

**"A PRIMARY MATHEMATICS TEACHING WITH
FINGER- METHOD ALPHA »**

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ATHENS 2019

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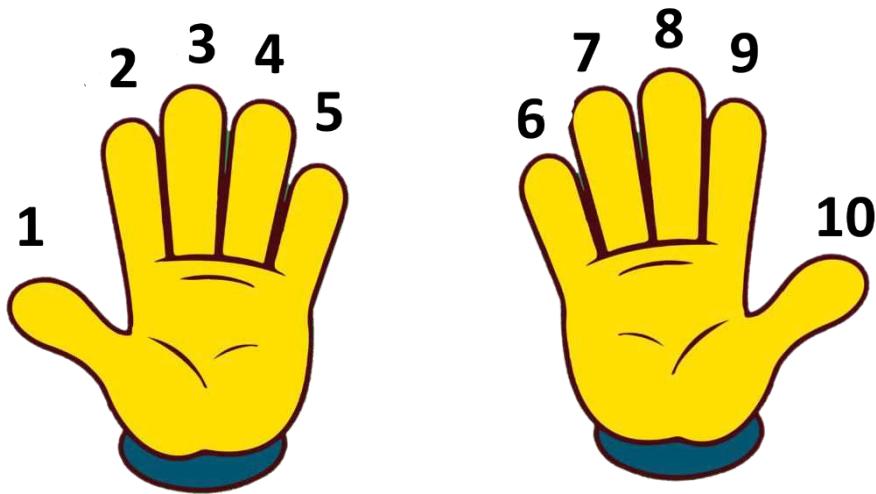
This paper describes in detail an alternative method of teaching mathematics of an elementary school, based solely on the use of fingertips. In more detail, the finger count is initially displayed from 1 to 20 and the display of the fingers of the crowd. Then, the teaching is analyzed in numbers up to 5, up to 10 and up to 20 (addition, subtraction, supplements, and pairs of additives with sums up to 10). Following which, there is a description of teaching multiplication and division, the four mathematical operations, problems and abacus. In addition, the required supervisory material is presented through

teaching records and finally the different logistical means which can be used to teach the method.

A. NUMBERS FROM 1 TO 20

1. Finger count to 10

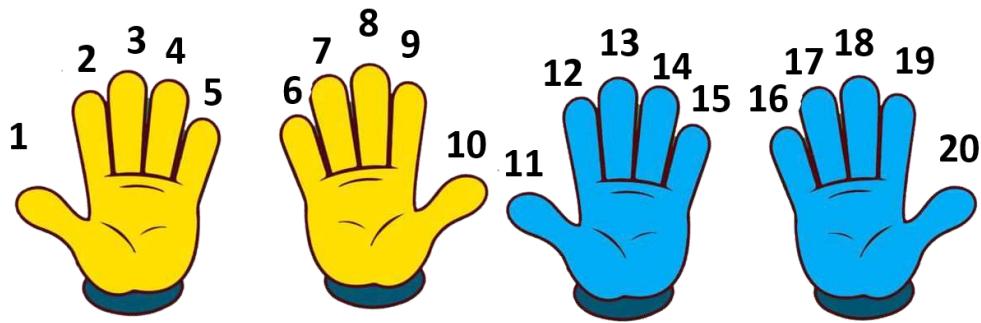
Finger count to 10 is based on the image that students have if they turn their palms to themselves. Each finger corresponds to a single number. So, the left thumb corresponds to the number "1", the left forefinger to the number "2", the left middle is the number "3", left ring finger in the number "4", the left little finger corresponds to the number "5", the right little finger on the number '6', the right ring finger corresponds to the number '7', the right middle to number '8', the right forefinger to number '9' and the right thumb to the number "10".



2. Finger count to 20

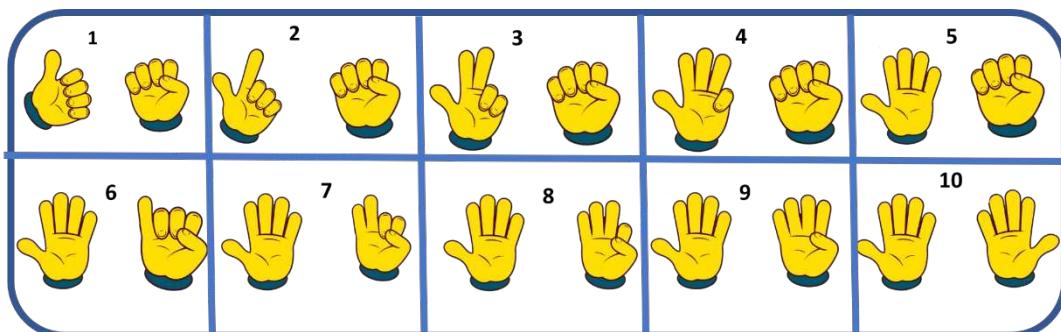
Finger count to 20 is done in a similar way as finger count to 10, that is, based on the image the students have if turn their palms to themselves. Every finger here corresponds to a single number. The pairs of the palms are two and the pairs of fingers, therefore, 20. It is recommended to use a different color for each pair. Thus, the left thumb of the first pair corresponds in the number "1", the left forefinger of the first pair is the number "2", the left middle of the first pair is the number "3", the left ring finger of the first pair is the number "4", the little finger of the first pair is the number "5", the right little finger is the number of first pair "6", the right ring finger of the first pair corresponds to the number "7", the right middle of the first pair is the number "8", the right forefinger of the first pair is the number "9", the right first pairs thumb is the number '10', his left

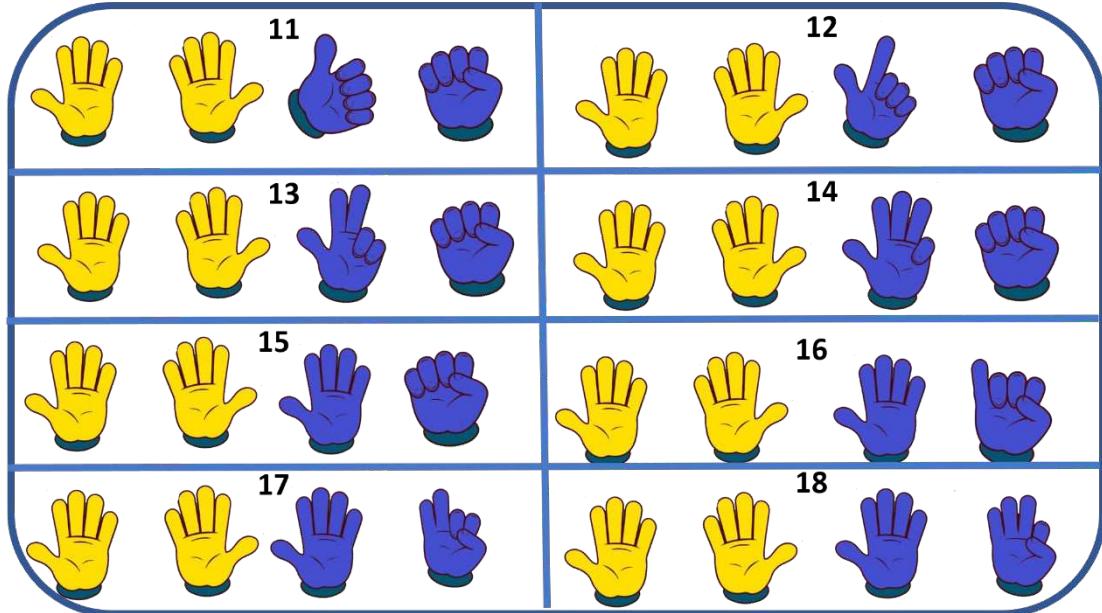
thumb in the second pair is the number '11', the left forefinger of the second pair is the number '12', the left middle of the second pair is the number '13', the left ring finger in the second pair of the number "14", the left little finger of the second pair is the number "15", second pair's right little finger in the number "16", the left one the second pair is the number '17', the left middle second pair in the number '18', the left forefinger of the second pair is the number '19' and the left thumb of the second pair is the number "20".



3. Illustration of the numbers as number of fingers from 1 to 20

The illustration of the numbers as number of fingers from 1 to 20, is made according to the pattern of finger counting. For example, the number "2" is represented by the thumb and forefinger lift of the left hand. Below are the representations of the numbers from 1 to 20.





B. TEACHING UP TO 5

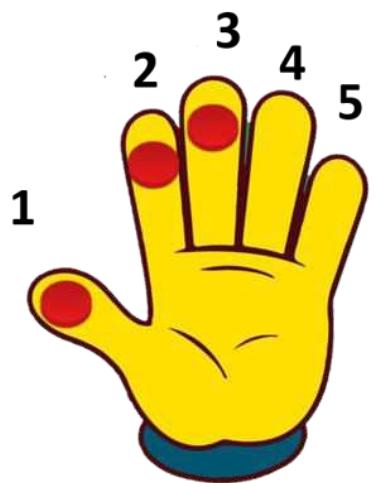
1. Number of fingers up to 5 and complement of 5

The student observes the picture, which shows 2 raised fingers of the left hand. They count and find out that it is 2. They also count 3 "Hidden" fingers, which are its complement 2, to create 5. So, $2 + 3 = 5$. The same can be done with their own fingers and for any number up to 5.

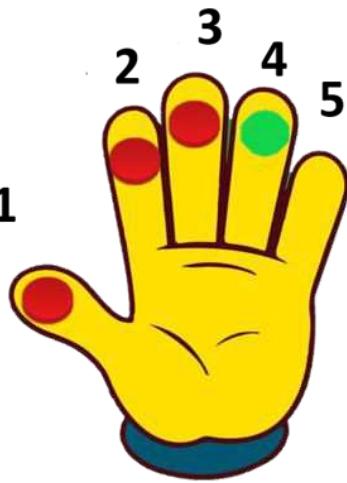


2. Addition up to 5

The student places as many red dots on the "fingers" as shown by the first additive and then sequentially green dots as indicated by the second additive. Then they find out the sum. For example, to find the sum $3 + 1$, there are placed 3 red dots (corresponding to the first image) in the positions "1", "2" and "3", and 1 green dot (corresponding to the second image) in the position "4". So it turns out that $3 + 1 = 4$. This process it can also be done with the students' fingers.



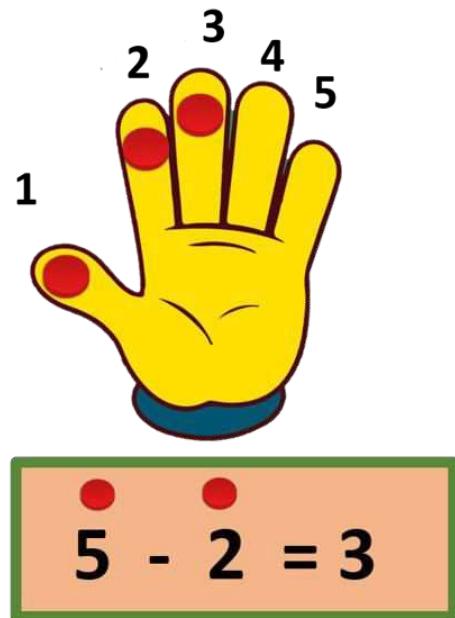
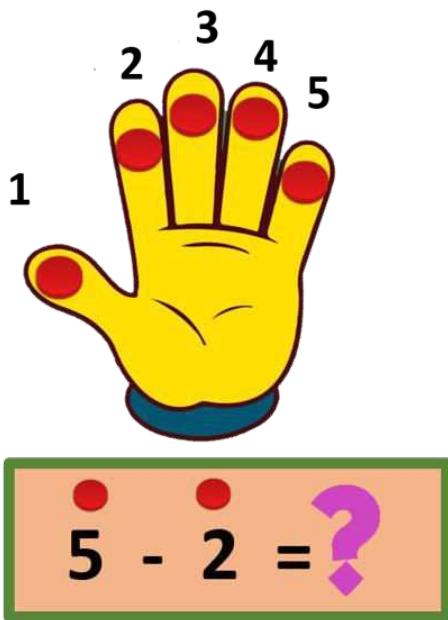
$$\bullet + \bullet = ?$$



$$\bullet + \bullet = 4$$

3. Subtraction up to 5

The student places as many red dots on the "fingers" as shown by the minuend, starting from his thumb on the left hand. From these, he removes as many dots as possible represented by the subtrahend. He then finds the result of subtraction. For example, for the effect of subtraction $5 - 2$ to be found, 5 red dots are inserted and then subtracted in rotation from the end 2 dots. This is the result. This process can also be done by using the students' fingers.



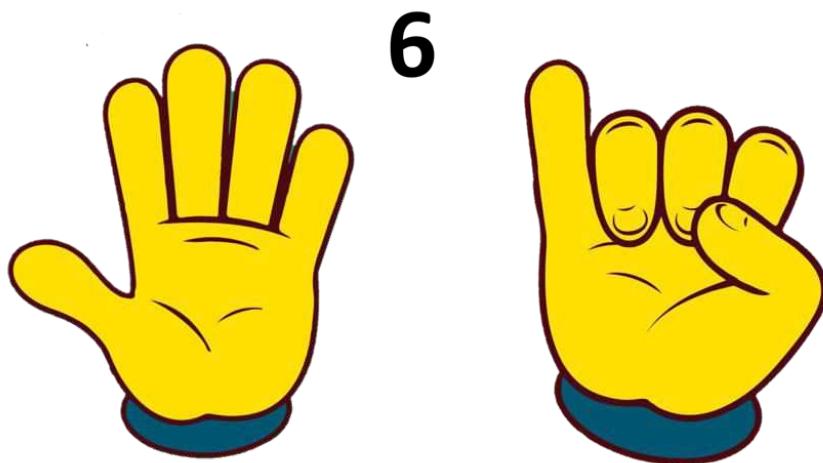
4. Pair of numbers up to 5

The student places as many red dots on the "fingers" as the number of pairs they wish to find. Then, by moving the blue bar successively between the fingers, creates all combinations of additives, which add up to the number. For example, to find all pairs of numbers having the sum of 4, the student places the blue bar outside and to the left of the Thumb and, by moving it successively to the right between the fingers, he finds the pairs of additives with sum 4 ($0 + 4$, $1 + 3$, $2 + 2$, $3 + 1$, $4 + 0$).



