Mesh Addition Based on the Depth Image (MABDI)

by

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B.S., Mechanical Engineering
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Abstract

Many robotic applications, especially those whose goal is to aid or assist through human-robot interaction, utilize a rich map of the world for reasoning tasks such as collision detection, path planning, or object recognition. Such map, and the method used to produce it, must take into consideration real-world constraints. Most mesh-based mapping algorithms resemble a "black box" and do not provide a mechanism to close the loop and make decisions about the incoming information. MABDI leverages the global mesh by finding the difference between what we expect to see and what we are actually seeing, and using this to classify the incoming measurements as novel or not. This allows the surface reconstruction method to be run only on data that has not yet been represented in the global mesh. The result is an algorithm that becomes computationally inexpensive once the environment is known, but can also react to new objects.

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Chapter 1

Introduction

1.1 Overview

Many robotic applications, especially those that involve human-robot interaction, often require a rich representation of the environment in order to perform such behavior as path planning and obstacle avoidance. In general, a rich representation, or map, is useful for providing situational awareness to an autonomous agent. A map is also important for applications such as teleoperation [1].

In robotics, map building in an unknown environment is referred to as the Simultaneous Localization and Mapping (SLAM) problem [2]. This label describes the fact that a methodology which solves the SLAM problem must simultaneously locate the robot in the environment as well as map the environment. The focus of this work is the mapping aspect of the SLAM problem. Fig. 1.1 gives a visualization of the goal.

The methodology to build a map is a continuously evolving subject in the field of robotics and computer graphics. Well known works of map building methods be-

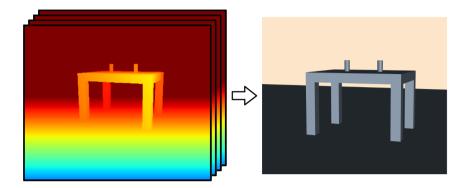


Figure 1.1: Goal is to create a map from depth images.

gan to be seen around 1987 [3]. Since then, the methods and the representations themselves have continued to evolve at an impressive rate. Growth in this field of research has been fueled by continuous advances in computing and sensing technologies. Over the years, sensors have continued to generate measurements at higher rates, higher resolution, and lower cost. RGB-D sensors are a new category of sensor that have recently gained extensive popularity in the robotics community due to their affordability and ability to generate a rich amount of data.

1.1.1 RGB-D Sensors

The popularity of RGB-D sensors began with the release and commercialization of the KinectTM by Microsoft. The arrival of the Kinect brought with it an inexpensive depth sensor that uses an active range system to generate a depth map of a given environment [4]. The Kinect and similar sensors, have come to be called RGB-D sensors. This class of sensors provide images which include both visual (RGB) and depth (D) values. Several works have taken advantage of this sensor technology in scenarios such as environmental mapping [5], 3D reconstruction [6], gesture recognition [7], and altitude control of aerial vehicles [8].

RGB-D sensors generally provide data at 30 frames per second and 640×480 resolution. Consequently, methods that use RGB-D data must handle over 9 million pixel values per second, if only using the depth information (D), and over 18 million if using both color (RGB) and depth (D). The magnitude of the amount of data output from RGB-D sensors create the need for mapping methods that are computationally inexpensive and also influence the type of data structure used to store the map.

1.1.2 Maps

There are different types of data structures that can define a map. All types have both intrinsic characteristics that impact the algorithms that generate them and constraints that must be considered for real-world applications. In addition, we are concerned with rich representation types, in contrast to sparse representation types [9], because rich types have the most use in applications such as human-robot interaction.

Table 1.1: Comparison of constraints for different map types.

	Supported	Computationally	Low Memory
		Inexpensive	Requirement
Point Clouds	X	X	-
Surfels	-	X	x
Implicit Functions	x	-	-
Mesh	x	x	x

When considering which type of map is best for real-world applications, we must consider the constraints imposed by each type:

 Supported - Is there software, tools, research, algorithms, etc., for this type of map?

- Computationally Inexpensive Can the algorithms run quickly on low cost computers (rather than specialized hardware)?
- Low Memory Requirement Can the algorithms run on hardware with a standard amount of RAM?

Table 1.1 compares the constraints of common map types. We can see, in general a mesh type map satisfies real-world constraints. Additionally, It has been used extensively by the gaming and graphics communities, and so benefits from an incredible amount of continued research and advances in hardware such as Graphics Processing Units (GPUs).

1.2 Goal

The goal of this work is to develop a mapping algorithm that can gracefully utilize the amount of data output from an RGB-D sensor. Additionally, the algorithm will make use of software tools and hardware that have been developed for mesh data structures. The algorithm will be able to make intelligent decisions on the data it receives based on the knowledge it has been building about the environment. The decisions will be driven by the leveraging the difference between what the algorithm is actually seeing and what it expects to see. The decisions will be generated using computationally inexpensive computer vision methods.

1.3 Contribution

Currently, one of the issues with mesh mapping techniques is they are generally "black box" methods. Meaning the data comes in from the sensor, those measurements are turned into a mesh, and then that mesh is appended to a global mesh.

Fig. 1.2 visualizes this common pipeline in black. The goal of this work is to design an algorithm to close the loop (as visualized in red) and allow the system to make decisions about the incoming data based on what it already knows.

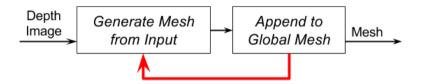


Figure 1.2: Common "black box" pipeline in black. The contribution of MABDI in red.

Chapter 2

Related Works

A major problem in robotics has been and continues to be: How can we create the "best" representation of an unknown environment? There are two main communities of researchers who been working on developing algorithms and methods to answer precisely this question. They are the robotics community and the computer graphics community and each community has a slightly different motivation for solving this problem. The robotics community is concerned with developing a real-time solution of generating representations in large environments. These representations are used by both fully autonomous and teleoperated systems. The common name which is used by the robotics community for this problem is Simultaneous Localization and Mapping or SLAM. The name SLAM refers to the problem of mapping and locating a robot in an unknown environment. Early methods generated very sparse representations of the world, as time and sensor technology progressed the representations became denser. A dense representation is desired for any system which must have good situational awareness of its environment. The computer graphics community is concerned with generating high quality representations of smaller environments and single objects. They generally refer to the problem as surface reconstruction. These representations are used by augmented reality, computer game object creation, 3D

printing, and more applications. In the following sections we will trace the development of representation generating methods in both communities. We will then discuss what is needed in future work.

2.1 SLAM

The problem of SLAM has been a primary focus of the robotics community for more than 25 years. A complete solution to the SLAM problem must be able to generate a representation of an unknown environment and track the robot in this new representation. In this body of literature the act of generating a representation is referred to as mapping. A good overview of the problem can be found in [10] and [11]. Each solution is designed to consider the goal application, type of sensor, computational constraints, and memory limits. All these factors influence the researcher's choice of what type of representation to use for the mapping procedure. In 2002 Thrun wrote a famous survey [2] of the SLAM literature which categorized existing algorithms on many traits including the representation. The representation choice of prior work can be roughly categorized into 3 types. The first type is characterized by some sort of list of 2D or 3D points and are usually considered to be sparse representations. Common names for these types are landmark locations and point clouds. The second type are considered to be more volumetric based and are often times considered to be a dense representation. Common names for these types are occupancy grid and Truncated Signed Distance Function (TSDF). The last type have the characteristic of being a surface representation and are also considered to be a dense representation. Common names for these types are surfels and mesh. In the following sections we will trace the history of each of the 3 types of representation that is seen in the SLAM literature.

2.1.1 Point Locations

One of the most well known and earliest solution to the SLAM problem, which uses a point location representation, was proposed by Smith et al. in 1990 [12]. The mathematical framework that he created was the origin of a family of solutions based on the Extended Kalman Filter (EKF). The representation he chose was simply a list of 2D landmark locations. Each location was part of a state matrix which was estimated at every iteration. A list of landmark locations was chosen because it allowed the method to have a low computational cost and use a small amount of memory, important factors in the days of early computing. There have been many improvements to the family of SLAM solutions which generate a list of point locations since Smith's work. One of the first practical implementations on a real robot was done by Thrun in 1998 [13]. In this work the SLAM problem was posed in an Expectation Maximization (EM) framework which is similar to the EKF framework in that landmark locations are saved in a state vector which is estimated at every iteration. In Thrun's work an occupancy grid map is generated as a post processing step from sonar measurements. The results showed that their representation could become more accurate over time by using new observations to improve the current estimate. This a highly desired ability of any representation generation method. The next step was the ability of these methods to include a loop closure procedure. A loop closure procedure was proposed by Gutmann in 1999 [14]. The key ability of the method was it could recognize when the robot was revisiting a prior location and adjust the entire representation with the constraint that the two points must coincide. In 2001 Dissanayake et al. [9] derived 3 theorems to theoretically prove the convergence of the SLAM problem. Their test platform used a millimeter-wave radar mounted on a vehicle and generated a list of 2D landmark locations. In 2001 Thrun et al. [15] cast the SLAM problem using particle filter techniques. Their results generated a 2D map and showed an increased robustness and lower computational

cost than prior methods. One of the key disadvantages of methods up to this point was that complexity scaled quadratically with the number of landmark locations. In 2002 Montemerlo et al. [16] created a SLAM solution named FastSLAM which was able to handle a much larger number of landmarks. They showed results with maps containing more than 50,000 points. Then, SLAM solutions using point locations became much more directed towards 3D.

Some of the first interesting works which represented the world as a list of 3D point locations were done by Thrun et al. in 2000 [17], Liu and Emery in 2001 [18], and Hähnel et al. in 2003. In these works the 2D landmark locations and robot position were estimated using very similar techniques from past work. Once this had been done the 3D laser scan data was simply appended to each estimated robot location. Then, a mesh was created by post processing the 3D point cloud. They utilized the fact that the laser collected the data in an incremental manner and simply connected neighboring 3D points. Finally, the mesh was simplified by looking for large planar sections and merging the corresponding mesh elements. One of the first SLAM solutions which used a single camera to generate a list of 3D points was done by Davison in 2003 [19]. Here he used a single camera to generate a very sparse list of 3D points. This method was limited to small environments. Future advances allowed representations of larger environments. In 2003 Thrun et al. [20] created a SLAM procedure which did not rely on having a structured environment and was applied to mapping large mines. In 2004 Howard et al. [21] created a SLAM systems based on a Segway platform equipped with a 3D laser which could map large areas of roughly 0.5 km on each side. One of the results showed a map with approximately 8 million points. In 2006 Cole and Newman [22] continued work in large-scale SLAM by increasing robustness and also generated maps with many 3D points using a laser sensor. In 2007 Clemente et al. created a large-scale SLAM system that used a single camera. The system had an advanced loop closing procedure based on visual features and created large maps of 3D points. In 2001 Klein and Murray [23] developed a

SLAM solution which used a single camera. The uniqueness of their method was the algorithmic structure. Their SLAM solution consisted of 2 separate processes: a tracking processes and a map building process. This algorithmic structure has become very common in many current SLAM solutions because of the advances in pose estimation technology. Klein and Murray were able to get very good results for a small environment and showed Augmented Reality (AR) applications. Many of the future advances of SLAM solutions, which generated 3D point sets, dealt with camera systems [24, 25, 26] and improved in speed and robustness. Many of the most current methods which produce a list of points are systems that use a relatively new type of sensor named a RGB-D sensor. One good example is a work that was produced in 2011 by Engelhard et al. [27]. In this work they used an algorithm named the Iterative Closest Point (ICP) [28] to align point clouds from coming from the RGB-D sensor into a large colored point cloud. The resulting maps were visually impressive. However, the map could not be adapted to new information and was not well suited for other applications, such as obstacle avoidance. These limitations are inherent in maps that consist of lists of points.

2.1.2 Volumetric

Many SLAM solutions generate a 2D volumetric representation of the world because they are especially advantageous in dealing with noisy sensors. Two of the first major works which generated a 2D volumetric representation were done in 1998 by Yamauchi et al. [29] and Schultz et al. [30]. These works generated a 2D occupancy grid which is a type of volumetric representation. Here the environment was divided into a 2D grid. Each square of the grid contained the probability that it was occupied with an object. All squares would be updated iteratively based on the current sensor readings. Occupancy grids, like any other volumetric-based representation, are limited by the amount of available memory. In 2002 Biswas et

al. [31] extended occupancy grid methods by allowing dynamic environments. This was done by looking at past "snapshots" of the map. In 2004 Eliazar and Parr [32] continued the advancement by decreasing computational cost and implemented a loop closure method.

There have been a few impressive SLAM solutions which generate a 3D volumetric representation. There are three major works which generated something very similar to a 3D occupancy grid which was saved as in a octree data structure [33, 34, 35, 36]. Each work had a slightly different name and procedure for generating the representation, but in general the representations divided the environment into cubes and had a scalar value representing the belief of a surface being there. Octrees were used to save memory by only having a fine resolution of cubes at places where there was a surface. There are many advantages to a 3D occupancy grid representation. When applied to obstacle avoidance and path planning algorithms. Also, the representation is very adaptable to new information. The major disadvantage is that the representation can not be visualized immediately. In order to render, an image must be generated at each desired viewpoint by ray tracing the volume. This can be a problem when using such method for applications such as teleoperation due to the computational cost of rendering. The current state of the art for generating a volumetric representation was done by Newcombe et al. in 2011 [6]. Their system used a RGB-D sensor and generated a 3D voxelized grid Truncated Signed Distance Function (TSDF) of the environment. For this type of representation each cube contains the value of the distance to the nearest surface. The sign of the value is based on which side of the surface the cube is relative to the sensor. This work has been the most capable at dealing with extremely noisy data and dynamic scenes. However, due to memory constraints the method can only represent environments that are about the size of a 4m cube. Also, it must be ray traced in order to be visualized.

2.1.3 Surface

One of the first major works which created a surface representation of the environment in real-time was done by Martin and Thrun in 2002 [37]. Their method utilized an EM framework to fit plane models to 3D point cloud data. Polygon mesh elements were then easily assigned to each plane. The main drive behind this work was to generate a map of the environment that uses a low amount of memory. Their method worked well for structured environments. One of the major limitations of their method, and other methods that only mesh large planar sections, is that the representation will only consist of planar sections and not capture the fine detail of the environment. In 2004 Viejo and Cazorla [38] developed a methodology for generating a mesh that can contain more information of the environment than large planar sections. Due to this ability, they termed their method to be "unconstrained." Essentially their method was based on a 3D Delaunay triangulation algorithm. Giesen surveyed Delaunay triangulation methods in [39]. Viejo and Cazorla were not able to obtain real-time results and, in fact, it has been seen that it is extremely difficult to run a 3D Delaunay triangulation in real time because of the numerous amount of distance calculations that are needed. One of the next major advances came from Weingarten and Siegwart in 2006 [40]. Their work also created a mesh that was only capable of capturing large planar surfaces. However, they showed increased robustness. In 2007 Pollefeys et al. published a work with multiple researchers [41, 42]. They developed a large urban mapping system consisting of a vehicle and eight camera systems. The processing was carried out by multiple CPUs and optimized with Graphics Processing Unit (GPU) calculations. In their work they used the camera systems to generate depth maps. The set of depth maps was then fused to create a new smaller set of depth maps that were more accurate. The depth maps in the smaller set were more accurate because the error was averaged out by using near-by depth maps. The smaller set was then used by a triangulation procedure to create

