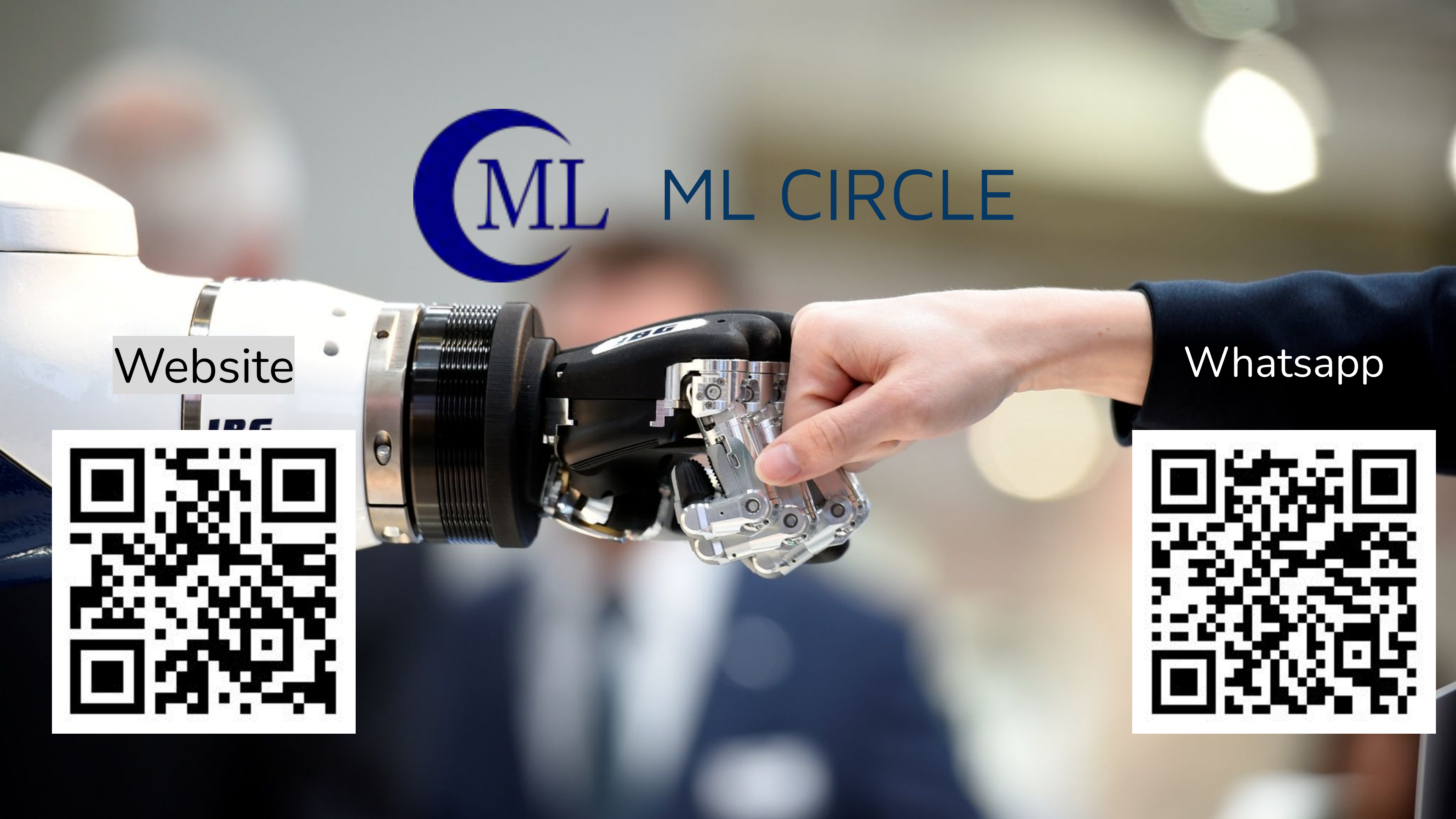

ML CIRCLE
Website
Whatsapp

Probability Theory

- ML circle Discussion Session (by Abhishek Varghese)

Takeaways

- Understand probability in depth and intuitively
- Understand the concept of Independence and its significance
- Understand what random variables really are and why do we need it
- Understand deeply all the terms/ keywords you may or maynot have known
    - Expectation
    - Correlation
    - Variance, etc

Soft takeaways :