C. TEACHING UP TO 10

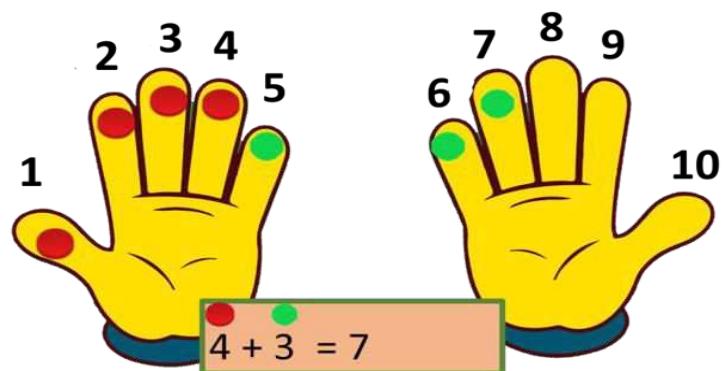
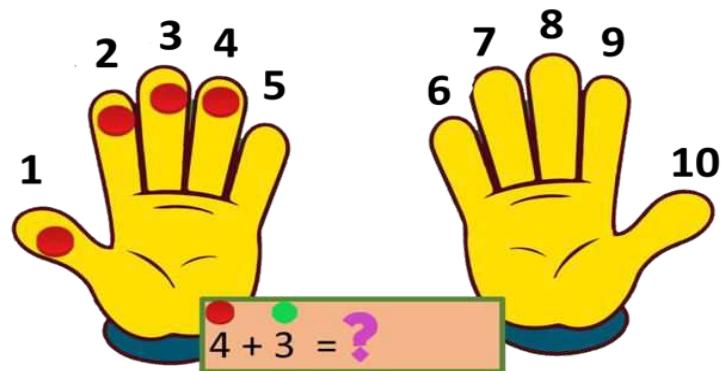
1. Number of fingers up to 10 and complement of 10



The student observes the image, which shows 6 up fingers. He counts them and finds that it is 6. He also counts the 4 "hidden" fingers, which are the complement of 6, to create the 10. So $6 + 4 = 10$. He can do the same with his own fingers and for any number up to 10.

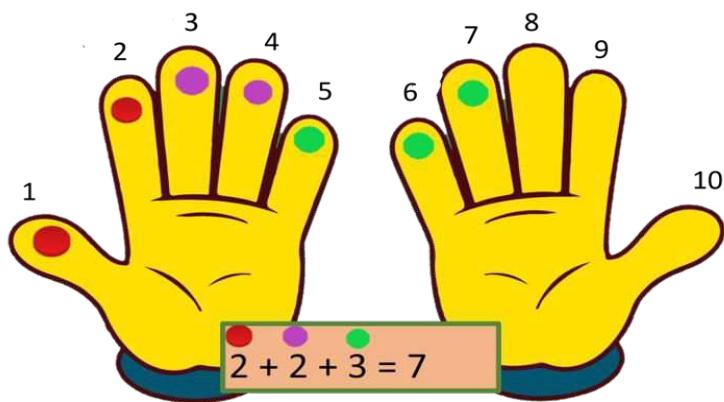
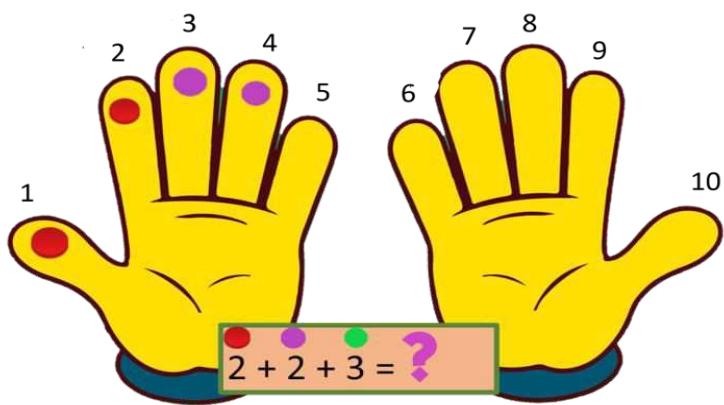
2. Addition up to 10

The student places as many red dots on the "fingers" as shown by the first additive and then successively as many green dots as indicated by the second additive. Then he finds the sum. This process can also be done by using students' fingers. For example, to find the sum of $4 + 3$, 4 red dots are placed (corresponding to the first additive) at positions "1", "2", "3" and "4", and 3 green dots (which correspond to the second additive) in the positions "5", "6" and "7". So, it turns out that $4 + 3 = 7$.



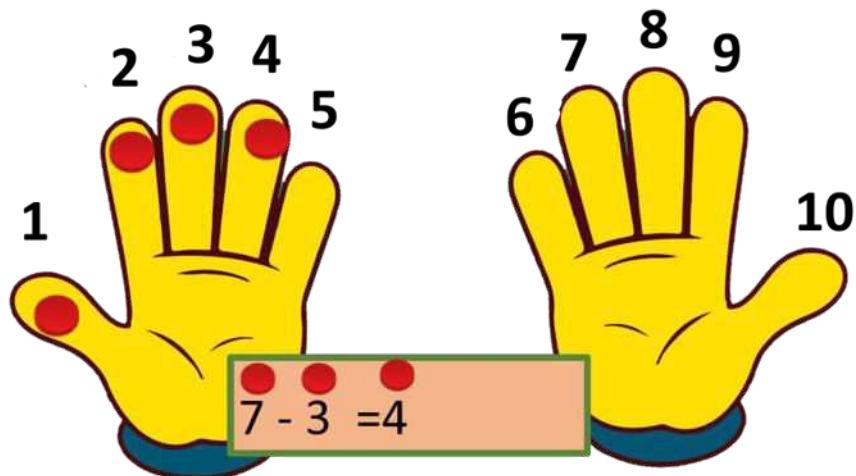
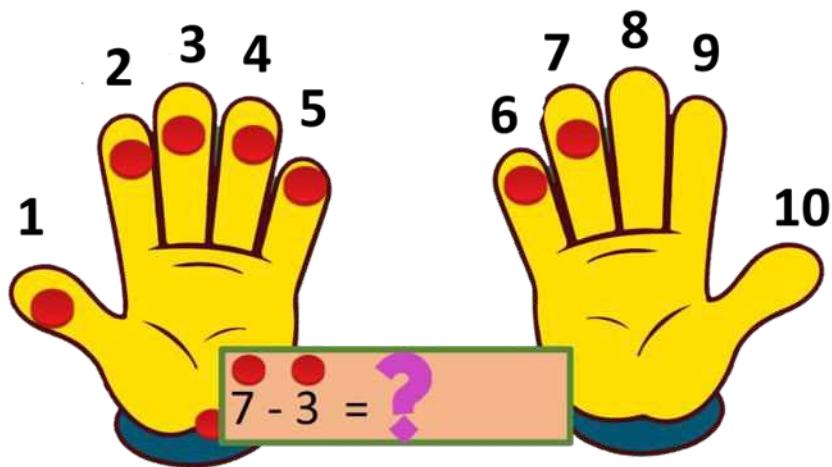
3. Addition of 3 numbers up to 10

The student places as many red dots on the "fingers" as the first additive indicates and then successively as many purple dots as indicated by the second additive. Then, he places as many green dots as indicated by the third additive. Then he finds the sum. This procedure can also be done with the use of fingers of students. For example, to find the sum $2 + 2 + 3$, the student inserts 2 red dots (corresponding to the first additive) at positions "1" and "2", 2 purple dots at positions "3" and "4" (corresponding to the second additive) and 3 green dots at positions, "5", "6" and "7" (corresponding to the third additive). So, it turns out that $2 + 2 + 3 = 7$.



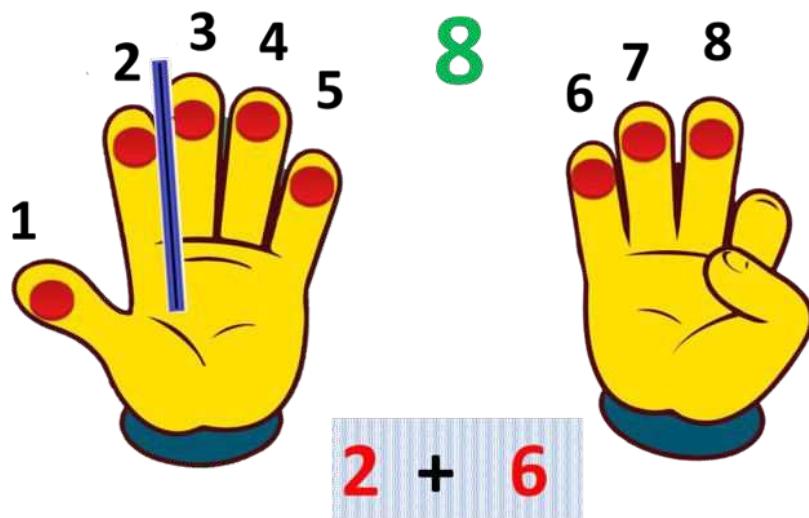
4. Subtraction up to 10

The student places as many red dots on the "fingers" as it is shown by the minuend, starting from his thumb on the left hand. From these, he removes as many dots as possible from the end indicated by the subtrahend. Then he finds the result. This process can also be done by using the students' fingers. For example, to find the effect of subtraction of $7 - 3$, 7 red dots are placed and successively three dots from the end are removed. This gives the result $7 - 3 = 4$.



5. Pair of numbers up to 10

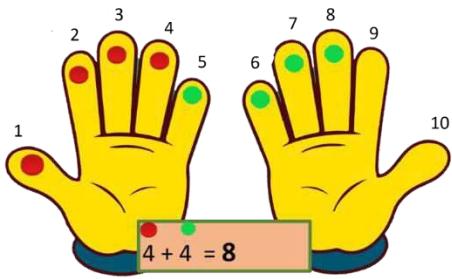
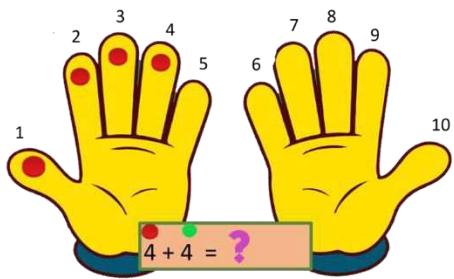
The student places as many red dots on the "fingers" as the number of additive pairs we want him to find. Then, move the blue bar out and to the left of the left palm and moving it successively to the right between the fingers, creates all the combinations of additives which have the sum of the number.



For example, to find all pairs of numbers, which are having a sum of 8, the student places the bar outside and to the left of the left palm and moving it successively to the right between fingers, finds pairs of additives with sum 8 ($0 + 8$, $1 + 7$, $2 + 6$, $3 + 5$, $4 + 4$, $5 + 3$, $6 + 2$, $7 + 1$, $8 + 0$).

6. Double additions up to 10

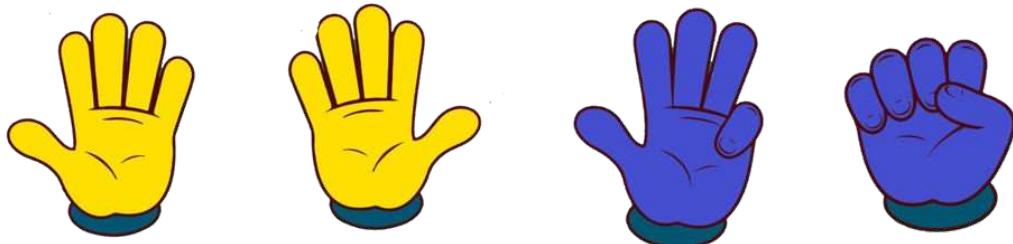
The student places as many red dots on the "fingers" as the first additive indicates, and successively, equal number of green dots. Then he finds the sum. For example, to find the sum of $4 + 4$, 4 red dots (which correspond to the first additive) and 4 green dots (which correspond to the second additive). So it turns out that $4 + 4 = 8$



D. TEACHING UP TO 20

1. Number of fingers up to 20 and complement of 20

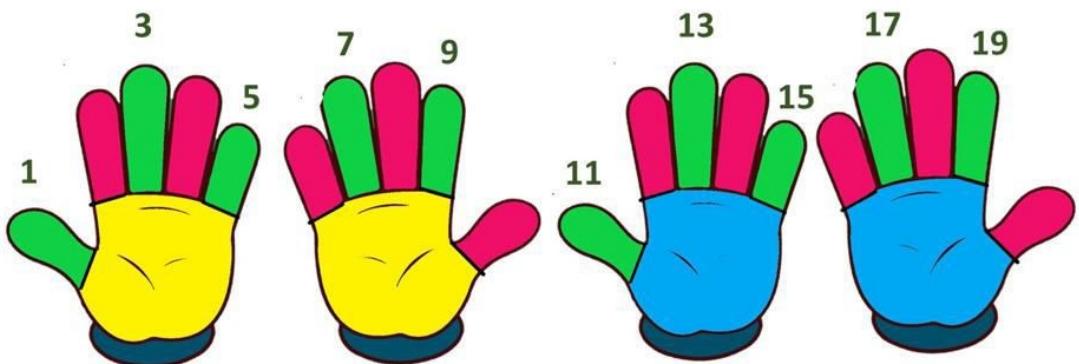
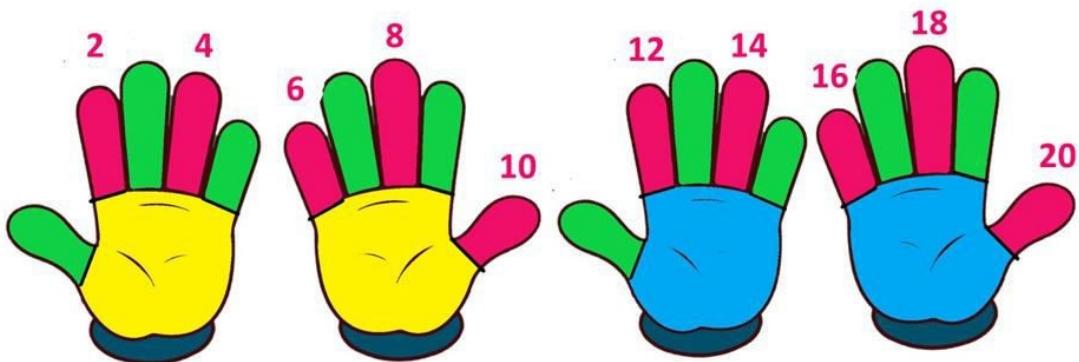
14



The student observes the picture, which shows 14 raised fingers. He counts them and finds out that they are 14. He also counts the 6 "hidden" fingers, which are the complement of 14, to create 20. So, $14 + 6 = 20$. He can do the same thing with his own fingers and for every number up to 20.

