Spectral Density Analysis of a Moored Ship Motion in Pohang New Harbor

Prashant Kumar and Kim kwang Ik

Department of Mathematics
Pohang University of Science and Technology

Contents

Introduction

- The Pohang New Harbor (PNH) is taken as model harbor for moored ship motion situated in the northeast part of Pohang city in South Korea.
- Occasionally, incoming waves to PNH have high amplitudes when they are generated by typhoons or by seasonal winds and currents, which are typically observed in Korean peninsula.
- Loading and unloading of the moored ships during the seasonal weather is a difficult task.
- Usually, irregular ocean waves are often characterized by a "wave spectrum", which describes the distribution of wave energy with frequency.

Motivation and Background

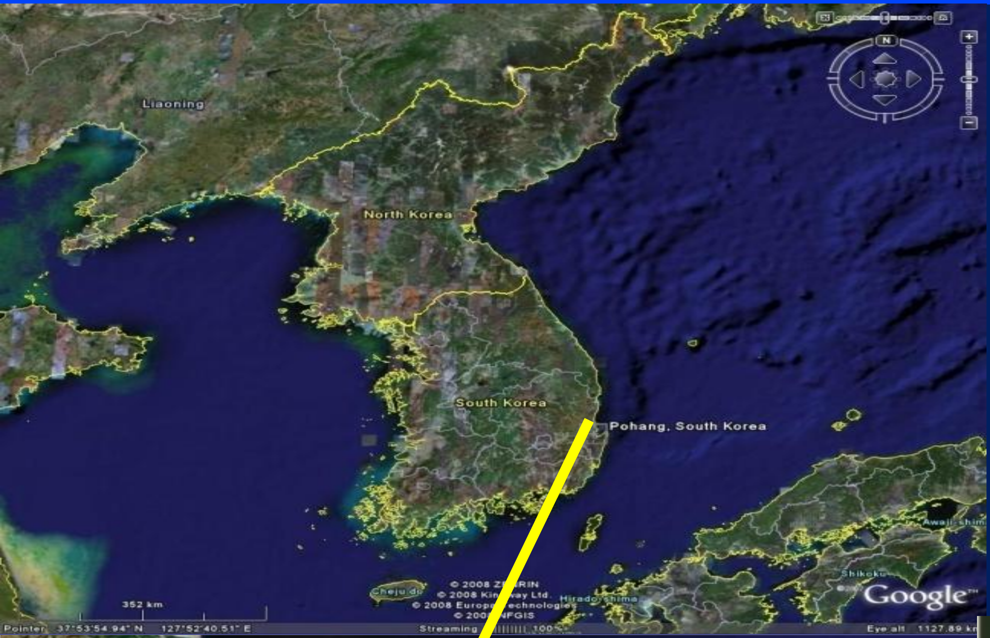
- To analyze the incident waves behavior for moored ship motion in the harbor.

- To protect the ship and offshore structures from high amplitude waves, for that the numerical simulation has been done for a moored ship motion.

- To understand roughly the physical mechanism of the wave spectra (spectral density) for a moored ship motion in the harbor.

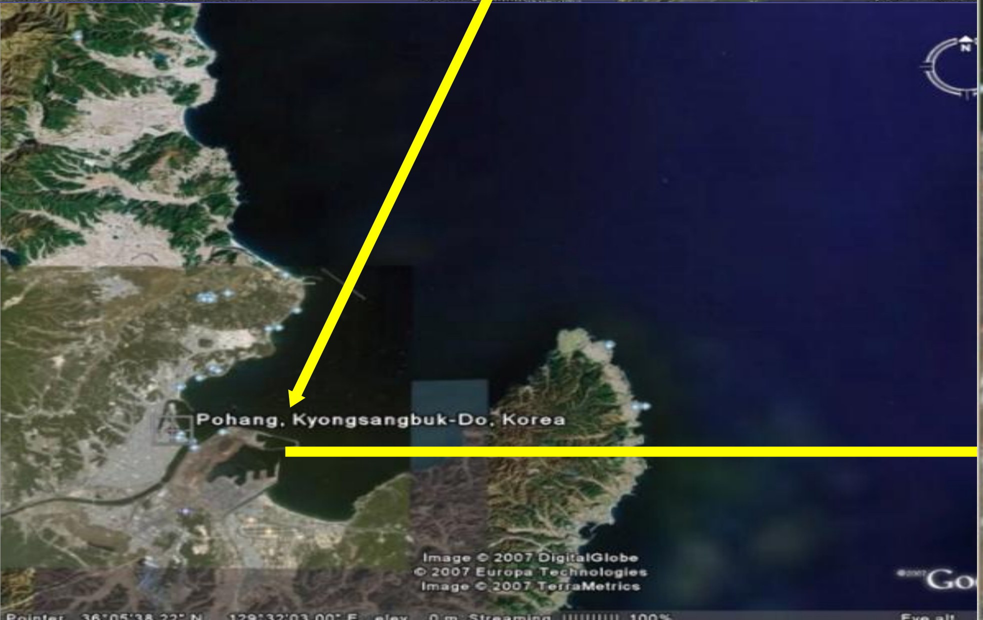
- Model has examined in the actual PNH and artificially modified PNH for moored ship motion.

➢ Motivation and Background



Pohang New Harbor

Berthing capacity: capable of handling 36 ships concurrently including 250,000DWT size ship
Cargo handling capacity : 47 million tons yearly



➢ Incident Waves to PNH

➢ Derivation of Helmholtz Equation

Navier-Stokes equation for mass conservation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0 \text{ ....(1)}$$

Applying constant density assumption,

for an incompressible fluid:

$$\nabla \cdot \vec{u} = 0, \text{ i.e. } \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \text{ .....(2)}$$

By the Definition of the velocity potential,

Laplace's equation is obtained:

$$\nabla . \vec{u} = \nabla^2 \phi = 0 \text{ ....(3)}$$

➢ Derivation of Helmholtz Equation

Velocity potential, ϕ which satisfies Laplace's equation, equation (3) subject to a number of prescribed boundary conditions, on solid boundaries:

$$\frac{\partial \vec{U}}{\partial n} = 0 \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

where $\vec{U}$ (u,v ,w) is velocity vector.

The form of the solution of the velocity potential

$$\phi(x, y, z; t) = \frac{1}{-i\omega}\eta(x, y)Z(z)\exp(-i\omega t) \ldots\ldots\ldots\ldots\ldots (5)$$

where ω is angular frequency,

$\eta(x, y)$ is termed as wave function.

➢ Derivation of Helmholtz Equation

Substituting equation $\left(5\right)$ into $\left(3\right)$ one obtains

$$\frac{1}{\eta}\left(\frac{\partial^2\eta}{\partial x^2}+\frac{\partial^2\eta}{\partial y^2}\right)=-\frac{1}{Z}\frac{\partial^2 Z}{\partial z^2}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

I f $\left(6\right)$ is to be equal to a constant, say $-\mathrm{k}^2$,

then the following set of equations obtained :

$$\frac{d^2 Z}{dz^2}-k^2 Z=0\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

$$\frac{\partial^2\eta}{\partial x^2}+\frac{\partial^2\eta}{\partial y^2}+k^2\eta=0\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

➢ Derivation of Helmholtz Equation

The boundary condition at the bottom and the linearized dynamics free surface condition are:

