Date 24.1.2020
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IP FORMULATIONS

COST OF PRODUCTION

fixed cost + variable cost

$K + Cx$

piecewise linear cost

if the production is below 4000 units, unit price is C_1
if the production is between 4000 and 9000, unit price is C_2.
if the production is above 9000 and below 15000, then cost is C_3.

$0 \le x_1 \le 4000$

$0 \le x_2 \le 5000$

$0 \le x_3 \le 6000$

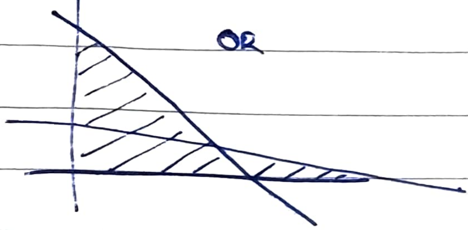
cost $= C_1 x_1 + C_2 x_2 + C_3 x_3$ $\quad (x = x_1 + x_2 + x_3)$

$4000 w_1 \le x_1 \le 4000$

$5000 w_2 \le x_2 \le 5000 w_1$

$0 \le x_3 \le 6000 w_2$

$f_i(x) \le b_i$ $\Big\}$



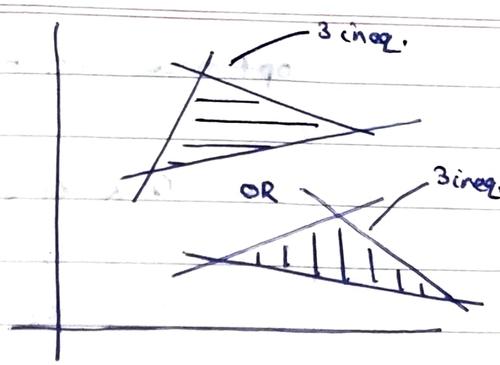
OR

$f_1(x) \le b_1 + y B$

$f_2(x) \le b_2 + y B$

$f_3(x) \le b_3 + y B$

$f_4(x) \le b_4 + (1-y) B$



Constraints on y.
assume constraint

$\cap$ convex = ✓ convex

$\cup$ convex = ✗ convex

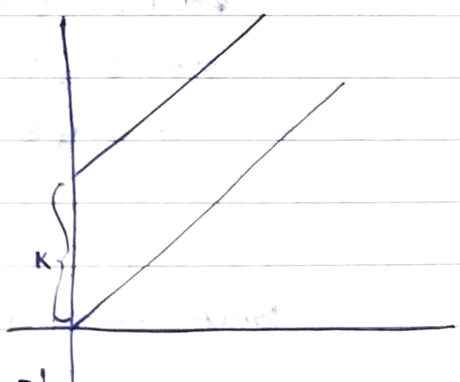
$x \le By$

x is non-zero when $y=1$

$K + Cx$

↓

$Ky + Cx \qquad y \in \{0, 1\}$

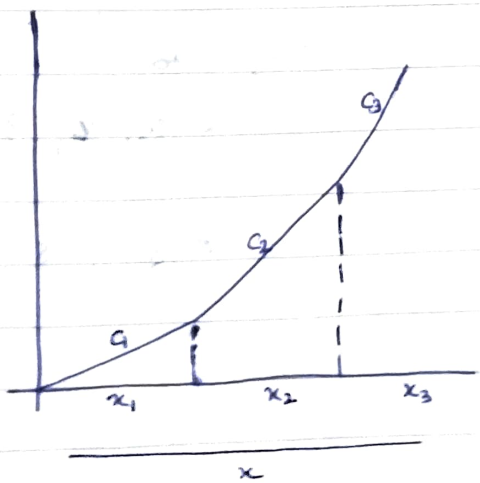


$x \le By \qquad$ x is non-zero when $y=1$

$0 \le x_1 \le 4000$

$0 \le x_2 \le 5000$

$0 \le x_3 \le 6000$



optimize $c_1x_1 + c_2x_2 + c_3x_3$.

st. $0 \le x_1 \le 4000$	$4000\,w_1 \le x_1 \le 4000$
$0 \le x_2 \le 5000$	$5000\,w_2 \le x_2 \le 5000\,w_1$
$0 \le x_3 \le 6000$	$0 \le x_3 \le 6000\,w_2$