a mesh of the environment. The mesh generation procedure was based on a work from 2002 by Pajarola et al. [43]. This method defines a mesh in the depth image. It starts from a very coarse mesh and continues to refine in areas of the depth image based on a confidence criteria. In the work of Weingarten and Siegwart these meshes which are defined for each fused depth image are then checked for overlaps and duplicates are removed to make a single large mesh. One of the major drawbacks of this approach is that the output mesh can not be adapted by measurements which come from revisited parts of the scene. Another major advancement came in 2008 from Poppinga et al. [44]. In this work they used a Time of Flight (ToF) camera to generate a mesh representation of the large planar structures in the environment. Here they also develop a procedure to determine a mesh in a depth image. They leverage the structure of the depth image to make the method computationally inexpensive. In their work they simply append the meshes which are created from each depth image into a global coordinate system. They obtain very good results from a simple method. Once again, the method is not adaptive to new information. Also, a mesh is created for each depth image instead of updating and maintaining a global mesh. A major advancement came from a famous work done by Newcombe and Davison in 2010 [45]. In this work they designed a method to create a mesh reconstruction from a single video camera. Their method used Structure From Motion (SFM) to obtain a sparse point cloud of the scene. Then an implicit function was fit to the point cloud using the methodology of Ohtake et al. [46]. A bundle of depth maps is then selected. From the bundle a single reference depth image is selected and a "base" model is constructed by sampling the implicit surface for vertices in the reference frame. The neighboring frames are used to better the "base" model and create a more accurate mesh. Each reference frame has its own mesh and all the meshes are put into a global coordinate system. Duplications are then detected and removed. Again, the representation is not adaptive to new information. In 2010 Stühmer et al. [47] advanced the field by publishing a method to generate very accurate depth maps

from several color images in real-time. They showed very impressive results but their method was not designed to maintain a representation in a global coordinate frame.

The next major advances in methods that generated surface representations of the environment, were based on RGB-D sensors. This type of sensor has become very popular since the release of the Kinect from Microsoft which was the first mass produced RGB-D sensor of its kind. RGB-D sensors are inexpensive and produce noisy 640x480 depth images at 30Hz. The RGB-D sensor has excited the robotics community because this has been the first time that depth data has been so readily accessible from such an inexpensive sensor. Therefore, these methodologies must be able to quickly deal with very high rates of information. One impressive work came from Henry et al. in 2012 [48]. In this work they designed a system which used a RGB-D sensor to build a map made of surfels (Surfels are circular disks which have a particular position and orientation and also a radial size based on confidence.). In order to generate and maintain the surfel map they used the work of Weise et al. [49]. The map consists of a large number of surfels. The surfel map can be updated given new registered depth images from the sensor. Decisions are made how to handle each measurement in the depth image based on the difference between an expectation generated using the current map and the actual readings from the sensor. Rendering a surfel map requires special methods [50] and is difficult to use in applications such as obstacle avoidance.

One of the next major advances is a highly-related work that was published by Whelan et al. in 2012 [51] and more recently in 2013 [52]. The system they developed was named Kintinuous and was able to produce a high quality mesh representation of the environment. Their hybrid system utilized the KinectFusion method [6] of Newcombe et al. to create a volumetric representation of the portion of the environment in front of the sensor. As the sensor moves, portions of the environment that leave the volume in front of the sensor are ray cast and turned into a mesh. They

obtain very impressive results but also mention a limitation of their system for future work. The limitation is that the mesh can not be updated once created, which is an issue when revisiting parts of the environment. One of the most impressive current works which has an adaptable mesh came from Cashier et al. in 2012 [53]. In this work, they were able to generate and update a mesh with new measurements from a ToF sensor. They used the difference between the existing model and the actual measurements to decide whether to adapt the mesh or add new elements. The mesh topology was not adaptive to the environment and their experiments only showed results of mapping a single flat wall with no robot movement. The system needs to be tested for object addition and removal.

2.2 Surface Reconstruction

The computer graphics field has spent considerable effort to develop methodologies for creating representations from sets of data. Generally, these sets of data are acquired from a sensor. Methodologies have progressed steadily and are often designed for a specific application. One of the original motivations was to generate surfaces from medical imaging data. This allows doctors to make better decisions because the data are presented in a more intuitive manner. Current applications include augmented reality and 3D printing. Older methodologies were not as concerned with speed and often times had a large computational cost. Also, the methodologies are often designed for single objects or small environments. Following the taxonomy of such well-known works as [54, 55], the field can be roughly divided into representations that are generated with volume-based techniques and those that use surface-based techniques. Methods that use volume-based techniques are characterized by spatially subdividing the environmental volume and are usually computationally expensive and require a large amount of memory. Methods that use surface-based

techniques generate the representation using surface properties of the input data. Both types of methods can have mechanisms to adapt the mesh to noisy or new information. In the following section we will trace the progression of the methodologies.

2.2.1 Volume-based

Volume-based methods spatially subdivide the volume into smaller parts and operations are performed on the mesh to either implicitly imply the surface or define the surface depending on the information contained in each subdivision. One of the first well-known works that used a volume-based technique was proposed by Lorensen and Cline in 1987 [3]. In this work they proposed a method named marching cubes which is still known for its reliability and simplicity and is used by applications which do not have a computational requirement. Marching cubes subdivides the space into cubes. The data contained in each cube dictate how the surface connectivity will be defined in that cube. Possible vertex locations are at the corners and along the edges. Once this has been done for all cubes the process is complete. One of the next major steps came from Hoppe et al. in 1992 [56] In this work they used the input points to define a Signed Distance Function (SDF) in 3D space and then meshed the zero-set to obtain the output mesh. A SDF is a spatial function which has the value of the distance to the nearest surface at each point. The sign is used to specify if the point is inside our outside of the surface relative to the sensor. The zero-set of the SDF is the surface where the values transition from positive to negative. Using a SDF has proven to be very effective and has been the core idea of many methodologies that came after this work of Hoppe et al., such as KinectFusion [6]. One of the next advances came from Edelsbrunner and Mücke in 1994 [57] with a method named alpha shapes. Here they used 3D Delaunay triangulation and the input point set to decompose the volume into a Delaunay tetrahedrization. This gives a triangulation

of the input set which involves all points. A sphere of radius alpha is then used to remove edges and vertices to obtain a mesh of user specified resolution. Many works have made use of 3D Delaunay triangulation to create a mesh. Methods which use 3D Delaunay on the input set have a large computational cost and often cannot be executed in real-time. The next valuable contribution came from Bloomenthal in 1994 [58] as open source software for surface polygonization of implicit functions. This was a stable and robust open source software that has been used in many wellknown algorithms [45]. Another major advance came from Curless and Levoy in 1996 [59]. In this work they also constructed a Truncated Signed Distance Function (TSDF). A TSDF is very similar to a SDF, the only difference is that distance values are truncated after they exceed a certain threshold. Their method was one of the first to be able to handle several registered range scans. Their work showed how well a TSDF can deal with several noisy scans by naturally integrating out the error. They obtained very good results but not even close to real-time. A speed up in processing time was achieved by Pulli et al. in 1997 [60] by utilizing octrees. They obtained good results and their method was used by Surmann et al. [61] in a well-known robotic mapping work. Another major advance came in 2001 from Zhao et al [62]. They used Partial Differential Equation (PDE) methods to obtain a final reconstruction that was of better quality than prior methods. In 2001 Carr et al. [63] created a volumetric method based on the radial basis function (RBF). Their method was able to successfully deal with holes and generate water tight models. A water tight model is useful for single object reconstruction. However, it is not desired for mapping large environments. One of the next major advances was published in 2003 by Ohtake et al. [46]. In this work they created a method which was faster than the work of Carr et al. [63] by implementing a hierarchical approach with compactly supported basis functions. Their work has been considered to be the state of the art for calculating an implicit function of a noisy point set and was used by Newcombe et al. [45]. Volume-based methods have been able to create high quality represen-

tations and work well for single objects and small environments. These methods must spatially divide the environmental volume and therefore have a high memory requirement.

