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# ABSTRACT REDOKSION CIGTUMS

ARS

$\langle A, \rightarrow \rangle$ where $\rightarrow \subseteq A \times A$

$\xrightarrow{0}$ identity

$\xrightarrow{i}$ ith composition $\longrightarrow$ $i>0$

$\xrightarrow{+}$ $\bigcup_{i>0} \xrightarrow{i}$

$\xrightarrow{*}$ $\bigcup_{i\in\mathbb{N}} \xrightarrow{i}$

$\xrightarrow{-1}$ inverse

$\leftrightarrow$ symmetric closure

$\xrightarrow{=}$ = $\rightarrow \cup \xrightarrow{0}$ reflexive closure

$\xleftrightarrow{*}$ reflexive, symmetric, transitive closure

## PROPERTIES OF RELATIONS

is <u>normalizing</u> if every element has a normal form.

$$\begin{tikzcd} a \arrow[dr] & & b \arrow[dl] \arrow[loop right] \\ & c & \end{tikzcd}$$

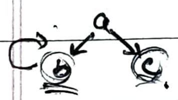
$b \rightarrow b \rightarrow \ldots$ (non terminating)

is <u>terminating</u> no infinite chains

is <u>confluent</u> $\forall a, b, c$ $a \xrightarrow{*} b$ & $a \xrightarrow{*} c$ $\Rightarrow$ $b \downarrow c$

b and c are joinable

$$\begin{tikzcd} & a \arrow[dl, "*"'] \arrow[dr, "*"] & \\ b \arrow[dr, dashed] & & c \arrow[dl, dashed] \\ & d & \end{tikzcd}$$




non-confluent

$a \to b$

confluent

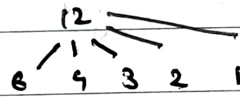
a

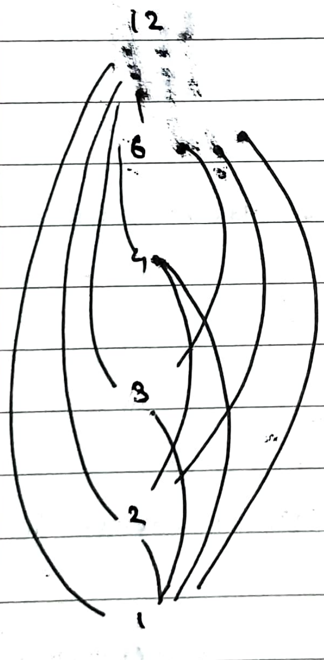
Confluent.

$(x-2)(x-3)=0$

divisors inc. 1 = confluent.

divisors exc. 1 = non conf.




$A = \{1, 2, 3 \ldots\}$

$a \to b$ iff

$a = nb$

$n = (1 \ldots a)$ non-con~~vergent~~fluent.

<u>convergent</u> iff terminating and confluent

$a_0 \to a_1 \to a_2 \ldots$ non-terminating confluent.

not necessarily to the same term.

termination $\Rightarrow$ normalizing.

confluence $\Rightarrow$ at most 1 normal form.

if $\rightarrow$ is confleent, then every element has at most one normal form.

proof: let $a \in A$ suppose

$a \xrightarrow{*} b$

$a \xrightarrow{*} c$

$\therefore b \downarrow c$

since $b$ & $c$, are both normal forms
the only way $b, c$ are joinable is if $b = c$

imp. lang.
ordering matters.

func. lang.
ordering not matters.

INDUCTION (WELL FOUNDED)

for N

$$\frac{P(0) \wedge P(k) \Rightarrow P(k+1) \quad \forall k}{\Rightarrow P(n) \quad \forall n}$$

let $A, \rightarrow$ be an ARS.
the well founded induction PRINCIPLE is

$$\frac{\forall a \in A ; (\forall b \in A \quad a \rightarrow b \Rightarrow P(b)) \Rightarrow P(a)}{\forall a \in A : P(a)}$$

natural nos.

$$\underbrace{(A, \to)}_{\alpha} \qquad \underbrace{(A, \xrightarrow{+})}_{\beta}$$

if $\alpha$ is terminating then $\beta$ is terminating

for ARS:

$$\frac{\forall a \ (\forall b \in A \ \ a \xrightarrow{+} b = P(b)) \Rightarrow P(a)}{\forall a \ P(a)} \qquad \text{(strong)}$$

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$e ::= \bar{n} \mid e + e$

$(\bar{3} + \bar{4}) + (\bar{5} + \bar{6})$

$(\bar{3}+\bar{4}) + (\bar{5}+\bar{6})$ — not deterministic.
$= (\bar{7} + (\bar{5}+\bar{6}))$ — rewriting
$= \bar{7} + \bar{11}$ — derivation
$= \bar{18}$

rewriting roles

$\bar{n}_1 + \bar{n}_2 \hookrightarrow \overline{n_1 + n_2}$ ADD

reduction

$$\frac{e \hookrightarrow e'}{e \to e'} \ \text{REWRITE}$$

$$\frac{e_1 \to e_1'}{e_1 + e_2 \to e_1' + e_2} \ \text{LEFT}$$

$$\frac{e_2 \to e_2'}{e_1 + e_2 \to e_1 + e_2'} \ \text{RIGHT}$$

$\forall e$

$\exists\ n \in \mathbb{N}$ s.t.

$e \xrightarrow{*} \bar{n}$