$$\partial\phi(x, y, z=-h; t)/\partial z = 0 \quad \text{..}(9a)$$

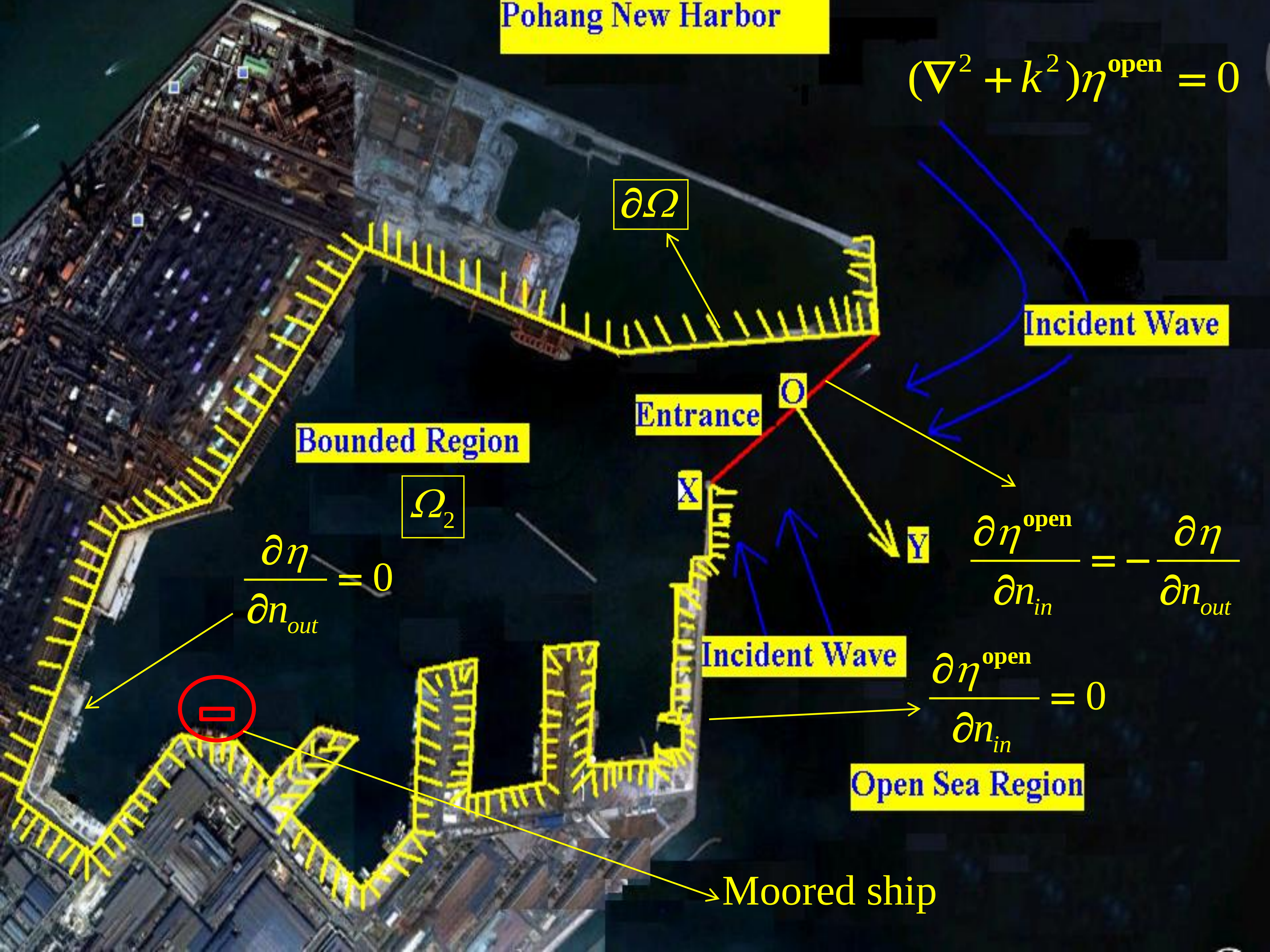
$$\eta(x, y; t) = A_i \eta(x, y) \exp(-i\omega t) = \frac{1}{g}\left(\frac{\partial\phi}{\partial t}\right)_{z=0,} \quad \text{...........}(9b)$$

The function $Z(z)$ which satisfied (7) and (9) can be found as

$$Z(z) = -\frac{A_i g \cosh k(h+z)}{\cosh kh} \quad \text{..}(10)$$

Velocity potential becomes :

$$\phi(x, y, z; t) = \frac{1}{i\omega}\frac{A_i g \cosh k(h+z)}{\cosh kh}\eta(x, y)\exp(-i\omega t) \text{...............}(11)$$



Pohang New Harbor
$(\nabla^2 + k^2)\eta^{\text{open}} = 0$
$\partial\Omega$
Incident Wave
O
Entrance
Bounded Region
X
Ω_2
Y
$\frac{\partial \eta^{\text{open}}}{\partial n_{in}} = -\frac{\partial \eta}{\partial n_{out}}$
$\frac{\partial \eta}{\partial n_{out}} = 0$
Incident Wave
$\frac{\partial \eta^{\text{open}}}{\partial n_{in}} = 0$
Open Sea Region
Moored ship

➢ Governing Equation for Open sea Region

The complete wave field can express in the open sea region as:

$$\eta^{\text{open}} = \eta^{\text{inc}} + \eta^{\text{ref}} + \eta^{\text{R}} \qquad ....(8)$$

where , η^{open} the wave field for open sea region, η^{inc}, incident wave field, η^{ref}, reflective wave field, due to the reflection from the coast of the harbor and η^{R} is the radiated wave field due to the wave scattered toward entrance from the local topography of the harbor.

An incident wave field can be expressed as:

$$\eta^{inc} = A\exp(ik(x\cos\theta_I + y\sin\theta_I)) \quad ....(9a)$$

The reflective wave field expressed as:

$$\eta^{ref} = A\exp(ik(x\cos\theta_I - y\sin\theta_I)) \quad ....(9b)$$

Where θ_I is the angle of incident wave towards entrance.

➢ Governing Equation for Open sea Region

On the coast line boundary

$$\frac{\partial}{\partial y}(\eta^{\mathrm{inc}} + \eta^{\mathrm{ref}}) = 0 \qquad \text{............(10)}$$

The normal flux vanish along the straight coast for radiated wave field:

$$\frac{\partial}{\partial y}(\eta^{\mathrm{R}}) = 0 \qquad \text{............(11)}$$

The ration boundary condition for large distance satisfied the following as:

$$\lim_{kr\to\infty}(kr)^{-1/2}\left(\frac{\partial}{\partial r} - ik\right)\eta^{\mathrm{R}} = 0 \qquad \text{............(12)}$$

The governing equation formed in open sea region in term of the radiated wave function given below as:

$$(\nabla^2 + k^2)\eta^{\mathrm{R}} = 0 \qquad \text{.............(13)}$$

➢ Weber Solution of Helmholtz Equation

Wave function in the harbor is defined as:

$$\eta(x,y)=c(i)\int_{\partial\Omega}\left[\eta(x_0,y_0)\frac{\partial}{\partial\vec{n}_{\text{out}}}\left(H_0^{(1)}(kr)\right)-H_0^{(1)}(kr)\frac{\partial}{\partial\vec{n}_{\text{out}}}\left(\eta(x_0,y_0)\right)\right]d\Omega,$$

where:

$$c(i)=\begin{cases}\text{-}i/4 & \text{if } (x,y)\in\Omega_2\\ \text{-}i/2 & \text{if } (x,y)\in\Omega_2\\ \text{-}i\pi/2\alpha & \text{if } (x,y)\in \text{coner point, } \alpha \text{ is corner angle.}\end{cases}$$

$\eta(x,y)$: **wave function inside the harbor,**

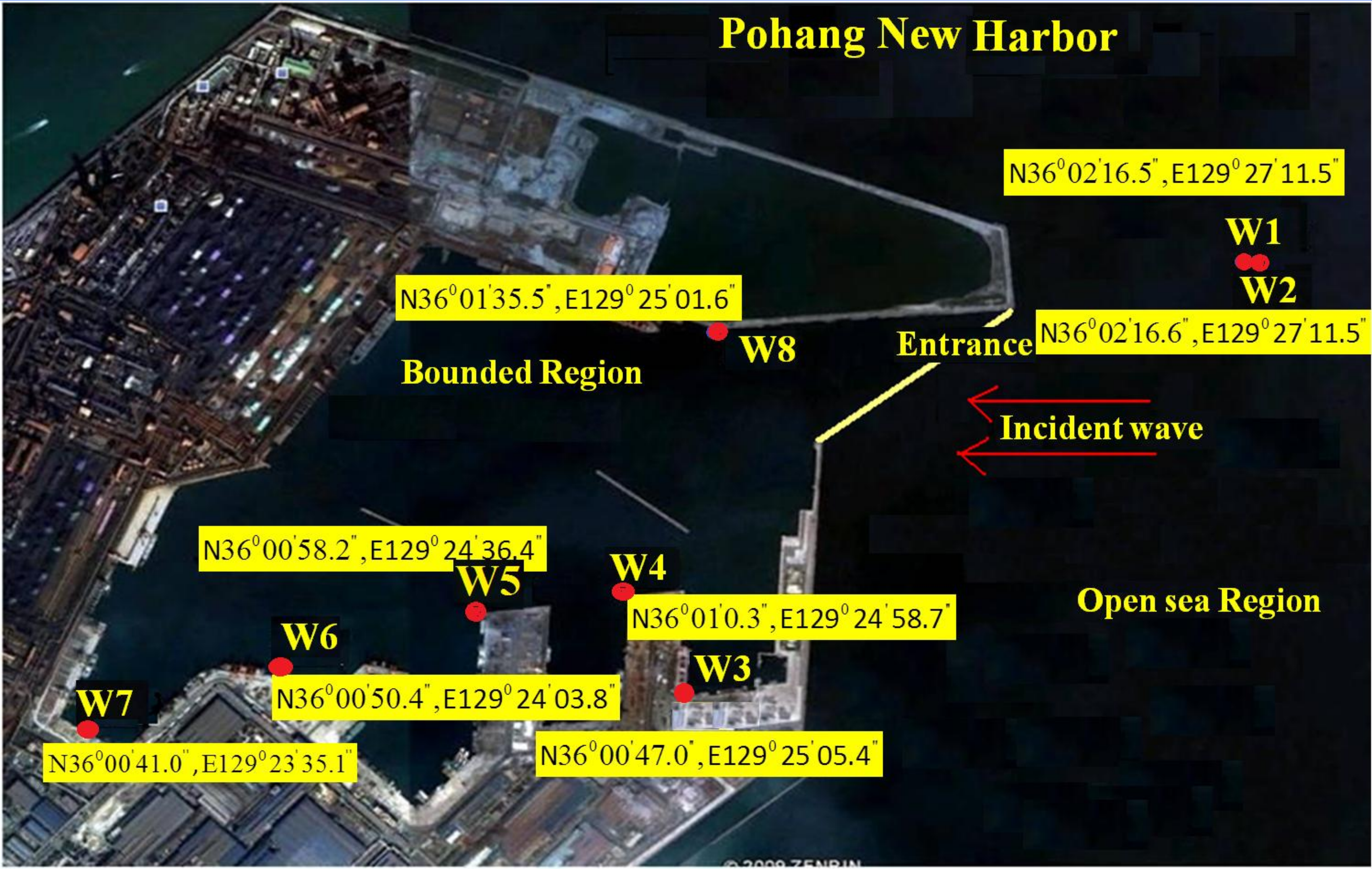
(x_0,y_0): **integration variable on the boundary,**

