MAX FLOW PROBLEM (REVIEW)

max $f(s,u) + f(s,v)$

subject to :

$f(s,u) = f(u,v) + f(u,t)$

$f(s,v) + f(u,v) = f(v,t)$

$0 \leq f(s,u) \leq 10$

$0 \leq f(s,v) \leq 5$

$0 \leq f(u,t) \leq 5$

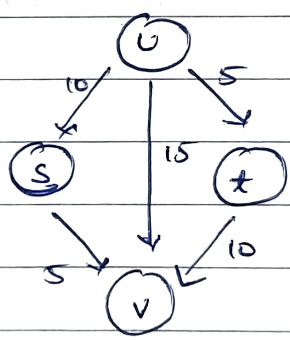
$0 \leq f(u,v) \leq 15$

$0 \leq f(v,t) \leq 10$

max $c^T x$

$Ax \leq b$

$x \geq 0$



LP IP

maxflow = min cut

$LP^* = IP^*$

duals

simplex algorithm

possible ?

min $c'^T x$

$A'x \geq b$

$x \geq 0$

same problem.

edge ⟶ $x_i \in \{0,1\}$

cut or not → 0 or 1

Some lucky integer progmming problems can be solved easily.

MINIMIZING NORMS

minimize $\|Ax-b\|_1$ l_1 norm $\|Ax-b\| = y$

subject to $\|x\|_\infty \leq 1$ max norm. $|y_1|+|y_2|+|y_3|$

$\min 1^T y$

rewrite the above as.

$$\min \sum_{i=1}^{n} y_i$$

subject to

$$-y_i \leq \sum_{j=1}^{n} (a_{ij} x_j) - b_i \leq y_i \quad i = 1, 2, \ldots m$$

$$-1 \leq x_j \leq 1 \quad j = 1, 2, \ldots n$$

what are c, A and b for this LP problem as per the standard form?

IP WITH BRANCH AND BOUND.

maximize $x_1 + x_2$

subject to $x_2 - x_1 \leq 2$

$8x_2 + 2x_1 \leq 19$

$x_1, x_2 \geq 0$

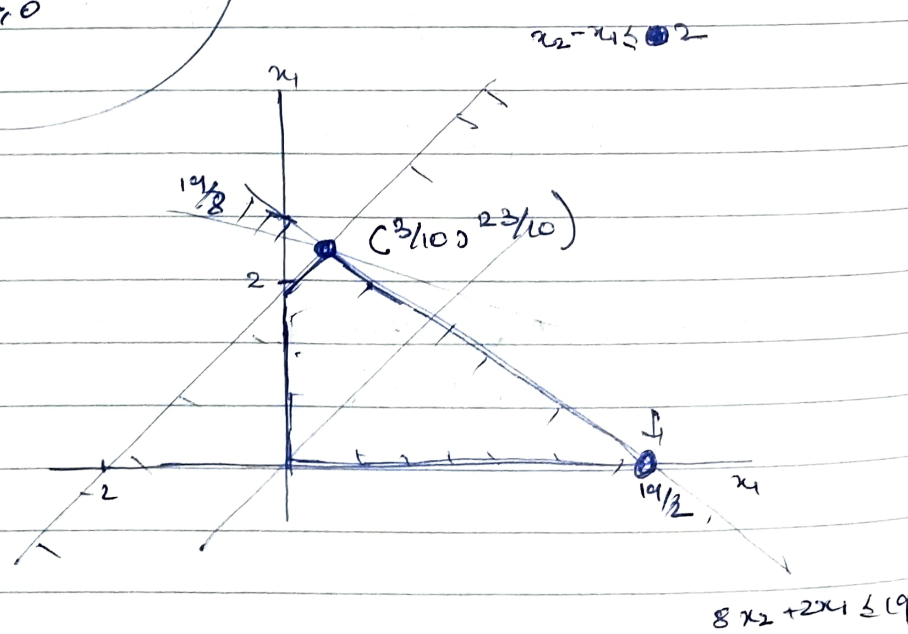
$x_1, x_2 \in \mathbb{Z}$.

max $c^T x$

$Ax \leq b$

$x \geq 0$

try to find problems using this pattern.



$x_2 - x_1 = 2 \quad \times 2$

$8x_2 + 2x_1 = 19 \qquad x_1 = x_2 - 2 = \left(3/10\right)$

$10x_2 = 23$

$\Rightarrow x_2 = \left(23/10\right)$

let us understand the issues.

