Isotropic smoothing of image via Heat equation

import library

In [ ]:
```
import numpy as np
import matplotlib.image as img
import matplotlib.pyplot as plt
from matplotlib import cm
import matplotlib.colors as colors
from skimage import color
from skimage import io
```

load input image

- filename for the input image is 'barbara_color.jpeg'

In [ ]:
```
I0 = io.imread('barbara_color.jpeg')
```

check the size of the input image

In [ ]:
```
# ++++++++++++++++++++++++++++++++++++++++++++++++
# complete the blanks
#
num_row     = I0.shape[0]
num_column  = I0.shape[1]
num_channel = I0.shape[2]
#
# ++++++++++++++++++++++++++++++++++++++++++++++++

print('number of rows of I0 = ', num_row)
print('number of columns of I0 = ', num_column)
print('number of channels of I0 = ', num_channel)
```

```
number of rows of I0 =  512
number of columns of I0 =  512
number of channels of I0 =  3
```

convert the color image into a grey image

In [ ]:
```
# ++++++++++++++++++++++++++++++++++++++++++++++++
# complete the blanks
#
I = color.rgb2gray(I0)

num_row     = I.shape[0]
num_column  = I.shape[1]
#
# ++++++++++++++++++++++++++++++++++++++++++++++++
```

```
print('number of rows of I = ', num_row)
print('number of columns of I = ', num_column)
```

```
number of rows of I =  512
number of columns of I =  512
```

normalize the converted image

- normalize the converted grey scale image so that its maximum value is 1 and its minimum value is 0

In [ ]:
```
# ++++++++++++++++++++++++++++++++++++++++++++++++
# complete the blanks
#
I = ( (I - np.min(I)) / (np.max(I) - np.min(I)) )
#
# ++++++++++++++++++++++++++++++++++++++++++++++++

print('maximum value of I = ', np.max(I))
print('minimum value of I = ', np.min(I))
```

```
maximum value of I =  1.0
minimum value of I =  0.0
```

define a function to compute the derivative of input matrix in x(row)-direction

- forward difference : $I[x+1, y] - I[x, y]$

In [ ]:
```
def compute_derivative_x_forward(I):

    D = np.zeros(I.shape)

    # ++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #

    xForward = I[1:, :]
    xForward = np.insert(xForward, -1, xForward[-1,:], axis=0)
    D = xForward - I

    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++

    return D
```

- backward difference : $I[x, y] - I[x-1, y]$

In [ ]:
```
def compute_derivative_x_backward(I):

    D = np.zeros(I.shape)

    # ++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #

    xBackward = I[:-1, :]
```

```
    xBackward = np.insert(xBackward, 0, xBackward[0,:], axis=0)
    D = I - xBackward

    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++
    
    return D
```

define a function to compute the derivative of input matrix in y(column)-direction

- forward difference : $I[x, y+1] - I[x, y]$

In [ ]:
```
def compute_derivative_y_forward(I):

    D = np.zeros(I.shape)

    # ++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #

    yForward = I[:, 1:]
    yForward = np.insert(yForward, -1, yForward[:, -1], axis=1)
    D = yForward - I

    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++

    return D
```

- backward difference : $I[x, y] - I[x, y-1]$

In [ ]:
```
def compute_derivative_y_backward(I):

    D = np.zeros(I.shape)

    # ++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #

    yBackward = I[:, :-1]
    yBackward = np.insert(yBackward, 0, yBackward[:,0], axis=1)
    D = I - yBackward

    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++

    return D
```

define a function to compute the laplacian of input matrix

- $\Delta I = \nabla^2 I = \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2}$
- $\Delta I = I[x+1, y] + I[x-1, y] + I[x, y+1] + I[x, y-1] - 4 * I[x, y]$

- ΔI = derivative_x_forward - derivative_x_backward + derivative_y_forward - derivative_y_backward