2. Even and Odd numbers

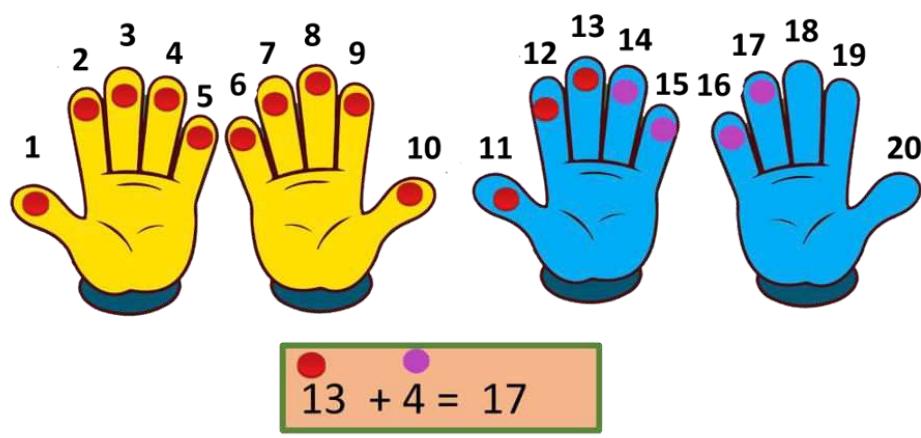
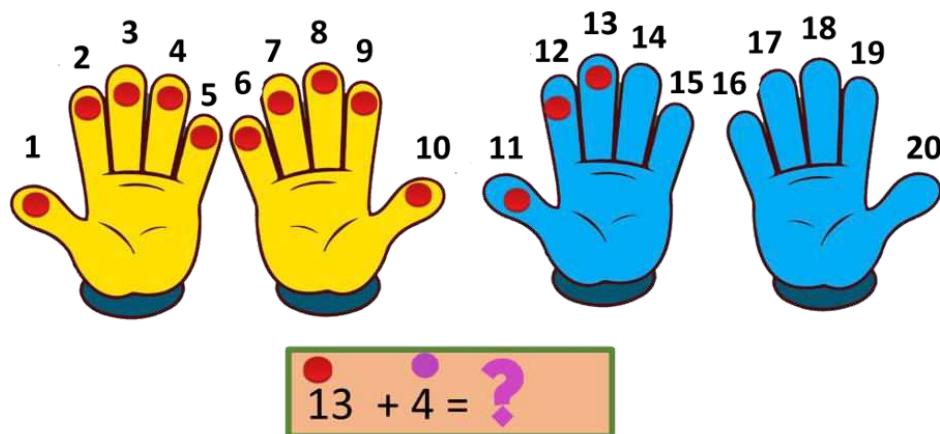
The 20 fingers are depicted with 2 different colors, the one color shows the odd numbers and the other color the even numbers. The student sequentially numbers even and odd numbers up to 20



3. Addition up to 20

Case a: The first additive greater than or equal to 10

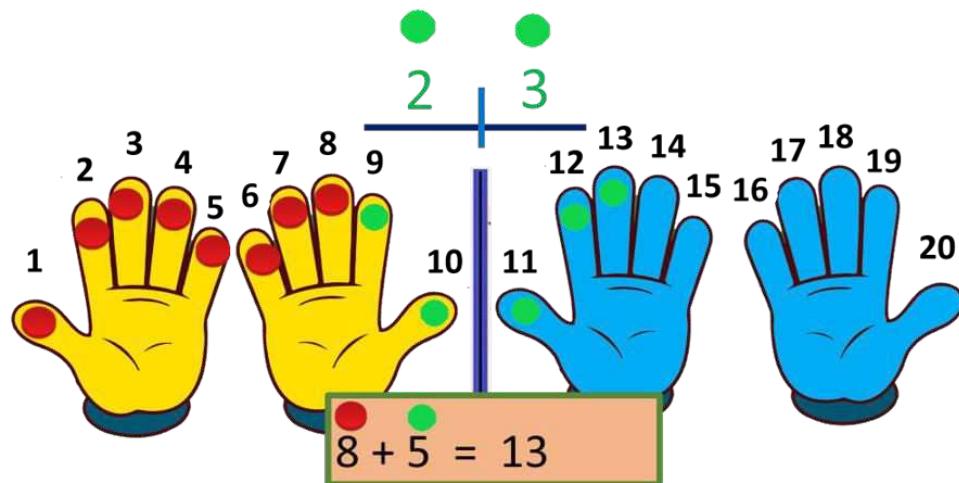
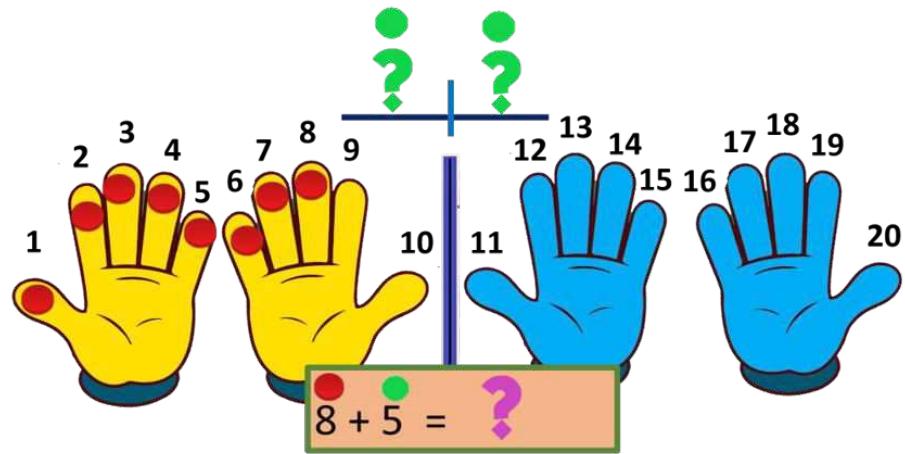
The student places as many red dots on the "fingers" as the first additive indicates, and then as many purple ones as it indicates is the second additive. Then he finds the sum. For example, to find the sum of $13 + 4$, there are 13 red dots placed (corresponding to the first additive) at corresponding positions and 4 purple dots (corresponding to the second additive) at the corresponding positions. So it turns out that $13 + 4 = 17$.



Case b: The first additive less than 10 and the aggregate greater than 10

The student places as many red dots on the "fingers" as the first additive indicates and then in as many green dots as indicated by the second additive. Then he finds out the sum. For example, to find the sum of $8 + 5$, the student places 8 red dots (corresponding to the first additive) and 5 green dots (corresponding to the second additive) at corresponding positions. 5 of the

second additive are broken down into $2 + 3$ and are marked in the appropriate positions. So it turns out that: $8 + 5 = (8 + 2) + 3 = 10 + 3 = 13$.

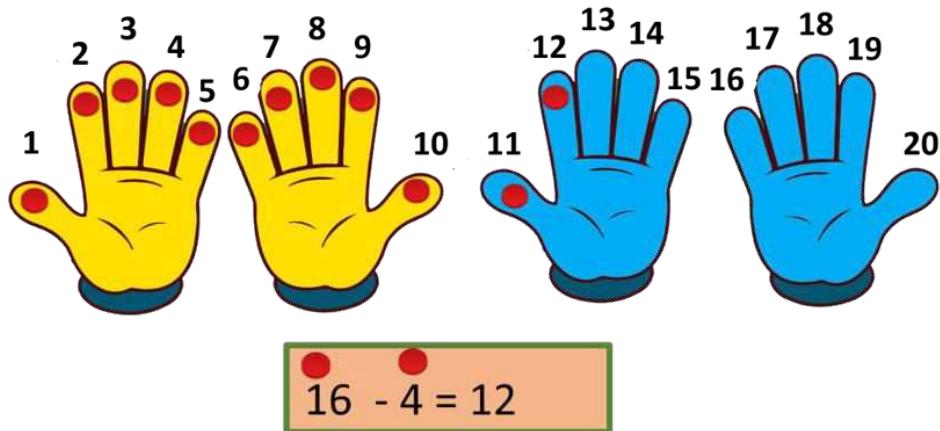
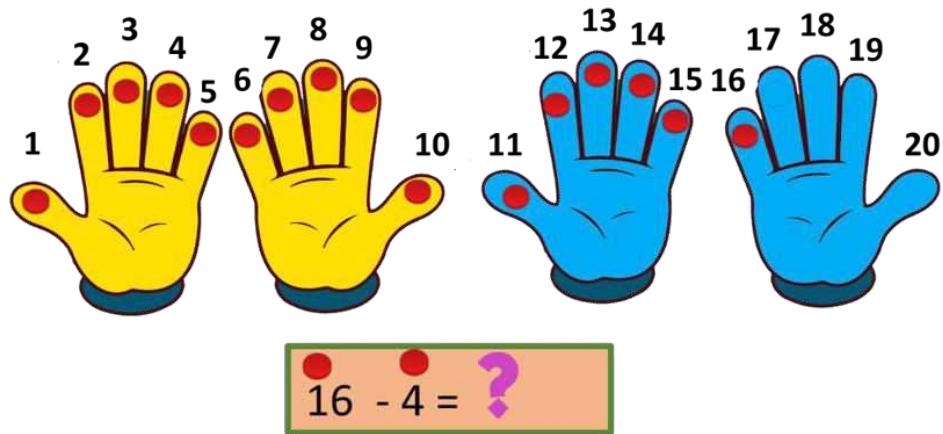


4. Subtraction up to 20

Case A: The minuend from 11 to 20 and the result of the subtraction greater or equal to 10

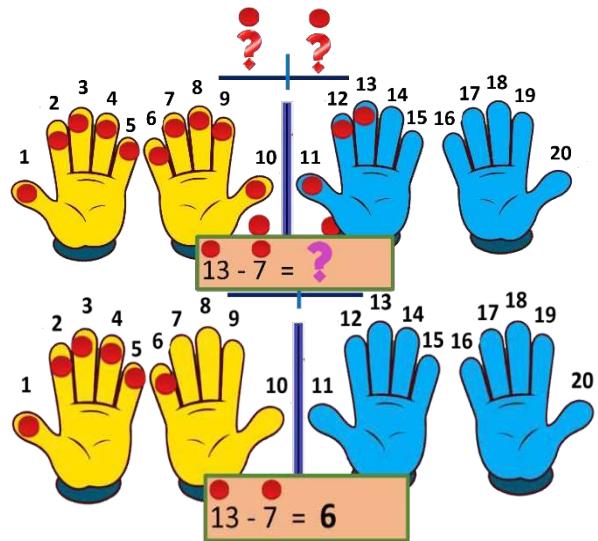
The student places as many red dots on the "fingers" as shown by the minuend, starting from his thumb on the left hand. From these, he removes as many dots as the subtrahend indicates. Then he finds the result. For example, in order to find the result of the subtraction $16 - 4$, 16 red dots are placed and then 4 dots are successively deducted from the end.

So it turns out that: $16 - 4 = 12$.



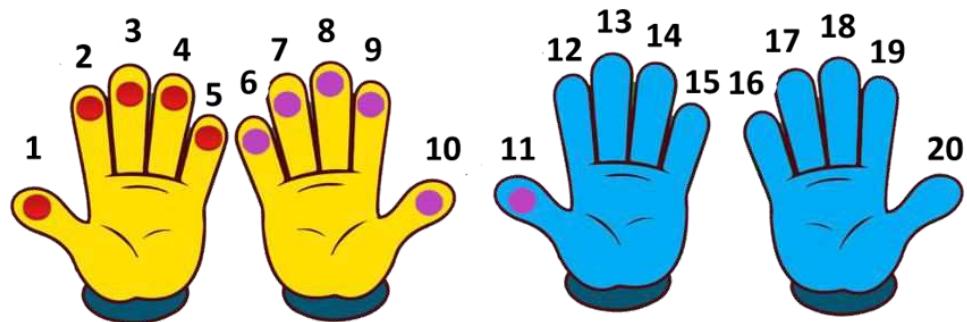
Case B: The minuend from 11 to 20 and the result of the subtraction less than 10

The student places as many red dots on the "fingers" as shown by the minuend, starting from his thumb on the left hand. From these, he removes as many dots as the subtrahend indicates. Then he finds the result of the subtraction. For example, to find out the result of the subtraction of $13 - 7$, places 13 red dots (corresponding to the minuend) at the corresponding positions and removes 7 dots in succession. 7 of the subtrahend are broken down into $4 + 3$ and are marked in the appropriate positions. Thus, it derives that: $13 - 7 = (13 - 3) - 4 = 6$