2.2.2 Surface-based

One of the first interesting and adaptive surface-based methods was published by Terzopoulos and Vasilescu in 1991 [64] and dealt with 2.5D data such as intensity and range images. The goal of their work was to create an adaptive mesh of an input image. The mesh was initialized as a 2D sheet of mesh elements with virtual springs along each edge. The stiffnesses of the virtual springs would then adjust based on the image information at that location. The mesh was able to adapt to be more dense in regions of higher intensity. In 1992 Terzopoulos and Vasilescu extended their methodology to 3D data [65]. In this work they used the distance between the mesh and the data to drive the vertices to be near the surface. In this early work they needed to initialize the mesh and control the subdivision of mesh elements to obtain a suitable resolution. In 1993 Hoppe et al. [66] published a method to optimize to resolution of a given mesh by posing the problem in an energy minimization framework. They minimized an energy function which tried to adequately represent the surface in the most concise mesh possible. One of the next advances in physical based adaptation of meshes came in 1993 from Huang and Goldof [67]. In this work they were able to adjust the size of the mesh elements to obtain a better resolution in areas of high frequency information using a physical based model. In addition, it was one of the first works to represent an object undergoing deformation. They method was able to perform tracking on simple simulation examples. Another advancement came in 1994 Rutishauser et al. [68] with a method specifically designed for incremental data. Their methodology worked with a sequential input set of range data and used a probabilistic framework to adjust the vertices of a mesh to the ex-

pected value given the prior observations. Their methodology also modeled the noise of the sensor with a sensor model. In 1994 Delingette [69] developed a methodology to generate a simplex mesh model of structured and unstructured 3D datasets. Elastic behavior of the mesh surface was modeled by local stabilizing functionals. Also, they implemented an iterative refinement process to refine the mesh in areas of high frequency information. One of the next steps was published by Turk and Levoy in 1994 [70]. Their method allowed overlapping meshes to be "zippered" into a single mesh surface. This ability is especially important for methods that generate a mesh for each depth image of the sensor and then need to combine all registered meshes into a single mesh. Their method is computationally expensive due to distance calculations. An interesting work came in 1995 from Chen and Medioni [71]. They devised an adaptive mesh methodology based on the inflation of a balloon. A mesh sphere was first initialized within the registered range measurements of the object. Virtual inflation forces were then used to expand the balloon until the mesh surface was a minimal distance from the range data. This method was limited to objects which are water tight. A major advancement came in 1999 from Bernardini et al., [72] in a method named the ball-pivoting algorithm. Their method is a good example of an advancing front method. These types of algorithms start with a seed mesh element and advance the boundary by adding new mesh elements in the immediate area of the boundary which is supported by measurements. Most advancing front algorithms differ in how it is decided to add new mesh elements. In the work of Bernardini et al. a virtual sphere of a user defined radius is rolled along the boundary of the mesh and new elements are added if the ball touches another measurement. Their methodology became popular because of its simplicity. One major disadvantage was that the generated mesh was a fixed topology. Another advancing front method came in 2001 from Gopi et al. [73, 54]. Here they sampled the input dataset to obtain a new dataset with a lower density of points in areas of lower frequency information. This effectively gave their method an adaptive topology. Next, a local

neighborhood was computed at each data point and projected to a plane tangent to the surface. The triangulation is then computed on this local tangent plane. They obtained impressive results on datasets of varying sample density and curvature. An interesting work was published in 2003 by Ivrissimtzis et al. [74]. Here they used a neural network model to adapt a mesh model to the data. They claimed that their method is computationally independent of the size of the input dataset because the dataset is only sampled by the method. There obtained good results. In 2004 Alexa et al. published a very interesting work to generate point set surfaces from an input dataset [75]. They use moving least squares (MLS) to locally approximate the surface with polynomials. The original dataset is then no longer used. Instead, they develop tools to sample the approximated surface to any resolution desired so that the end result is another point set of user specified resolution lying closer to the surface than the input dataset. One drawback is they had to develop their own methodology to render a point set. In 2005 Scheidegger et al. used the work of Alexa et al. to develop an advancing front methodology to generate concise meshes of high accuracy. Their main contribution was to augment an advancing front algorithm with global information so that the triangle size could adapt gracefully to any change. They obtained very impressive results. Most methodologies in Surface Reconstruction had been solely concerned with object or small environment recreation and have computational or memory requirements which do not work well with large environments. One of the first successful methods intended for large environments was published in 2009 by Marton et al. [76]. Their methodology was an advancing front algorithm which worked on a point set which was sampled from the MLS surface of the original point set. They were able to obtain impressive and near real-time results on datasets of large environments. They also developed a method to deal with revisited parts of the scene by determining the overlapping area and reconstructing only the updated part of the surface mesh. While this is a step forward, it would be better if the mesh surface converged to the actual surface with an increasing number of measurements.

To support dynamic scenes they developed mechanisms to decouple and reconstruct the mesh quickly. They only discussed these mechanisms in theory and had no results of how these mechanisms work. While Surface-based Surface Reconstruction techniques have developed impressively, several key issues arise when applying these techniques to mapping large environments.

2.3 Summary

The fields of Robotics and Computer Vision have developed many exciting methodologies to construct representations from a noisy input dataset. However there is still work to be done to obtain the ideal reconstruction method. A mesh is clearly a desirable type of representation. An ideal method should be able to generate and maintain a mesh representation efficiently. Also, many existing methods do not leverage the inherent structural information contained within the depth image. There are imaging processing techniques that could be used to answer some of the remaining problems in surface reconstruction, such as the need for adaptive topology and the need to decide how each measurement should be used to update the existing mesh. Henry et al. [48] has already investigated using the difference between the expected and actual measurements to guide the decision of how to use each measurement. However, their work was intended for surfels and needs to be extended to meshes. A method to generate a representation is needed which is computationally and memory efficient and can better adapt the representation to new information.

Chapter 3

Approach

3.1 Algorithmic Design

The algorithmic structure of MABDI can be seen in system diagram shown in Fig. 3.1. Table 3.1 gives a description of the main variables.

Table 3.1: Basic description of the main variables

Variable Name	Description
D	Depth image from RGB-D sensor
P	Pose of the sensor
D_n	Parts of D that are $novel$
S	Novel surface generated from D_n
M	Global mesh

The system diagram is very similar to Fig. 1.2 with the exception of the Classification component, shown in blue. This Classification component is MABDI's contribution to the state-of-art in mesh based mapping algorithms, and is what gives MABDI the ability to make decisions about the incoming data. The Classification component consists of two components:

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1. Generate Expected Depth Image E - Here we take the global mesh M, render it using computer graphics, and use the depth buffer of the render window to create a depth image E of what we expect to see from our sensor. This method requires the current pose P of the actual sensor (simulated for our experiments).