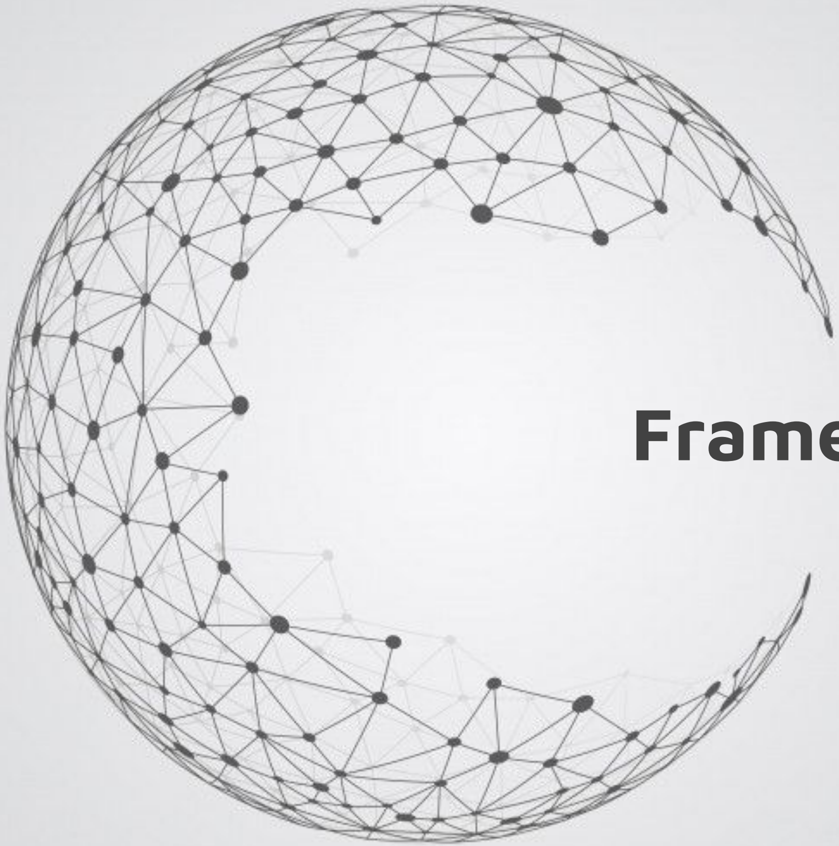
- A push to think on a concept until you deeply understand it

Probability Theory

Session 1

Philosophical Thinking

Models



Frameworks to build a model

°C

$$F = (9/5 * C) + 32$$

°F

Foundations of

Probability Theory

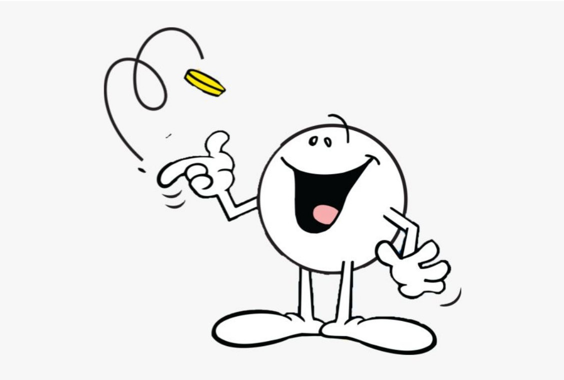
The classical definition (and its limitations)

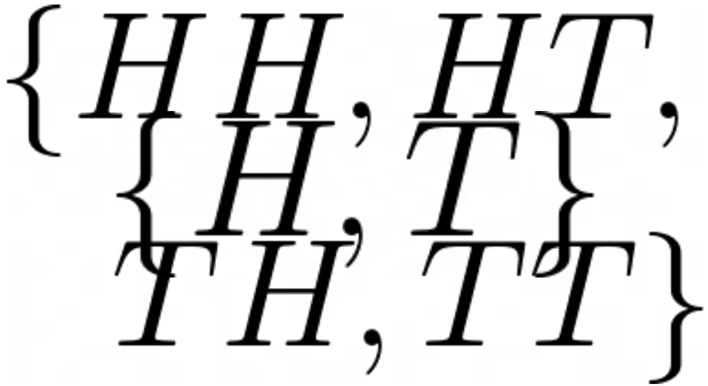
$$P(A) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

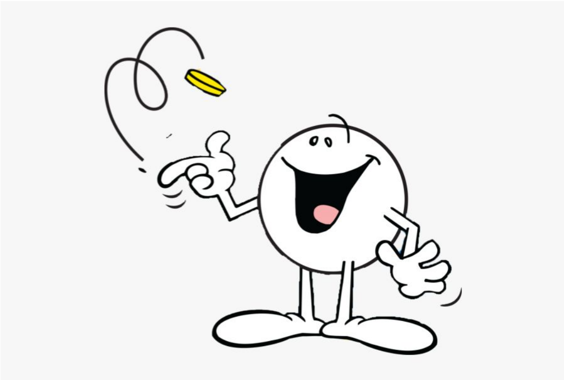
- where, all outcomes are equally likely