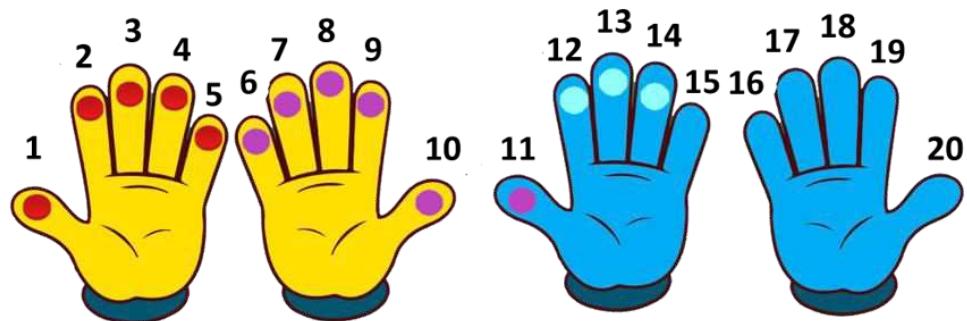


5. Addition of 3 numbers up to 20

The student places red dots on the "fingers" successively as many as the first additive indicates, as many purple dots as the second additive indicates and as many green dots as the third additive indicates. Thus, we obtain the sum. For example, to find the sum of $5 + 6 + 3$, successively 5 red dots are placed (corresponding to the first additive), 6 purple dots (that correspond to the second additive), and 3 blue dots (that correspond to the third additive). So it turns out to be $5 + 6 + 3 = 14$



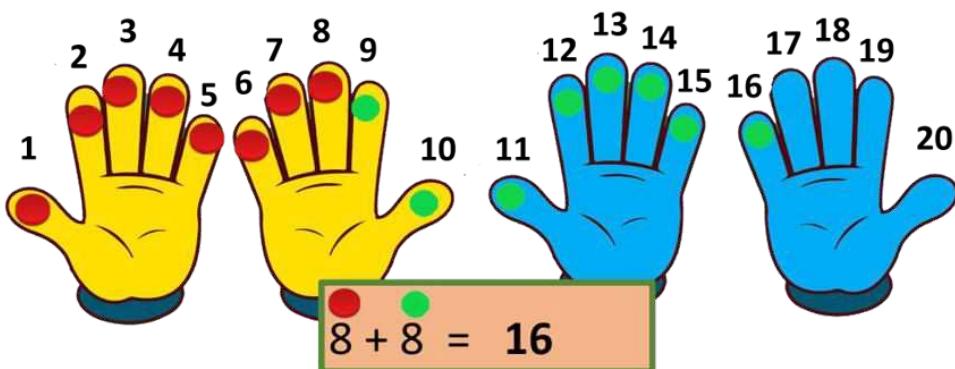
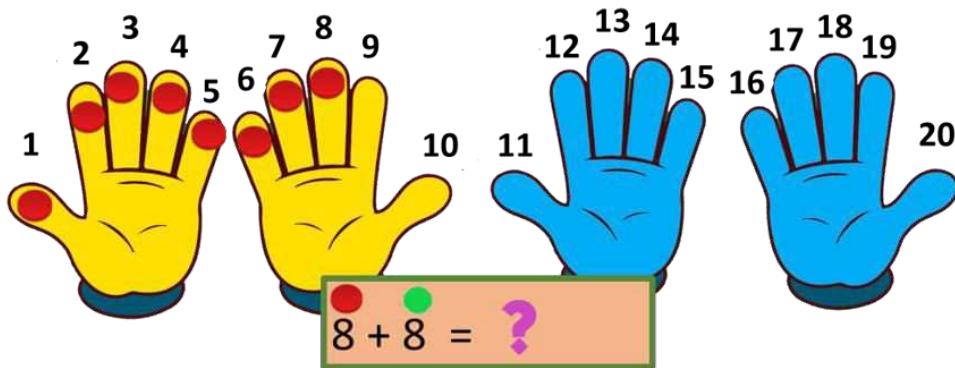
$$5 + 6 + 3 = ?$$



$$5 + 6 + 3 = 14$$

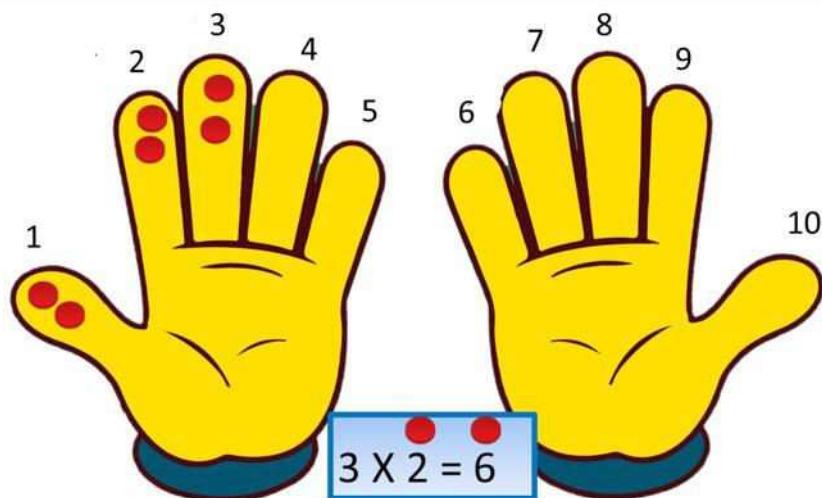
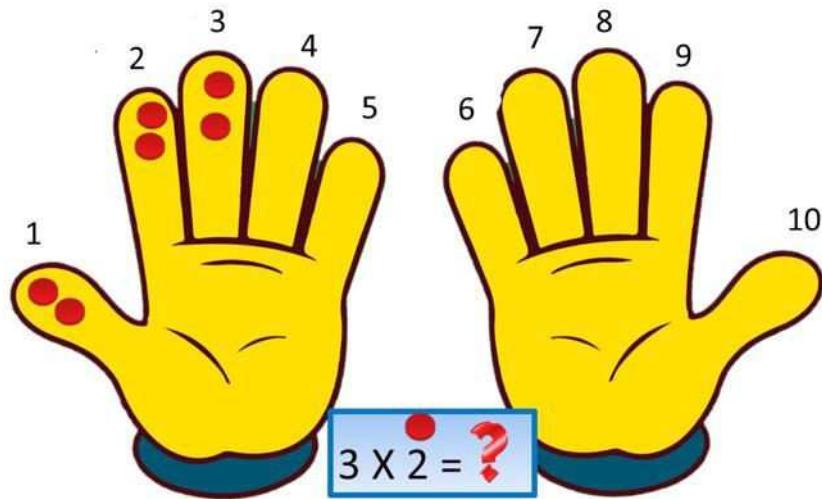
6. Double addition up to 20

The student places as many red dots on the "fingers" as the first additive indicates, and then, equal number from the green dots. Then he finds the sum. For example, in order to find the sum of $8 + 8$, there are placed 8 red dots (which correspond to the first additive) and 8 green dots (which correspond to the second additive). So it turns out that $8 + 8 = 16$.



E. MULTIPLICATION

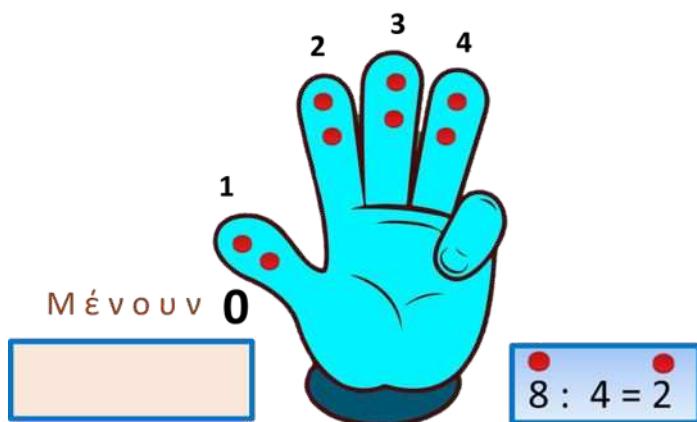
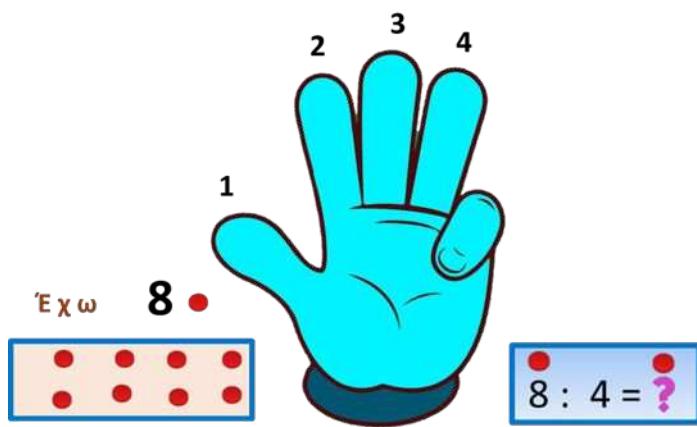
The student places as many red dots on each "finger" as the 'multiplier' indicates. Adding the red dots, he finds the result of the multiplication. For example, for multiplying 3 X 2, on 3 "Fingers" they place 2 red dots and then finds the product by adding the red dots.



F. DIVISION

1. Division of partition

The student places successively, one by one, as many red dots as the dividend indicates on as many fingers as the divisor shows. The number of dots on each finger shows the quotient and the number of dots which remain shows the remainder of the division. For example, the 8 dots (dividend) are shared in 4 fingers (divisor) one by one fairly. The number "2" of the dots on each finger indicates the quotient and the number "0" indicates the remainder of the division. So it turns out that $8 : 4 = 2$ with 0 remainder.

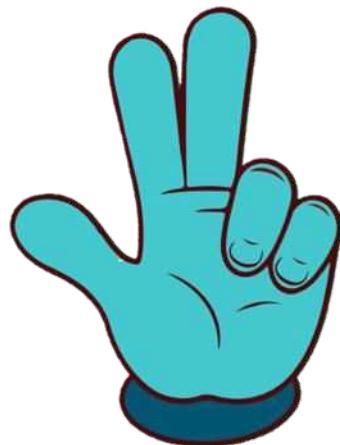


2. Division of counting

The student divides the divisor in twos dots or threes dots or fours dots, etc. Then, he places on the fingers the twos dots or the triples of dots or the fours of dots etc. that have been created. The number of fingers indicates the quotient and the number of dots left over indicates the remainder. For example, with dividend 15 and divisor 5 (five), 3 (number of fingers) indicates the quotient and 0 the remainder of the division. So, $15 : 5 = 3$ and remainder 0.

$$15 : \text{○} = ?$$

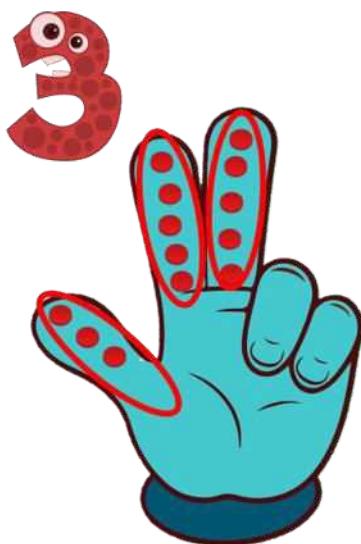
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$$15 : \text{○} = 3$$

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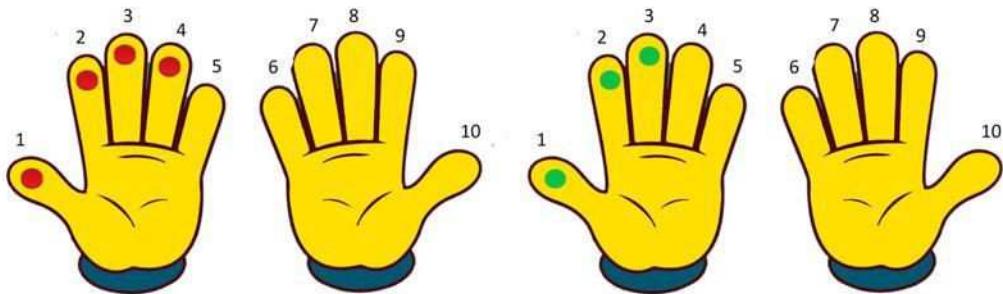
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G. PROPERTIES OF MATHEMATICAL OPERATIONS

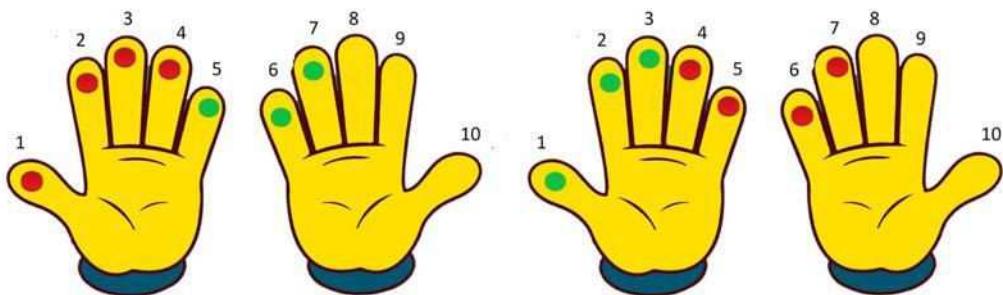
1. Commutative property of addition

The student places as many red dots on the "fingers" as shown by the first additive and then successively as many green dots as indicated by the second additive. It changes the order of additives and understands that the result remains the same. For example, they have 4 red dots and 3 green dots have the sum of 7 dots. Changing the order of additives he will find the same sum.