2. Classify Depth Image D - Here we classify the actual depth image D (simulated for our experiments) by first taking the absolute difference between E and D and thresholding. If the differences are small, those points are thrown away and if the differences are large, those points are kept as D_n . The idea behind this is, if the difference is large, the measurements are coming from a part of the environment that has not been seen before, i.e. novel. The implication of this assumption is that this version of MABDI cannot handle object removal. It is worth noting that MABDI can be extended to handle object removal by using the sign of the difference between E and D instead of the absolute value.

The system diagram in Fig. 3.1 also shows the Input and the Surface Reconstruction components. The Input component has been simulated for our experiments. More details of this simulation will be covered in Section ??. The Surface Reconstruction component of the MABDI algorithm can be implemented with any viable surface reconstruction method. Our implementation utilizes the structural information contained within the depth image. We will discuss this in more detail in the next section.

3.2 Implementation

3.2.1 Surface Reconstruction

A depth image is not simply a set of unorganized points, but has inherent structural information as well. This characteristic of the depth image allows us to define a topology on the 2 dimensional depth image that is preserved when projected to real-world coordinates. Our surface reconstruction method defines a topology that is visualized in Fig. 3.2. Here elements of the mesh are shown in light blue and novel points D_n are shown as blue dots. The red dot signifies a point that is not from the set D_n . Our implementation removes all mesh elements that include such points, as seen in the figure.

After initial experimentation with our method, we found erroneous elements could be defined that connect points who lie on completely different surfaces. To remove these elements, a step was added that uses imaging techniques in the form of a convolution filter. A 2 dimensional, backward differencing convolution filter is passed over the depth image. This filter has a large response at points where the difference between neighboring pixels is large. Remembering pixel values signify depth, points with large differences between themselves and their neighbor are marked as invalid and all elements that use them are removed.

Our surface reconstruction method was chosen for its ability to be implemented simply and run quickly. One consequence of our method is that the resulting surface S can have a large amount of elements. For example, if all points from D are classified as novel (this happens on the first frame), S can contain over 600,000 elements. This assumes D has no invalid points, e.g., out of the sensor's range. Many surface reconstruction methods have been developed to create a surface more intelligently, as discussed in Section 2. For example, the advancing front method developed by

Chapter 3. Approach

Marton et al. [76] is capable of creating surfaces with fewer elements by utilizing robust resampling methods. One great thing about the MABDI algorithm is that the method developed by Marton et al. can be used in place of our surface reconstruction method. Also, due to our implementation's modular software design, the entire code base would not need to be changed in order to accomplish this. We will discuss the software design in the next section.

3.2.2 Software Design

From a software perspective, the major difficulty of implementing the MABDI algorithm was found to be creating both the simulated depth image D and the expected depth image E. In addition, managing the complexity of the data pipeline needed to run the algorithm and the simulation of the sensor proved to be difficult. Thankfully, Kitware, who is a leading edge developer of open-source software, created the Visualization Toolkit (VTK) [77, 78]. At the time of this writing the VTK Github repository has over 60,000 commits and is contributed to by supporters such as Sandia National Labs [79].

VTK is aptly designed for the implementation of MABDI for many reasons. Perhaps the most important is the concept of a vtkAlgorithm (often called a Filter). This allows a programmer to create a custom and modular processing pipeline by defining classes that inherit vtkAlgorithm and then defining the connections between these classes. For example, you could have a pipeline that reads an image from a source (component 1), performs edge detection (component 2), and then renders the image (component 3).

Using the concept of VTK filters, the individual elements of MABDI can be succinctly defined in individual classes. With that in mind, we can see in Fig. 3.3 the layout used in our implementation of MABDI. vtkImageData and vtkPolyData

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are VTK types used to represent an image and mesh respectively. The elements shown in blue in Fig. 3.3 are the core components of the MABDI algorithm and are implemented as custom VTK filters. Their source code is included in Appendix A. Here we will discuss all components in more detail:

- Source Classes with the prefix Source define the environment that is used for the simulation and provide a mesh in the form of a vtkPolyData.
- FilterDepthImage Render the incoming vtkPolyData in a window and output the depth buffer from the window as a vtkImageData. The output additionally has pose information of the sensor.
- FilterClassifier Implements the true innovation of MABDI, takes the difference between the two incoming depth images (vtkImageData) and outputs a new depth image where the data that is not novel is marked to be thrown away.
- FilterDepthImageToSurface Performs surface reconstruction on the novel points. For more detail see Section 3.2.1. The surface is output as a vtkPoly-Data.
- FilterWorldMesh Here we simply append the incoming novel surface to a growing global mesh that is also output as a vtkPolyData.

MABDI is implemented in Python and uses VTK. Our implementation is distributed under the BSD license and is available on Github at the address below:

At the time of this writing, it consists of over 1,400 lines. The code that implements the MABDI algorithm itself is around 750 lines.