In [ ]:
```
def compute_laplace(I):

    laplace = np.zeros(I.shape)

    # ++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #

    derivative_x_forward = compute_derivative_x_forward(I)
    derivative_x_backward = compute_derivative_x_backward(I)
    derivative_y_forward = compute_derivative_y_forward(I)
    derivative_y_backward = compute_derivative_y_backward(I)

    laplace = derivative_x_forward - derivative_x_backward + derivative_y_forward -

    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++

    return laplace
```

define a function to compute the heat equation of data I with a time step

- $I = I + \delta t * \Delta I$

In [ ]:
```
def heat_equation(I, time_step):

    I_update = np.zeros(I.shape)

    # ++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #

    I_update = I + time_step * compute_laplace(I)

    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++

    return I_update
```

run the heat equation over iterations

In [ ]:
```
def run_heat_equation(I, time_step, number_iteration):

    I_update = np.zeros(I.shape)

    for t in range(number_iteration):
        # ++++++++++++++++++++++++++++++++++++++++++++++++
        # complete the blanks
        #
        if t == 1 :
            I_update = heat_equation(I, time_step)
        else :
            I_update = heat_equation(I_update, time_step)
```

```
        #
        # ++++++++++++++++++++++++++++++++++++++++++++++++
    return I_update
```

---

functions for presenting the results

In [ ]:
```
def function_result_01():

    plt.figure(figsize=(8,6))
    plt.imshow(I0)
    plt.show()
```

In [ ]:
```
def function_result_02():

    plt.figure(figsize=(8,6))
    plt.imshow(I, cmap='gray', vmin=0, vmax=1, interpolation='none')
    plt.show()
```

In [ ]:
```
def function_result_03():

    L = compute_laplace(I)

    plt.figure(figsize=(8,6))
    plt.imshow(L, cmap='gray')
    plt.show()
```

In [ ]:
```
def function_result_04():

    time_step   = 0.25
    I_update    = heat_equation(I, time_step)

    plt.figure(figsize=(8,6))
    plt.imshow(I_update, vmin=0, vmax=1, cmap='gray')
    plt.show()
```

In [ ]:
```
def function_result_05():

    time_step           = 0.25
    number_iteration    = 128

    I_update = run_heat_equation(I, time_step, number_iteration)

    plt.figure(figsize=(8,6))
    plt.imshow(I_update, vmin=0, vmax=1, cmap='gray')
    plt.show()
```

In [ ]:
```
def function_result_06():

    time_step           = 0.25
    number_iteration    = 512
```

```
    I_update = run_heat_equation(I, time_step, number_iteration)

    plt.figure(figsize=(8,6))
    plt.imshow(I_update, vmin=0, vmax=1, cmap='gray')
    plt.show()
```

In [ ]:
```
def function_result_07():

    L = compute_laplace(I)

    value1 = L[0, 0]
    value2 = L[-1, -1]
    value3 = L[100, 100]
    value4 = L[200, 200]

    print('value1 = ', value1)
    print('value2 = ', value2)
    print('value3 = ', value3)
    print('value4 = ', value4)
```

In [ ]:
```
def function_result_08():

    time_step   = 0.25
    I_update    = heat_equation(I, time_step)

    value1 = I_update[0, 0]
    value2 = I_update[-1, -1]
    value3 = I_update[100, 100]
    value4 = I_update[200, 200]

    print('value1 = ', value1)
    print('value2 = ', value2)
    print('value3 = ', value3)
    print('value4 = ', value4)
```

In [ ]:
```
def function_result_09():

    time_step           = 0.25
    number_iteration    = 128

    I_update = run_heat_equation(I, time_step, number_iteration)

    value1 = I_update[0, 0]
    value2 = I_update[-1, -1]
    value3 = I_update[100, 100]
    value4 = I_update[200, 200]

    print('value1 = ', value1)
    print('value2 = ', value2)
    print('value3 = ', value3)
    print('value4 = ', value4)
```

In [ ]:
```
def function_result_10():

    time_step           = 0.25
    number_iteration    = 512

    I_update = run_heat_equation(I, time_step, number_iteration)

    value1 = I_update[0, 0]
    value2 = I_update[-1, -1]
    value3 = I_update[100, 100]
```

