ECN-311: Principles of Digital Communication

Department of Electronics and Communication Engineering
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Tutorial 1

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Instructions for students:

- This tutorial consists of 12 questions.
- Please submit it for CWS assessment on or before the due date.

Problem 1. Consider a noisy telephone line transmitting a signal with a signal-to-noise ratio (SNR) of 30 dB. Find the channel capacity.

Problem 2. A communication channel with bandwidth B and certain signal-to-noise ratio, $SNR >> 1$ has capacity C. How does SNR change if the channel capacity is increased by twice the bandwidth?

Problem 3. For a given signal-to-noise ratio, $SNR = \lambda$, the capacity of a noisy communication channel is given as

$$C = Blog_2(1 + \lambda B^{-1})$$

where B denotes the bandwidth of the channel. If the bandwidth of the channel becomes infinite, Can you comment on the capacity of the channel?

Problem 4. A communication system always encounters one of three possible interference waveforms: F_1, F_2, or F_3. The probability of each interference is 0.8, 0.16, and 0.04, respectively. The communication system fails with probabilities 0.01, 0.1, and 0.4 when it encounters F_1, F_2, and F_3, respectively. Given that the system has failed, find the probability that the failure is a result of F_1, F_2, or F_3, respectively.

Problem 5.
a) Suppose a box of diodes consists of 12 good diodes and 4 faulty diodes. If 5 diodes are randomly selected, one at a time, without replacement, determine the probability of obtaining the sequence of diodes in the order of good, faulty, good, good, faulty.

b) From a lot of 20 integrated circuits (ICs), 5 ICs are defective. A sample of 4 ICs is drawn. Let X be a number of defective ICs drawn. Obtain probability mass function if:
i) ICs are drawn with replacement.
ii) ICs are drawn without replacement.
Can you calculate P$(0 < X < 4)$ for both cases?

Problem 6. A non-symmetric binary communications channel is shown in the figure below: Assume the input is "0" with probability p and "1" with probability (1-p).

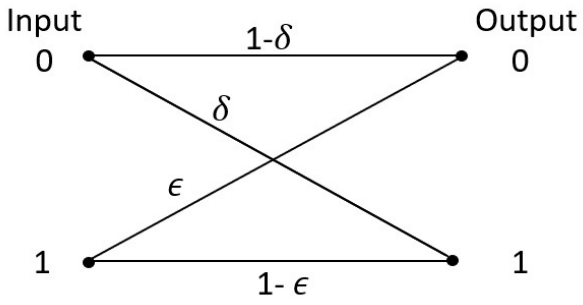

Figure 1: A binary non-symmetric channel

i) Can you guess the condition for which the given channel becomes symmetric?
ii) For $p = 0.8$, $\epsilon = 0.05$ and $\delta = 0.01$, find the probability that the output is 0.
iii) Given that the output is 1, which input is more probable for the same p, ϵ, and δ as in part ii)?

Problem 7. The transmission time, X of the messages in a communication system has an exponential distribution such that

$$P[X > x] = e^{-\lambda x}, x > 0.$$

i) Find the cdf and the pdf of X.
ii) Use Markov's inequality to bound P$(X \geq 3)$ if $\lambda = 1$.
iii) Use Chebyshev's inequality to bound P$(X \geq 3)$ for same $\lambda = 1$ and compare it with actual value.
iv) Prove the memoryless property of the exponential distribution.

Problem 8. The noise voltage in an electric circuit can be modeled as a Gaussian random variable with mean equal to zero and variance equal to 10^{-8}.

i) What is the probability that the value of the noise exceeds 10^{-4}?
ii) What is the probability that the noise value is between -2x10^{-4} and 10^{-4}?

Problem 9.
a) Find the mean and variance of the uniform discrete random variable that takes on values in the set {1, 2,..., L} with equal probability.

b) An optical communication system employs a photodetector whose output is modeled as a Poisson(λ) random variable X. Find the mean and the variance of X.

Problem 10. Suppose that a smartphone attempts to transmit a packet to a base station using wireless communications. Due to the unreliability of the wireless channel, the base station successfully receives the packet with a probability $\frac{2}{3}$, independent of outcomes of prior transmissions. The smartphone knows whether or not the packet is successfully received by the base station right after the transmission. If the packet is not received, then the smartphone immediately retransmits.
i) Find the probability that a packet is successfully delivered to the base station in exactly 10 attempts.
ii) Suppose the smartphone has 3 packets to send, one at a time. Find the probability that there are all 3 packets are successfully delivered to the base station in exactly 10 attempts.

Problem 11. Consider a communication system that transmits a data packet of 1024 bits. Each bit can be in error with the probability of 10^{-2}. Find the (approximate) probability that more than 30 of the 1024 bits are in error.

Problem 12 A binary transmission system sends a "0" bit by transmitting a -v voltage signal and a "1" bit by transmitting a +v voltage. The received signal is corrupted by Gaussian Noise and given by:

$$Y = X + N,$$

where X is a transmitted signal, and N is a noise voltage with pdf $f_N(x)$. If P["1"]=p=1-P["0"]. Find the pdf of Y.