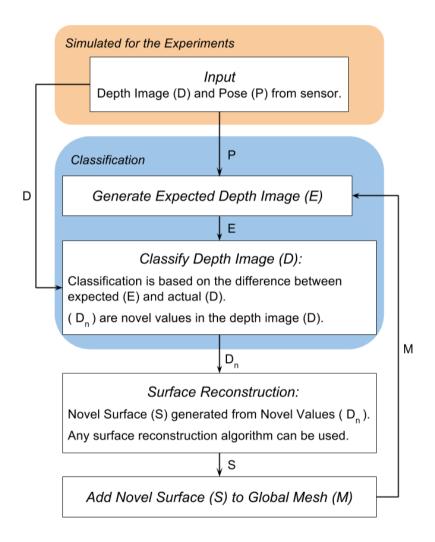


Figure 3.1: MABDI system diagram

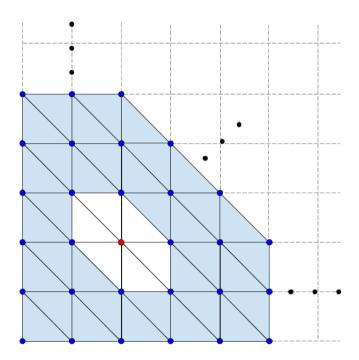


Figure 3.2: Topology defined on the depth image

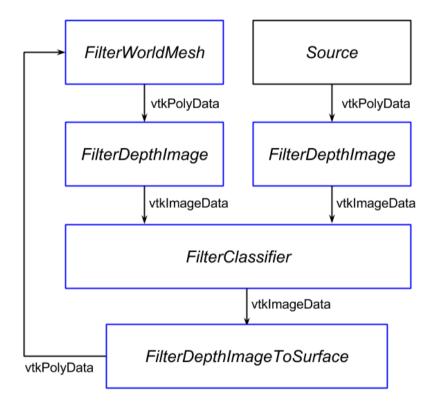


Figure 3.3: MABDI software diagram

Appendix A

MABDI code

A.1 FilterDepthImage.py

FilterDepthImage.py

```
import vtk
  from vtk.util.vtkAlgorithm import VTKPythonAlgorithmBase
  from vtk.util import numpy_support
  from vtk.numpy_interface import dataset_adapter as dsa
  import numpy as np
  from timeit import default_timer as timer
  import logging
  class FilterDepthImage(VTKPythonAlgorithmBase):
13
      Create a depth image of a scene
14
      This class uses the geometric information of the scene (vtkPolyData) and the
16
      orientation of the depth sensor.
19
      def __init__(self,
20
```

```
21
                     name='none',
                     offscreen=False,
22
                     noise = 0.0,
23
                     depth_image_size=(640, 480)):
25
           :param name: default='none'
26
             Used for the logging statements.
28
           :param offscreen:
             Create the render window that is used to produce the depth image offscreen
20
           :param noise:
30
             Noise to add to depth image.
31
           :param depth_image_size:
             Size of the depth image.
33
           11 11 11
34
           VTKPythonAlgorithmBase.__init__(self,
36
                                              nInputPorts=0,
37
                                              nOutputPorts=1, outputType='vtkImageData')
           self.\_name = name
39
40
41
           if type(noise) == bool:
                if noise:
42
                    \mathtt{noise} \, = \, 0.002
43
                else:
                    noise = 0.0
45
           self.\_noise = noise
46
           # vtk render objects
48
           self._ren = vtk.vtkRenderer()
49
           self._renWin = vtk.vtkRenderWindow()
           self._iren = vtk.vtkRenderWindowInteractor()
52
           # wire them up
           self._renWin.AddRenderer(self._ren)
           self._iren.SetRenderWindow(self._renWin)
55
56
           # offscreen rendering
57
58
           if offscreen:
                self._renWin.SetOffScreenRendering(1)
59
60
61
           # kinect intrinsic parameters
           # https://msdn.microsoft.com/en-us/library/hh438998.aspx
62
```

```
63
           self._renWin.SetSize(depth_image_size)
           self._ren.GetActiveCamera().SetViewAngle(60.0)
64
           self._ren.GetActiveCamera().SetClippingRange(0.8, 4.0)
65
           self._iren.GetInteractorStyle().SetAutoAdjustCameraClippingRange(0)
67
           # have it looking down and underneath the "floor"
68
           # so that it will produce a blank vtkImageData until
70
           # set_sensor_orientation() is called
           self.ren.GetActiveCamera().SetPosition(0.0, -20.0, 0.0)
71
           self._ren.GetActiveCamera().SetFocalPoint(0.0, -25.0, 0.0)
73
           # calculate image bounds
74
           self.imageBounds = [0, 0, 0, 0]
           viewport = self._ren.GetViewport()
76
           size = self._renWin.GetSize()
           self.imageBounds[0] = int(viewport[0] * size[0])
           self._imageBounds[1] = int(viewport[1] * size[1])
79
           self.imageBounds[2] = int(viewport[2] * size[0] + 0.5) - 1
80
           self.imageBounds[3] = int(viewport[3] * size[1] + 0.5) - 1
82
       def set_polydata(self, in_polydata):
83
           What this filter will render and consequently produce a depth image of.
85
           :param in_polydata: vtkAlgorithm that produces a vtkPolyData
86
           logging.info('')
89
           mapper = vtk.vtkPolyDataMapper()
           mapper.SetInputConnection(in_polydata.GetOutputPort())
91
92
           actor = vtk.vtkActor()
           actor.SetMapper(mapper)
95
           self._ren.AddActor(actor)
           self._iren.Initialize()
98
           self._iren.Render()
99
100
       def set_polydata_empty(self):
           Use to initialize this filter with an empty vtkPolyData
103
104
           logging.info('')
105
```

```
106
            polydata = vtk.vtkPolyData()
107
108
109
            mapper = vtk.vtkPolyDataMapper()
            mapper. SetInputDataObject (polydata)
110
111
            actor = vtk.vtkActor()
113
            actor.SetMapper(mapper)
114
            self._ren.AddActor(actor)
115
            self._iren.Initialize()
117
            self._iren.Render()
119
       def set_sensor_orientation(self, in_position, in_lookat):
120
            :param in_position: Position of sensor in world coordinates.
            :param in_lookat: Where the sensor is looking in world coordinates.
            logging.info('position{} lookat{}'.format(in_position, in_lookat))
125
126
            self._ren.GetActiveCamera().SetPosition(in_position)
            self._ren.GetActiveCamera().SetFocalPoint(in_lookat)
128
            self._iren.Render()
129
       def get_vtk_camera(self):
            return self._ren.GetActiveCamera()
132
       def get_width_by_height_ratio(self):
134
            return float (self._renWin.GetSize()[0]) / float (self._renWin.GetSize()[1])
136
       def kill_render_window(self):
            ** ** **
138
            Kill render window that this instance owns. Only to be used when the user
139
            is sure the filter will not be run again.
140
141
            # http://stackoverflow.com/questions/15639762/close-vtk-window-python
142
            self._renWin.Finalize()
143
144
            self._iren.TerminateApp()
            del self._renWin, self._iren
145
146
147
       def RequestInformation(self, request, inInfo, outInfo):
            logging.info('')
148
```

```
149
           size = self._renWin.GetSize()
           extent = (0, size[0] - 1, 0, size[1] - 1, 0, 0)
150
           info = outInfo.GetInformationObject(0)
           info.Set(vtk.vtkStreamingDemandDrivenPipeline.WHOLE_EXTENT(),
                     extent, len(extent))
153
           return 1
154
156
       def RequestData(self, request, inInfo, outInfo):
           logging.info('{{}}'.format(self._name))
           start = timer()
159
           # get the depth values
           vfa = vtk.vtkFloatArray()
           ib = self._imageBounds
162
           self._renWin.GetZbufferData(ib[0], ib[1], ib[2], ib[3], vfa)
163
           # add noise
165
           if self._noise is not 0.0:
               nvfa = numpy_support.vtk_to_numpy(vfa)
               nvfa += self._noise * nvfa * np.random.normal(0.0, 1.0, nvfa.shape)
168
               vfa = dsa.numpyTovtkDataArray(nvfa)
169
           # pack the depth values into the output vtkImageData
171
           info = outInfo.GetInformationObject(0)
           ue = info.Get(vtk.vtkStreamingDemandDrivenPipeline.UPDATE_EXTENT())
           out = vtk.vtkImageData.GetData(outInfo)
174
           out.GetPointData().SetScalars(vfa)
175
           out.SetExtent(ue)
           # append meta data to the vtkImageData containing intrinsic parameters
178
           out.sizex = self._renWin.GetSize()[0]
179
           out.sizey = self.renWin.GetSize()[1]
180
           out.viewport = self._ren.GetViewport()
181
           vtktmat = self._ren.GetActiveCamera().GetCompositeProjectionTransformMatrix(
182
                self._ren.GetTiledAspectRatio(),
183
                0.0, 1.0)
184
           vtktmat.Invert()
185
           out.tmat = self._vtkmatrix_to_numpy(vtktmat)
186
           end = timer()
188
           logging.info('Execution time {:.4f} seconds'.format(end - start))
189
190
191
           return 1
```

```
def _vtkmatrix_to_numpy(self, matrix):
193
194
            Copies the elements of a vtkMatrix4x4 into a numpy array.
196
            :param matrix: The matrix to be copied into an array.
197
            :type matrix: vtk.vtkMatrix4x4
199
            :rtype: numpy.ndarray
            ,, ,, ,,
200
            m = np.ones((4, 4))
201
            for i in range(4):
202
                for j in range(4):
203
                    m[i, j] = matrix.GetElement(i, j)
204
205
            return m
```