$$4\bullet + 3\bullet = ?$$

$$3\bullet + 4\bullet = ?$$

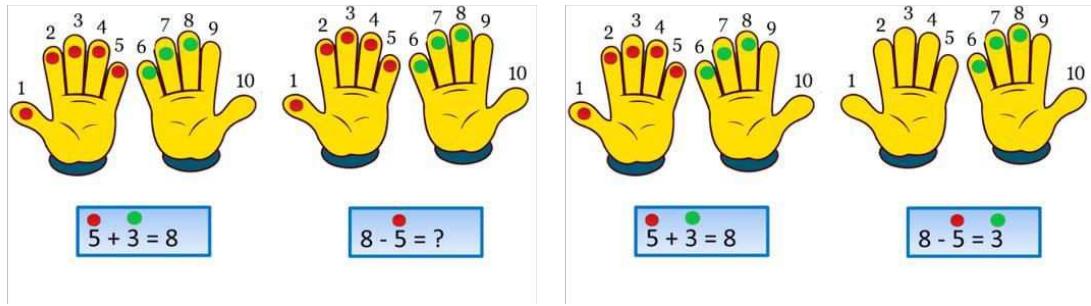
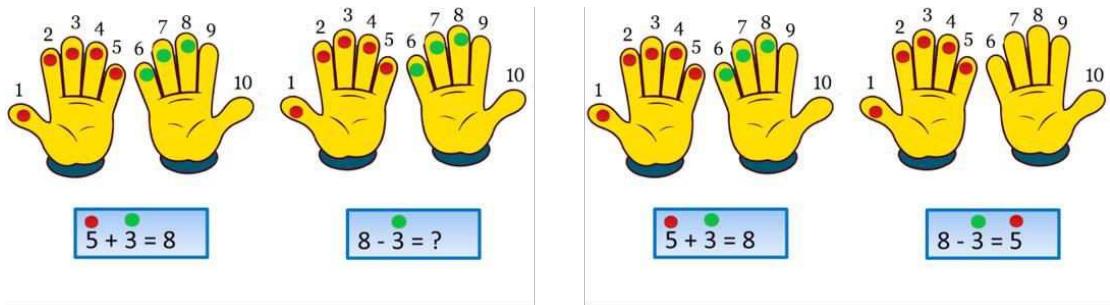


$$4\bullet + 3\bullet = 7$$

$$3\bullet + 4\bullet = 7$$

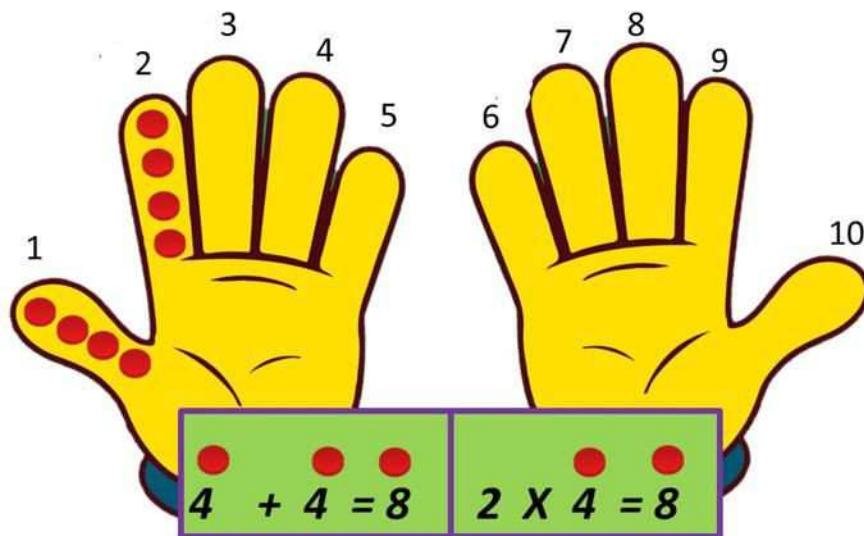
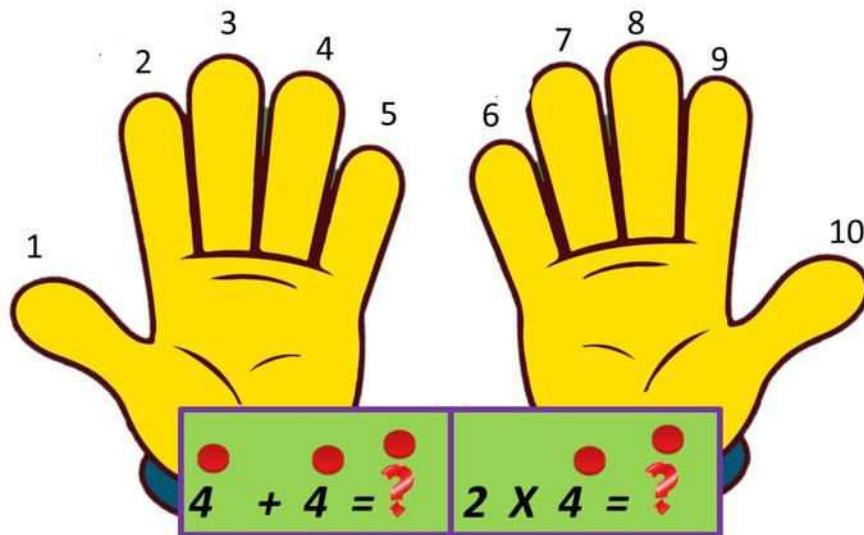
2. Addition and subtraction as reversed operations

The student places as many red dots on the "fingers" as the first additive indicates and then, as many green dots as the second additive indicates. Then, he removes the red dots and notices the green dots that are left. The same is repeated by subtracting the green dots from the sum and he observes the red dots that are left. For example, 5 red dots and 3 green dots have a sum of 8 dots. From 8 dots, by removing, the 5 red dots, remains 3 green dots and from the 8 red dots, subtracting 3 green dots, there are 5 red dots.



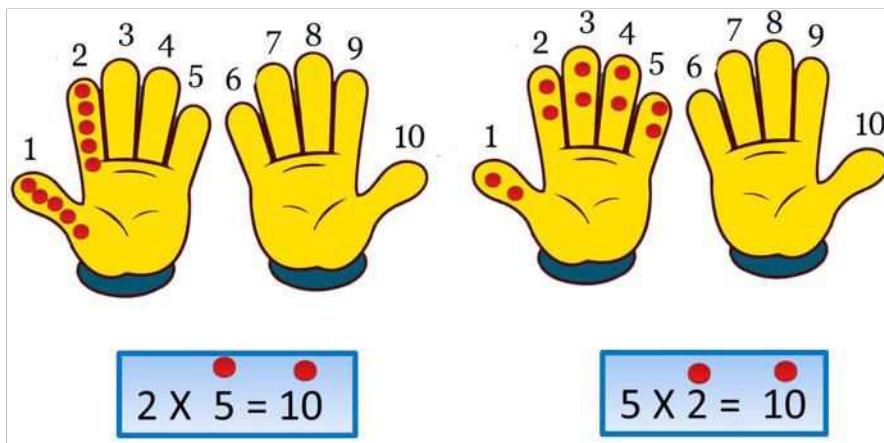
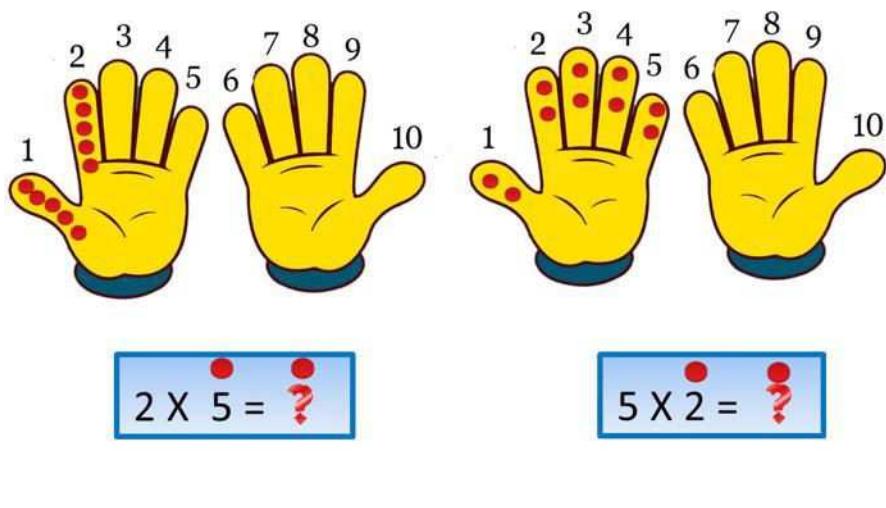
3. Multiplication as repeatable addition

The student places as many red dots on each "finger" as shown by each additive (the additives are the same). Adding the red dots, he will find the sum. They will find the same result if they multiply the number of "fingers" (multiplier) with the numbers of red dots on each "finger" (multiplicand). For example, $4 + 4$ and 2×4 have the same result, by placing 4 red dots on 2 "fingers". So it turns out that $4 + 4 = 8$ and $2 \times 4 = 8$.



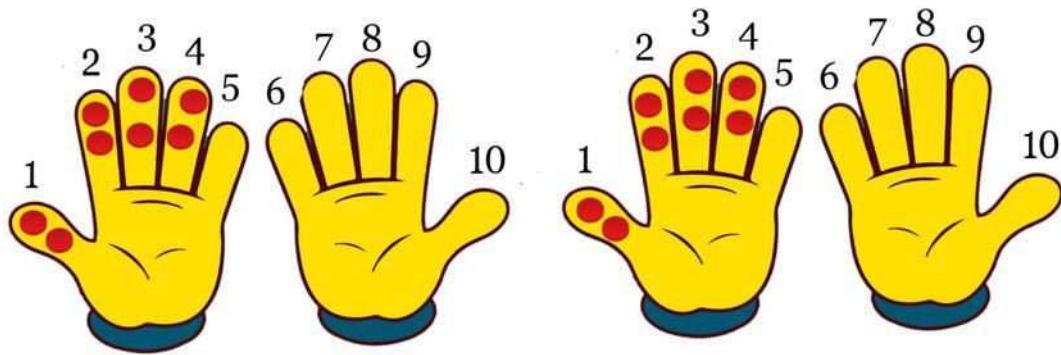
4. Commutative property of multiplication

The student places as many red dots on each "finger" as the multiplicand indicates on as many "fingers" as the multiplier indicates and finds the result. In a second pair of hands the student does the opposite. That is, by inverting the multiplier with the multiplicand from the first pair of hands, he finds the same result. For example, in the first pair of hands, placing 5 red dots on 2 "fingers", will give him that $2 \times 5 = 10$. Then, by placing 2 red dots on 5 "fingers", he will also find that $5 \times 2 = 10$.



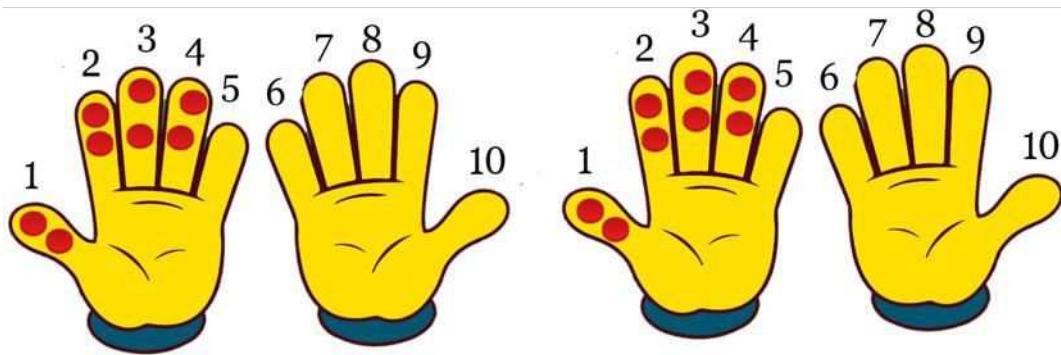
5. Division and multiplication as reversed operations

The student, performing a division without remainder on the “fingers” points out that the same picture shows the counterpart multiplication, as a reverse act of division. For example, in division $8: 4 = 2$, is true that $4 \times 2 = 8$.



$$8 : 4 = 2$$

$$4 \times 2 = ?$$

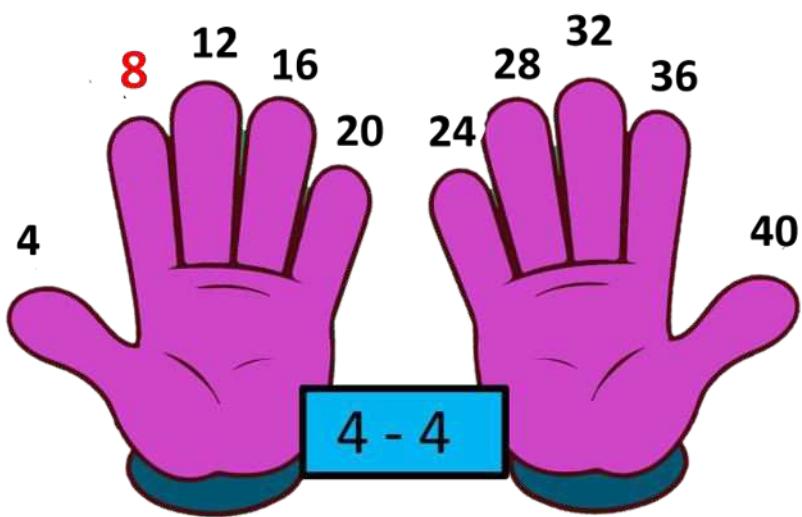
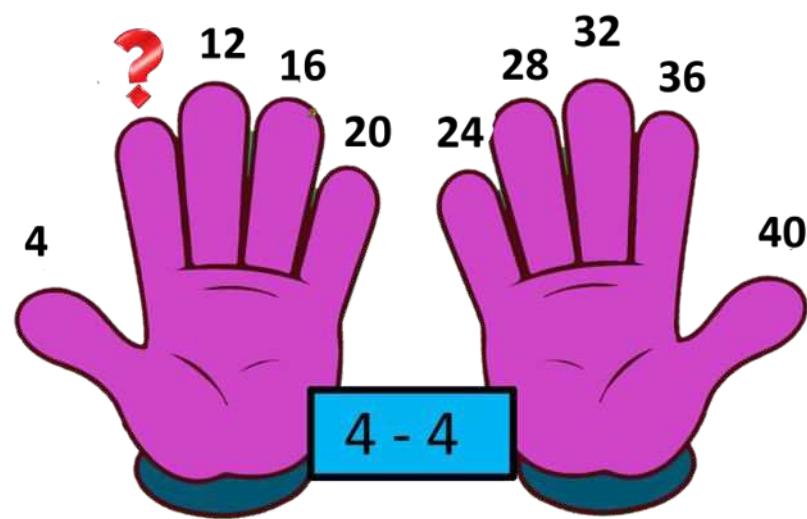


