

# Logistic Regression

## import library

```
In [ ]: import numpy as np
import matplotlib.pyplot as plt
import matplotlib.colors as colors
from matplotlib import ticker, cm
```

## load training data

```
In [ ]: fname_data = 'assignment_08_data.csv'

data      = np.genfromtxt(fname_data, delimiter=',')
number_data = data.shape[0]

point_x = data[:, 0]
point_y = data[:, 1]
label    = data[:, 2]

print('number of data = ', number_data)
print('data type of point x = ', point_x.dtype)
print('data type of point y = ', point_y.dtype)

point_x_class_0 = point_x[label == 0]
point_y_class_0 = point_y[label == 0]

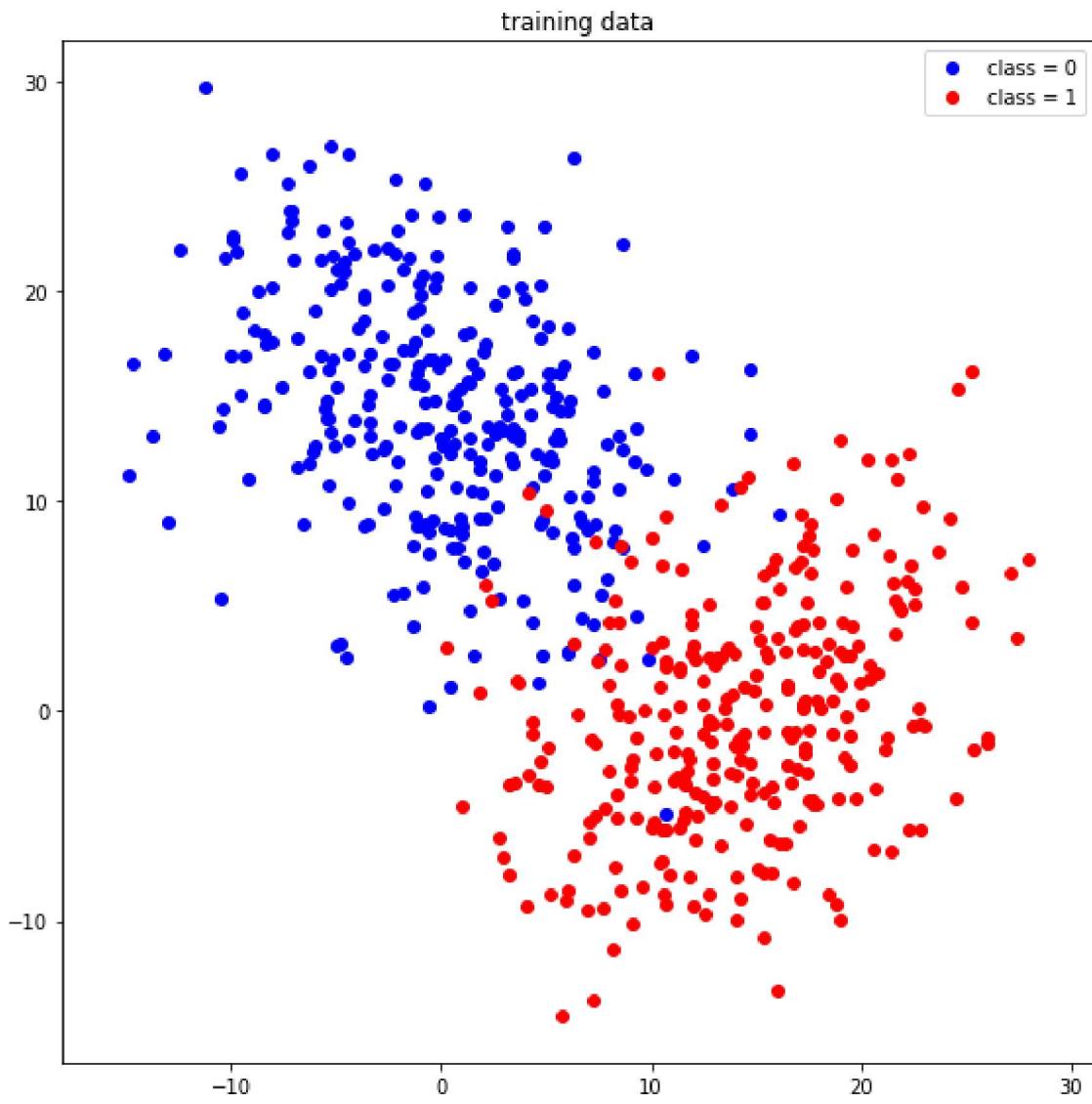
point_x_class_1 = point_x[label == 1]
point_y_class_1 = point_y[label == 1]
```

```
number of data = 600
data type of point x = float64
data type of point y = float64
```