$r=\sqrt{(x-x_0)^2+(y-y_0)^2},$

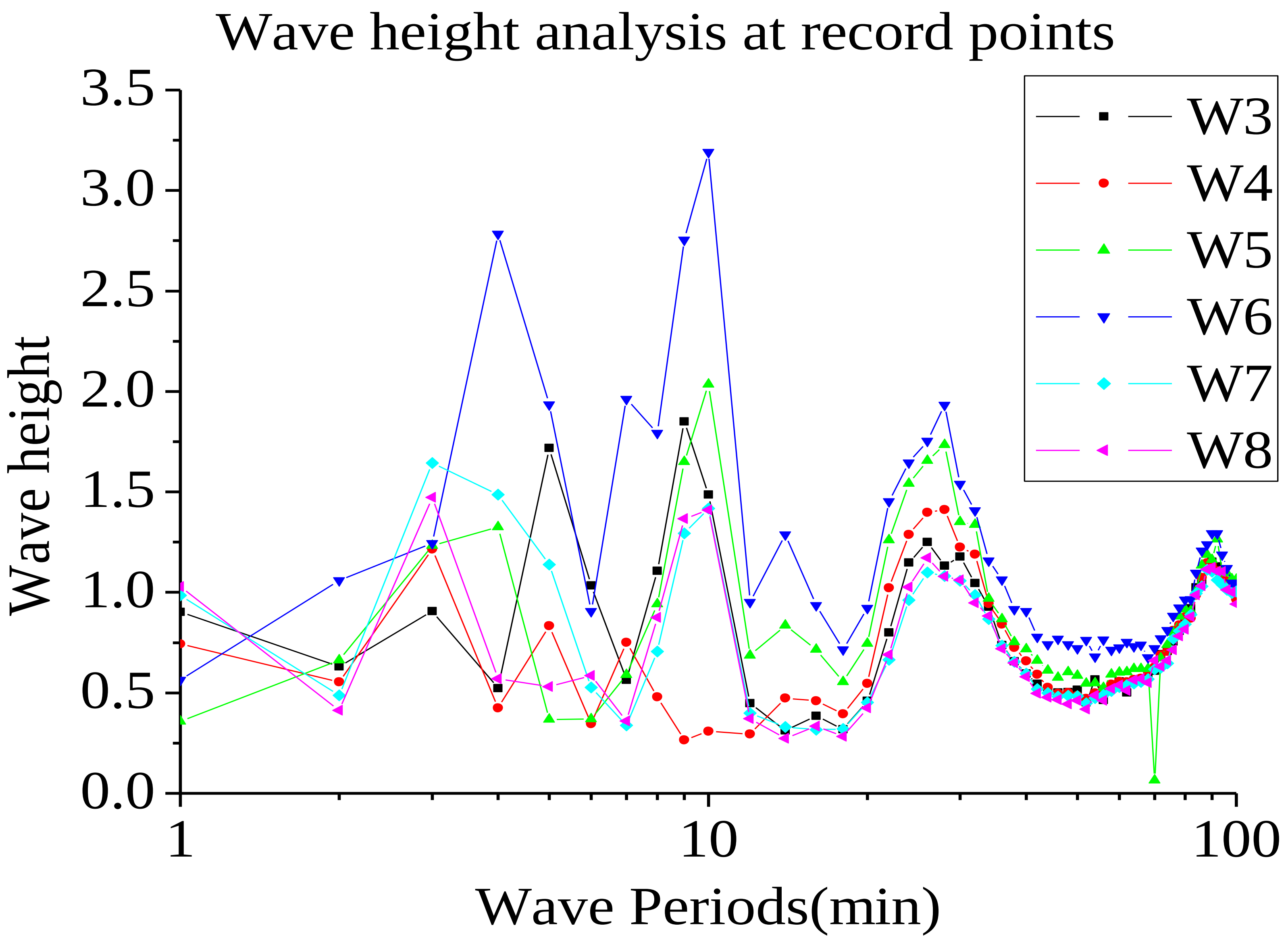
$H_0^{(1)}(kr)$ **is the Hankel function of zeroth order of first kind,**

