HW4

10.1 (10) what can we say about matrix B if $AB = BC$ and matrices A and C have no common eigenvalues?

in mathematics, in the field of control theory, a sylvester equation is a matrix equation of the form

$$AX + XB = C$$

a sylvester equation has a unique solution for X exactly when there are no common eigen values of A and $-B$.

when $C = 0$,

$$AX = -XB$$

$$\Rightarrow AX = X\underbrace{(-B)}_{B'}$$

$\therefore$ there is a unique soln. for X when there are no common eigen values of A and B'

or in our case A and C.

classmate

Date 21.4.2020
Page 2

(9.3) Show that a square matrix is singular iff 0 is an eigenvalue of A.

a singular matrix squeeze all of vector space to a point, like a black hole. so it makes sense for all vectors ~~in~~ to be squished as well, hence 0 must be an eigen value of A.

or,

A is singular

$\Rightarrow \det(A) = 0$

$\Rightarrow \det(A - 0 \cdot I) = 0$

$\Rightarrow$ 0 is eigenvalue of A.

(8.1) find the sequence of basic feasible solutions to arrive at the optimal point for the following linear program.

max. $z = 2x_1 + 3x_2$

st. $5x_1 + 25x_2 \leq 40$

$x_1 + 3x_2 \geq 20$

$x_1 + x_2 = 20$

$x_1, x_2 \geq 0$

$\downarrow$ to min. problem.

min. $z = -2x_1 - 3x_2$

st. $5x_1 + 25x_2 \leq 40$

$-x_1 - 3x_2 \leq -20$

$x_1 + x_2 = 20$

$\downarrow$ substuting x_1

$$\min.\ z = -2(20 - x_2) - 3x_2$$
$$= -40 + 2x_2 - 3x_2$$
$$= -40 - x_2$$

using slack variables x_3, x_4

$$5x_1 + 25x_2 + x_3 = 40$$

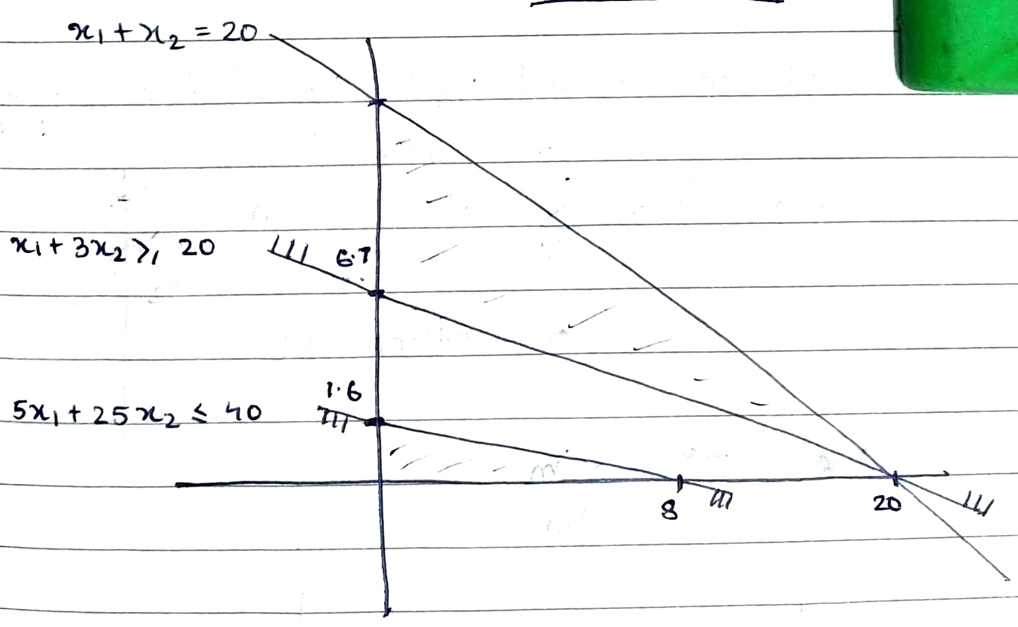
$$\Rightarrow 5(20 - x_2) + 25x_2 + x_3 = 40$$

$$\Rightarrow 100 - 5x_2 + 25x_2 + x_3 = 40$$

$$\Rightarrow 20x_2 + x_3 = -60 \quad \text{(invalid)}$$

$x_2, x_3 \geq 0$

the solution must be infeasible.



yes it is infeasible.

(7.4) for the following LP, express the optimal value and the optimal solution in term of the problem parameters. if the optimal solution is not unique, it is sufficient to give one optimal solution.

the variable is $x \in \mathbb{R}^n$

$$\max.\ c^T x$$

$$\text{st.}\quad d^T x \le K$$

$$0 \le x_i \le 1 \qquad i = 1 \ldots n$$

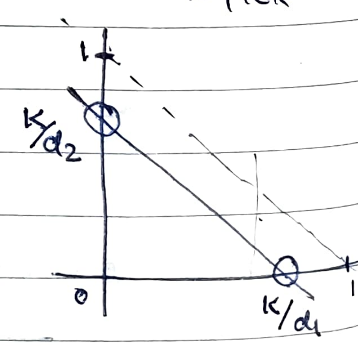
K is a constant. components of d are +ve.

let $m = \{ m \mid c_m \geq c_i \ \forall i \}$ which corner to pick

$\min(K/d_i, 1)$

then $x_i^* = \dfrac{K}{d_i}$ if $i == m$

$\qquad\quad 0$ otherwise



then $z^* = \dfrac{K c_m}{d_m}$