## plot the data

```
In [ ]: f = plt.figure(figsize=(8,8))

plt.title('training data')
plt.plot(point_x_class_0, point_y_class_0, 'o', color='blue', label='class = 0')
plt.plot(point_x_class_1, point_y_class_1, 'o', color='red', label='class = 1')
plt.axis('equal')
plt.legend()
plt.tight_layout()
plt.show()
```



## define the linear regression function

- $\theta = (\theta_0, \theta_1, \theta_2)$
- point =  $(1, x, y) \in \mathbb{R}^3$

```
In [ ]: def compute_linear_regression(theta, point):
    # ++++++
    # complete the blanks
    #
    value = np.inner(point, theta)
    #
    # ++++++
    return value
```

## define sigmoid function with input

- $z \in \mathbb{R}$

```
In [ ]: def sigmoid(z):

    # ++++++
    # complete the blanks
    #

    value = 1 / (1 + np.exp(-z))

    #
    # ++++++

    return value
```

## define the logistic regression function

- $\theta = (\theta_0, \theta_1, \theta_2) \in \mathbb{R}^3$
- point =  $(1, x, y) \in \mathbb{R}^3$

```
In [ ]: def compute_logistic_regression(theta, point):

    # ++++++
    # complete the blanks
    #

    f = compute_linear_regression(theta, point)
    value = sigmoid(f)

    #
    # ++++++

    return value
```

## define the residual function

- $\theta = (\theta_0, \theta_1, \theta_2) \in \mathbb{R}^3$
- point =  $(x, y) \in \mathbb{R}^2$
- label =  $l \in \{0, 1\}$

```
In [ ]: def compute_residual(theta, point, label):

    # ++++++
    # complete the blanks
    #

    h = compute_logistic_regression(theta, point)
    residual = - ((label * np.log(h)) + ((1 - label) * np.log(1 - h)))

    #
    # ++++++

    return residual
```

## define the loss function for the logistic regression

- $\theta = (\theta_0, \theta_1, \theta_2) \in \mathbb{R}^3$
- point =  $(1, x, y) \in \mathbb{R}^3$
- label =  $l \in \{0, 1\}$

```
In [ ]: def compute_loss(theta, point, label):
    # ++++++
    # complete the blanks
    #

    n = point.shape[0]
    r = compute_residual(theta, point, label)
    loss = np.sum(r) / n

    #
    # ++++++
    return loss
```

## define the gradient of the loss with respect to the model parameter $\theta$

- $\theta = (\theta_0, \theta_1, \theta_2) \in \mathbb{R}^3$
- point =  $(1, x, y) \in \mathbb{R}^3$
- label =  $l \in \{0, 1\}$

```
In [ ]: def compute_gradient(theta, point, label):
    # ++++++
    # complete the blanks
    #

    n = point.shape[0]
    h = compute_logistic_regression(theta, point)
    gradient = np.sum((h.reshape(n, 1) - label.reshape(n, 1)) * point, axis = 0) / n

    #
    # ++++++
    return gradient
```

## initialize the gradient descent algorithm

```
In [ ]: num_iteration = 5000 # USE THIS VALUE for the number of gradient descent iterations
learning_rate = 0.001 # USE THIS VALUE for the learning rate

theta = np.array((0, 0, 0))
theta_iteration = np.zeros((num_iteration, theta.size))
loss_iteration = np.zeros(num_iteration)

number_point_class_0 = len(point_x_class_0)
number_point_class_1 = len(point_x_class_1)

point_class_0 = np.ones((number_point_class_0, 3))
point_class_1 = np.ones((number_point_class_1, 3))
```

```

point_class_0[:, 1] = point_x_class_0
point_class_0[:, 2] = point_y_class_0

point_class_1[:, 1] = point_x_class_1
point_class_1[:, 2] = point_y_class_1

label_0      = np.zeros(number_point_class_0)
label_1      = np.ones(number_point_class_1)

point = np.concatenate((point_class_0, point_class_1), axis=0)
label = np.concatenate((label_0, label_1), axis=0)

print('shape of point_class_0 : ', point_class_0.shape)
print('shape of point_class_1 : ', point_class_1.shape)
print('shape of label_0 : ', label_0.shape)
print('shape of label_1 : ', label_1.shape)

print('shape of point : ', point.shape)
print('shape of label : ', label.shape)