$\vec{n}_{\text{out}}$: **the outward normal vector on the vertical side wall.**

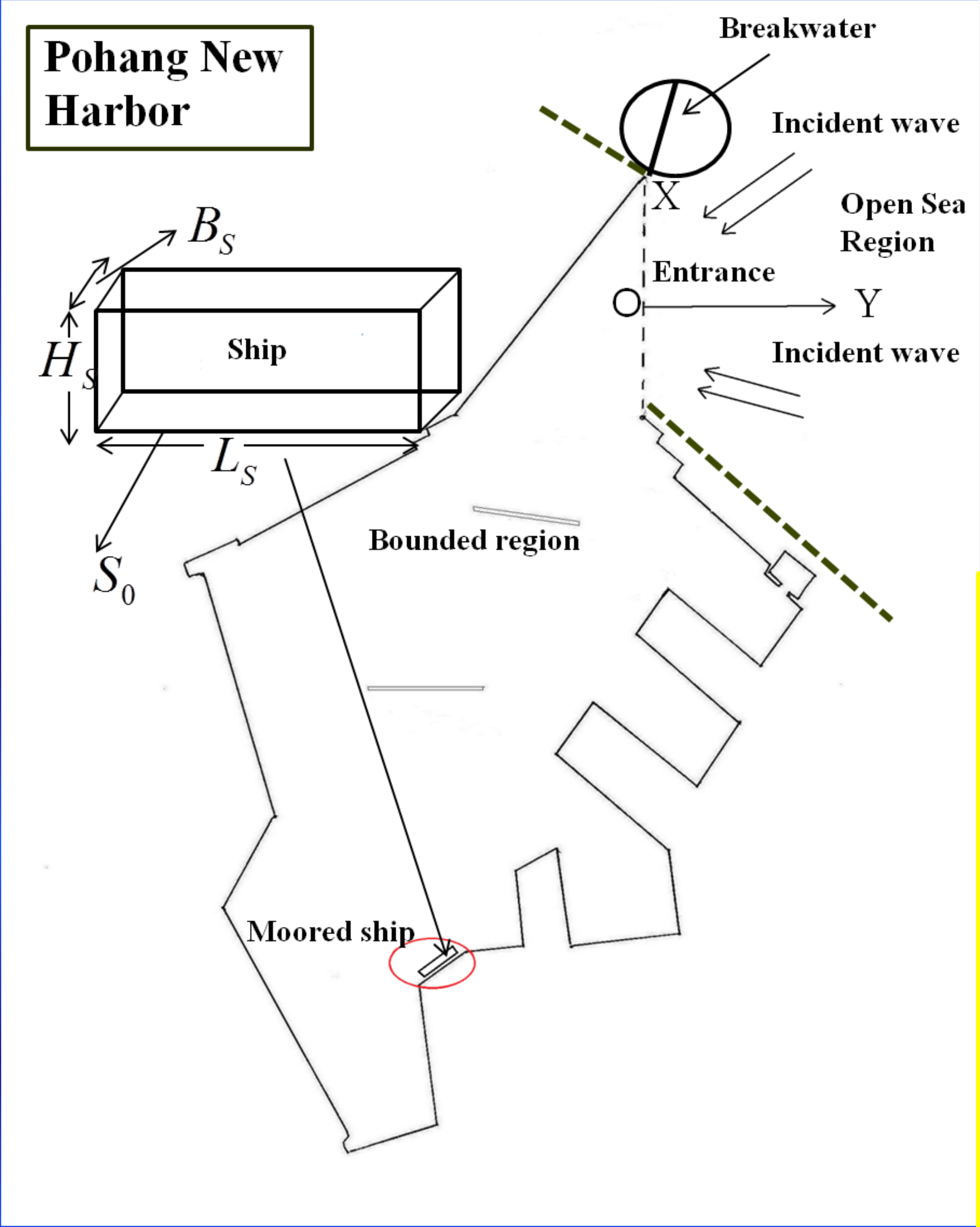
➢ Location of the Moored Ship Motion

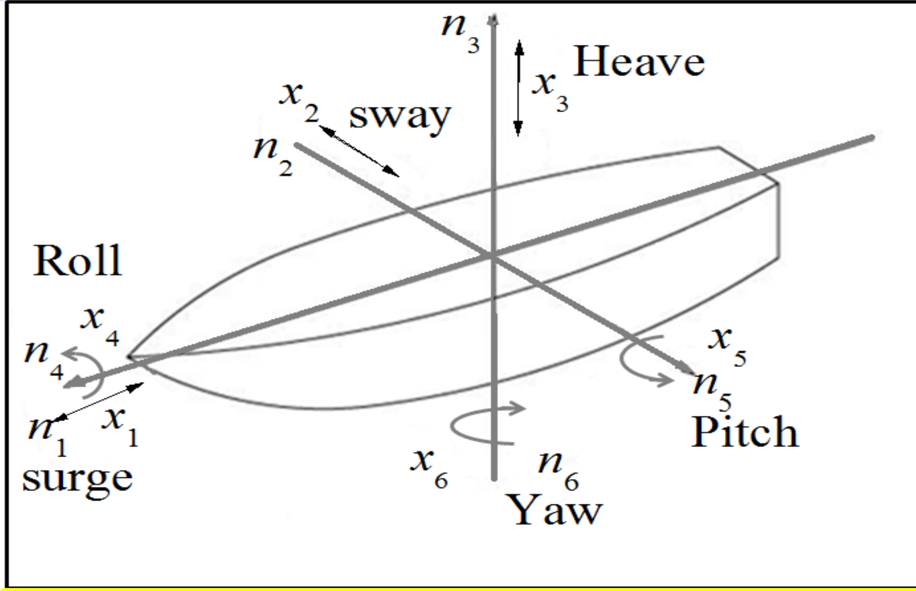


➢ Wave Height Analysis at Record Points inside the PNH



➢Moored Ship Motion





Translations: Surge, Sway and Heave Motion, is the linear vertical (front/back), linear lateral (side-to-side) and the linear longitudinal (up/down) motion respectively

Rotational: Roll, Pitch and Yaw is when the vessel rotates about the longitudinal (front/back), transverse (side to side) and vertical (up and down) axis.

➢ Governing Equation in the Ship Region

The potential can be expressed by the summation of diffraction potential and radiation potential.

$$\phi^1(x,y,z) = \mathrm{Re}[\phi_0(x,y,z) + \phi_i(x,y,z)e^{-i\omega t}] \quad ....(14)$$

The governing equation and the boundary condition for each potential is

$$\nabla^2\phi_i + \frac{\partial^2\phi_i}{\partial^2 z} = 0,\ in\ \ the\ region - I\,(i = 0,1,2,3,4,5,6)$$

$$\frac{\partial^2\phi_i}{\partial^2 z} - \frac{\omega^2}{g}\phi_i = 0\ \ on\ S_F\,(i = 0,1,2,3,4,5,6) \quad ....(15)$$

➢ Governing Equation in the Ship Region

The Boundary condition on the Ship region

$$\frac{\partial \phi_0}{\partial z} = 0 \quad on \quad S_0 \qquad \text{............}(16)$$

$$\frac{\partial \phi_i}{\partial n} = -\omega n_i\,, \; on \;\; the \; region \;\; S_0 (i = 1, 2, 3) \qquad \text{.........}(17)$$

$$\frac{\partial \phi_i}{\partial n} = -i\omega(\vec{r} \times \vec{n})_{i-3}\,, \quad on \quad S_0 (i = 4, 5, 6) \qquad \text{...........}(18)$$

$$\frac{\partial \phi_i}{\partial n} = 0, \quad on \;\; \partial\Omega \cup S_0 \; (i = 0, 1, 2, 3, 4, 5, 6) \qquad \text{......}(19)$$

➢ Hydrodynamic Forces on Floating the Ship

The hydrodynamic forces and moments acting on the floating ship surface is calculated by integration on the surface.

$$f_{ij} = -\rho \iint_{S_0} \frac{\partial \phi_i}{\partial n} \phi_i dS = \omega^2 a_{ij} - i\omega b_{ij} \quad \text{..........(20)}$$

Where a_{ij} are added mass coefficients and b_{ij} are damping coefficients .

Wave exciting force is calculated by integration on the surface

$$F_{ex,\,i} = \mathrm{Re}\{A e^{i\omega t} X_i\} \quad \text{......................(21)}$$

Where, A is amplitude of the incident wave. X_i is defined as

$$X_i = -\rho \iint_{S_0} \phi_0 \frac{\partial \phi_i}{\partial n} dS \quad \text{........................(22)}$$

➢ Hydrodynamic Forces on Floating the Ship

The equation of motion with six degree of freedom is

$$\sum_{j=1}^{6}[(M_{ij}+a_{ij})\xi_j+b_{ij}\xi_j+C_{ij}\xi_j]=F_{ex,i},(i=1,2,3...6) \quad ....(23)$$

$$\sum_{j=1}^{6}[-\omega^2(M_{ij}+a_{ij})+i\omega b_{ij}+C_{ij}]\xi_j=AX_i,(i=1,2,3...6) \quad ....(24)$$

T_{ij} is defined as follows

$$T_{ij}=[-\omega^2(M_{ij}+a_{ij})+i\omega b_{ij}+C_{ij}] \quad ....(25)$$

the transfer function or responce amplitude operator

$$Z_j(\omega,\theta)=\frac{\xi_j}{A}=\sum_{i=1}^{6}[T_{ij}]^{-1}X_i, \quad ....(26)$$

where ω is the frequency of incident wave and θ is the incident angle.

Spectral Density

- **The spectral density can be defined as an image that identifies the relative wave energy present at all frequency or periods at a fixed location for a predefined time period, regardless of the energy's directional heading.**
- **The power spectral density function (PSD) shows the strength of the variations (energy) as a function of frequency. In other words, it shows at which frequencies variations are strong and at which frequencies variations are weak.**
- **The discretized wave height at specific record points inside the harbor with respect to time series defined as:**

$$\zeta_{\mathrm{j}} = \eta(x, y, t_j), \textbf{ where } t_{\mathrm{j}} = j\Delta t,\ j = 1, 2, 3 \ldots N - 1$$

➢ Spectral Density

The digital Fourier transform Z_n of a wave record ζ_j equivalent as:

$$Z_{\mathrm{n}} = \frac{1}{N}\sum_{j=0}^{N-1}\zeta_{\mathrm{j}}\exp(-i2\pi\, jn/N) \qquad \text{............(27)}$$

