Mid Semester Examination September 2024
PAS3300C : Probability and Statistics for Computer Science
B.Tech (IT, CS) III Semester

Time: 2 hrs   Max.Marks: 30

- This question paper contains 6 questions printed in one page. Answer all questions. Each question carries 5 marks.

- Scientific calculators are allowed.

1. The annual rainfall at Shillong is modeled as a normal random variable with a mean of $\mu = 68.9$ inches, and a standard deviation of $\sigma = 16.4$. What is the probability that this year's rainfall will be at least 82 inches?
   Note: Use appropriate values from those given: $P(Z < 0.54) = 0.70540$, $P(Z < 0.63) = 0.73565$, $P(Z < 0.79) = 0.78524$, $P(Z < 0.86) = 0.80511$

2. The suburban train arrives at the station near Sunny's home every quarter hour starting at 6:00 AM. He walks into the station every morning between 7:10 and 7:30 AM to attend his classes at University. The time in this interval is a uniform random variable. What is the probability distribution function of the time Sunny has to wait for the first train to arrive?

3. Find the moment generating function of a negative binomial random variable X and hence show that the mean and the variance of X is $\frac{r}{p}$ and $\frac{r(1-p)}{p^2}$.

4. Alpha airline sells 200 tickets for its flight to Lakshadweep on an airplane that has only 198 seats as they know from past experience that 1 percent of purchasers of airline tickets don't turn up at the time of departure of their flight. Determine the probability that everyone who appears for the departure of this flight will have a seat.

5. Consider a circle of radius 5cm. A point is chosen in the circle in such a way that all regions of the circle with equal area are equally likely to contain that point. If centre of the circle is the origin and (X, Y) is the chosen point in the circle it follows that the (X, Y) is near any point on the circle and the joint pdf of X and Y is

$$f(x, y) = \begin{cases} \alpha & x^2 + y^2 \leq 25 \\ 0 & x^2 + y^2 > 25 \end{cases} \text{ for some } \alpha.$$

   (a) Find the value of α

   (b) Find the marginal density functions of X and Y

   (c) Find the probability that the distance of the selected point from the origin is less than or equal to 2cm.

6. Let $f(x, y) = 21x^2 y^3, 0 < x, y < 1, 0$ elsewhere be the joint pdf of X and Y

   (a) Find the conditional mean and variance of X given $Y = y, 0 < y < 1$.

   (b) Find the distribution of $Z = E(X/Y)$

   (c) Determine $E(Z), Var(Z)$ and compare these values with $E(X), Var(X)$ respectively.