ICB2023040.

INDIAN INSTITUTE OF INFORMATION TECHNOLOGY LUCKNOW
MID- SEMESTER EXAMINATION 2024
SUBJECT: PROBABILITY & STATISTICS

Course Code: PAS3300C
Time: 2 Hrs

Semester: III
Max. Marks: 30

All questions are compulsory and carries 3 marks each.

1. Let Ω be the set of all non-negative integers and S the class of all subsets of Ω. In each of the following cases, P defines a probability on (Ω, S), for $A \in S$, let

$$P(A) = \sum_{x \in A} \frac{e^{-\lambda}\lambda^x}{x!}, \lambda > 0.$$

Let $A = \{\text{all integers} > 2\}$, $B = \{\text{all non-negative integers} < 3\}$, and $C = \{\text{all integers } x, 3 < x < 6\}$. Find $P(A \cap B), P(B \cap C), P(A \cap C)$.

2. Urn 1 contains one white and two black marbles, urn 2 contains one black and two white marbles, and urn 3 contains three black and three white marbles. A die is rolled. If a 1, 2, or 3 shows up, urn 1 is selected; if a 4 shows up, urn 2 is selected; and if a 5 or 6 shows up, urn 3 is selected. A marble is then drawn at random from the urn selected. Let A be the event that the marble drawn is white. If U, V, W, respectively, denote the events that the urn selected is 1, 2, 3, then find $P(V|A)$.

3. Suppose there are n types of coupons and that each new coupon collected is, independent of previous selections, a type i coupon with probability $p_i (\sum_{i=1}^{n} p_i = 1)$. Suppose k coupons are to be collected. If A_i is the event that there is at least one type i coupon among those collected, then for $i \neq j$, find

   i. $P(A_i)$
   ii. $P(A_i \cup A_j)$
   iii. $P(A_i \cap A_j)$

4. Calculate $Var(x)$ if X represents the outcome when a fair die is rolled.

5. A communication system consists of n components, each of which will, independently, function with probability p. The total system will be able to operate effectively if at least one-half of its components function. For what values of p is a 5-component system more likely to operate effectively than a 3-component system?

6. A purchaser of electrical components buys them in lots of size 10. It is his policy to inspect 3 components randomly from a lot and to accept the lot only if all 3 are non-defective. If 30% of the lots have 4 defective components and 70% have only 1, what proportion of lots does the purchaser reject?

7. The amount of time in hours that a computer functions before breaking down is a continuous random variable with pdf given by

$$f(x) = \begin{cases} \lambda e^{-x/100} & x \geq 0 \\ 0 & x < 0. \end{cases}$$

What is the probability that

   i. a computer will function between 50 and 150 hours before breaking down?
   ii. it will function for fewer than 100 hours?

8. A stick of length 1 is split at a point U having pdf $f(u) = 1, 0 < u < 1$. Determine the expected length of the piece that contains the point $p, 0 \leq p \leq 1$.

9. Find the mean of a Gamma distributed random variable with parameters $(r, \lambda), r, \lambda > 0$, using the moment generating function.

10. If X is a normal random variable with mean μ and variance σ^2, then find the pdf of $Y = e^X$.