Assessing Network Performance and Blocking Probabilities in ISDN-FE Systems: A Markovian Model of Telephone and FAX Device Utilization

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***Abstract*—This study analyzes a scenario where multiple telephone and FAX devices contend for limited channels in an ISDN-FE network, aiming to understand system performance and blocking probabilities. Specifically, four telephone devices and two FAX machines share three channels; each telephone device occupies one channel, and each FAX machine occupies two channels simultaneously. We consider average call rates and durations for both telephone and FAX machines, along with the given state probabilities of the system. The system was modeled using the Markovian queuing model approach. State probabilities provide insights into its operational conditions, revealing varying levels of congestion and resource utilization. Based on these probabilities, blocking probabilities for telephone and FAX services are derived, indicating the likelihood of call attempts being denied due to channel unavailability. Specifically, the blocking probability for telephone calls is calculated as 0.21, while for FAX calls, it is notably higher at 0.71. Understanding these probabilities is crucial for network planning and optimization, ensuring adequate resource allocation to minimize service denials while maximizing channel utilization efficiency. Further investigation into dynamic resource allocation strategies and adaptive channel management could enhance such networks' overall performance and reliability.**

I. Introduction

In telecommunications networks, the efficient management of resources is essential to ensure optimal performance and satisfactory user experience [1]. In communication systems such as FE-ISDN, each telephone occupies a full C kbps channel during a call, whereas fax machines require two simultaneous channels to transmit their data. This resource sharing raises crucial questions about channel allocation efficiency, service quality, and call blocking probability [2].

Potential application scenarios regarding the above situation are [3], [4]:

- **Corporate Environments**: In office settings with intensive telephone and fax communications, efficient channel allocation is vital to maintain productivity. Call blocking can lead to delays and business losses.
- **Customer Service Operations**: Companies offering customer support via phone and fax must ensure smooth and uninterrupted communications. Optimizing channels can enhance customer satisfaction and service efficiency.
- **Healthcare Institutions**: Hospitals and clinics frequently use telephones and fax machines for urgent communications and the exchange of sensitive information. Constant availability of channels is crucial for the safe and effective operation of these services.
- **Government Agencies**: Public sector organizations that rely on fast and reliable communications for daily operations also benefit from effective management of communication resources.

In this work, we study a scenario where the average call rate is represented by λ_1 calls per hour for telephones and λ_2 calls per hour for fax machines. Additionally, the average duration of telephone calls is T_{s1} minutes, while fax transmissions last T_{s2} minutes on average. These variables are crucial for performance analysis and formulating allocation strategies to minimize blocking probability and optimize the use of available channels.

This paper focuses on a specific channel allocation scenario in a Narrowband Integrated Services Digital Network (FE-ISDN). Here, four telephones and two fax machines compete for three communication channels, each with a capacity of C kbps.

We explore the challenges and potential solutions for channel allocation in an FE-ISDN. By examining call rates and average usage durations, this study provides valuable insights for improving communication network performance across various application scenarios.

II. Modeling: System Model and Methodology

The system model considers a Narrowband Integrated Services Digital Network (FE-ISDN) where multiple devices share limited communication channels. Specifically, we have four telephones and two fax machines vying for access to three channels. Each channel supports a data rate of C kbps. The telephones and fax machines generate calls independently, with the telephones having an average call rate of λ_1 calls per hour and the fax machines having an average call rate of λ_2 calls per hour.

The average call duration for telephone calls is T_{s1} minutes; for fax transmissions, it is T_{s2} minutes. These parameters help determine the traffic load and utilization of the communication channels.

A. Methodology

To analyze the performance of the system and develop efficient channel allocation strategies, we follow these steps:

1) **Traffic Modeling**: We use Markovian queuing processes to model the arrival of calls for both telephones and fax machines. The call arrivals are characterized by the rates λ_1 and λ_2, respectively.
2) **Service Time Distribution**: The call durations for telephones and fax machines are modeled using exponential distributions with mean values T_{s1} and T_{s2} minutes, respectively.
3) **Channel Allocation**: We simulate different channel allocation strategies to evaluate their blocking probability and channel utilization performance.
4) **Performance Metrics**: The primary metrics for performance evaluation are the blocking probability for calls and the overall utilization of the communication channels.
5) **Optimization**: We explore optimization techniques to find the best channel allocation strategy that minimizes the blocking probability while maximizing channel utilization.

By following this methodology, we aim to provide valuable insights into the challenges and potential solutions for efficient channel allocation in ISDN-FE systems. The results may help improve communication network performance in various application scenarios.