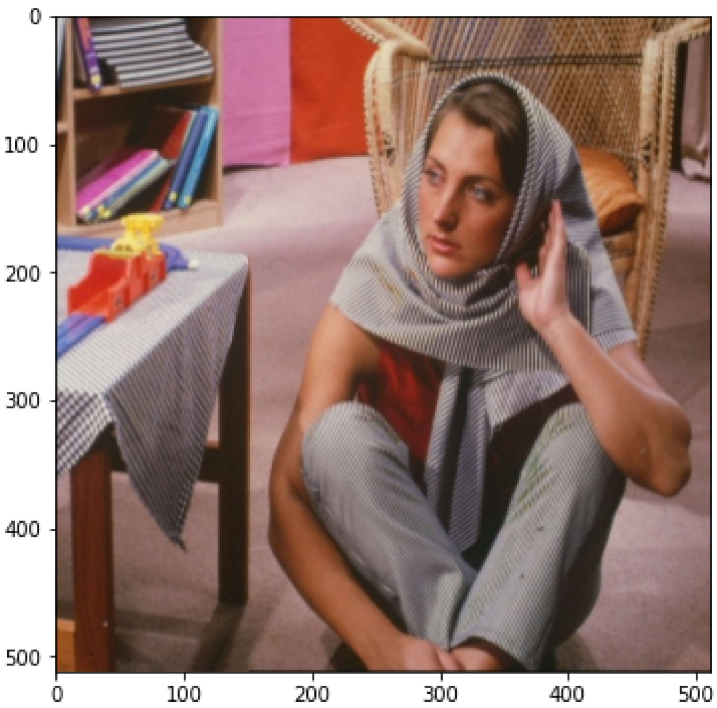
```
    value4 = l_update[200, 200]

    print('value1 = ', value1)
    print('value2 = ', value2)
    print('value3 = ', value3)
    print('value4 = ', value4)
```

---

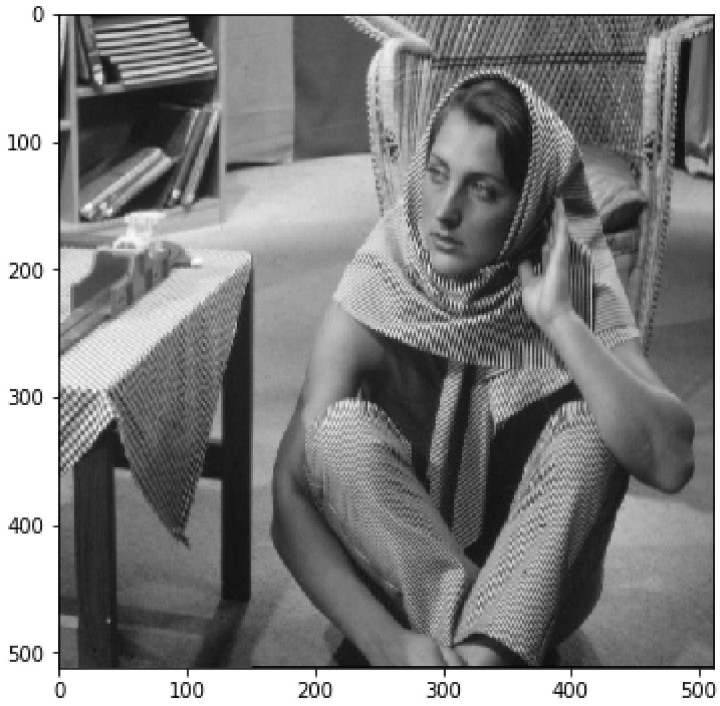
results

In [ ]:
```
number_result = 10

for i in range(number_result):
    title = '## [RESULT {:02d}]'.format(i+1)
    name_function = 'function_result_{:02d}()'.format(i+1)

    print('************************************************')
    print(title)
    print('************************************************')
    eval(name_function)
```

