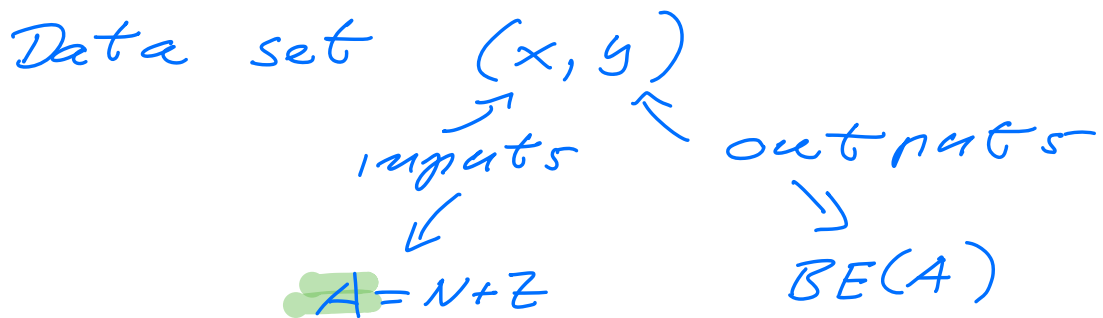


Regression, Linear Regression.



Model

$$BE(A) \rightarrow \tilde{y}(x)$$

$$\tilde{y}_i = a_1 x_i + a_2 x_i^{2/3} + a_3 x_i^{-1/3} + a_4 x_i^{-1}$$

$$BE(N, Z) = a_1 A - a_2 A^{2/3} - a_3 \frac{Z^2}{A^{1/3}} - a_4 \frac{(N-Z)^2}{A}$$

$$i = 0, 1, 2, \dots, 269$$

P = features (4 in total)

$$P = a_1, a_2, a_3, a_4$$

unknown parameters

$$\tilde{y} = \begin{bmatrix} \tilde{y}_0 \\ \tilde{y}_1 \\ \vdots \\ \vdots \end{bmatrix} \quad \text{Model}$$

$$y = \begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_{n-1} \end{bmatrix} \quad \text{Data.}$$

$$\tilde{y} = \underset{\substack{\uparrow \\ \text{Design} \\ \text{matrix}}}{X} a \quad a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

$$a \rightarrow \Theta(\beta) \quad \text{optimal parameter}$$

$$\Theta \in \mathbb{R}^p$$

$$X \in \mathbb{R}^{n \times p}$$

$$y, \tilde{y} \in \mathbb{R}^n$$

$$\tilde{y} = X\Theta \in \mathbb{R}^n$$

$$\sim \begin{bmatrix} 0 & \overset{p=1}{x_0} & \overset{2}{x_0^{2/3}} & \overset{3}{x_0^{-1/3}} & \overset{4}{x_0^{-1}} \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & x_1 & x_1^2 & x_1^3 & x_1^4 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{n-1} & x_{n-1}^2 & x_{n-1}^3 & x_{n-1}^4 \end{bmatrix}$$

all entries
are known.

$$\tilde{y} = X\theta$$

linear in $\theta \Rightarrow$
Linear Regression.

Minimize:

$$MSE = \frac{1}{n} \sum_{i=0}^{n-1} (y_i - \tilde{y}_i)^2$$

$$\Rightarrow \hat{\theta} = (X^T X)^{-1} X^T y$$

↑
optimal

$$\left[\frac{\partial MSE}{\partial \hat{\theta}} = 0 \right] \text{ tomorrow,}$$