```

```

shape of point_class_0 : (300, 3)
shape of point_class_1 : (300, 3)
shape of label_0 : (300,)
shape of label_1 : (300,)
shape of point : (600, 3)
shape of label : (600,)

```

## run the gradient descent algorithm to optimize the loss function with respect to the model parameter

```

In [ ]: for i in range(num_iteration):

    # ++++++
    # complete the blanks
    #

    theta    = theta_iteration[i - 1] - (learning_rate * compute_gradient(theta_iteration[i - 1], point, label))
    loss     = compute_loss(theta, point, label)

    #
    # +++++++

    theta_iteration[i, :] = theta
    loss_iteration[i]     = loss

theta_optimal = theta

```

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## functions for presenting the results

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```

In [ ]: def function_result_01():

    input1 = np.array([0.1, 0.2, 0.3])

```

```
    input2 = np.array([[1, 2, 3], [1, -2, -3]])  
  
    value = compute_linear_regression(input1, input2)  
  
    print(value)
```

```
In [ ]: def function_result_02():  
  
    input1 = np.array([0.1, 0.2, 0.3])  
    input2 = np.array([[1, 2, 3], [1, -2, -3]])  
  
    value = compute_logistic_regression(input1, input2)  
  
    print(value)
```

```
In [ ]: def function_result_03():  
  
    input1 = np.array([0.1, 0.2, 0.3])  
    input2 = np.array([[1, 2, 3], [1, -2, -3]])  
    input3 = np.array([0, 1])  
  
    value = compute_residual(input1, input2, input3)  
  
    print(value)
```

```
In [ ]: def function_result_04():  
  
    input1 = np.array([0.1, 0.2, 0.3])  
    input2 = np.array([[1, 2, 3], [1, -2, -3]])  
    input3 = np.array([[0], [1]])  
  
    value = compute_loss(input1, input2, input3)  
  
    print(value)
```

```
In [ ]: def function_result_05():  
  
    input1 = np.array([0.1, 0.2, 0.3])  
    input2 = np.array([[1, 2, 3], [1, -2, -3]])  
    input3 = np.array([[0], [1]])  
  
    value = compute_gradient(input1, input2, input3)  
  
    print(value)
```

```
In [ ]: def function_result_06():  
  
    plt.figure(figsize=(8,6))  
    plt.title('loss')  
  
    plt.plot(loss_iteration, '—', color='red')  
    plt.xlabel('iteration')  
    plt.ylabel('loss')  
  
    plt.tight_layout()  
    plt.show()
```

```
In [ ]: def function_result_07():  
  
    plt.figure(figsize=(8,6)) # USE THIS VALUE for the size of the figure  
    plt.title('model parameter')
```

```

plt.plot(theta_iteration[:, 0], '--', color='red', label=r'$\theta_0$')
plt.plot(theta_iteration[:, 1], '--', color='green', label=r'$\theta_1$')
plt.plot(theta_iteration[:, 2], '--', color='blue', label=r'$\theta_2$')

plt.xlabel('iteration')
plt.legend()

plt.tight_layout()
plt.show()

```

## plot the linear regression values over the 2-dimensional Euclidean space and superimpose the training data

```

In [ ]: def function_result_08():

    X = np.arange(-20, 35, 0.1) # USE THIS VALUE for the range of x values in the co
    Y = np.arange(-20, 35, 0.1) # USE THIS VALUE for the range of y values in the co

    [XX, YY] = np.meshgrid(X, Y)

    # ++++++
    # complete the blanks
    #
    fig = plt.figure(figsize=(8,8))
    ax = fig.add_subplot(111)
    plt.title('linear regression values')

