Name: Karan Agarwal R. No.: LIT2023011

Indian Institute of Information Technology Lucknow

Mid Semester Examination

Regular/Back

Discrete

Mathematics for CS I (MCS4300C)

BTech (IT, CS, CSAI & CSB) 4^{th} Semester

Date: 24 Feb 2025 **QPS:** Dhananjoy Dey

Max Marks: 30 **Max Time:** 2 Hours

Answer all of the following questions. The use of mobile phones and calculators is strictly prohibited in the examination hall. Each and every step of your calculation should be shown on the answer sheet with justification.

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1. What is the truth value of the following statement? Write your answer with justification. 2

$$\forall\, xP(x) \Rightarrow \exists\,!\, xP(x)$$

2. If f and $f \circ g$ are one-to-one, does it follow that g is one-to-one? Justify your answer. 2

3. Prove or disprove that if a and b are rational numbers, then a^b is also rational. 2

4. How many bytes contain either exactly four consecutive 0s or exactly four consecutive 1s? 4

5. Consider a function $f : \{0,1\}^{16} \to \{0,1\}^8$. What is the largest k such that in any set of 2000 inputs, there are atleast k inputs that f maps to the same value? 5

6. Prove that there is no surjective function from $\mathbb{N}$ onto its power set $\mathcal{P}(\mathbb{N})$. 5

7. Find $\begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}^{-1}$ in $GL(2, \mathbb{Z}_7)$. 4

8. (a) Find $\langle S_i \rangle$ using the following Cayley table of D_4, where each $S_i \subset D_4$ for $1 \leq i \leq 4$. Justify your answer.
   (i) $S_1 = \{\rho_2\}$, (ii) $S_2 = \{H, V\}$, (iii) $S_3 = \{\rho_2, \rho_3\}$ (iv) $S_4 = \{H, D\}$ 4

$\circ$	ρ_0	ρ_1	ρ_2	ρ_3	H	V	D	D'
ρ_0	ρ_0	ρ_1	ρ_2	ρ_3	H	V	D	D'
ρ_1	ρ_1	ρ_2	ρ_3	ρ_0	D'	D	H	V
ρ_2	ρ_2	ρ_3	ρ_0	ρ_1	V	H	D'	D
ρ_3	ρ_3	ρ_0	ρ_1	ρ_2	D	D'	V	H
H	H	D	V	D'	ρ_0	ρ_2	ρ_1	ρ_3
V	V	D'	H	D	ρ_2	ρ_0	ρ_3	ρ_1
D	D	V	D'	H	ρ_3	ρ_1	ρ_0	ρ_2
D'	D'	H	D	V	ρ_1	ρ_3	ρ_2	ρ_0

(b) Find all elements $g \in D_4$ such that $g^2 = \rho_0$. 2