MORE ON LP RELAXATION

maximum weight bipartite matching

given a bipartite graph $G(V, E)$ with $|X| = |Y|$

$$\max \sum_{e \in E} w_e x_e$$

st $\sum_{v \in e} x_e = 1 \quad \forall \; v \in V_j$

$x_e \in \{0, 1\} \quad \forall \; e \in E$

min vertex cover

find min $|V'|$, $V' \subset V$

such that $\forall (u, v) \in e \qquad e \in E$

either $u \in V'$ or $v \in V'$ or both.

$$\min. \sum_{v \in V} x_v$$

st. $x_u + x_v \geq 1 \quad \forall \; (u, v) \in E$

$x_u \in \{0, 1\}$.

maximum independent set

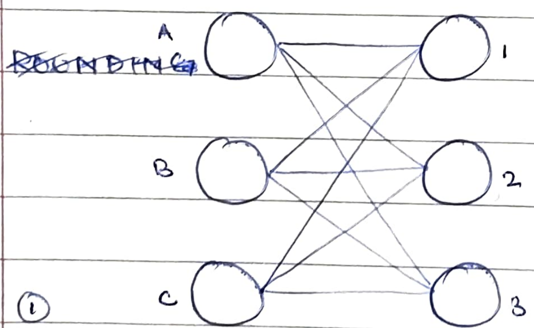
$$\max_{v \in V} \; x_v$$

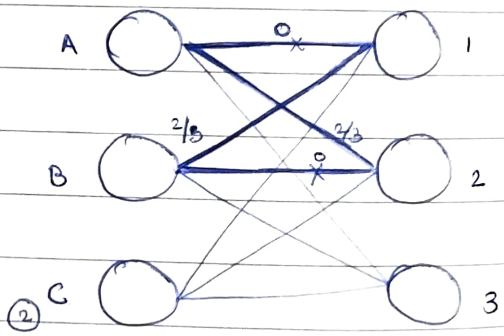
s.t. $\quad x_u + x_v \leq 1 \quad \forall \; (u, v) \in E$

$$x_u \in \{0, 1\}$$

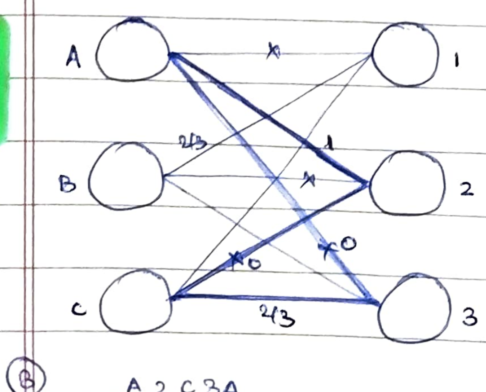
i.e., maximum size set such that no 2 vertices in it are connected by an edge.

$w_e = 1 \qquad x_e = 1/3$

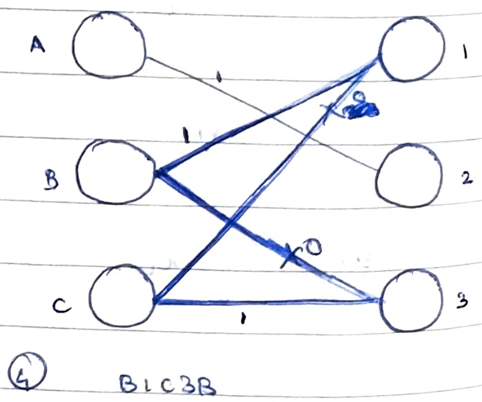




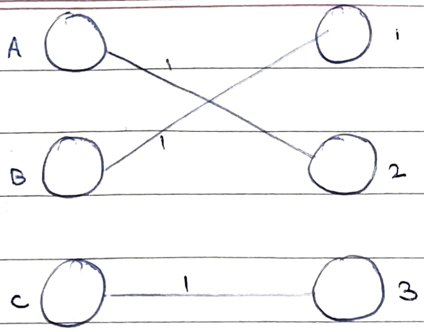
B1A2B



A2C3A



B1C3B



(5)

ROUNDING FOR OTHER TWO PROBLEMS

minimum vertex cover

$x_v \in [0, 1]$ (LP-R) Z^*_{LP} x^*

$S_{LP} = \{ x^*_v \geq 1/2 \} \longrightarrow$ vertex cover may not be optimal.

$S_{LP} = \{ v \in V \mid x^*_v \geq 1/2 \}$

$$|S_{LP}| = \sum_{v \in S_{LP}} 1 \leq \sum_{v \in S_{LP}} 2 \cdot x^*_v$$

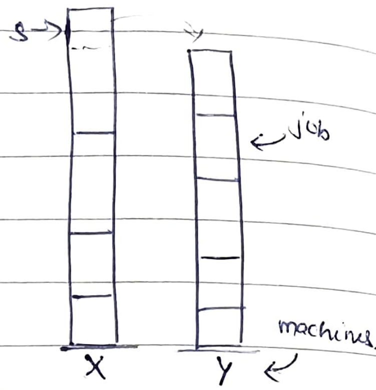
$$\leq \sum_{v \in V} 2 \cdot x^*_v \quad 2\,LP$$

$$\leq 2 \sum_{v \in V} y_v = 2|S_{OPT}|$$

$|S_{OPT}| \leq |S_{LP}| \leq 2|S_{OPT}|$

maximum independent set : no useful bounds.
(depends on size of graph)

LP RELAXATION FOR 2 MACHINES



2 machines and J jobs.

d_{ij} → time for job j to run on machine i.

we know that max time is the time taken by the machine ends last.

$$\min_{t,\, x_{ij}} \quad t$$

$$\text{s.t.} \quad \sum_j x_{ij} d_{ij} \leq t \qquad \forall i \in M$$

$$\sum_i x_{ij} = 1 \qquad \forall j \in J \quad ; \quad x_{ij} \in \{0, 1\}.$$

if we LP-relax $x_{ij} \in \{0, 1\} \rightarrow [0, 1]$

only 1 job will get split. why?
let s be the job that gets split.
we assign s to m_1 if $x_{1s} > x_{2s}$ otherwise, assign to m_2.

if this leads to Tapprox and T^* is the optimal soln, then.

$$T_{approx} \leq 2T^*$$

$T_{approx} \leq T' + T_s$

$T' \leq T^*$ $\quad$ $T_s \leq T^*$

$T_{approx} \leq 2T^*$

where T' be optimal soln. before assigning S to one machine.

WHEN DOES AN LP LEAD TO INTEGER SOLUTION ?

note:
the constraints $Ax \leq b$ ~~ca~~ can be converted to $A'x \leq b'$ with additional variables.

a matrix is totally unimodullar (TU) if all its square submatrices have a determinant in $\{-1, 0, 1\}$.

note:
if A is TU
$A' = [A, I]$ is also TU

if A is TU
and b is integral, LP gives integral soln.

Date 31.1.2020
Page 6,

WHEN DOES AN LP LEAD TO INTEGER SOLUTION?

for BPG, A is TU

consider a matrix A $(|V| \times |E|)$.
A has only two 1 in every column.
we can prove by induction. sketch,

Q is 1×1. Q is TU. why?

if sub matrices of $(I-1) \times (I-1)$ are TU,
Q then Q of $I \times I$ is also TU. why?

Q has a column of no 1.
Q has a column of one 1,
all columns of Q have 2 1s.

numerical example.