)$, and we are asked to fit (or find) a line (say $ax+b=y$) that minimizes an "error" (in predicting y) i.e., find a and b by.

$$\min_{a,b} \sum_{i=1}^{N} |y_i - (ax_i + b)|$$

our objective is to find a and b corresponding to the optimal line $y = ax + b$

we can rewrite the problem as,

$$\min \sum_{i=1}^{N} e_i$$

- what are the constraints.
- what are c, A, b for this LP (standard form)
- what about L1, L0, L2, $L\infty$ norms?

$$\max \; c_1x_1 + c_2x_2 + \dots c_nx_n \qquad (c^Tx)$$

subject to

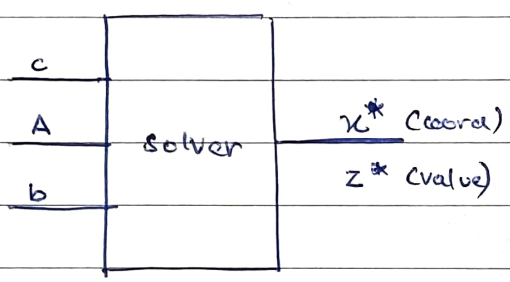
$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots a_{1n}x_n &\leq b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots a_{2n}x_n &\leq b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots a_{mn}x_n &\leq b_m \end{aligned} \qquad (Ax \leq b)$$

$x_i \geq 0$

$x \in \mathbb{R}^n$ LP

$x \in \mathbb{Z}^n$ IP

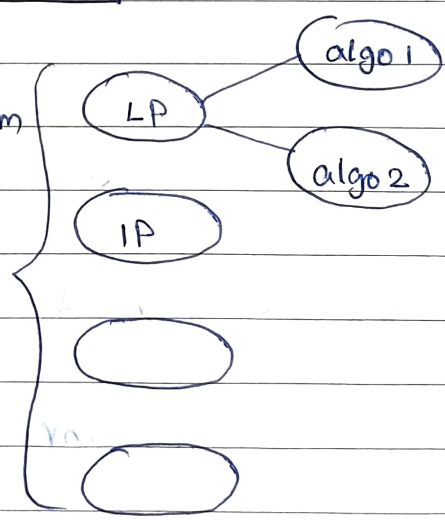
QP ...

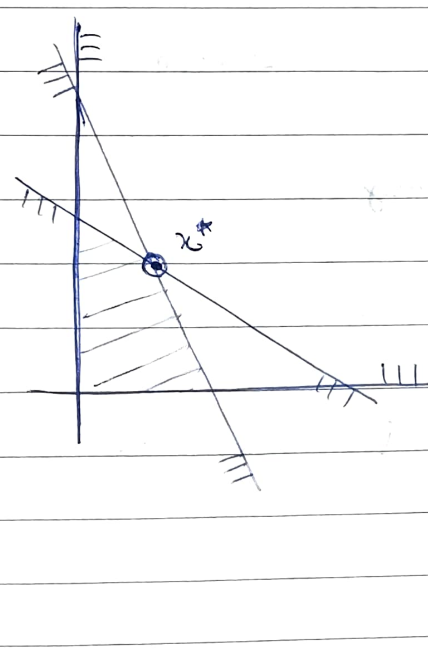



to learn:
art of converting real world problems to LP.