A.2 FilterClassifier.py

FilterClassifier.py

```
import vtk
  from vtk.util.vtkAlgorithm import VTKPythonAlgorithmBase
  from vtk.util import numpy_support
  from timeit import default_timer as timer
  import logging
  class FilterClassifier(VTKPythonAlgorithmBase):
      vtkAlgorithm with 2 inputs of vtkImageData and an output of vtkImageData
      Input: Depth images
      Output: Classified depth image
      def __init__(self , param_classifier_threshold = 0.01):
17
          : param\_classifier\_threshold: \ default = 0.01
18
            Threshold to determine when the difference in the depth images is too big
            and is therefore a novel measurement.
20
          :return:
21
```

```
,, ,, ,,
22
23
           VTKPythonAlgorithmBase. __init__(self,
24
2.5
                                             nInputPorts=2, inputType='vtkImageData',
                                             nOutputPorts=1, outputType='vtkImageData')
26
           self.\_param\_classifier\_threshold = param\_classifier\_threshold
29
           self._postprocess = []
30
           self._postprocess_im1 = []
31
           self._postprocess_im2 = []
32
           self._postprocess_difim = []
33
      def set_postprocess(self, do_postprocess):
35
           self._postprocess = do_postprocess
36
      def get_depth_images(self):
38
           11 11 11
39
           Get the depth images. User has to call set_postprocess(True) first.
           :return: Depth images
41
             return [0] - actual
49
             return[1] - expected
             return [2] - threshold absolute difference
44
45
           return self._postprocess_im1, self._postprocess_im2, self._postprocess_difim
47
      def RequestInformation(self, request, inInfo, outInfo):
48
           logging.info(',')
50
           # input images dimensions
51
           info = inInfo [0]. GetInformationObject (0)
           ue1 = info.Get(vtk.vtkStreamingDemandDrivenPipeline.UPDATE_EXTENT())
           info = inInfo[1]. GetInformationObject(0)
54
           ue2 = info.Get(vtk.vtkStreamingDemandDrivenPipeline.UPDATEEXTENT())
           if ue1 != ue2:
56
               logging.warning('Input images have different dimensions. {} {} '.format(
57
      ue1, ue2))
59
           extent = ue1
           info = outInfo.GetInformationObject(0)
60
           info.Set(vtk.vtkStreamingDemandDrivenPipeline.WHOLE_EXTENT(),
61
62
                     extent , len(extent))
63
```

```
64
           return 1
65
       def RequestData(self, request, inInfo, outInfo):
66
           logging.info('')
           start = timer()
60
           # in images (vtkImageData)
71
           inp1 = vtk.vtkImageData.GetData(inInfo[0])
           inp2 = vtk.vtkImageData.GetData(inInfo[1])
72
           # convert to numpy arrays
74
           dim = inp1.GetDimensions()
7.5
           im1 = numpy_support.vtk_to_numpy(inp1.GetPointData().GetScalars())\
                .reshape(dim[1], dim[0])
77
           dim = inp1.GetDimensions()
78
           im2 = numpy_support.vtk_to_numpy(inp2.GetPointData().GetScalars())\
                .reshape(dim[1], dim[0])
80
81
           # difference in the images
           # im1 is assumed to be from the actual sensor
83
           # im2 is what we expect to see based on the world mesh
84
           # Anywhere the difference is small, throw those measurements away
           # by setting them to one. By doing this FilterDepthImageToSurface
86
           # will assume they lie on the clipping plane and will remove them
87
           difim = abs(im1 - im2) < self._param_classifier_threshold
           if self._postprocess:
89
                self._postprocess_im1 = im1.copy()
90
               self._postprocess_im2 = im2.copy()
               self._postprocess_difim = difim.copy()
92
           imout = im1
93
           imout[difim] = 1.0
95
           info = outInfo.GetInformationObject(0)
96
           ue = info.Get(vtk.vtkStreamingDemandDrivenPipeline.UPDATEEXTENT())
           # output vtkImageData
99
           out = vtk.vtkImageData.GetData(outInfo)
100
           out.SetExtent(ue)
           (out.sizex, out.sizey, out.tmat, out.viewport) = \
               (inpl.sizex, inpl.sizey, inpl.tmat, inpl.viewport)
           out.GetPointData().SetScalars(
104
               numpy\_support.numpy\_to\_vtk(imout.reshape(-1)))
106
```

```
end = timer()
logging.info('Execution time {:.4f} seconds'.format(end - start))
return 1
```

