Regression, Linear Regression.

Data set (x, y)

inputs $\swarrow$ $A = N+Z$

outputs $\searrow$ $BE(A)$

<u>Model</u>

$BE(A) \to \tilde{y}(x)$

$$\tilde{y}_i = a_1 x_i + a_2 x_i^{2/3} + a_3 x_i^{-1/3} + a_4 x_i^{-1}$$

$$BE(N,Z) = a_1 A - a_2 A^{2/3} - a_3 Z^2/A^{1/3} - a_4 (N-Z)^2/A$$

<u>$i = 0, 1, 2, \ldots 269$</u>

p = features (4 in total)

p = $\underline{a_1, a_2, a_3, a_4}$

unknown parameters

$$\tilde{y} = \begin{bmatrix} \tilde{y}_0 \\ \tilde{y}_1 \\ \vdots \end{bmatrix}$$ Model

$$\begin{bmatrix} \vdots \\ y_{n-1} \end{bmatrix}$$

$$y = \begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_{n-1} \end{bmatrix}$$ Data.

$$\tilde{y} = \underset{\substack{\uparrow \\ \text{Design matrix}}}{X} a \qquad a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

$$a \rightarrow \Theta(\beta)$$

optimal parameter

$$\Theta \in \mathbb{R}^p$$

$$X \in \mathbb{R}^{n \times p}$$

$$y, \tilde{y} \in \mathbb{R}^n$$

$$\tilde{y} = X\Theta \in \mathbb{R}^n$$

$$X = \begin{array}{c} \\ 0 \\ 1 \end{array} \begin{array}{cccc} p=1 & 2 & 3 & 4 \\ \hline x_0^{1} & x_0^{2/3} & x_0^{-1/3} & x_0^{-1} \\ \ldots \end{array}$$

$$X = \begin{array}{c} \\ 2 \\ \vdots \\ n-1 \end{array} \begin{bmatrix} x_1 & x_1^{2/3} & x_1^{-1/3} & x_1^{-1} \\ \vdots & & & \\ & & & \\ x_{n-1} & x_{n-1}^{2/3} & x_{n-1}^{-1/3} & x_{n-1}^{-1} \end{bmatrix}$$

all entries are known.

$$\boxed{\tilde{y} = X\theta}$$

linear in θ $\Rightarrow$ linear Regression.

Minimize:

$$MSE = \frac{1}{n}\sum_{i=0}^{n-1}\left(y_i - \tilde{y}_i\right)^2$$

$$\Rightarrow \quad \hat{\theta} = \left(X^T X\right)^{-1} X^T y$$

optimal

$$\left[\frac{\partial MSE}{\partial \hat{\theta}} = 0\right]$$ Tomorrow.