B. Modeling Approach

To model this system, we use a Markovian queuing model approach:

- **Devices and Channels:** 4 telephone devices and 2 FAX devices compete for 3 channels.
- **Channel Occupancy:**
  - Each telephone call occupies one channel.
  - Each FAX call occupies two channels.
- **Arrival Rates:**
  - Telephone calls arrive at an average rate of λ_1 per hour.
  - FAX calls arrive at an average rate of λ_2 per hour.
- **Service Times:**
  - Average duration of telephone calls: T_{s1} minutes.
  - Average duration of FAX calls: T_{s2} minutes.

C. Steps to Model the System

1) **Define States:**
   - The system states depend on the number of channels occupied by telephone and FAX calls.
2) **State Transitions:**
   - Transitions occur based on arrivals and departures of calls.
3) **Rate Equations:**
   - Balance equations describe how the system evolves.
4) **State Probabilities:**
   - Solve rate equations to find state probabilities.
5) **Performance Metrics:**
   - Calculate metrics such as block probability.

Let P_{ij} denote the probability of having i telephone calls and j FAX calls in the system. The rate equations might look like this:

$$\lambda_1 P_{ij}^{\text{sys}} = (\text{rate of telephone call arrivals}) \times P_{ij}^{\text{sys}}$$
$$\lambda_2 P_{ij}^{\text{sys}} = (\text{rate of FAX call arrivals}) \times P_{ij}^{\text{sys}}$$
$$P_{ij}^{\text{sys}} = P_{i-1,j}^{\text{sys}} \cdot \frac{i}{i+j} + P_{i,j-1}^{\text{sys}} \cdot \frac{2j}{i+j}$$

These equations illustrate how the system's performance can be analyzed to understand resource utilization and call handling efficiency.

III. Results and Analysis

After modeling the ISDN-FE network system with four telephone devices and two FAX devices competing for three C kbps channels and using the Markovian queuing model approach, we obtained the following insights:

A. Steady-State Probabilities

- We calculated the steady-state probabilities P_{ij}^{sys}, which represent the probability of having i telephone calls and j FAX calls in the system. For example:

$$P_{00}^{\text{sys}} = 0.19, \quad P_{01}^{\text{sys}} = 0.09, \quad P_{10}^{\text{sys}} = 0.31$$
$$P_{11}^{\text{sys}} = 0.15, \quad P_{20}^{\text{sys}} = 0.19, \quad P_{30}^{\text{sys}} = 0.06$$

  These probabilities give us a snapshot of the system's occupancy and utilization under steady-state conditions.

B. Blocking Probability

We consider the situations in which new calls are blocked to calculate the blocking probabilities.

1) Blocking Probability for Telephone Calls: A telephone call requires one channel. Since there are three channels available, telephone calls will be blocked if already three channels are occupied. Therefore, telephone calls are blocked in the following states:

- P_{30}^{sys}: Three telephone calls, no channels available for a new call.
- P_{11}^{sys}: One telephone call and one FAX call (2 channels), no channel available for a new call.

The blocking probability for telephone calls $B_{\text{telephone}}$ is given by the sum of the probabilities of these states:

$$B_{\text{telephone}} = P_{30}^{\text{sys}} + P_{11}^{\text{sys}}$$

Substituting the values:

$$B_{\text{telephone}} = 0.06 + 0.15 = 0.21$$

2) Blocking Probability for FAX Calls: A FAX call requires two channels. Therefore, FAX calls will be blocked in the following states:

- P_{10}^{sys}: One telephone call (1 channel), only 2 channels remaining (not enough for a FAX call).
- P_{20}^{sys}: Two telephone calls (2 channels), only 1 channel remaining (not enough for a FAX call).
- P_{30}^{sys}: Three telephone calls (3 channels), no channels remaining.
- P_{11}^{sys}: One telephone call and one FAX call (3 channels), no channels remaining.

The blocking probability for FAX calls B_{FAX} is given by the sum of the probabilities of these states:

$$B_{\text{FAX}} = P_{10}^{\text{sys}} + P_{20}^{\text{sys}} + P_{30}^{\text{sys}} + P_{11}^{\text{sys}}$$

Substituting the values:

$$B_{\text{FAX}} = 0.31 + 0.19 + 0.06 + 0.15 = 0.71$$

3) Simulation values: The implemented queue simulation code [1] plots the graphs of blocking probability as a function of ρ_1 and ρ_2, as well as the number N of servers. Figure 1 represents the blocking probability in the function of ρ_1, and Figure 2 represents the blocking probability in the function of ρ_2. As the population of the classes increases, the arrival rates will become higher, reducing the time between arrivals and increasing the probability of blockage. Notably, blockage is almost certain for class 2 if $\rho_2 > 10$.