A.3 FilterDepthImageToSurface.py

FilterDepthImageToSurface.py

```
import vtk
2 from vtk.util.vtkAlgorithm import VTKPythonAlgorithmBase
  from vtk.util import numpy_support
  from vtk.numpy_interface import dataset_adapter as dsa
  from Utilities import DebugTimeVTKFilter
  import numpy as np
  from scipy import ndimage
  from timeit import default_timer as timer
  import logging
12
13
  class FilterDepthImageToSurface(VTKPythonAlgorithmBase):
      vtkAlgorithm with input of vtkImageData and output of vtkPolyData
      This filter first defines a connectivity on the depth image that is like a
18
      checkerboard but with two triangles in each square. It then throws away all
19
      farther than the param_farplane_threshold and all points with a large difference
20
      between neighbors (controlled with param_convolution_theshold)
21
      Input: Depth image
      Output: Mesh created by projecting depth image
23
      def __init__(self,
26
                    param_farplane_threshold = 1.0,
27
                    param_convolution_threshold = 0.01):
29
           Algorithm setup and define parameters.
30
```

```
31
           :param param_farplane_threshold: default=1.0
             Values on the depth image range from 0.0-1.0. Points with depth values
32
      greater
33
             than param_farplane_threshold will be thrown away.
           :param param_convolution_threshold: default = 0.01
34
             Convolution is used to determine pixel neighbors with a large difference.
35
      If
36
             there is one, the point will be thrown away. This threshold controls
      sensitivity.
           ,, ,, ,,
37
38
           VTKPythonAlgorithmBase. __init__(self,
39
                                             nInputPorts=1, inputType='vtkImageData',
                                             nOutputPorts=1, outputType='vtkPolyData')
41
49
           self._param_farplane_threshold = param_farplane_threshold
           self.param_convolution_theshold = param_convolution_threshold
44
45
           self.\_sizex = []
           self.\_sizey = []
47
           self._viewport = []
48
50
           self._display_pts = []
           self._viewport_pts = []
51
           self._world_pts = []
53
           self._points = vtk.vtkPoints()
54
           self._polys = vtk.vtkCellArray()
           self._polydata = vtk.vtkPolyData()
56
           self._polydata.SetPoints(self._points)
           self._polydata.SetPolys(self._polys)
59
           self._extract = vtk.vtkExtractPolyDataGeometry()
60
           DebugTimeVTKFilter(self._extract)
61
           self._extract.SetInputData(self._polydata)
62
           planefunc = vtk.vtkPlane()
63
           planefunc. SetNormal (0.0, -1.0, 0.0)
           planefunc. SetOrigin (0.0, -1.0, 0.0)
65
66
           self._extract.SetImplicitFunction(planefunc)
67
      def RequestData(self, request, inInfo, outInfo):
68
69
           logging.info('')
70
```

```
71
           start = timer()
72
           # input (vtkImageData)
           inp = vtk.vtkImageData.GetData(inInfo[0])
75
           # if the vtkImageData size has changed or this is the first time
76
           # save new size info and initialize containers
78
           if (self._sizex, self._sizey, self._viewport) != (inp.sizex, inp.sizey, inp.
       viewport):
               (self._sizex, self._sizey) = (inp.sizex, inp.sizey)
                self._viewport = inp.viewport
80
                self._init_containers()
81
           # the incoming depth image
83
           di = numpy_support.vtk_to_numpy(inp.GetPointData().GetScalars())\
84
                .reshape((self._sizey, self._sizex))
86
           # add z values to viewport_pts based on incoming depth image
87
           self.viewport_pts[2, :] = di.reshape(-1)
89
           # project to world coordinates
90
           self._world_pts = np.dot(inp.tmat, self._viewport_pts)
           self._world_pts = self._world_pts / self._world_pts[3]
92
93
           """ Remove invalid points """
95
           # index to pts outside sensor range (defined by vtkCamera clipping range)
96
           outside_range = ~(di < self._param_farplane_threshold)</pre>
           # find pixel neighbors with large differences in value
99
           # http://docs.scipy.org/doc/scipy/reference/tutorial/ndimage.html
           kh = np.array([[1, -1], [0, 0]])
           edges_h = abs(ndimage.convolve(di,
103
                                            mode='nearest',
104
                                            origin = -1)) > self.param_convolution_theshold
105
           kv = np.array([[1, 0], [-1, 0]])
106
           edges_v = abs(ndimage.convolve(di,
107
108
                                            mode='nearest',
109
                                            origin =-1)) > self.param_convolution_theshold
           # combine all the points found to be invalid
112
```

```
# and set them to a value underneath the "floor of the environment"
113
            # http://stackoverflow.com/a/20528566/4068274
114
            invalid_index = np.logical_or.reduce((outside_range.reshape(-1),
116
                                                     edges_h.reshape(-1),
                                                     edges_v.reshape(-1))
117
            self.world_pts[0:3, invalid_index] = np.array([[0.0], [-2.0], [0.0]])
118
120
            """ Update and set filter output """
191
            # update vtkPoints
122
            vtkarray = dsa.numpyTovtkDataArray(self._world_pts[0:3, :].T)
123
            self._points.SetData(vtkarray)
124
125
            # update output (vtkPolyData)
126
            out = vtk.vtkPolyData.GetData(outInfo)
            self._extract.Update()
            logging.info('Number of triangles: {}'.format(self._extract.GetOutput().
129
       GetNumberOfCells()))
            out.ShallowCopy(self._extract.GetOutput())
            end = timer()
            logging.info('Execution time {:.4f} seconds'.format(end - start))
134
            return 1
        def _init_containers(self):
            logging.info('Initializing arrays for projection calculation.')
138
            tstart = timer()
139
140
            # helper variables (width, height)
141
            (w, h) = (self._sizex, self._sizey)
142
143
            """ display points (list of all pixel coordinates) """
144
145
            self.\_display\_pts = np.ones((2, w * h))
146
            self.\_display\_pts[0, :], self.\_display\_pts[1, :] = \
147
                zip(*[(j, i) for i in np.arange(h) for j in np.arange(w)])
148
149
            """ viewport points """
            # https://github.com/Kitware/VTK/blob/52
151
       \mathtt{d}45496877b00852a08a5b9819d109c2fd9bfab/Rendering/Core/vtkCoordinate.h\#L2664bfab/Rendering/Core/vtkCoordinate.h
            self._viewport_pts = np.ones((4, self._display_pts.shape[1]))
153
```

```
self.\_viewport\_pts\,[0\,,\ :]\ =\ 2.0\ *\ (self.\_display\_pts\,[0\,,\ :]\ -\ w\ *\ self\,.
154
       _viewport[0]) / \
                (w * (self._viewport[2] - self._viewport[0])) - 1.0
            self.\_viewport\_pts[1, :] = 2.0 * (self.\_display\_pts[1, :] - h * self.
156
       _viewport[1]) / \
                (h * (self.\_viewport[3] - self.\_viewport[1])) - 1.0
159
           """ new world points (just initializing the container) """
160
            self._world_pts = np.ones(self._viewport_pts.shape)
161
162
           """ cells (list of triangles created by connecting neighbors in depth image
163
       space ) """
164
           # connectivity on the depth image is almost like a checkerboard pattern
165
           # except with two triangles in every checkerboard square
           nt = (2*w)*(h-1) # number of triangles
167
            cells = np.zeros((3, nt), dtype=np.int)
168
           i = 0
            while i < (nt/2):
                if ((i+1)\% w) != 0: # if on the side of the image skip
171
                    cells[:, 2*i] = (i, i+1, w+i)
                    cells[:, 2*i+1] = (i+1, w+i+1, w+i)
173
                i += 1
174
           # remove columns with zeros (the ones we skipped in the while loop)
            index = np.where(cells.any(axis=0))[0] # all columns that are non zero
177
            cells = cells[:, index]
179
           # turn our connectivity list into a vtk object (vtkCellArray)
180
            for tpt in cells.T:
181
                self._polys.InsertNextCell(3)
182
                self._polys.InsertCellPoint(tpt[0])
183
                self._polys.InsertCellPoint(tpt[1])
184
                self._polys.InsertCellPoint(tpt[2])
185
            self._polydata.SetPolys(self._polys)
186
187
           # time me
188
189
           tend = timer()
            logging.info('Initializing arrays for projection calculation {:.4f} seconds'
190
       .format(tend - tstart))
```

A.4 FilterWorldMesh.py

FilterWorldMesh.py

```
import vtk
  from vtk.util.vtkAlgorithm import VTKPythonAlgorithmBase
  from vtk.numpy_interface import dataset_adapter as dsa
  import numpy as np
  import matplotlib.pyplot as plt
  from itertools import cycle
  from timeit import default_timer as timer
  import logging
19
  class FilterWorldMesh(VTKPythonAlgorithmBase):
14
16
      vtkAlgorithm with input vtkPolyData and output vtkPolyData
      Input: Surface to be added to the global mesh
17
      Output: The global mesh
      def __init__(self , color=False):
20
21
           :param color: default=False
             Color every new surface of the global mesh a different color.
23
          :return:
2.4
26
          VTKPythonAlgorithmBase. __init__(self,
                                            nInputPorts=1, inputType='vtkPolyData',
                                            nOutputPorts=1, outputType='vtkPolyData')
29
           self._worldmesh = vtk.vtkAppendPolyData()
32
          # colormap for changing polydata on every iteration
33
          # http://matplotlib.org/examples/color/colormaps_reference.html
           self._color = color
35
           if self._color:
36
               gist_rainbow_r = plt.cm.get_cmap(name='gist_rainbow_r')
               mycm = gist_rainbow_r(range(160, 260, 5))[:, 0:3]
38
               self._colorcycle = cycle(mycm)
39
```

```
40
      def RequestData(self , request , inInfo , outInfo):
41
           logging.info('')
42
           start = timer()
          # input polydata
45
          # have to make a copy otherwise polys will not show up in the render
47
          # even though GetNumberOfCells() says they should be there
           tmp = vtk.vtkPolyData.GetData(inInfo[0])
48
           inp = vtk.vtkPolyData()
           inp.ShallowCopy(tmp)
50
51
          # change color of all cells
           if self._color:
53
               ncells = inp.GetNumberOfCells()
54
               c = self._colorcycle.next()
               vtkarray = dsa.numpyTovtkDataArray(np.tile(c, (ncells, 1)))
56
               inp.GetCellData().SetScalars(vtkarray)
          # add to world mesh
59
           self._worldmesh.AddInputData(inp)
60
           self._worldmesh.Update()
62
           logging.info('Number of cells: in = {} total = {}'
63
                        . format(inp.GetNumberOfCells(),
                                 self._worldmesh.GetOutput().GetNumberOfCells()))
65
66
           # output world mesh
           out = vtk.vtkPolyData.GetData(outInfo)
           out.ShallowCopy(self._worldmesh.GetOutput())
69
           end = timer()
71
           logging.info('Execution time {:.4f} seconds'.format(end - start))
72
73
           return 1
74
```

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