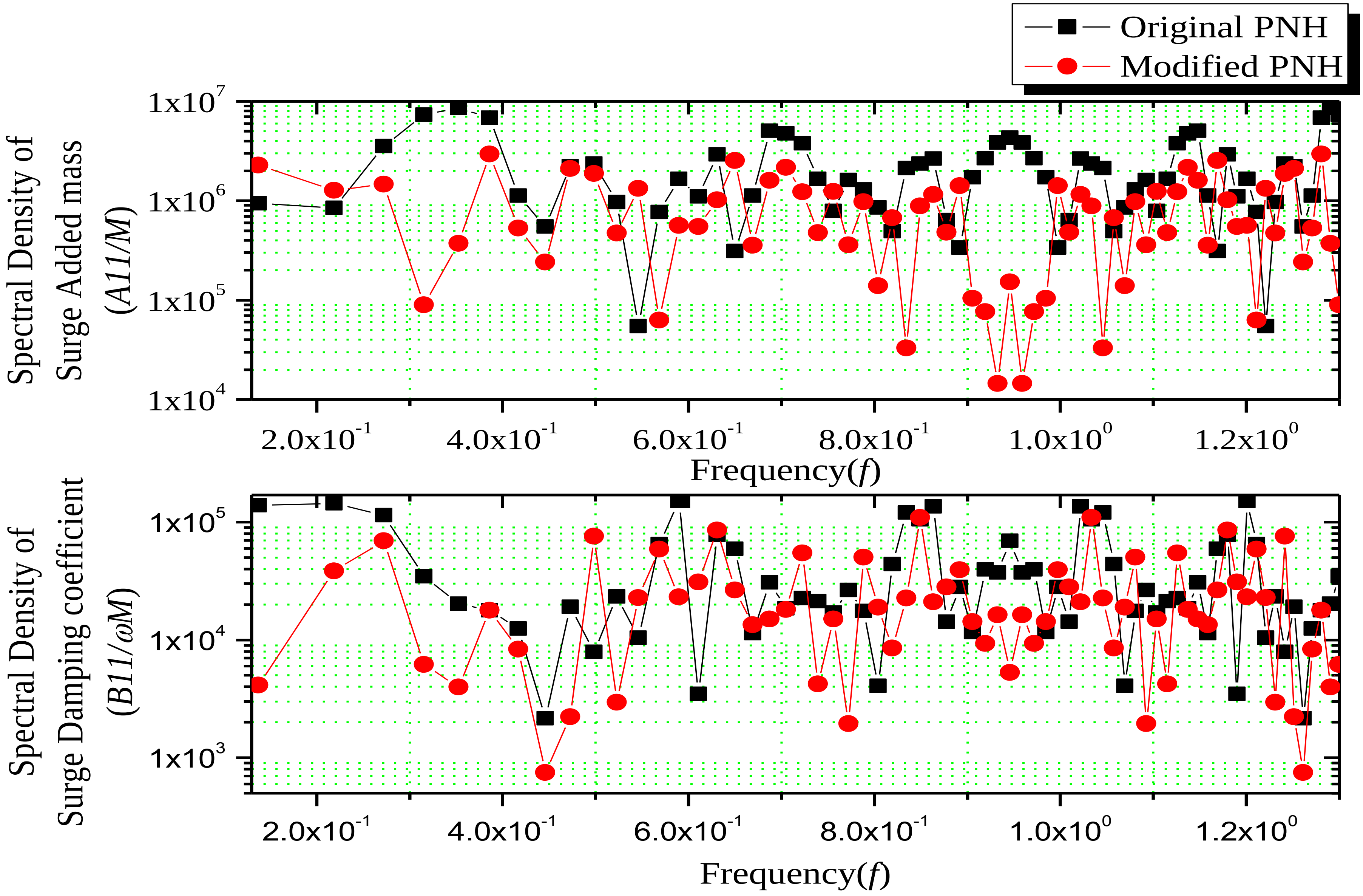
For *j=0,1,2…N-1 and n=0,1,2,3….N-1* is called the Fourier transform of $\zeta_j(t)$. The real amplitude can be calculated as

$$A_n = \sqrt{RE(Z_n)^2 + \mathrm{Im}(Z_n)^2} \qquad \text{...............(28)}$$

In the numerical calculation, the FFT algorithm of MATLAB has been used, which implements the Fourier transform of Eq. (27). In addition, amplitude only associated with the frequency interval $(1/T)$ is estimated. The wave spectrum (spectral density is measured as

$$S(f_i) = \frac{1}{\Delta f}\frac{A_i^2}{2} \qquad \text{.....................(29)}$$

➢ Spectral density analysis for moored ship motion for original and modified PNH



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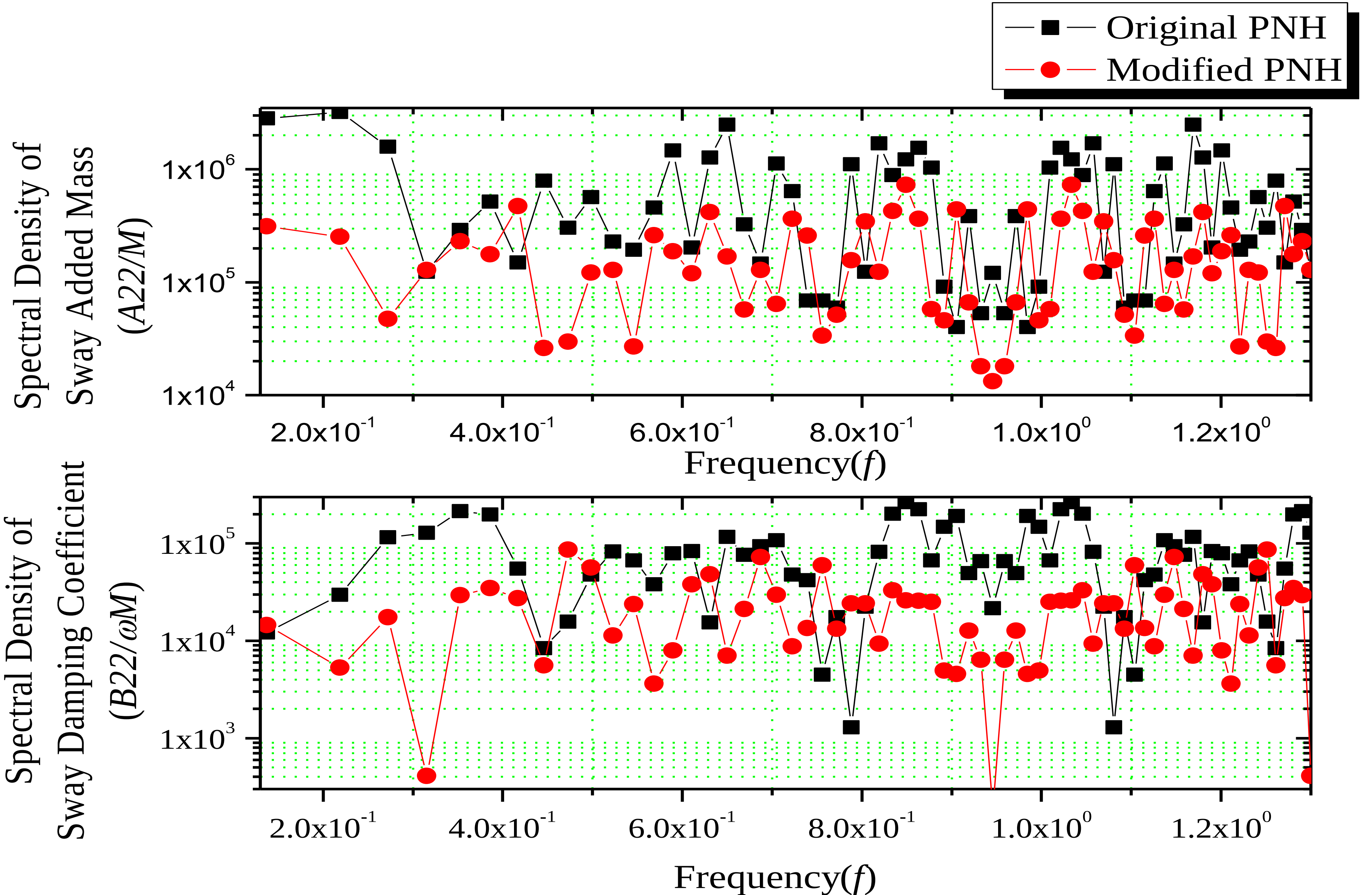

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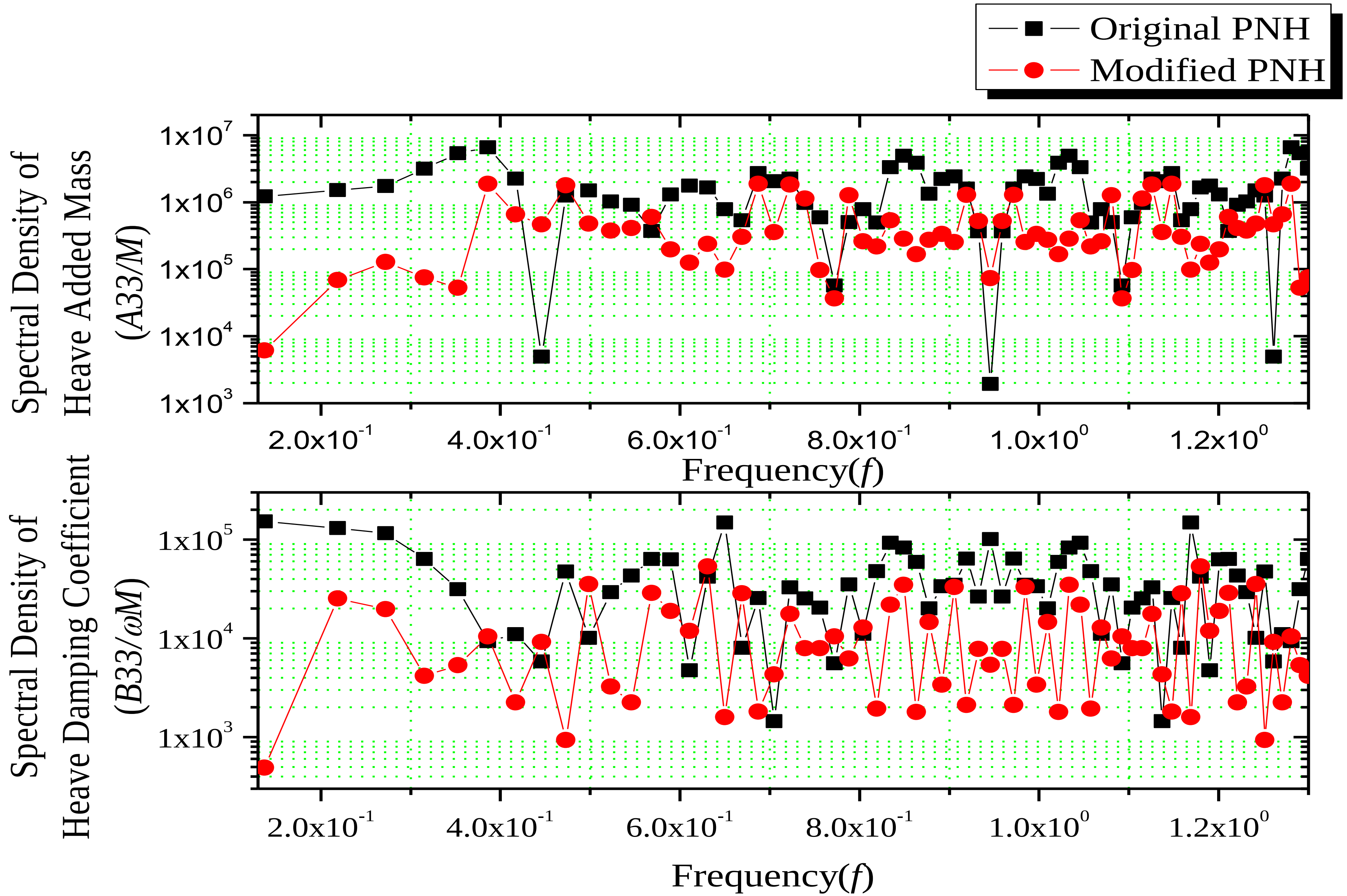