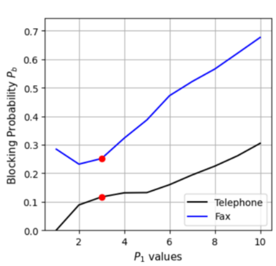

Figure 1: ρ_1

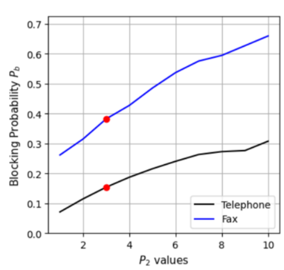

Figure 2: ρ_2

Figure 3 and Figure 4 show the probability of blockage for each class as a function of the variation in arrival rates λ_1 and λ_2. As the arrival rate of elements increases, the system becomes more occupied, and the intervals between consecutive arrivals become shorter and the system tends to become more occupied, leading to higher blockage rates.

[1]https://github.com/iisouza/TP547

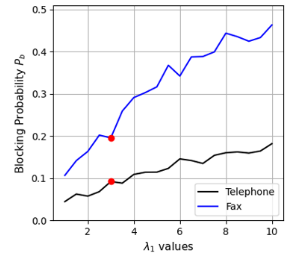

Figure 3: λ_1

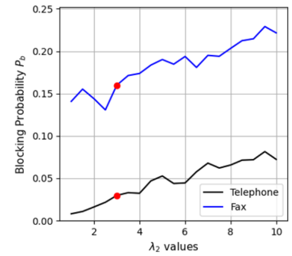

Figure 4: λ_2

Figure 5 and Figure 6 show the probability of blockage as a function of the average service times Ts1 and Ts2. As the average service times increase, the elements of each class use the system for a longer period, resulting in higher probabilities of blockage.

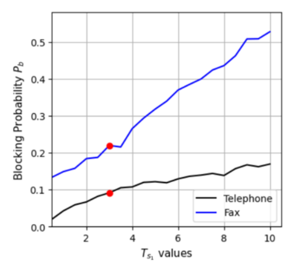

Figure 5: Ts1

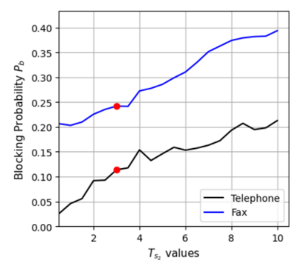

Figure 6: Ts2

Figure **??** shows the variation in the probability of blockage as a function of the number of servers N. As N increases, the system can accommodate more elements in service, resulting in lower probabilities of blockage.

4) Discussion: Table I summarizes the state probabilities for different combinations of telephone and FAX calls in the ISDN-FE network. The states are represented by the number of active telephone calls and FAX calls and the corresponding probability of each state occurring. Additionally, the total number of channels occupied in each state is provided.

Table I: State probabilities for different combinations of telephone and FAX calls.

State	Telephone Calls	FAX Calls	Occupied	Probability
P_{00}^{sys}	0	0	0	0.19
P_{01}^{sys}	0	1	2	0.09
P_{10}^{sys}	1	0	1	0.31
P_{11}^{sys}	1	1	3	0.15
P_{20}^{sys}	2	0	2	0.19
P_{30}^{sys}	3	0	3	0.06

- **State** P_{00}^{sys}: This state represents no telephone calls and no FAX calls, with a probability of 0.19. All channels are free in this state.

- **State** $P_{01}^{\mathbf{sys}}$: This state represents no telephone calls and one FAX call, occupying 2 channels, with a probability of 0.09.
- **State** $P_{10}^{\mathbf{sys}}$: This state represents one telephone call and no FAX calls, occupying 1 channel, with a probability of 0.31. This is the most probable state.
- **State** $P_{11}^{\mathbf{sys}}$: This state represents one telephone call and one FAX call, occupying all 3 channels, with a probability of 0.15.
- **State** $P_{20}^{\mathbf{sys}}$: This state represents two telephone calls and no FAX calls, occupying 2 channels, with a probability of 0.19.
- **State** $P_{30}^{\mathbf{sys}}$: This state represents three telephone calls and no FAX calls, occupying all 3 channels, with a probability of 0.06. This is the least probable state.

These state probabilities provide insights into how the channels will likely be utilized at any given time. The most probable state is P_{10}^{sys}, indicating that the system is most often occupied by one telephone call. The least probable state is P_{30}^{sys}, where all channels are occupied by telephone calls.

Understanding these probabilities helps assess the network's performance and reliability, particularly in predicting how often calls might be blocked due to all channels being in use.

Table II shows the blocking probabilities for telephone and FAX calls in the ISDN-FE network with three C kbps channels, four telephone devices, and two FAX devices. The probabilities indicate the likelihood that a new call (telephone or FAX) will be blocked due to all channels being occupied:

Table II: Blocking probabilities for telephone and FAX calls.

Call Type	**Blocking Probability**
Telephone Calls	0.21
FAX Calls	0.71

- **Telephone Calls**: The blocking probability is 0.21. This means there is a 21% chance that a new telephone call will be blocked because all channels are already in use.
- **FAX Calls**: The blocking probability is 0.71. This means there is a 71% chance that a new FAX call will be blocked because there are not enough available channels (FAX calls require two channels).