$$8 : 4 = 2$$

$$4 \times 2 = 8$$

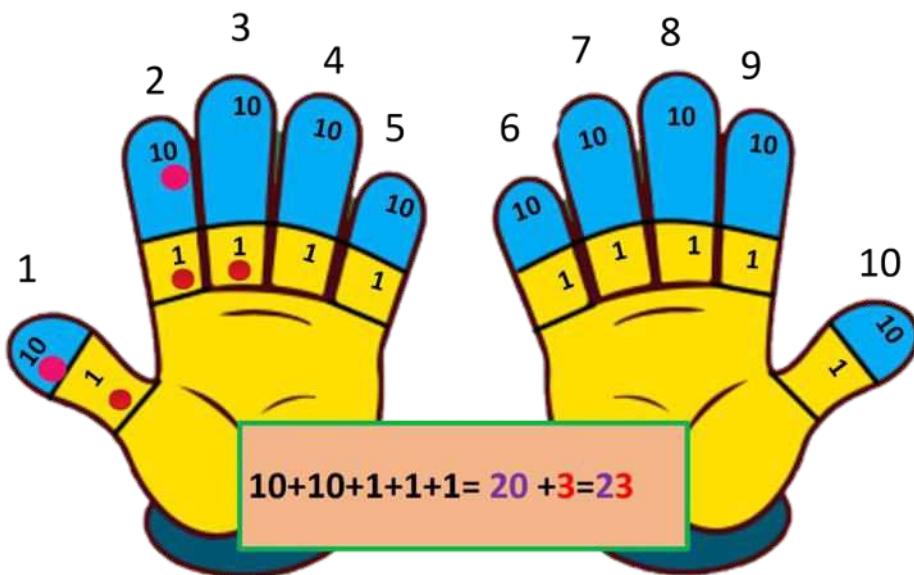
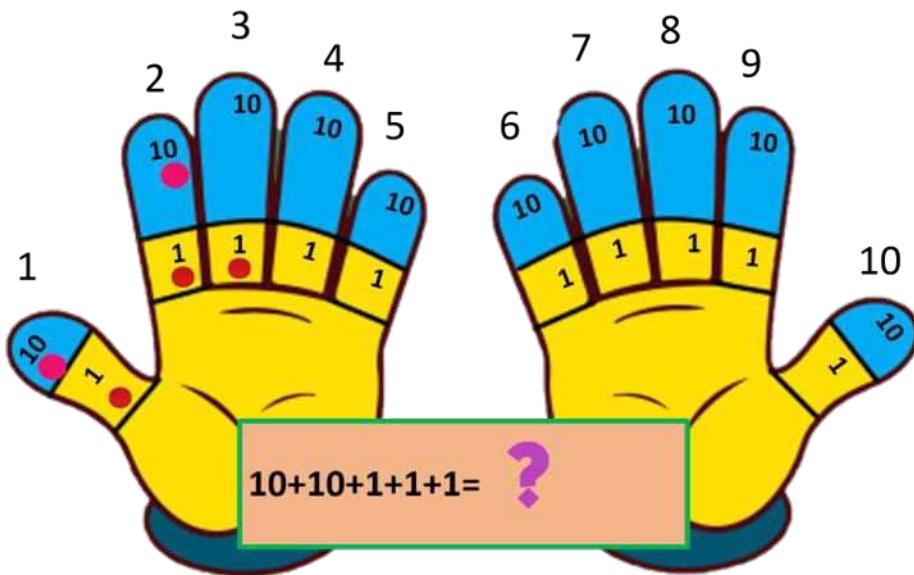
6. Multiples of 2,3,4,5,6,7,8,9,10

Each of the ten fingers corresponds to each of the multiples of one of the numbers: 2,3,4,5,6,7,8,9 or 10. For example, multiples of 4 are: 4, 8, 12, 16, 20, 24, 28, 32, 36, and 40.



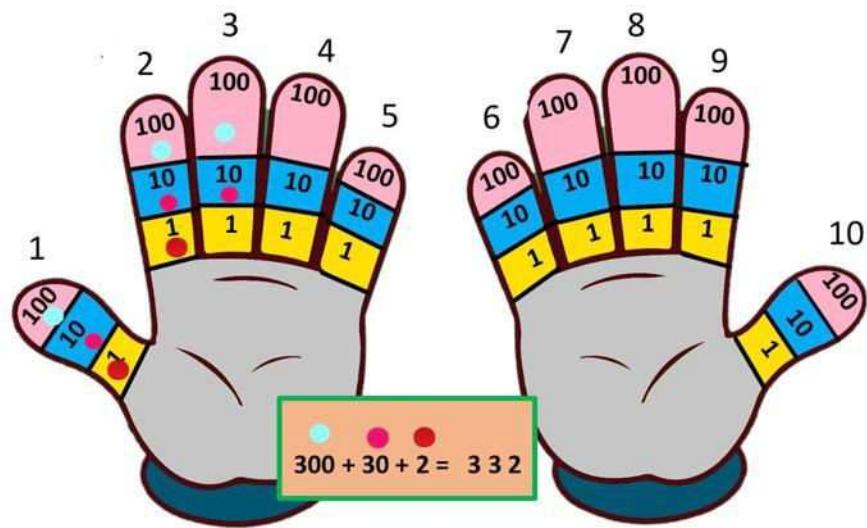
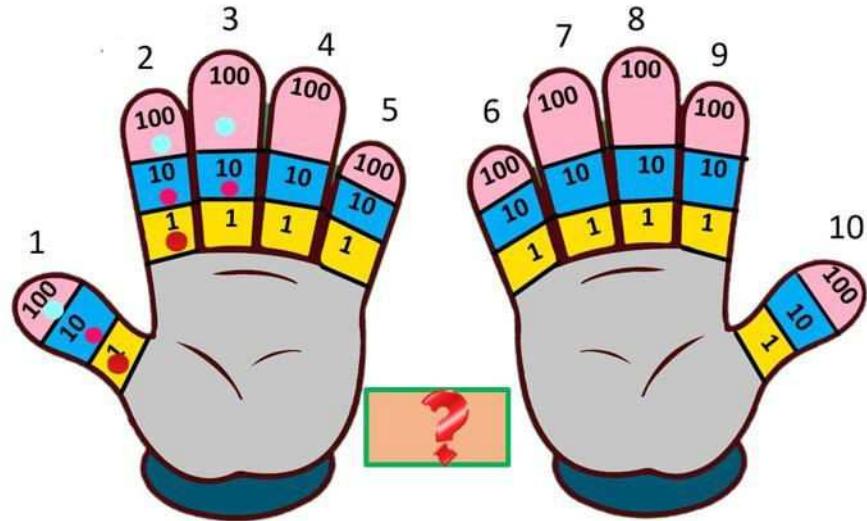
H. ABACUS 1. Abacus up to 100

Each of the 10 fingers is divided into 2 parts by 2 different colors. Tens and units are denoted in different color. Placing the appropriate dots of 2 different colors on the corresponding points, a number is shown as 100.



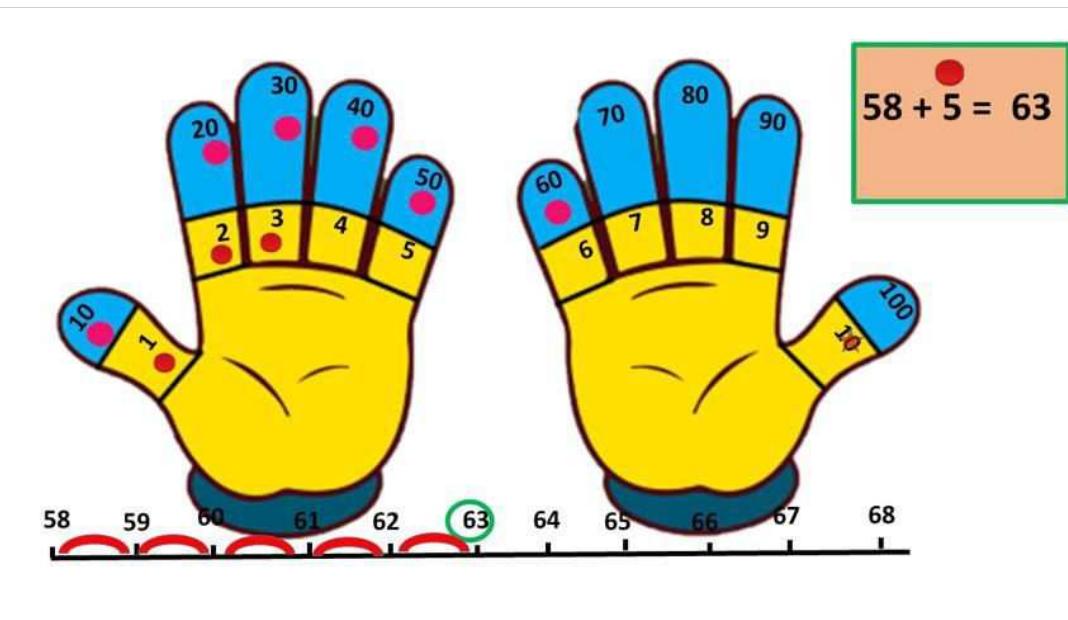
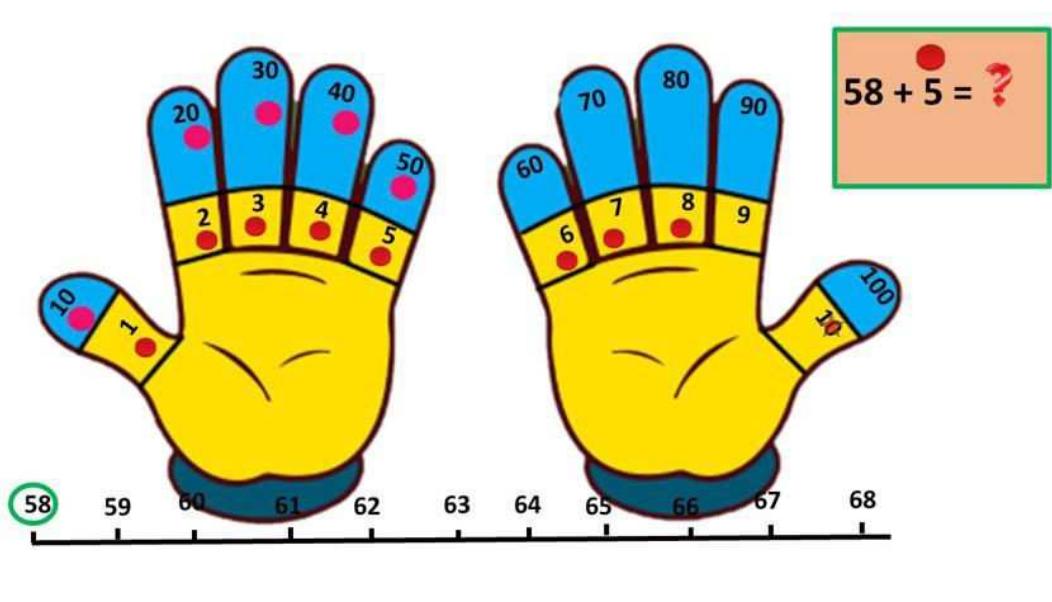
2. Abacus up to 100

All fingers are divided into 3 parts with 3 different colors. Each color corresponds to units or tens or hundreds. By placing dots of 3 different colors on the corresponding points shows a number as 1000.



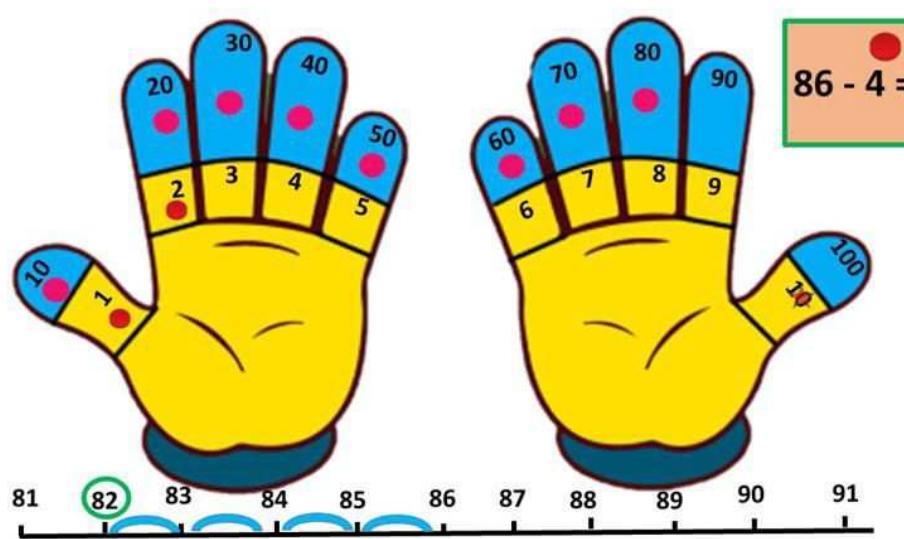
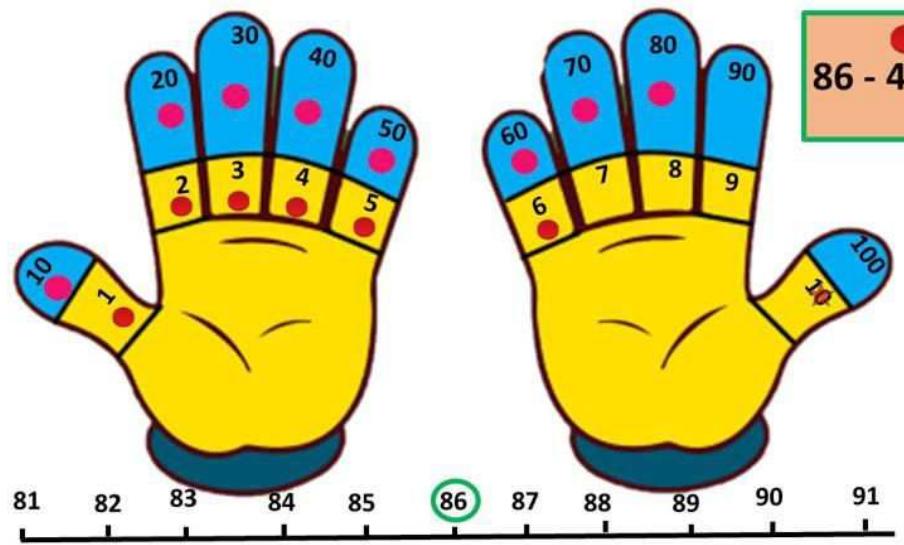
3. Addition by using abacus up to 100

The student places the corresponding dots that corresponds to first additive and then place the corresponding dots successively for the second additive. Then they find the sum. For example $58 + 5 = 63$.



