PROBABILITY and STATISTICS
BATCH B1 GROUP G2

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Question 1

A certain population consists of 20% children, 30% adolescents, and 50% adults. The probabilities that a member of this population catches the flu are 0.45 for a child, 0.2 for an adolescent, and 0.15 for an adult.
(a) What is the probability that a randomly selected member of this population has the flu?
(b) What is the probability that a randomly selected person with the flu is an adult?

Solution

(a) Given: $P(\text{children}) = 0.2$, $P(\text{adolescents}) = 0.3$, $P(\text{adult}) = 0.5$

Also: $P(\text{flu} \mid \textit{children}) = 0.45$

$P(\text{flu} \mid \textit{adolescents}) = 0.2$

$P(\text{flu} \mid \textit{adult}) = 0.15$

By Total Probability Theorem :

$$P(\text{E}) = \sum_{k=1}^{n} \text{P}(A_k)\text{P}(A \mid A_k)$$

$$= P(\text{children}) \times \text{P}(\textit{flu} \mid \textit{children}) +$$

$$P(\text{adolescents}) \times \text{P}(\textit{flu} \mid \textit{adolescents}) +$$

$$P(\text{adult}) \times \text{P}(\textit{flu} \mid \textit{adult})$$

$$= 0.2 \times 0.45 + 0.3 \times 0.2 + 0.5 \times 0.15$$

$$= 0.09 + 0.06 + 0.075$$

$$= 0.225$$

Solution

(b) So, by using Baye's theorem, we get:

$$P(\text{adult} \mid \textit{flu}) = \text{P}(\textit{flu} \mid \textit{adult}) \times \text{P}(\textit{adult}) / \text{P}(\textit{flu})$$

$$= \frac{0.15 \times 0.5}{0.225} \qquad (\textit{from} : \textit{part} - a)$$

So, $P(\text{adult} \mid \textit{flu}) = \frac{1}{3}$

Question 2

Let X be a discrete random variable with support

$S_X = \{0, 1, 2, 3, 4\}, P(\{X = 0\}) = P(\{X = 1\}) = \frac{1}{10}$

$P(\{X = 2\}) = P(\{X = 3\}) = P(\{X = 4\}) = \frac{4}{15}$.

Find the distribution function of X and sketch its graph.

Solution

Given:- $S_X = \{0, 1, 2, 3, 4\}, P(\{X = 0\}) = P(\{X = 1\}) = \frac{1}{10}$

$P(\{X = 2\}) = P(\{X = 3\}) = P(\{X = 4\}) = \frac{4}{15}$.

Now, distribution function is given by :

$$F_X(x) = \left\{ \begin{array}{ll} 0 & \text{for } x < 0 \\ \frac{1}{10} & \text{for } 0 \leq x < 1 \\ \frac{1}{10} + \frac{1}{10} = \frac{1}{5} & \text{for } 1 \leq x < 2 \\ \frac{1}{5} + \frac{4}{15} = \frac{7}{15} & \text{for } 2 \leq x < 3 \\ \frac{7}{15} + \frac{4}{15} = \frac{11}{15} & \text{for } 3 \leq x < 4 \\ \frac{11}{15} + \frac{4}{15} = 1 & \text{for } x \geq 4 \end{array} \right\}$$

Graph of distribution function is: