Indian Institute of Information Technology, Lucknow

भारतीय सूचना प्रौद्योगिकी संस्थान, लखनऊ

End Semester Examination November 2024

PAS3300C : Probability and Statistics for Computer Science

B.Tech (IT, CS) III Semester

Time: **3 hrs** Max.Marks: **75**

- This question paper contains 9 questions printed in two pages. Answer all questions.
- Scientific calculators are allowed.
- Statistical Table values required for computation is given in Page 2.

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1. A coffee vending machine is set so that the amount of coffee dispensed is a rândom variable with a mean of 200 milliliters and a standard deviation of 15 milliliters.
What is the probability that the average amount dispensed in a random sample of size 36 is at least 204 milliliters? **(6 Marks)**

2. The specifications of a trailer part has a critical dimension and that the process of manufacturing these parts is considered to be under control if the true variation among the specifications of the parts is given by a standard deviation not greater than $\sigma = 0.60$ thousandth of an inch. To keep a check on the process, random samples of size $n = 20$ are taken periodically, and it is regarded to be "out of control" if the probability that S^2 will take on a value greater than or equal to the observed sample value is 0.01 or less .
Is the process in/out of control if the standard deviation of such a periodic random sample is $s = 0.84$ thousandth of an inch? **(6 Marks)**

3. In 16 mileage test runs of a vehicle, the fuel consumption of an engine averaged 16.4 liters with a standard deviation of 2.1 liters. Is the claim true that the average fuel consumption of this engine is 12.0 liters per hour. **(7 Marks)**

4. Consider an image having value coded as $IRIS$ is transmitted from Arak to Biskra. The value received at Biskra is normally distributed with mean IRIS and variance 4. That is, if $IRIS$ is sent, then the value received is $IRIS + N$ where N , representing noise, is normal with mean 0 and variance 4. To reduce error, suppose the same value is sent 9 times. If the successive values received are 5, 8.5, 12, 15, 7, 9, 7.5, 6.5, 10.5, give a 95 percent confidence interval for true message value of $IRIS$. **(8 Marks)**

5. Take the data from the previous Question no. 4 and compute a 95 percent confidence interval for true message value of IRIS, but with σ^2 being unknown. Is your interval larger/smaller than the previous one? Explain the main reason for this in one or two lines. **(8 Marks)**

6. Cloud seeding, also known as artificial rain, is a weather modification technique that aims to enhance precipitation by introducing substances into clouds to stimulate rainfall. We have 26 log-rainfall measurements of unseeded clouds and the same for 26 observations of seeded clouds in such an experiment denoted by X_i and Y_i respectively. If the following data is available: $\overline{X}_m = 5.13$, $\overline{Y}_m = 3.99$, $S_X^2 = 63.96$ and $S_Y^2 = 67.39$, then test the null hypothesis that the two variances are the same against the alternative hypothesis that the two variances are different at level of significance $\alpha_0 = 0.05$. **(10 Marks)**

7. The grades of students scored in a public speaking course is graded as follows:

Grade	A	B	C	D	F
f	14	18	32	20	16

Test the hypothesis, at the 0.05 level of significance, that the distribution of grades is uniform. **(10 Marks)**

8. Precision Agriculture leverages advanced digital technologies and will play a significant role in the third modern farming revolution. It effectively minimizes inputs, labor, and time sustainably, maximizes productivity and profitability, ensures sustainability, and reduces environmental impact. To test the effectiveness of a new technology 2 before being introduced the average yield of crops from the existing technology 1 and the one after applying new technology is compared. Test the hypothesis at a 0.05 significance level. Pilot plant data yields $n_1 = 8$, $\overline{x_1} = 91.73$, $S_1^2 = 3.89$, $n_2 = 8$, $\overline{x_2} = 93.75$, and $S_2^2 = 4.02$. **(10 Marks)**

9. The random variable X, representing the number of cashew nuts in a plum cake, has the following probability distribution

x	4	5	6	7
P(X=x)	0.2	0.4	0.3	0.1

a) Find the mean μ and the variance σ^2 of X.

b) Find the mean $\mu_{\overline{X}}$ and the variance $\sigma_{\overline{X}}^2$ of the mean $\overline{X}$ for random samples of 36 plum cakes.

c) Find the probability that the average number of cashews in 36 plum cakes will be less than 5.5. **(10 Marks)**

Statistical Table Values

$P(z < 1.6) = 0.9452$, $P(z < 1.96) = 0.975$, $P(z < 1.33) = 0.9082$

$t_{0.05,15} = 1.753$, $t_{0.01,15} = 2.602$, $t_{0.025,8} = 2.306$, $t_{0.025,14} = 2.145$

$\chi^2_{0.05,4} = 9.488$, $\chi^2_{0.01,19} = 36.191$, $\chi^2_{0.05,19} = 30.144$

$f_{0.025}(25, 25) = 2.302$