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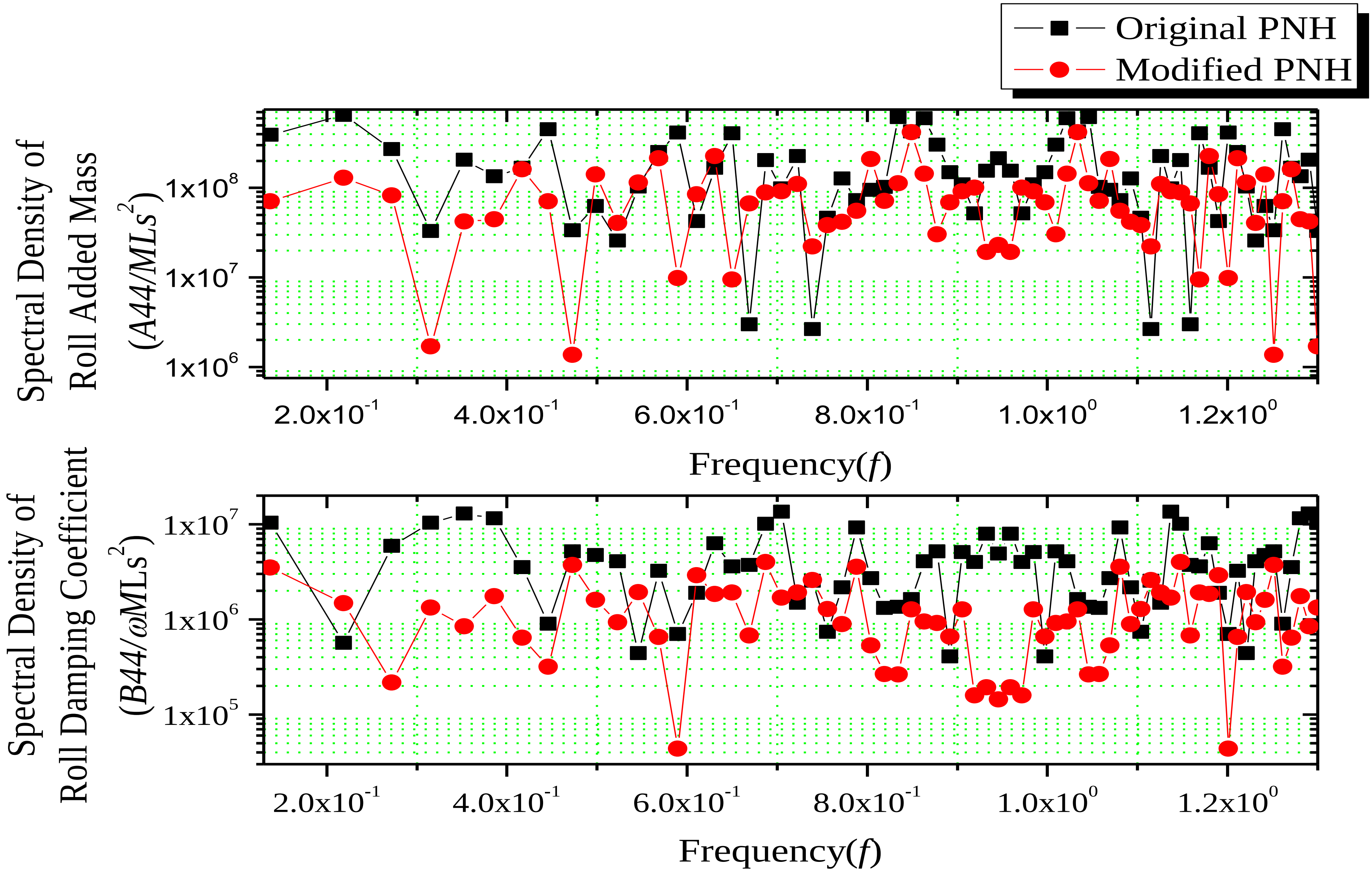

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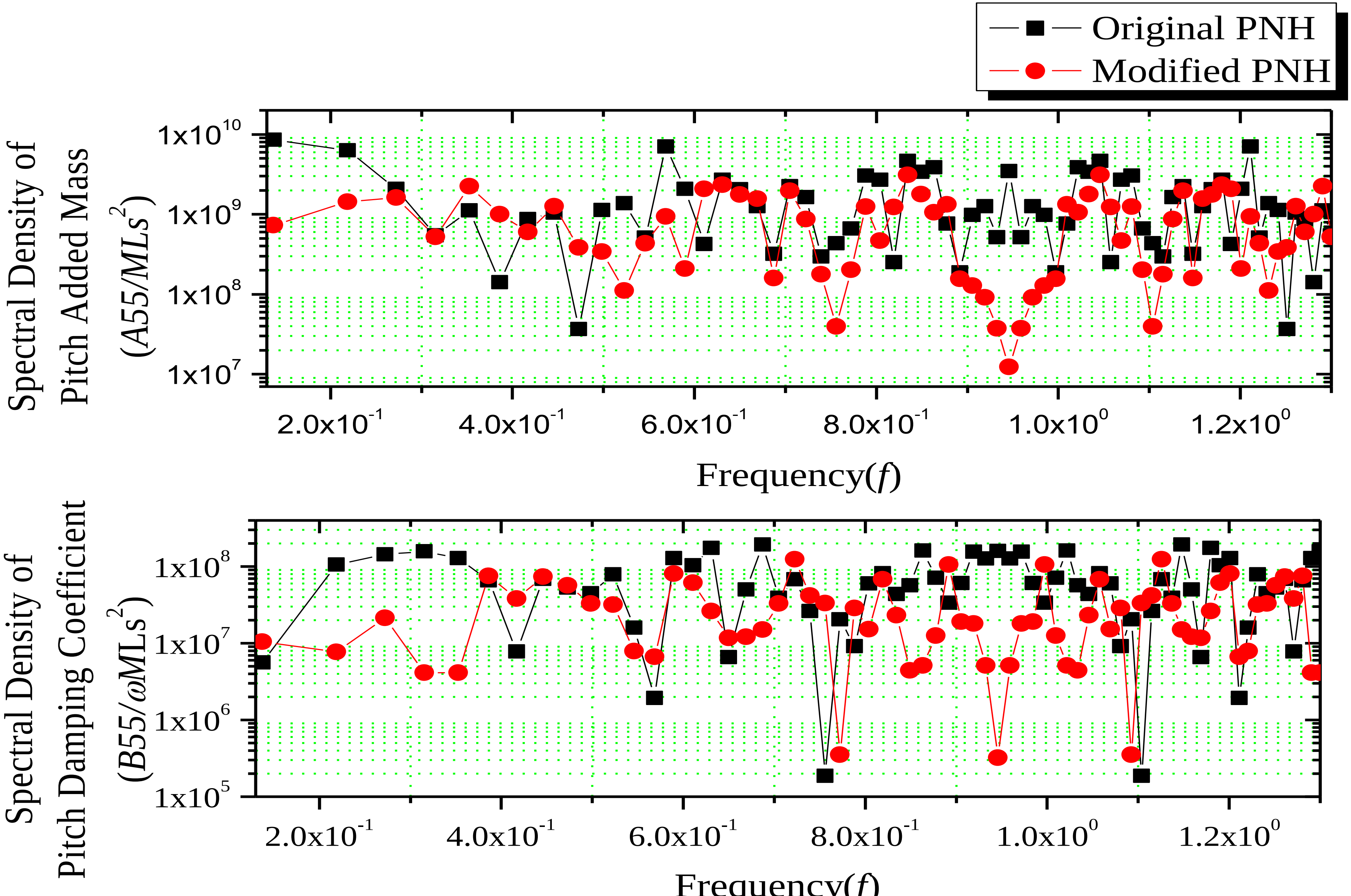