Ingredients for constructing mathematical theory of Probability

1) Sample Space (Ω): collection of possible outcomes of a random experiment

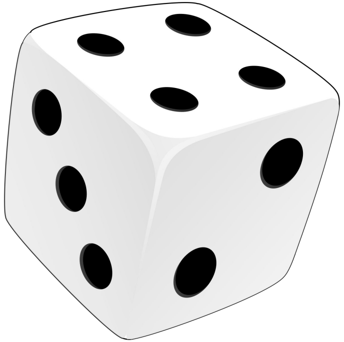






Ingredients for constructing mathematical theory of Probability

1) Sample Space (Ω): collection of possible outcomes of a random experiment



$$\{N \text{ is Even}, \; N \text{ is Odd}\}$$

Ingredients for constructing mathematical theory of Probability

2) Event : associated with an experiment corresponds to a question about the experiment, that has a yes or no answer.

Ingredients for constructing mathematical theory of Probability

$$\boxed{\{HH, HT, TH, TT\}}$$

$$\#heads \leq 1$$

$$A = \{HT, TH, TT\}$$

The event A will occur in a given performance of the experiment if and only if the outcome of the experiment corresponds to one of the points of A.

Ingredients for constructing mathematical theory of Probability

$$A = \{HT, TH, TT\}$$

$$B = \{HH, TH, HT\}$$

$$A \cap B = \{TH, HT\}$$

$$A \cup B = \{HH, TT, TH, HT\}$$

Ingredients for constructing mathematical theory of Probability

3. $\mathcal{F}$ - Sigma Field of Ω : Collection of subsets of Ω that follow the following rules -

$$\Omega \in \mathcal{F}$$

$$A \in \mathcal{F} \Rightarrow A^c \in \mathcal{F}$$

$$A_1, A_2, A_3, \ldots \in \mathcal{F} \Rightarrow \bigcup_{i=1}^{\infty} A_i \in \mathcal{F}$$

Ingredients for constructing mathematical theory of Probability

3. $\mathscr{F}$ - Sigma Field of Ω :

Event Space - Set of all possible events generated from a sample space.

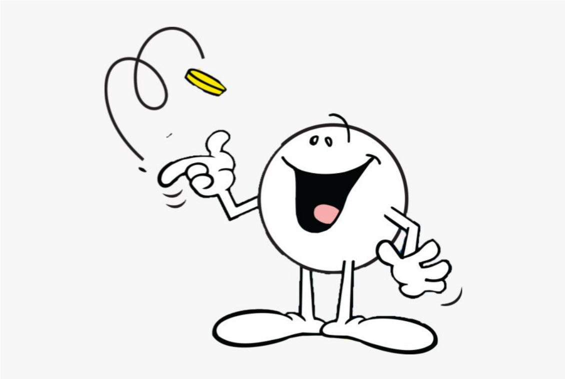
Event Space is a sigma field.

Ingredients for constructing mathematical theory of Probability

4. P - Probability Measure on $\mathcal{F}$: Mapping of elements (say A) from $\mathcal{F}$ to the real space, which we denote by P(A). This mapping should follow the following :
   - $P(\Omega) = 1$
   - $P(A) \geq 0 \qquad \text{for every } A \in \mathcal{F}$

- $P(A_1 \cup A_2 \cup A_3 \cup \cdots) = P(A_1) + P(A_2) + P(A_3) + \cdots$
  $\text{for all disjoint } A_1, A_2, A_3, \cdots \in \mathcal{F}$

Definition



A probability Space is a triple (Ω,$\mathscr{F}$,P),

where

- Ω is a set,
- $\mathscr{F}$ is a sigma field of subsets of Ω, and
- P is probability measure of $\mathscr{F}$ i.e. A function from set of events to the real space [0,1] following the constraints given in previous slide.

Randomness is not a Fundamental Force of Nature.

- Upto debate

Exercise

A probability Space is a triple (Ω,$\mathscr{F}$,P),

where

- Ω is a set (Sample Space or all possible observations),
- $\mathscr{F}$ is a sigma field of subsets of Ω, and
- P is probability measure of $\mathscr{F}$ i.e. A function from set of events to the real space [0,1] following the constraints given in previous slide.