① $w_1 = 0 \quad w_2 = 0$

$0 \le x_1 \le 4000$

$0 \le x_2 \le 0$

$0 \le x_3 \le 0$

② $w_1 = 1 \quad w_2 = 0$

$4000 \le x_1 \le 4000$

$0 \le x_2 \le 5000$

$0 \le x_3 \le 0$

③ $w_1 = 1 \quad w_2 = 1$

$4000 \le x_1 \le 4000$

$5000 \le x_2 \le 5000$

$0 \le x_3 \le 6000$

④ $w_1 = 0 \quad w_2 = 1$

$0 \le x_1 \le 4000$

$5000 \le x_2 \le 0$ ★ infeasible

$0 \le x_3 \le 6000$

BALA'S ALGORITHM FOR BIP (BRANCH & BOUND FOR BIP)

$$\min \; z = \sum_i c_i x_i \; ; \qquad x_i \in \{0, 1\}$$

Subject to

$$\sum a_{ij} x_j \ge b_i \quad \forall \; i \in 1, 2 \ldots m$$

$$0 \le c_1 \le c_2 \le c_3 \ldots \le c_n$$

⋮

Start expanding tree with x_1, then move to x_2 and so on. at each step, evaluate z, check if any constraint is violated. when you find a feasible point, you start expanding down that tree. also prune a branch if Znode is not best enough w.r.t. best z you saw till now.

BALA'S ALGORITHM - EXAMPLE

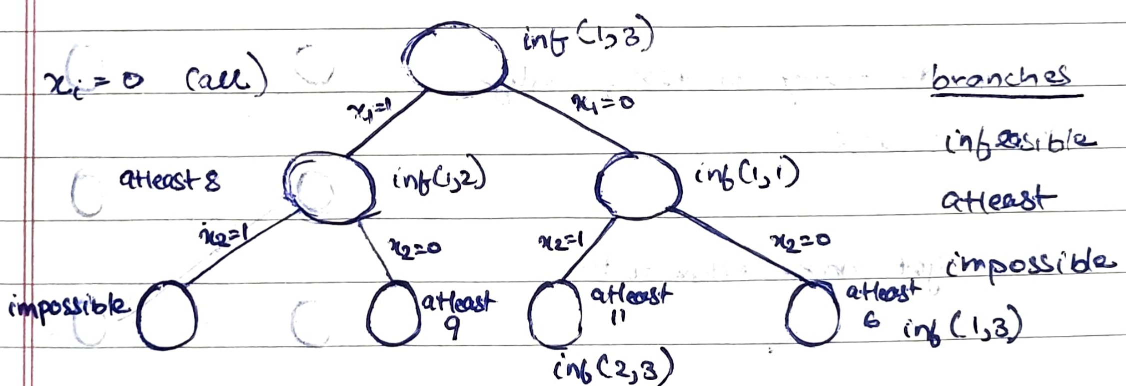
minimize $\quad z = 3x_1 + 5x_2 + 6x_3 + 9x_4 + 10x_5 + 10x$

st. $\quad -2x_1 + 6x_2 - 3x_3 + 4x_4 + x_5 - 2x_6 \geq 2$

$\quad -5x_1 - 3x_2 + x_3 + 3x_4 - 2x_5 + x_6 \geq -2$

$\quad 5x_1 - x_2 + 4x_3 - 2x_4 + 2x_5 - x_6 \geq 3$

$$x_i \in \{0, 1\} \quad \forall i \in 1, 2, \ldots 6.$$



$C_1 = \times$

$C_2 = \checkmark \qquad x_2 = 1$

$C_3 = \times \qquad x_3 = 1$

MAXIMUM WEIGHT BIPARTITE MATCHING

given a bipartite graph $G(V,E)$ with equal no. of nodes $|X| = |Y|$ on each side with edges $X \rightarrow Y$ only, a perfect matching $M \subset E$ is such that each vertex in X as well as Y only appears once in M.

we want to find M with the maximum weight:

$\max \sum_{e \in M}$

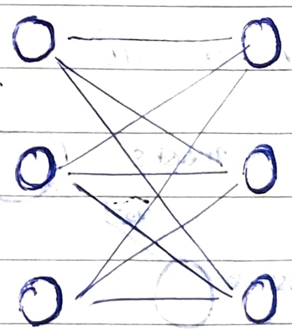
$$\max \sum_{e \in E} w_e x_e$$

st. $$\sum_{v \in E} x_e = 1 \quad \forall v \in V$$

$$x_e \in \{0,1\} \quad \forall e \in E$$

at every vertex, only 1 edge.

get max. edge wt.



MORE GRAPH PROBLEMS

min. vertex cover

find min $|V'|$, $V' \subset V$ such that
$\forall (u,v) \in e$, $e \in E$, either $u \in V'$ or $v \in V'$, or both.

in english: smallest set such that atleast one end of every edge is a member of the set.

IP formulation:

$$\min. \sum_{v \in V} x_v$$

st $\quad x_u + x_v \geq 1 \quad \forall (u,v) \in E$

$$x_v \in \{0,1\}.$$

maximum independent set

$\max_{v \in V} x_v$ such that $x_u + x_v \leq 1$

$\forall (u,v) \in E$, $x_v \in \{0,1\}$

i.e., maximum size set such that not two vertices in it are connected by and edge.