    ZZ = np.array([np.ones(XX.shape), XX, YY])
    linear_regression = theta_optimal[0]*ZZ[0] + theta_optimal[1]*ZZ[1] + theta_opt
    plt.scatter(XX, YY, c=linear_regression, cmap=plt.cm.bwr)
    plt.grid()

    classifier = np.where((linear_regression < 0.01) & (linear_regression > -0.01))
    plt.plot(XX[classifier], YY[classifier], '--', color='black')

    ax.plot(point_x_class_0, point_y_class_0, '.', color='blue', label='class = 0')
    ax.plot(point_x_class_1, point_y_class_1, '.', color='red', label='class = 1')
    ax = plt.axis('equal')

    plt.legend()
    plt.colorbar()

    plt.tight_layout()
    plt.show()
    #
    # ++++++

```

## plot the logistic regression values over the 2-dimensional Euclidean space

```

In [ ]: def function_result_09():

    X = np.arange(-20, 35, 0.1) # USE THIS VALUE for the range of x values in the co
    Y = np.arange(-20, 35, 0.1) # USE THIS VALUE for the range of y values in the co

    [XX, YY] = np.meshgrid(X, Y)

```

```

# ++++++
# complete the blanks
#
fig = plt.figure(figsize=(8,8))
ax = fig.add_subplot(111)
plt.title('logistic regression values')

ZZ = np.array([np.ones(XX.shape), XX, YY])
z = theta_optimal[0]*ZZ[0] + theta_optimal[1]*ZZ[1] + theta_optimal[2]*ZZ[2]
logistic_regression = 1 / (1 + np.exp(-z))
plt.scatter(XX, YY, c=logistic_regression, cmap=plt.cm.seismic)
plt.grid()

ax.plot(point_x_class_0, point_y_class_0, '.', color='blue', label='class = 0')
ax.plot(point_x_class_1, point_y_class_1, '.', color='red', label='class = 1')
ax = plt.axis('equal')

plt.legend()
plt.colorbar()