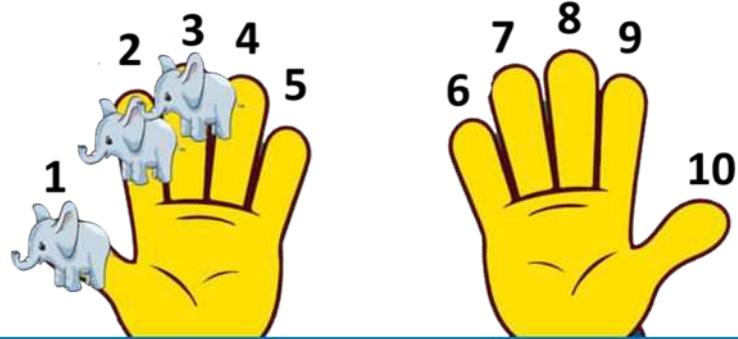
4. Subtraction by using abacus up to 100

The student places the dots corresponding to the minuend and then deducts as many dots as possible according to the subtrahend. The new number created corresponds to the result of the subtraction operation. For example $86 - 4 = 82$.

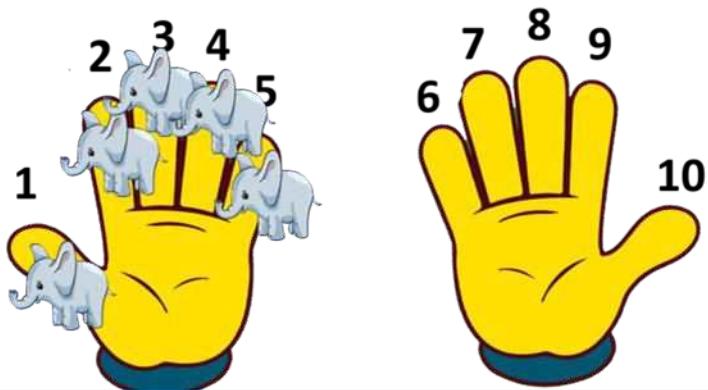


I. MATHEMATICAL PROBLEMS

Example of addition problems

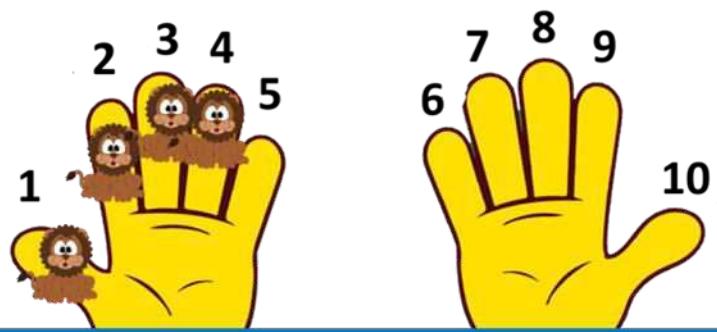


$$3 + 2 = ?$$

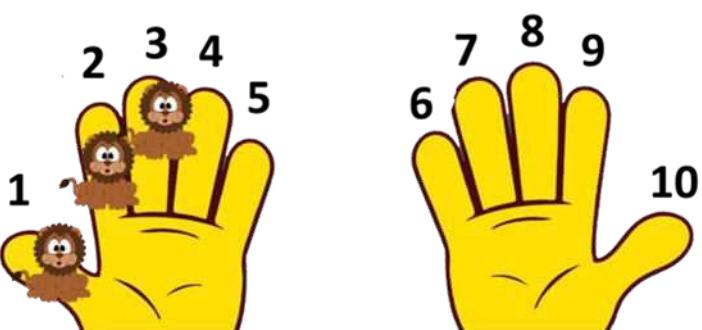


$$3 - 2 = 5$$

Example of subtraction problems

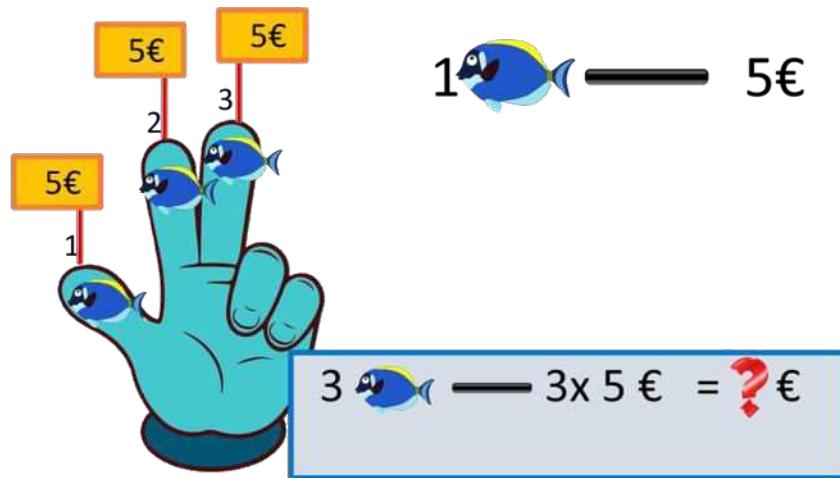
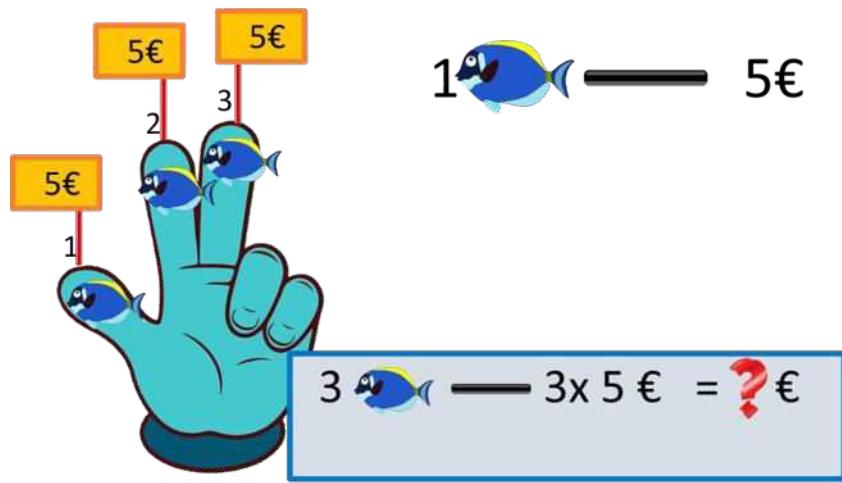


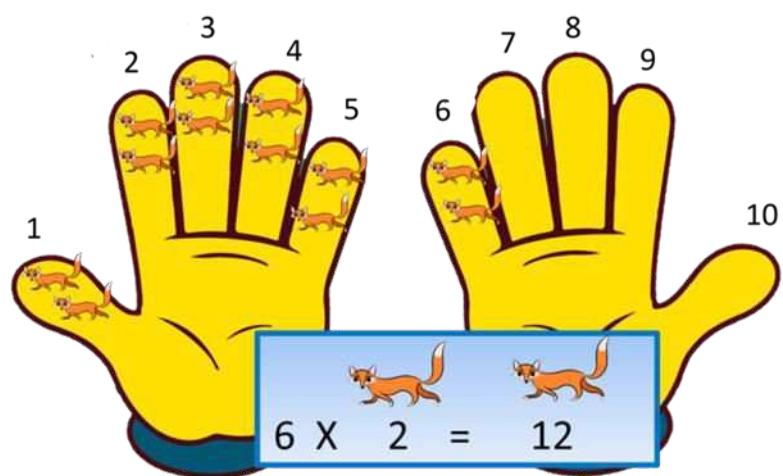
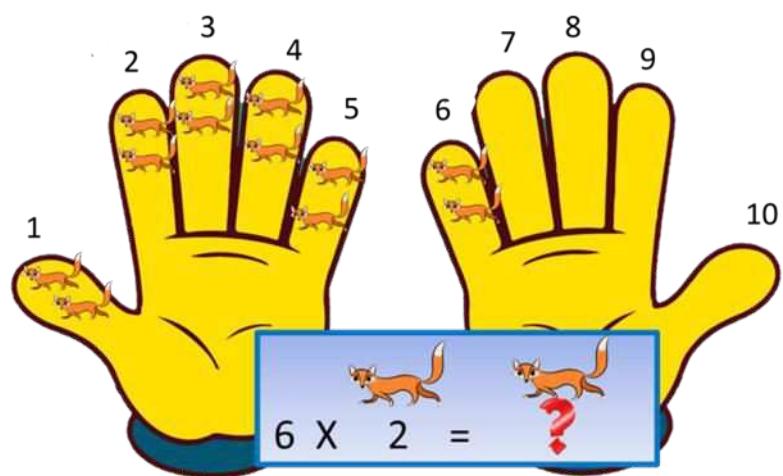
$$4 \text{ lions} - 1 \text{ lion} = ?$$



$$4 \text{ lions} - 1 \text{ lion} = 3 \text{ lions}$$

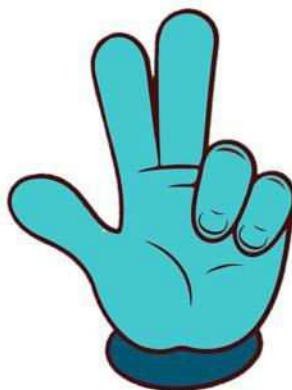
Examples of multiplication problems





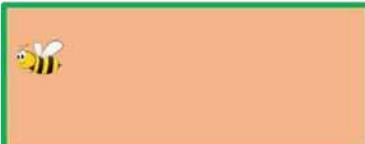
Examples of division problems

$$7 : \text{
$$7 \text{ $$$$

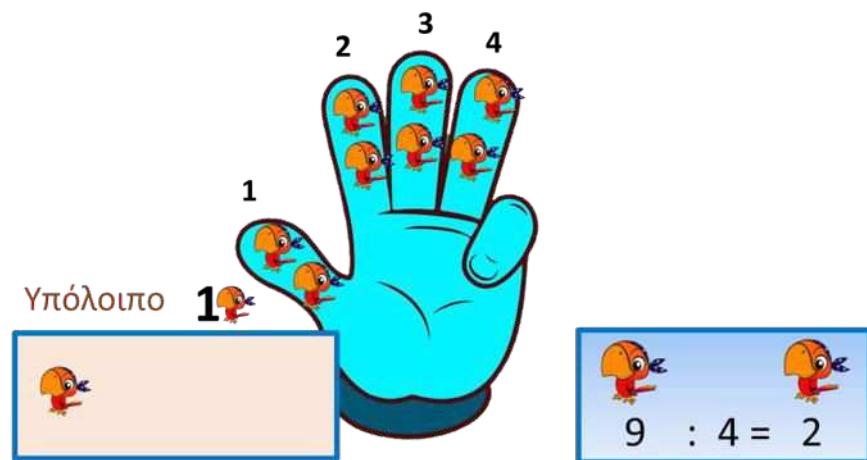
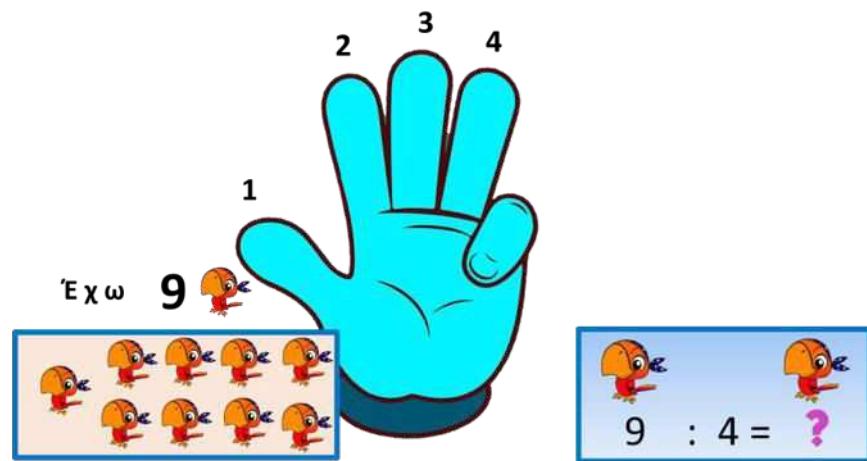


