LP RELAXATION

- LP and IP formulation
- graphical method to solve
- branch and bound for IP
- bala's methods for BIP

GRAPH PROBLEMS (REVISIT)

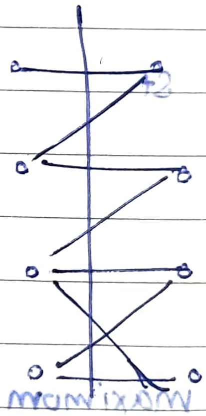
maximum weight bipartite matching

given a bipartite graph $G(V, E)$ with $|X| = |Y|$

$$\max \sum_{e \in E} w_e x_e$$

$$\text{s.t.} \sum_{v \in e} x_e = 1 \quad \forall v \in V;$$

$$x_e \in \{0, 1\} \quad \forall e \in E$$



edges only from X-Y not among themselves

$|X| = |Y|$

min vertex cover

find $\min |V'|$, $V' \subseteq V$ such that $\forall (u,v) \in e$, $e \in E$,

either $u \in V'$ or $v \in V'$, or both.

$$\min \sum_{v \in V} x_v \quad \text{subject to}$$

$$x_u + x_v \geq 1 \quad \forall (u,v) \in E; \quad x_v \in \{0, 1\}.$$

maximum independent set

$$\max \sum_{v \in V} x_v$$

st. $x_u + x_v \leq 1 \quad \forall (u,v) \in E, \quad x_u \in \{0,1\}$

i.e., maximum size set such that no two vertices in it are connected by an edge.

LP RELAXATION

convert IP to LP

call this as LP relaxation of original problem.

opt (LP) $\leq$ opt (IP) (for a minimization problem)

if $x_{LP} \in \mathbb{Z}$ then we got lucky and in this case both opt (LP) and opt (IP) are same.

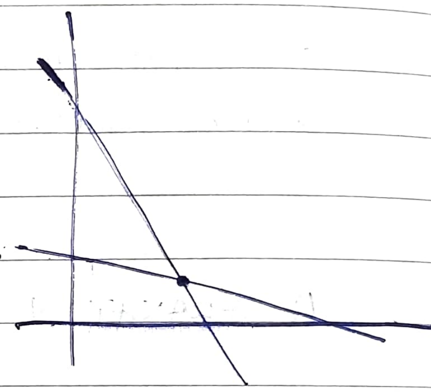
formulate a rounding procedure that transforms x_{LP} into an integral solution x' such that cost $(x') \leq c *$ cost (x_{LP}).

then we say x' is a c-approximate optimal solution to the original problem.

We give ~~our~~ final answer as x'.

its crucial to be able to get x', given x_{LP} and it is important that we understand how good the approximation is (c value).

in case LP is infeasible, what ~~is~~ does this tell about feasibility of IP?



1. solve int. constraint.
2. solve LP.
3. how bad approx. (rounded) is. (approximation algorithms).
4. get another way of solving IP. (how do you design algo with LP relaxation).

ROUNDING IN BPG MATCHING

what does it mean if infeasible? bound?

assume LP gives you $x_e \in [0,1]$

if an edge is not in $\{0,1\}$, then there should be another edge ~~in~~ in every vertex with the same ~~direction~~ situation.

if we round 0.9 to 1.0, then there should be another edge that needs to reduce by 0.1 at the vertex.

let the edges change by ϵ, $x_i^* + \epsilon$, there will be $x_j^* + \epsilon$

there are cycles of such non-saturated edges. let the new weights be y.

$$w(y) = w(x^*) + \epsilon \sum_i (-1)^i w_{e_i} = w(x^*) + \epsilon \Delta$$

where Δ is $\sum_{i=1}^{t} (-1)^i w_{e_i}$

Since x^* is optimal, Δ has to be zero.

repeat this for all cycles. we will reach integer soln.

$$\max_x \sum w_i x_i \qquad x_e \in \{0, 1\} \qquad // 1 \text{ if selected.}$$

$$\sum x_e = 1 \qquad \forall v \in V$$

LPR

$$\downarrow$$

$$x_e \in [0, 1] \qquad \text{(real interval, relaxed const.)}$$

$$\downarrow$$

$$x^* \qquad \text{round to real soln.}$$

$$Z^*_{IP} = Z^*_{LP}$$

$$\underbrace{\sum_e w_e y_e}_{J(y)} = \underbrace{\sum w_e x_e^*}_{J(x^*)} + (\epsilon \Delta) \longrightarrow \underline{\underline{0}}$$

$$y_e^* \longleftarrow x_e^*$$

using rounding.

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ROUNDING FOR OTHER TWO PROBLEMS

minimum vertex cover

$$S_{LP} = \{ v \in V \mid x_v^* \geq \frac{1}{2} \}$$

$$|S_{LP}| = \sum_{v \in S_{LP}} 1 \leq \sum_{v \in S_{LP}} 2 \cdot x_v^* \leq 2 \sum_{v \in V} y_v = 2 |S_{OPT}|$$

therefore,

$$|S_{OPT}| \leq |S_{LP}| \leq 2 |S_{OPT}|$$

maximum independent set

no useful bounds!!