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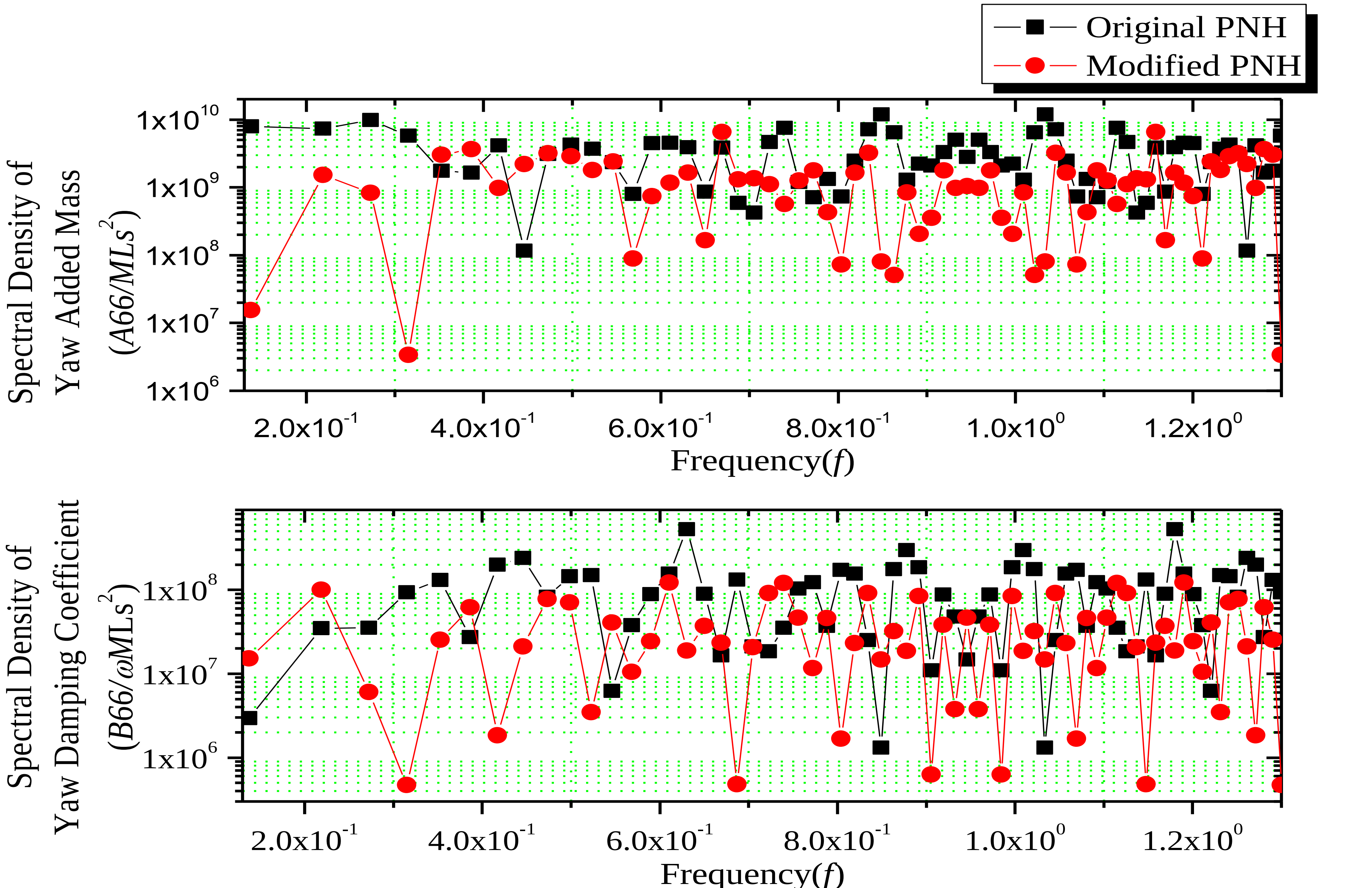

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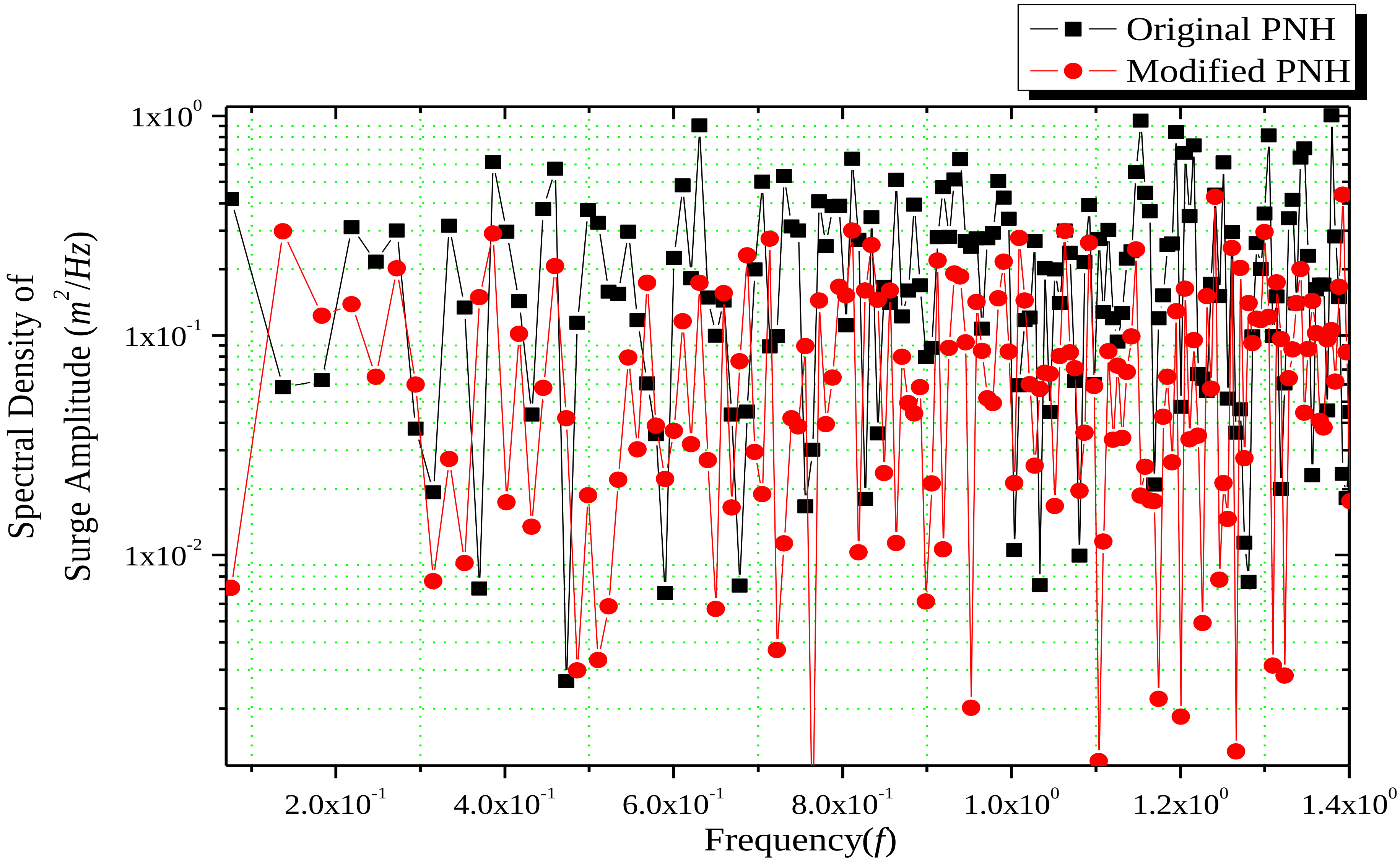