$$7 : \text{$$

$$1 \text{ $$







J. FLASH CARDS

1



2



3



4



5



7



6



8

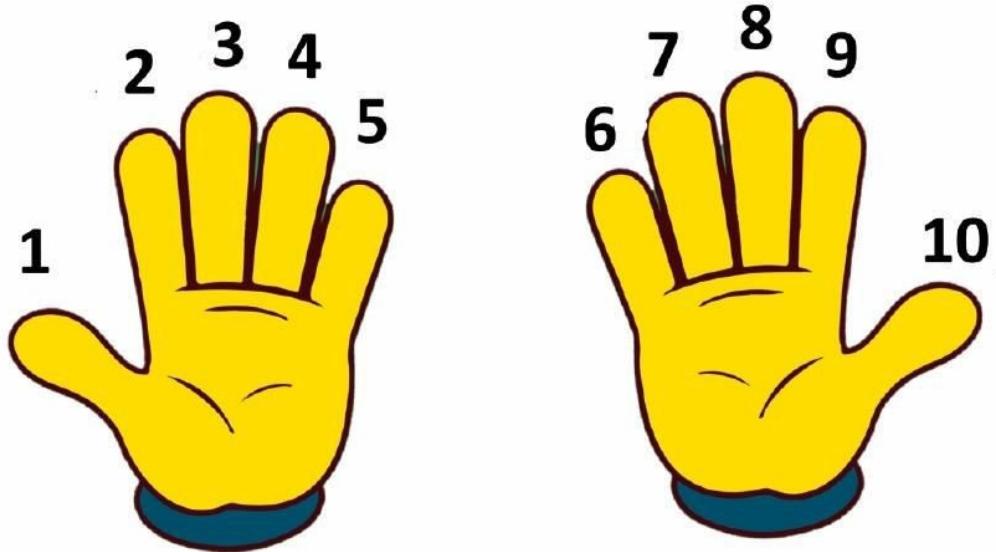


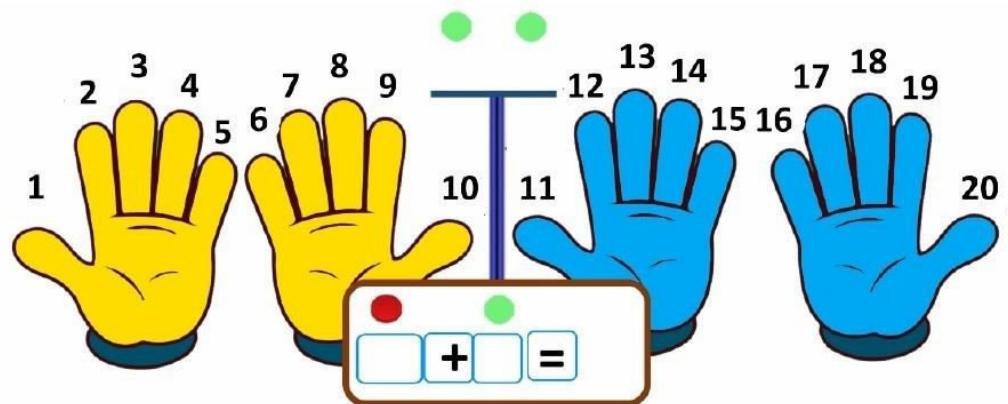
9

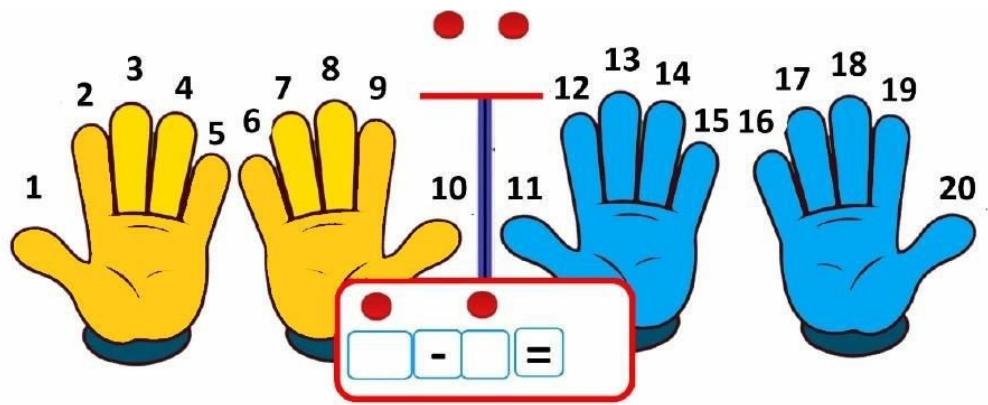


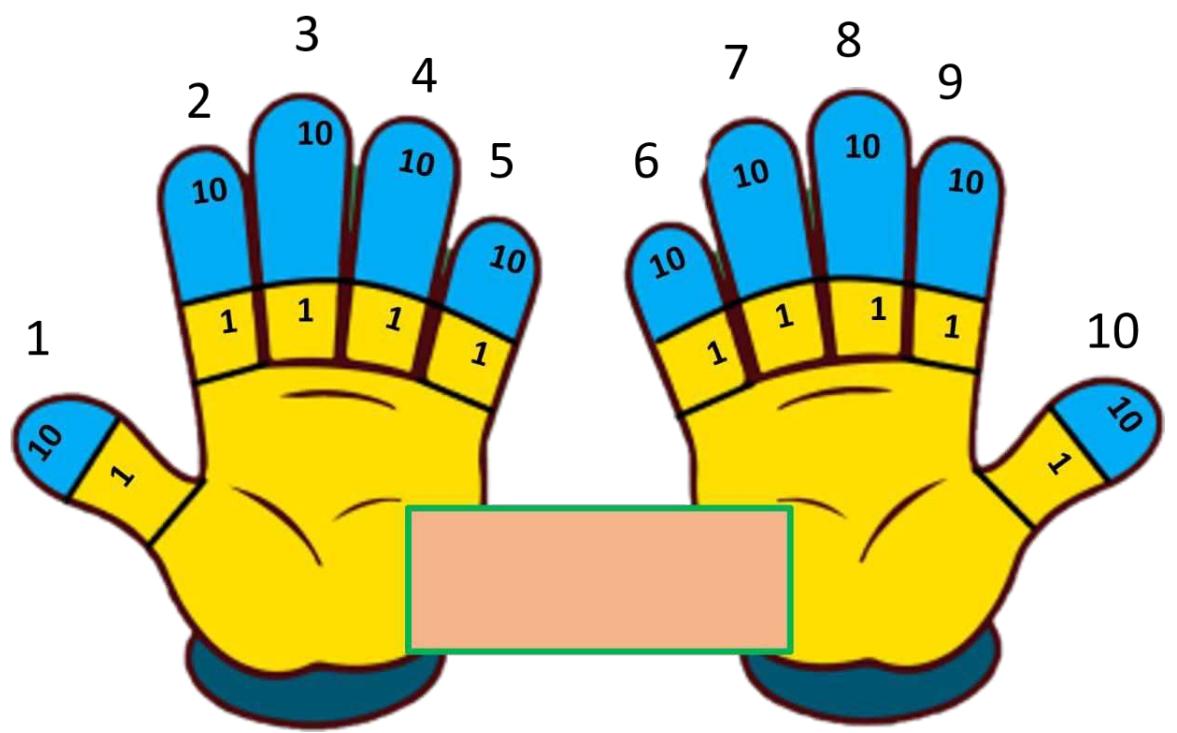
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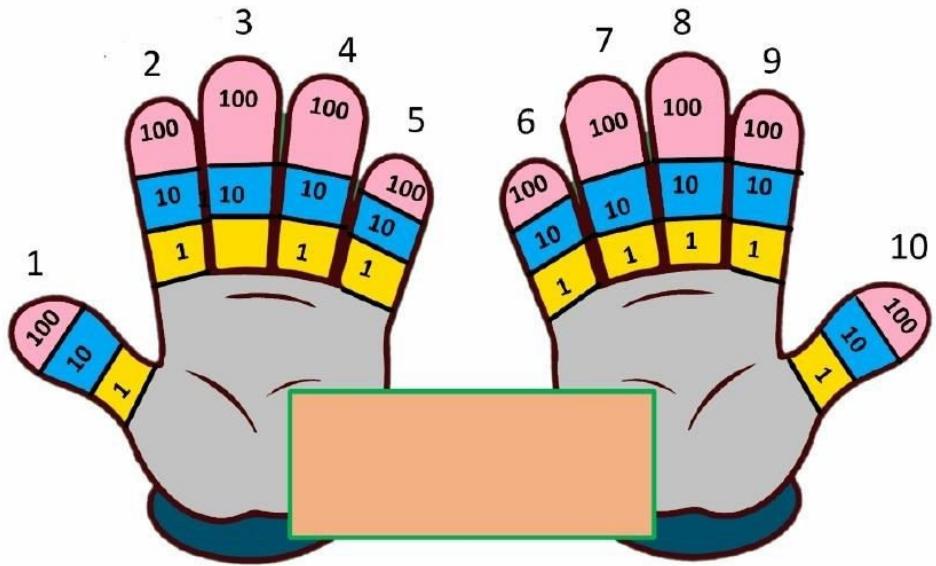












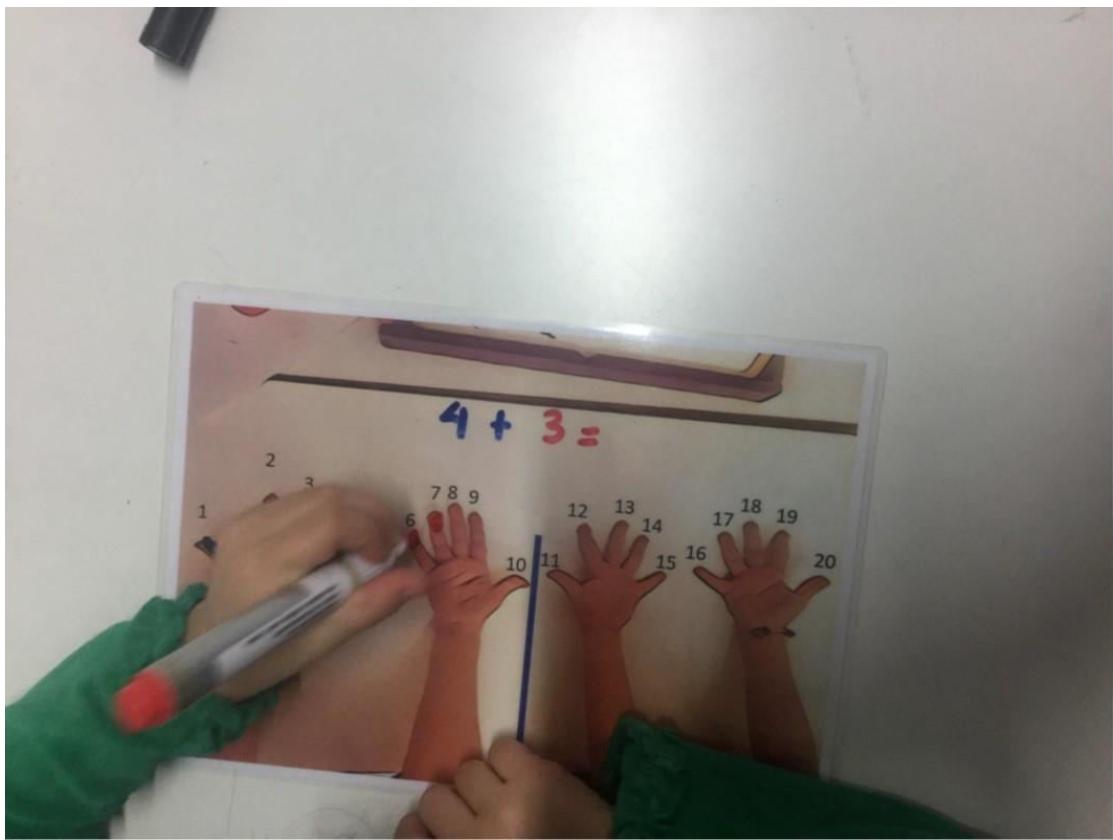
K. TEACHING MATERIALS

The first method can be administered with 3 different ways

1. Using laminated photocopy A4, 2 or 3 color markers of different colors for whiteboard, an eraser for whiteboard and a pencil or a small stick.

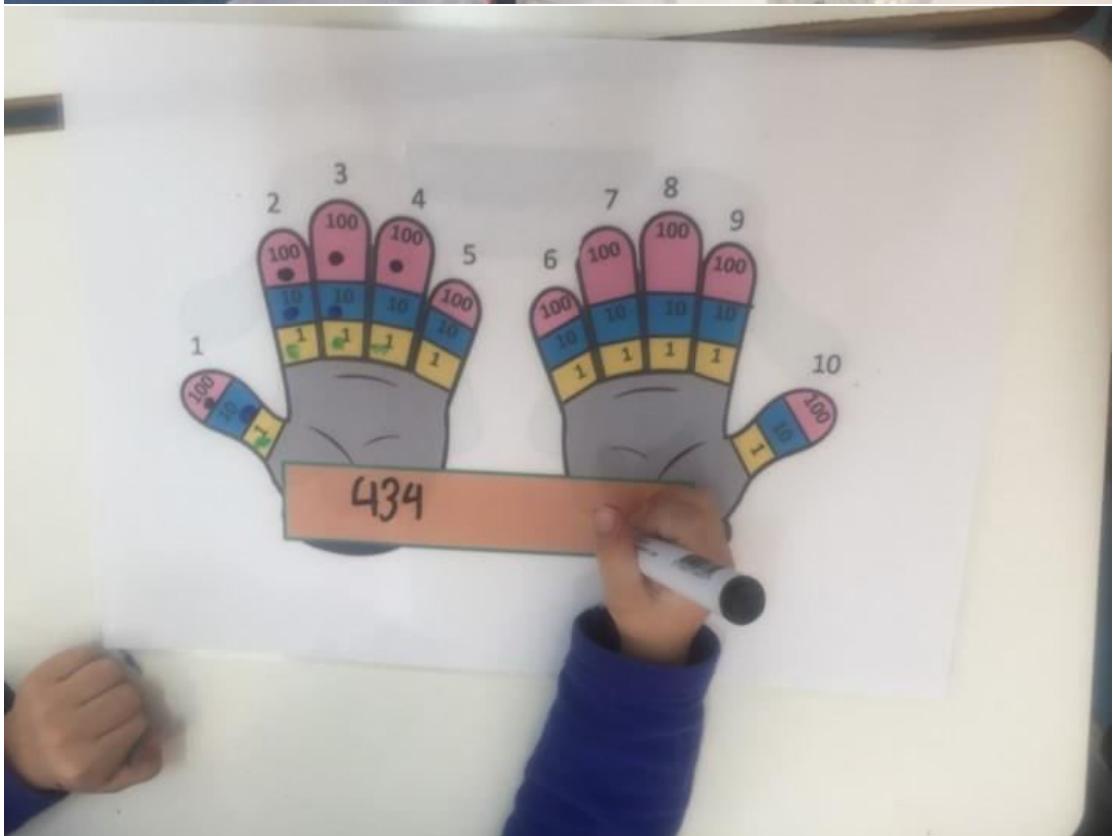






2. By using A2 laminated colored photocopy, substances of 2 or 3 different colors and a pencil or a small stick.









3. As an application from the website <https://www.firstgradesmath.com>, from the windows store with the title **Maths Made Easy - Method ALPHA** (for PC, LAPTOP as

and in a classroom using a projector) and from the play store for android devices with the title **Math Made Easy –Method ALPHA**