real problem

why difficult




LP problem can be either

① unique

② infeasible

③ unbounded

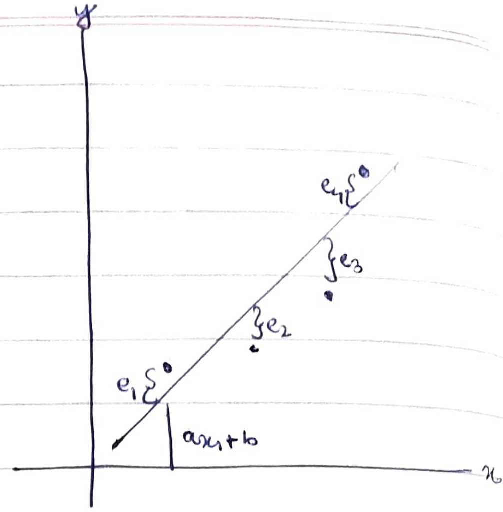
$$\min_{a,b} |y_i - (ax_i + b)|$$

$$\min_{x} \sum e_i$$

$\downarrow$

L1 norm: $|y_i - (ax_i + b)| \le e_i$

$e_i \geq 0$




$\Rightarrow$ $\min \sum e_i$ $\qquad \min_{a,b,e_i} \sum_{i=1}^{N} e_i$

L2 norm: $\min_{a,b} (y_i - (ax_i + b))^2$

easy — L1 norm: absolute error $\downarrow$

easy — L2 norm: mean square error $\downarrow$

hard — L0 norm: max. no. of zero error $\uparrow$

? — $L_\infty$ norm: ~~least~~ max. error, i.e. ~~least~~ $(\max e_i)\downarrow$

# PATTERN CLASSIFICATION AS LP

we are given $N_1$ positive examples and $N_2$ negative examples. how do we find a separating line that also maximizes a margin / distance from the line.

maximize $\delta$ subject to

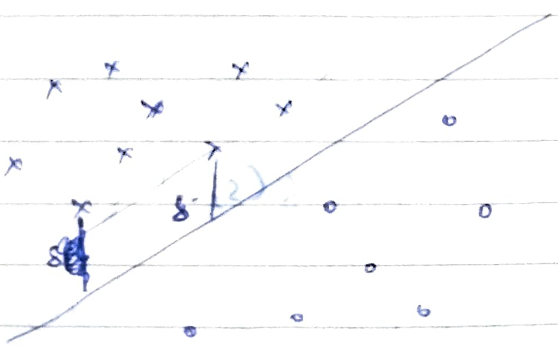
$$y_i^+ \geq a x_i^+ + b + \delta \quad \forall\ i = 1 \ldots N_1$$

$$y_i^- \leq a x_i^- + b - \delta \quad \forall\ i = 1 \ldots N_2$$

- again find the c, A, b for this LP problem as per standard form?

- note that the distances have been measured along Y-axis; not the orthogonal distance from the line $y = ax + b$

- what if we keep the LHS and RHS as is for both equations but flip the inequality signs. does anything change?



# MAX FLOW PROBLEM

- max flow passing from a source node S to a destination node T in a graph G (V, E) is the min. capacity which when removed from the network results in zero flow from S to T.

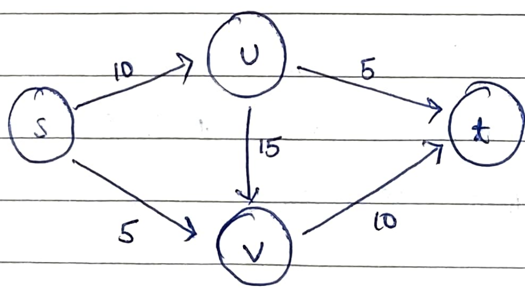
- think of this as the maximum amount of water that can go from A to B in a pipe network in a city.

comp. architect.

- you have seen this in your algorithms class as ford-fulkerson algorithm.

- this is exactly equal to the min S-T cut problem: a cut on the minimum sum of edge weights such that S and T are on opposite sides of the cut.

- review: maxflow - mincut theorm.




max. $f(s,u) + f(s,v)$

(total flow from source node).

$$\max \; f(s,u) + f(s,v)$$

subject to

$$f(s,u) = f(u,v) + f(u,t)$$

$$f(s,v) + f(u,v) = f(v,t)$$

$$0 \le f(s,u) \le 10$$

$$0 \le f(s,v) \le 5$$

$$0 \le f(u,t) \le 5$$

$$0 \le f(u,v) \le 15$$

$$0 \le f(v,t) \le 10$$

find c, A, b for the given graph.