$\boxed{x \in \mathbb{R}^n} \Rightarrow LP^*$ easy.

$\uparrow$

$\boxed{x \in \mathbb{Z}^n} \Rightarrow IP^*$ assume somehow we could solve.

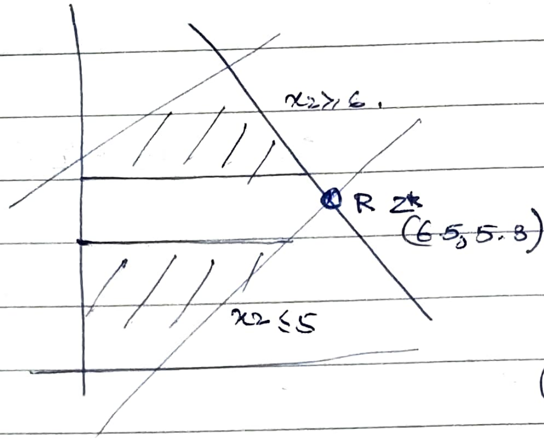
try solving without integer constraint first.
LP relaxation.

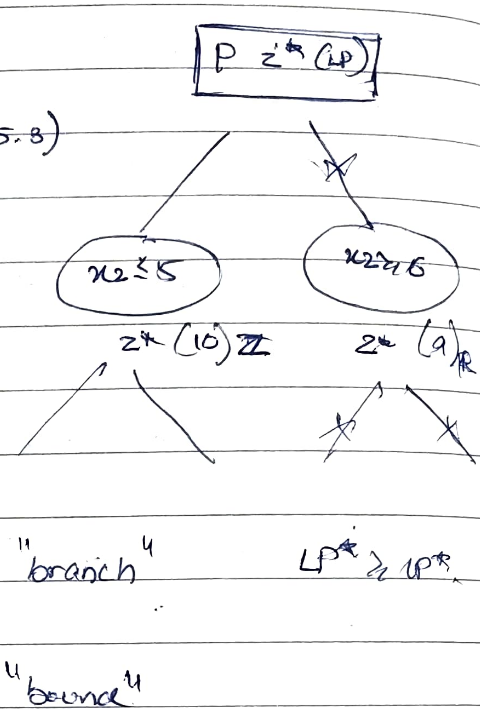
$\boxed{LP^* \geq IP^*}$

$\boxed{P}$ original problem.

$x_1 \geq 9 \qquad x_1 \leq 9$

$z^* = 9$





until we find an integer

BRANCH AND BOUND

consider the IP problem.

maximize $c^T x$ subject to $Ax \leq b$, $x \in \mathbb{Z}$

① initialize constraints as $L = \{Ax \leq b\}$.

② initialize $x^- = \phi$, $I = -\infty$ here x^- and I are the optimal solution and value resp.

③ while $L \neq \phi$

3.1 Pick a subproblem. maximize $C^T x$, $A'x \leq b'$ and solve the LP. also delete the subproblem constraint. from L.

3.2 let x^* be the optimum solution to the LP.

3.3 if $x^* \in \mathbb{Z}$ and $C^T x > I$. the set $\bar{x} = x^*$ and $I = C^T x$

3.4 If $x_j^* \notin \mathbb{Z}$ and $C^T x^* > I$. then add 2 subproblems in L for $x_j^* \notin Z$

- max $C^T x$ such that $A'x \leq b'$ and $x_j \leq \lfloor x_j^* \rfloor$ and add to L.

- max $C^T x$ such that $A'x \leq b'$ and $x_j \geq \lceil x_j^* \rceil$ and add to L.

BRANCH and bound
is used to solve hard problems.

LP & IP fastest way to get a solution to a problem.

classmate
Date 17.1.2020
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$y = 3x + 10$

1 (a) $x + y = 1$

2 (c)

3 (a)

4 (b)

paper rolls width 3m.

(i) 97 rolls of width 135 cm
(ii) 610 rolls of width 108 cm
(iii) 395 rolls of width 93 cm.
(iv) 211 rolls of width 42 cm.

① 2×135	x_1 roll in 1st way
② $1 \times 135 + 1 \times 108 + 1 \times 42$.	x_2 rolls in 2nd way
	$\min \sum x_i$
	$2x_1 + x_2 \geq 97$.

⋮

4 constraints.

$x_i \in \mathbb{Z}$