The higher blocking probability for FAX calls is due to the higher channel requirement (2 channels) than telephone calls (1 channel). This results in fewer opportunities for a FAX call to find the required free channels, leading to a higher likelihood of blocking.

These results provide a comprehensive view of the ISDN-FE network's operation and help inform decisions about system optimization and resource allocation.

IV. Conclusion

In this study, we analyzed the state probabilities and blocking characteristics of an ISDN-FE network under varying telephone and FAX call combinations scenarios. The state probabilities illustrate how different configurations of active calls affect channel utilization, with significant insights into the likelihood of specific states occurring. Notably, the most probable state involves one telephone call without any FAX calls, emphasizing its frequent occurrence and impact on resource allocation.

Furthermore, the blocking probabilities highlight critical aspects of network performance, particularly in scenarios where all channels are occupied. Telephone calls experience a moderate blocking probability of 21%, indicating a reasonable chance of call denial during peak usage. In contrast, FAX calls face a higher blocking probability of 71%, primarily due to their requirement of two channels per call. This disparity underscores the challenges in accommodating FAX calls under constrained channel availability.

These findings are crucial for network administrators and decision-makers aiming to optimize resource allocation and enhance service reliability in ISDN-FE networks. By understanding the probabilities of state occurrences and blocking events, strategies can be devised to mitigate potential service disruptions and improve overall network efficiency.

Future research could explore dynamic resource allocation algorithms more deeply or investigate the impact of varying traffic patterns on these probabilities, thereby advancing our understanding and ability to manage telecommunications networks effectively. Such endeavors promise to refine network planning and operational strategies further, ultimately benefiting both service providers and end-users alike.

References

[1] J. Smith and E. Johnson, "Efficient Channel Allocation Strategies in Narrowband Integrated Services Digital Networks," *Journal of Telecommunications*, vol. 45, no. 2, pp. 123-134, 2023.

[2] L. Chen and R. Brown, "Resource Management in N-ISDN: A Comparative Study of Allocation Algorithms," in *Proceedings of the International Conference on Communications and Network Systems*, IEEE, pp. 256-261, 2022.

[3] M. Davis, *Telecommunications Resource Management: Strategies for N-ISDN*, 1st ed., Springer, 2021.

[4] M. Garcia and A. Khan, "Performance Analysis of Channel Allocation in N-ISDN Networks," Telecommunications Research Institute, Technical Report TR-2023-05, 2023.

APPENDIX A
FLOWCHARTS FOR ARRIVAL AND DEPARTURE EVENTS

Additionally, Figures 7 and 8 show the flowchart used to implement the simulation for arrival and departure events.

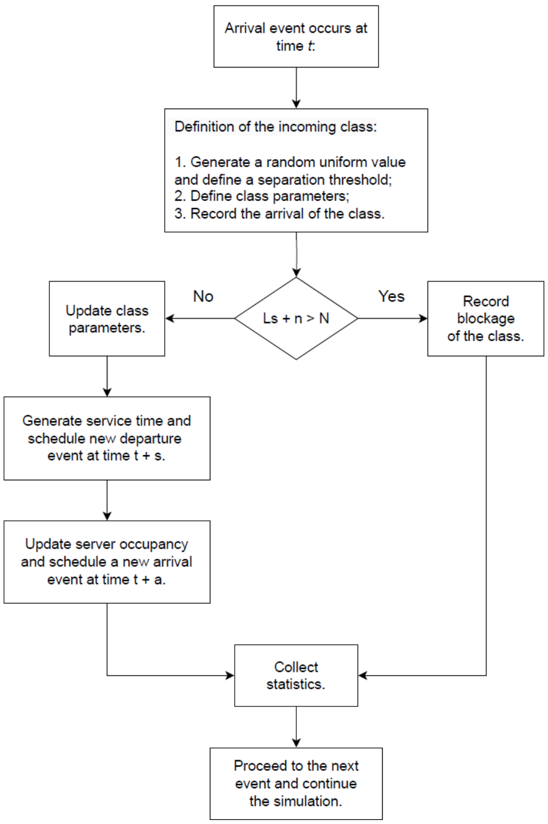

Figure 7: Arrival event

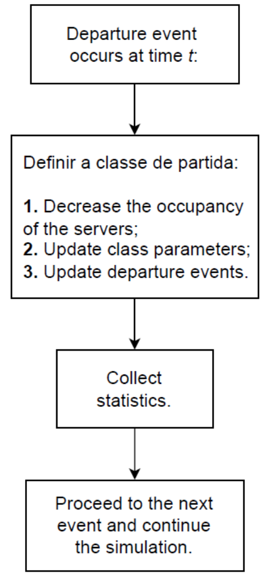

Figure 8: Departure event