Probability Space for the experiment where a dice is rolled and coin is tossed?

Exercise

A probability Space is a triple (Ω,$\mathscr{F}$,P),

where

- Ω is a set (Sample Space or all possible observations),
- $\mathscr{F}$ is a sigma field of subsets of Ω, and
- P is probability measure of $\mathscr{F}$ i.e. A function from set of events to the real space [0,1] following the constraints given in previous slide.

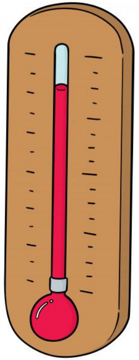
Probability Space for the experiment where a coin is tossed on pluto? (Roll the dice too because why not)

Reflection

Independence

Total number of teeth in a room = 64

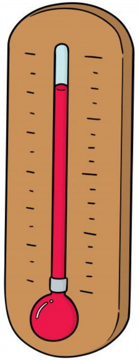
A



Temperature outside > 20

B

Independence



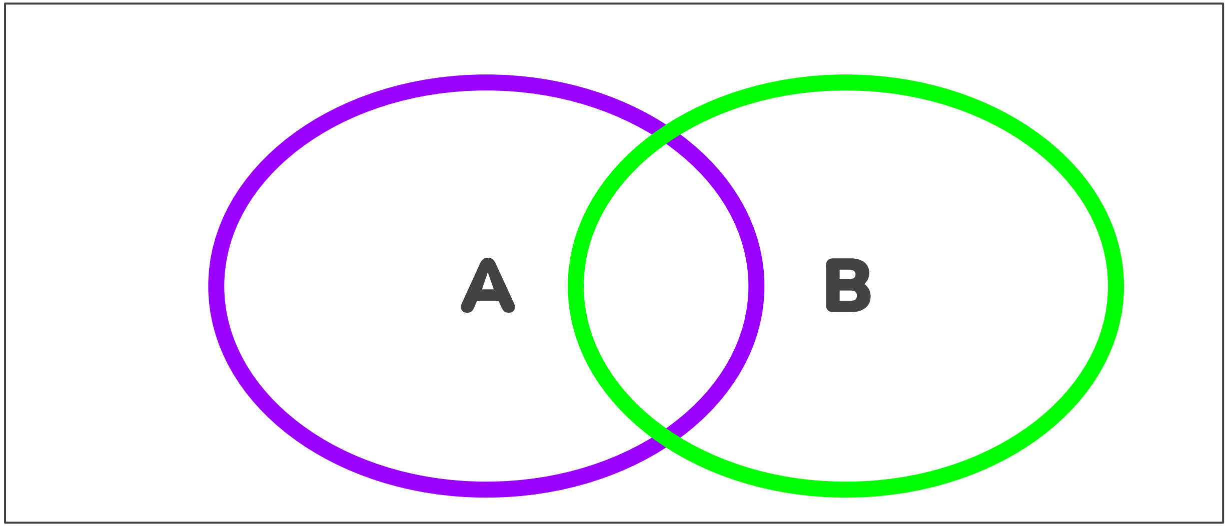
Total number of teeth in a room = 64

P(A) = 0.3

Temperature outside > 20