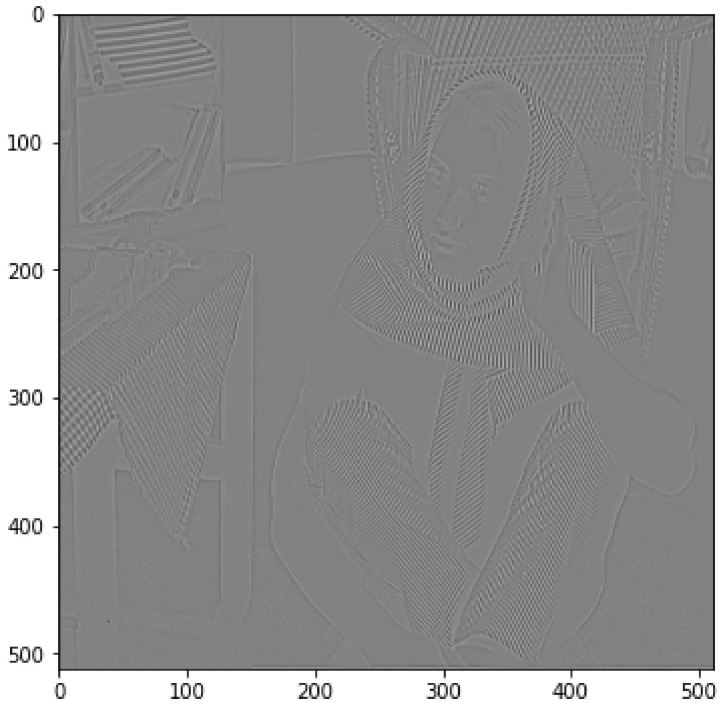
```
************************************************
## [RESULT 01]
************************************************
```



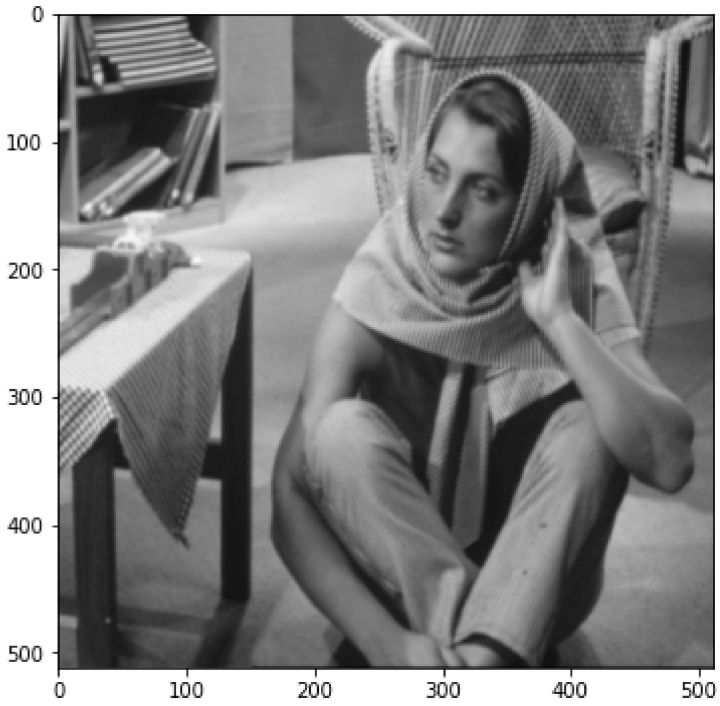

```
************************************************
## [RESULT 02]
************************************************
```



