

Name: Om Swamy

R. No.: 1272023002

Indian Institute of Information Technology Lucknow

End Semester Examination

Regular/Back

Mathematics for CS I (MCS4300C)

BTech (IT, CS, CSAI & CSB) 4th Semester

Date: 21 May 2025

Max Marks: 80

QPS: Dhananjay Dey

Max Time: 3 Hours

Answer all of the following questions. The use of mobile phones and calculators is strictly prohibited in the examination hall.

Each and every step of your calculation should be shown on the answer sheet with justification.

1. Determine whether the statement is true or false: if $m \mid n$ and $n \mid m \Rightarrow m = n$, where $m, n \in \mathbb{Z}$. 1
2. Prove that $n^2 + 1$ is not a multiple of 3 if $n \in \mathbb{Z}$. 5
3. Prove that the number of diagonals of a convex polygon with n vertices is $\frac{1}{2}n(n-3)$, for $n \geq 4$ using mathematical induction. 5
4. How many 4-digit odd numbers are there with no leading zeroes and no repeated digits? 4
5. For any given set of 201 positive integers bounded by 300, show that we can always choose two of them whose ratio is a power of 3. 5
6. Compute $(a^6 + a^5 + a^4 + a^3 + a^2 + a + 1)$ for each $a \in \mathbb{Z}_7^*$ with $a \neq 1$. 2
7. Find the element $g \in \mathbb{Z}_7^*$ s/t $g = 3^{-220}$. 3
8. Consider the function $f : \mathbb{Z}_{12} \rightarrow \mathbb{Z}_{12}$ where $x \mapsto 4x$, $\forall x \in \mathbb{Z}_{12}$. Determine whether f is homomorphic and/or isomorphic. 3

21
20

33
44

60
101 x 40
42
41

298
100
298

39
100
1 multiple
where is

9. Consider the function $f : D_4 \rightarrow \mathbb{R}^*$

$$f(\alpha) = \begin{cases} 1 & \text{if } \alpha \text{ is a rotation,} \\ -1 & \text{if } \alpha \text{ is a reflection.} \end{cases}$$

Explain whether f is a homomorphism or not. Prove that the subgroup consisting of only rotation is a normal subgroup of D_4 . 3+2

10. Let G be a group with 55 elements. Prove that there exists an element of G with order 11. 5

11. (a) Find the multiplicative inverse of $2 + 3i$ in the ring $\mathbb{Z}_5[i] = \{a + bi : a, b \in \mathbb{Z}_5\}$. 3

(b) Find a non-zero $\alpha \in \mathbb{Z}_5[i]$ such that $\alpha\beta = 0$, where $\beta = 2 + i$. 3

12. Find the multiplicative inverse of $(3x + 1) + \langle x^2 - 1 \rangle \in \mathbb{Z}_7[x]/\langle x^2 - 1 \rangle$. 5

13. Find the three distinct solutions of the equation: $54321x + 9876y = 3$ 6+1+1

14. Solve the following system of congruence equations:

$$x \equiv 5 \pmod{6}, x \equiv 3 \pmod{10}, \text{ and } x \equiv 8 \pmod{15}. \quad 5$$

15. Solve the congruence equation: $x^7 \equiv 29 \pmod{2^5}$ 4, 1

16. Find the value of $5207^{763} \pmod{55}$ 4, 1

4 17. What are the last three digits of 1783^{2002} ? 4, 4

18. Apply Miller-Rabin Algorithm using base 2 to test whether the number 341 is composite or not. 5