P(B) = 0.4

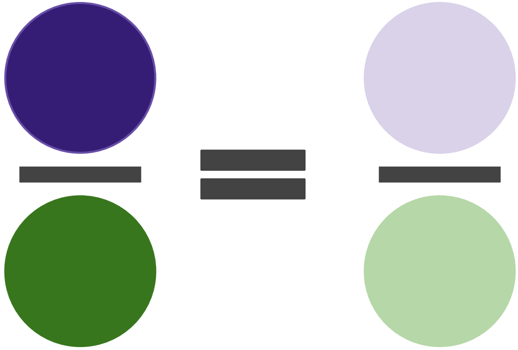
Independence

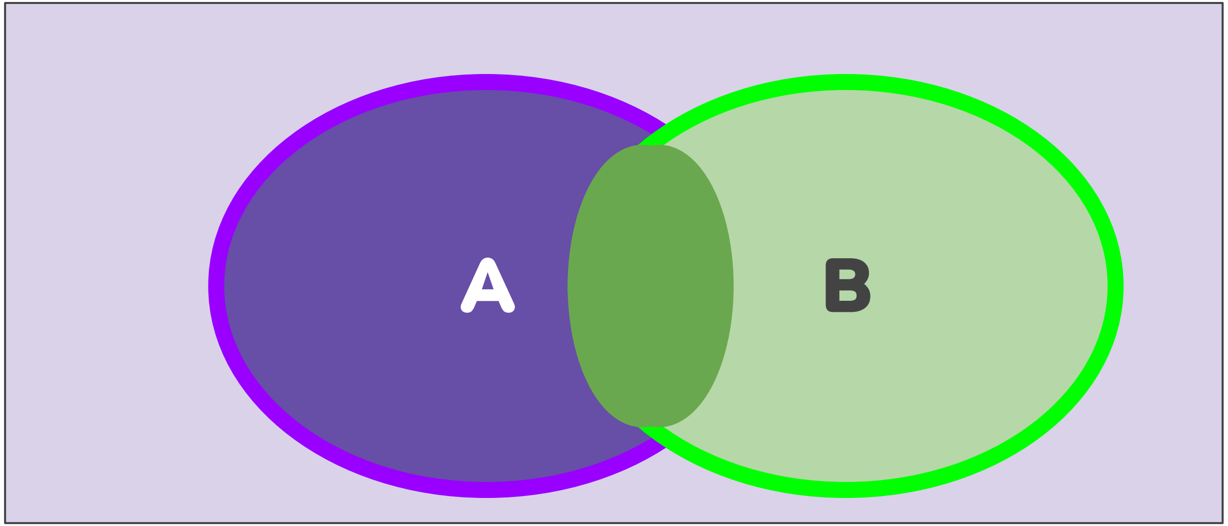


Independence

$P(A) = 0.3$

$P(B) = 0.4$





Independence

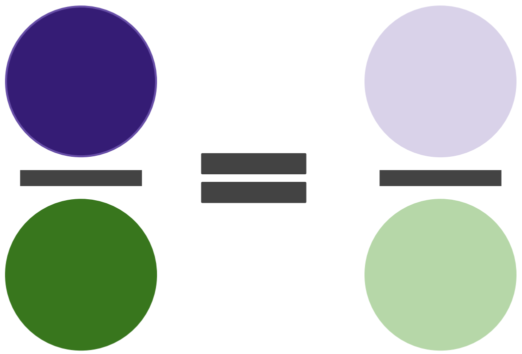
$P(A) = 0.3$

$P(B) = 0.4$

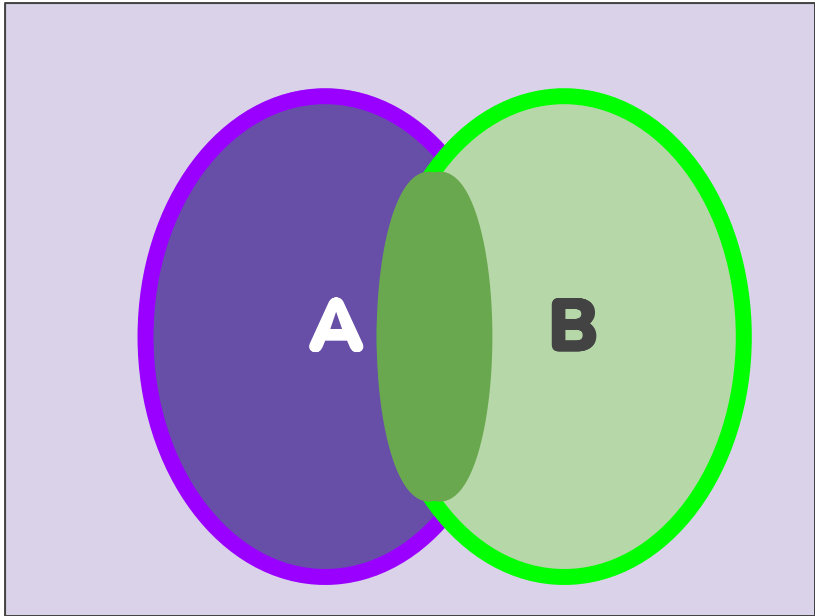
A occurs

A'

B'



B occurs



Ingredients for constructing mathematical theory of Probability

4. P - Probability Measure on $\mathscr{F}$: Mapping of elements (say A) from $\mathscr{F}$ to the real space, which we denote by P(A). This mapping should follow the following :

- $P(\Omega) = 1$
- $P(A) \geq 0 \qquad \text{for every } A \in \mathcal{F}$
- $P(A_1 \cup A_2 \cup A_3 \cup \cdots) = P(A_1) + P(A_2) + P(A_3) + \cdots$ for all disjoint $A_1, A_2, A_3, \cdots \in \mathcal{F}$

Independence

$$P(A \cap B) = P(A) * P(B)$$

Closing Remarks