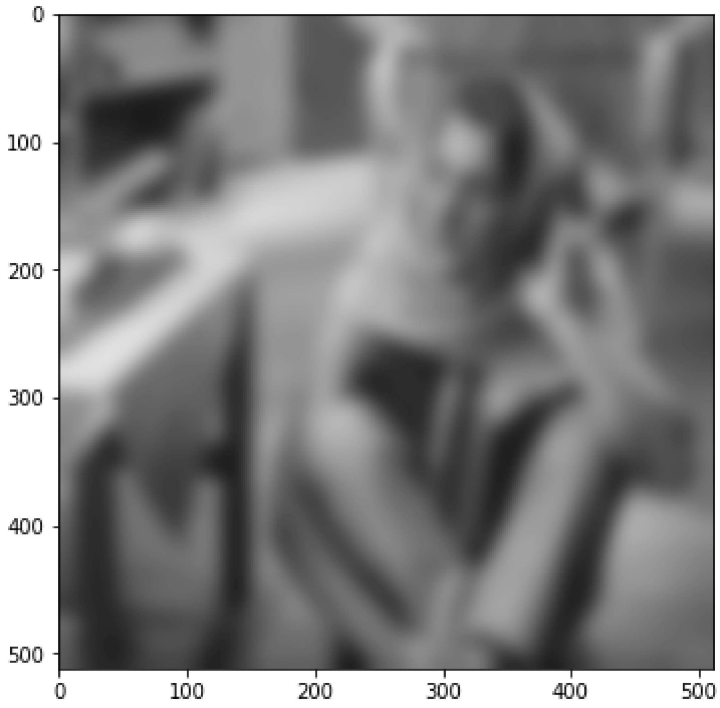
****
[RESULT 03]
****



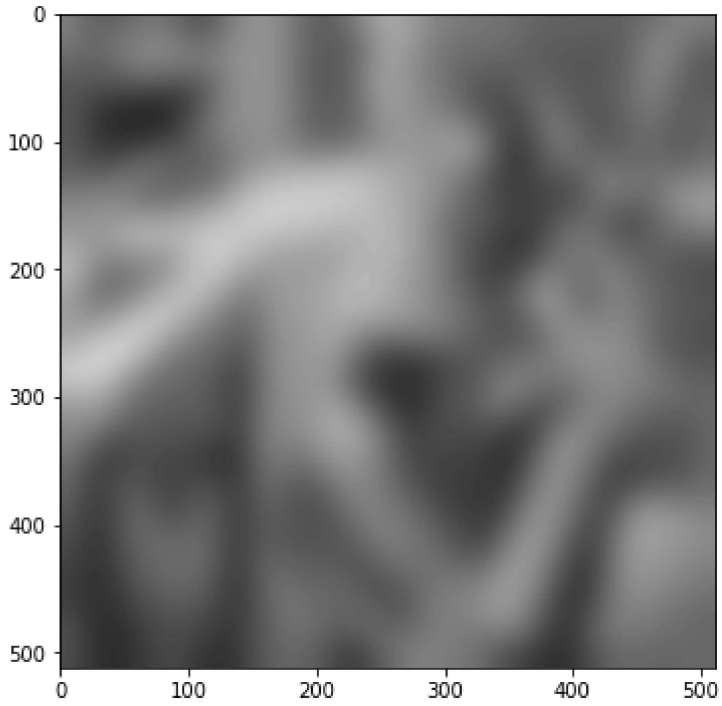
****
[RESULT 04]
****



****
[RESULT 05]
****



****
[RESULT 06]
****




```
**************************************************
## [RESULT 07]
**************************************************
value1 =  0.38873832902073646
value2 =  0.3503440089528004
value3 =  -0.1205387483392179
value4 =  -0.05899756869858064
**************************************************
## [RESULT 08]
**************************************************
value1 =  0.11620040458063048
value2 =  0.15748602900643
value3 =  0.5094184630641501
value4 =  0.592768775993217
**************************************************
## [RESULT 09]
**************************************************
value1 =  0.5893400746741129
value2 =  0.3999788214980097
value3 =  0.36696709016679235
value4 =  0.6015629132075617
**************************************************
## [RESULT 10]
**************************************************
value1 =  0.49014178827682775
value2 =  0.40789553966022124
value3 =  0.3499543186671801
value4 =  0.6313508207438923
```

In [ ]:

In [ ]: