IP FORMULATIONS

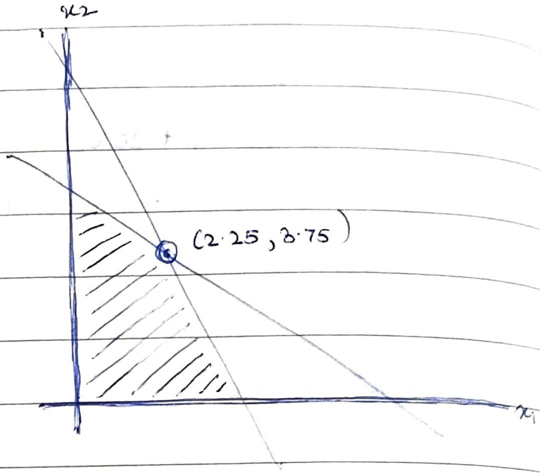
P: max. $5x_1 + 8x_2$

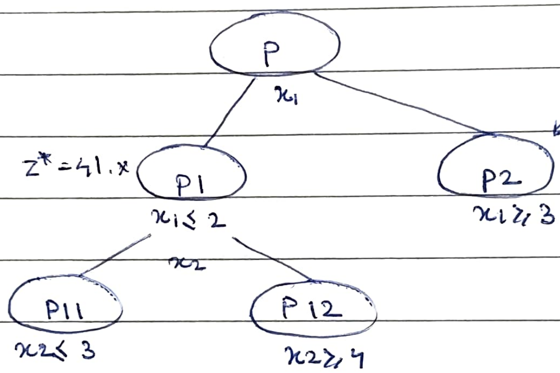
st: $x_1 + x_2 \leq 6$

$5x_1 + 9x_2 \leq 45$

$x_1, x_2 \geq 0$

$x_1, x_2 \in \mathbb{Z}$





$x^* = (2, 3)$ $z^* = 34$

IP

no need to further expand

max $c^T x$

$Ax \leq b$

Knapsack

Some max. wt. you can carry

more costly items there.

real LP

integer IP

binary BIP

mixed MIP

MIP FORMULATION - FUNCTION OF K DISCRETE VARIABLES

say you have to go from A to B; you can go by bus, bike or car. you are allowed to spend ₹5 if you choose bus, ₹10 if you choose bike, & ₹20 if you choose car.

we introduce a set of new variables y_i. let y_i denote whether you choose vehicle$_i$ and x_i denote the amount spent on vehicle$_i$.

the constraints can be written as:

$$y_1 + y_2 + y_3 = 1 \qquad y_i \in \{0,1\} \quad i = 1 \ldots 3$$
$$x_1 + x_2 + x_3 = 5y_1 + 10y_2 + 20y_3$$

notice that y_i's need to be 0 or 1 but no such constraint on x_i's.

SETTING UP A WAREHOUSE

let f_i is the fixed operating cost of warehouse i. c_{ij} is the per unit operating cost of warehouse i plus transportation cost for shipping from warehouse i to customer j.

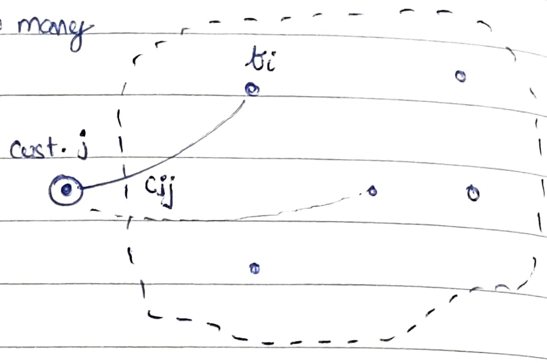
$y_i = 1$, if warehouse is opened; goods can be shipped only if its opened.

d_j is the demand of customer j.

$$\text{minimize} \sum_i \sum_j c_{ij} x_{ij} + \sum_i f_i y_i$$

complete the problem statement and write the entire constraints.

x = from warehouse, how many



$\sum x = d$

↳ demand.

$\sum_i x_{ij} = d_j$

$\sum_j x_{ij} \le y_i d_j$

min. supply cost + operating.

$\sum_{j=1}^{n} x_{ij} - y_i(\sum_j d_j) \le 0$

supply from warehouse > 0 only if warehouse open.

total customer demand satisfied.

travelling salesman

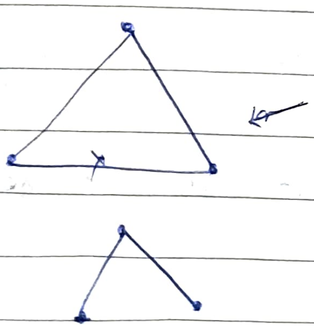
$x_{ij} \in \{0, 1\}$

min. $\sum c_{ij} x_{ij}$

$\sum_i x_{ij} = 1$

$\sum_j x_{ij} = 1$

forest will also get accepted.



CONSTRAINTS

multiple choice problems:

$$\sum_{i=1}^{n} y_i \leq 1 \quad \text{or} \quad = 1$$

a specific constraint is satisfied:

$$f(x_1, x_2, \ldots x_n) < b$$

how do we make a constraint to be trivially true/satis?
either of the constraints to be satisfied?

$$f(x_1, x_2, \ldots x_n) - By < b_1$$

$$f(x_1, x_2, \ldots x_n) - B(1-y) < b_2$$

what does it mean when y is 0 and 1 respectively?
when only m out of the n constraints to be true?
when only one set of the multiple sets of constraints to be true?

$B \leftarrow$ max no. (∞)

$f_1(x) \leq b_1 + y_1 B$ $\quad y_i = 1$ trivially true

$f_2(x) \leq b_2 + y_2 B$ $\quad y_i = 0$ constraint is active classmate

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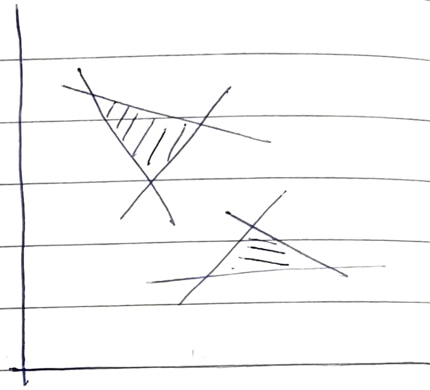
$y_1, y_2 \in \{0, 1\}$ assume many such constraints.

$y_1 + y_2 = 1$ $\Sigma y_i = 1$ 100 constraints.

$\Sigma y_i = n-1$ only i is inactive.

exactly 1 true

atleast 1 true



$f_i(x) \leq b_i$ } set n_1

$f_j(x) \leq b_j$ } set n_2

$y_1 + y_2 = 1$

not convex either or

IP FORMULATIONS : COST OF PRODUCTION

fixed cost + variable cost

$K + Cx$

piecewise linear cost

if the production is below 4000 units, unit price is C_1.

if prod. between 4000, 9000, unit price C_2.

if prod. above 9000, below 15000, cost is C_3.

$0 \leq x_1 \leq 400$ $\quad$ cost $= C_1 x_1 + C_2 x_2 + C_3 x_3$

$0 \leq x_2 \leq 5000$ $\quad$ $4000 w_1 \leq x_1 \leq 4000$

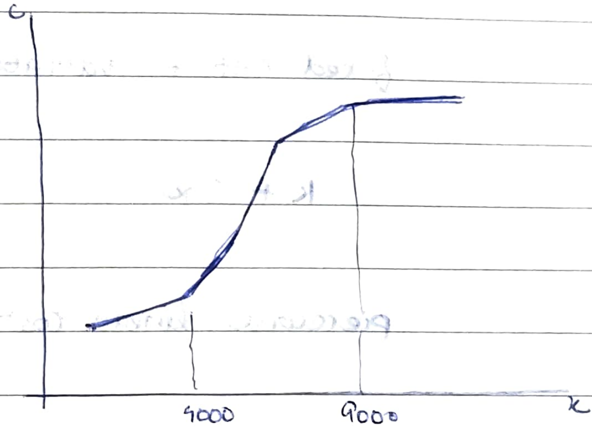
$0 \leq x_3 \leq 6000$ $\quad$ $9000 w_2 \leq x_2 \leq 9000 w_1$

$0 \leq x_3 \leq 6000 w_2$

max. $C^T x$ → cost of manufac.

x not const. over period.

$f_i(x) \le b_i$



$C^T x$ no longer linear fn.

non-linear → piecewise linear

$C_1 x$ if $x \le 4K$

$C_2 x$ if $x \ge 4K \le 9K$

$4000 w_1 \le x_1 \le 4000$

$5000 w_2 \le x_2 \le 5000$ $w_1, w_2 \in \{0, 1\}$

$0 \le x_3 \le 6000$