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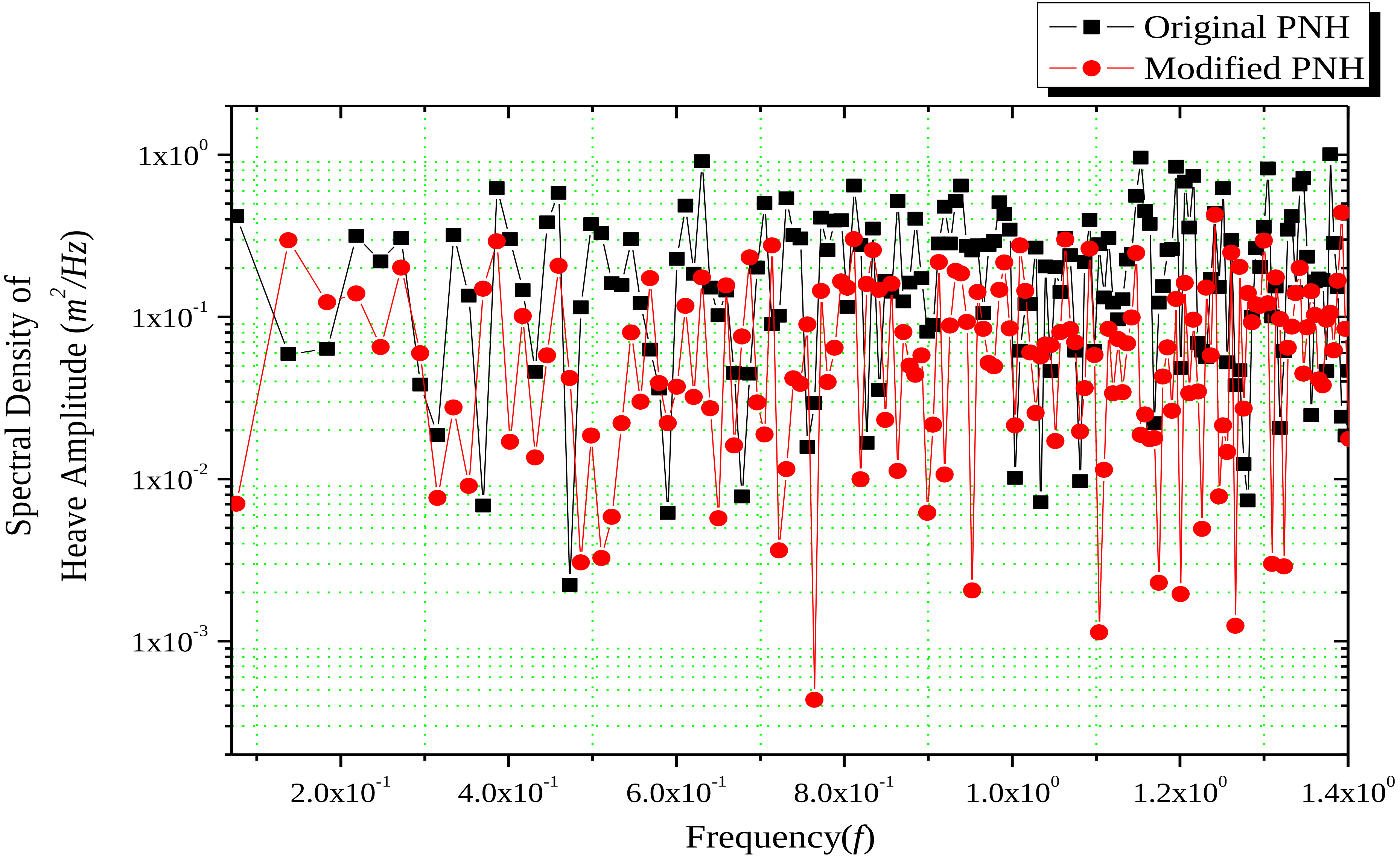

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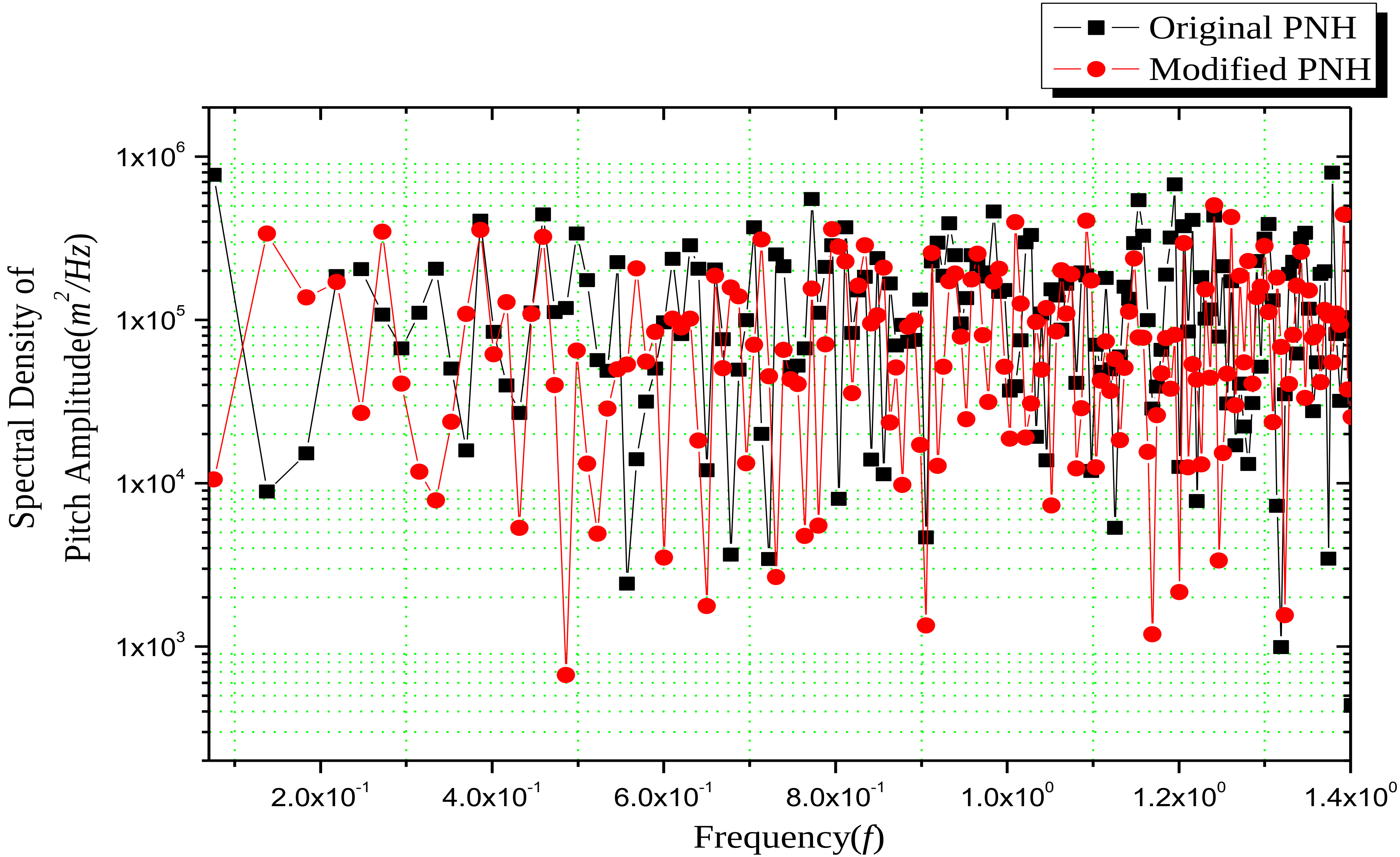

➢ Discussion & Conclusion

- For many today's practical problems, scope of aforethought discourse is sufficient and far more accurate the methods required like the consideration of the nonlinearity to analyze the moored ship motion.

- Consideration of linear incident waves of general wave spectrum or waves with multidirectional dimension requires for better models.

- We need to go towards the better precision and accountability of additional parameters to continue the greater accuracy and more genuine representation of dominant conditions.

- The geometry of the harbor and amplitude of incident wave can be taken more precisely for better approximation.

- Location of the breakwater also key factor to reduce oscillation inside harbor.

➢ Discussion & Conclusion

- The Spectral density analysis provided the crucial information about the energy distribution (wave height) with respect to the frequency variation at key location in and outside the harbor.

- We have got the improved moored ship motion by adding breakwater at the entrance of PNH despite the dissimilarity of harbor geometry.

- In the modified PNH, we have obtain spectral density has been reduced for various mode of ship motion (surge, sway, heave, roll, pitch and yaw) compared to original PNH.

- Adding break water can significantly affect the resonance inside the harbor, it will the direction of incident wave play significant role for resonance inside the PNH.

- This spectral analysis model for moored ship motion can be implement on any arbitrary shape harbor.

➢ Future Work

- To Analyze the moored ship motion inside the Pohang New Harbor by considering the non linear waves.
- Some more tactics like adding breakwaters with appropriated location have been introduced more precisely for the improvement for safe moored ship motion.
- To check six different mode of ship motion surge, sway, heave, roll pitch and yaw motion at various key locations inside the PNH to find the most suitable location for safe mooring during the seasonal weather.

Thank you

for

your attention