plt.tight_layout()
plt.show()
#
# ++++++

```

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## results

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```

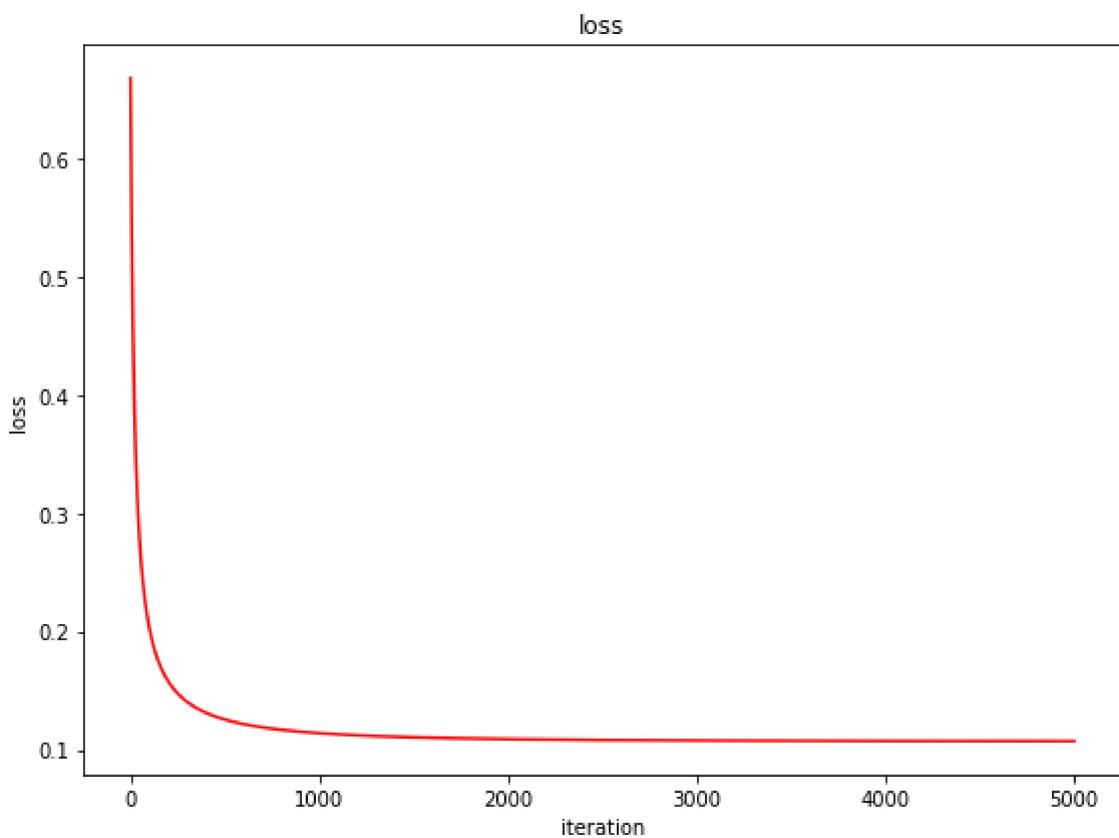
In [ ]: number_result = 9

for i in range(number_result):
    title = '## [RESULT {:02d}]'.format(i+1)
    name_function = 'function_result_{:02d}{}'.format(i+1)

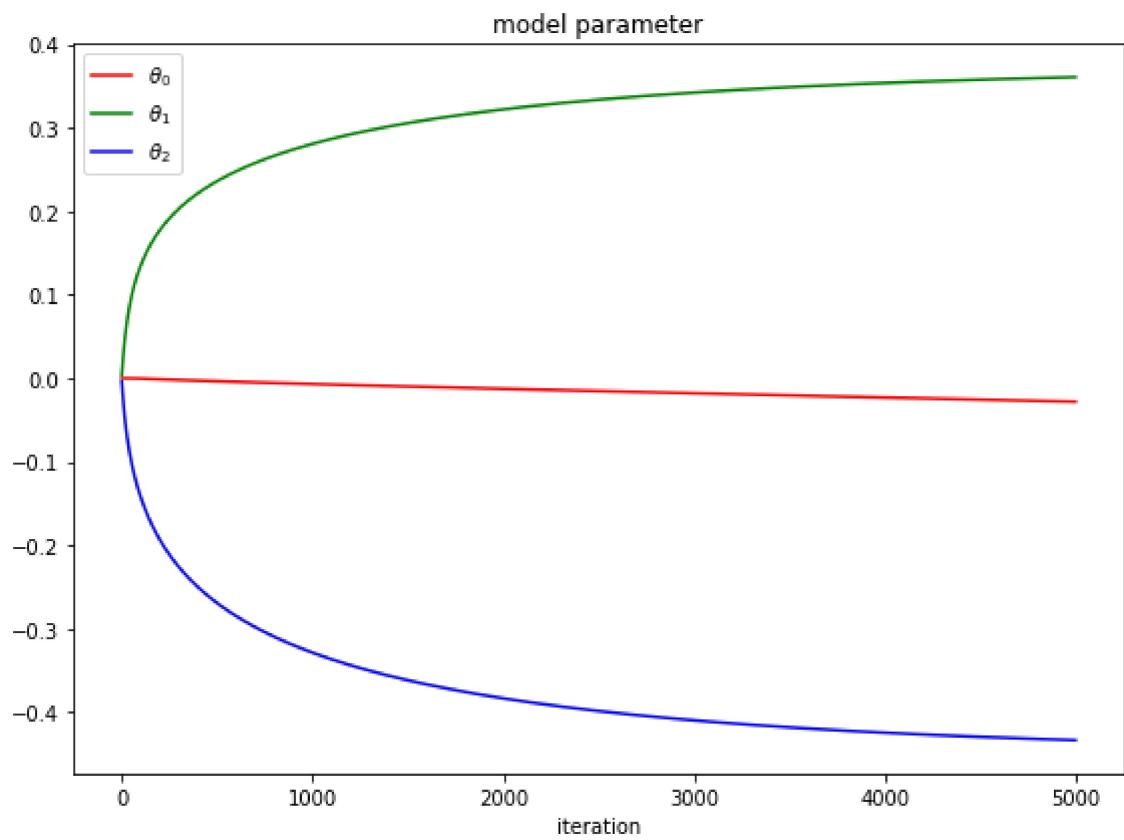
    print('*'*50)
    print(title)
    print('*'*50)
    eval(name_function)

```

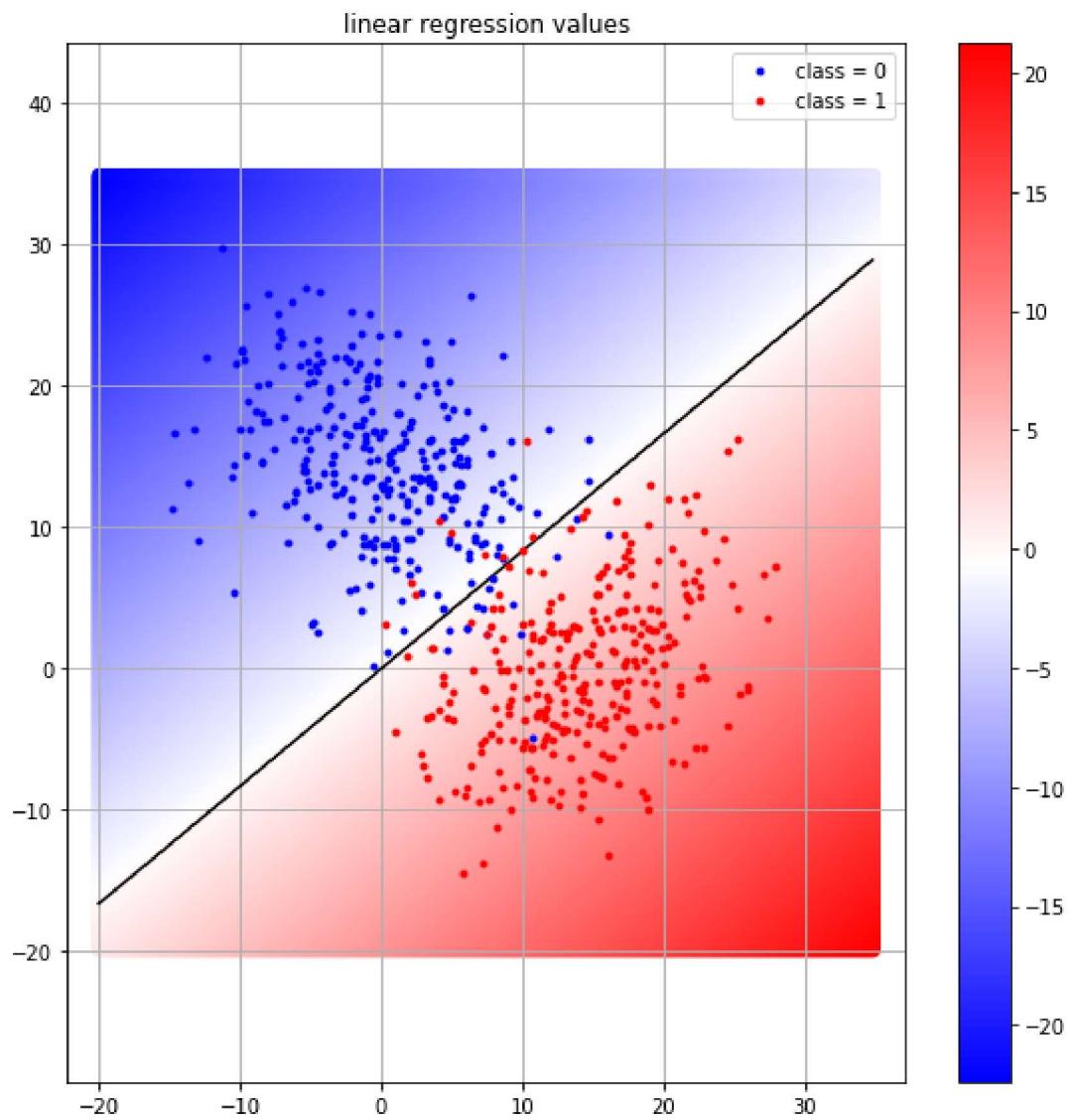
```
*****
## [RESULT 01]
*****
[ 1.4 -1.2]
*****
## [RESULT 02]
*****
[0.80218389 0.23147522]
*****
## [RESULT 03]
*****
[1.62041741 1.46328247]
*****
## [RESULT 04]
*****
1.783699877256482
*****
## [RESULT 05]
*****
[0.01682955 1.57070867 2.35606301]
*****
## [RESULT 06]
*****
```



```
*****
## [RESULT 07]
*****
```



```
*****  
## [RESULT 08]  
*****
```



\*\*\*\*\*  
## [RESULT 09]  
\*\*\*\*\*

logistic regression values

