BENCHMARKING OF RIGETTI ARCHITECTURE

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Abstract.

1. Introduction

Consider the following scenario. There is an unknown measurement device and the only thing we know about it is that it performs one of two known measurements, call them $\mathcal{S}$ and $\mathcal{T}$. We put a state into the device and our goal is to decide which of the measurements is performed. We aim to identify the assumptions needed for perfect discrimination of $\mathcal{S}$ and $\mathcal{T}$. Further, we want to construct the optimal discrimination scheme for this task. In the case when perfect distinctions is not possible, we would like to bound from above the probability of correct discrimination as well as derive a scheme which enables a correct guess with the optimal probability.

The second field of our interest is finding the optimal strategy for the discrimination. In other words, we would like to know which state should be used to provide the greatest possible probability of correct discrimination.

Of course, there is another possibility. As we are dealing with quantum states, we can utilize entanglement. Hence, we input one part of the entangled state into the unknown measurement device and later use the other part to strengthen the inference. The scheme of this process is presented in Fig. 1.

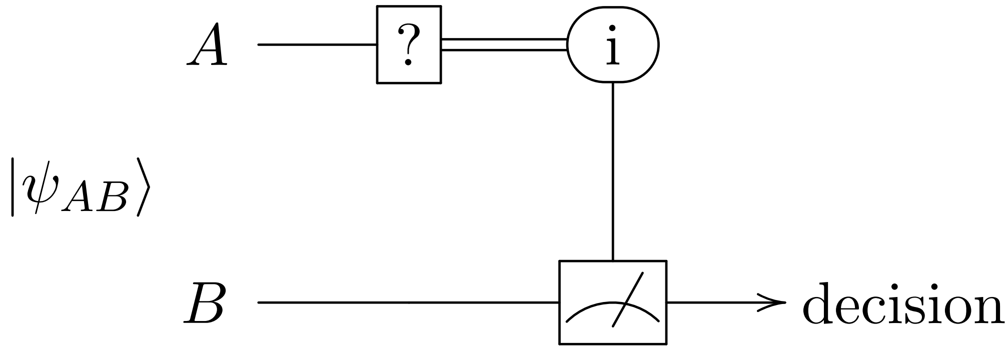

FIGURE 1. A schematic representation of the setup for distinguishing measurements using entangled states. One of two known measurements $\mathcal{S}$ or $\mathcal{T}$ is performed on part A of the input state $|\psi_{AB}\rangle$. We use the output label i and perform a conditional binary measurement $\mathcal{R}_i$ on part B. By the use of its output we formulate our guess, that is we decide weather the measurement performed on part A was $\mathcal{S}$ or $\mathcal{T}$.

2. Problem formulation

Consider two von Neumann measurements $\mathcal{P}_U$ and $\mathcal{P}_{\mathbf{1}}$ for

$$U = H_2 \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix} H_2^\dagger, \tag{1}$$

where U is unitary matrix parameterized by angle $\phi \in [0, 2\pi)$ and H_2 is Hadamard matrix.

2.1. **Holevo-Helstrom theorem.** The celebrated result by Helstrom gives an upper bound on the probability of correct distinction between two von Neumann measurements $\mathcal{P}_U$ and $\mathcal{P}_{\mathbf{1}}$ in terms of their distance with the use of the diamond norm

$$p \le \frac{1}{2} + \frac{1}{4}\|\mathcal{P}_U - \mathcal{P}_{\mathbf{1}}\|_\diamond. \tag{2}$$

Theorem 1. *(Tw. 1 from [1]) Let $U \in \mathcal{U}_d$ and let $\mathcal{P}_U$ and $\mathcal{P}_{\mathbf{1}}$ be two von Neumann measurements. Let also $\mathcal{DU}_d$ be the set of diagonal unitary matrices of dimension d. Then*

$$\|\mathcal{P}_U - \mathcal{P}_{\mathbf{1}}\|_\diamond = \min_{E \in \mathcal{DU}_d} \|\Phi_{UE} - \Phi_{\mathbf{1}}\|_\diamond, \tag{3}$$

where Φ_U is unitary channel.

2.2. **Discrimination of unitary channels.** Before we proceed to presenting our main results, we need to briefly discuss the problem of discrimination of unitary channels. This can be done without the usage of entangled input. In order to formulate the condition for perfect discrimination of unitary channels we introduce the notion of numerical range of a matrix $A \in M_d$, denoted by $W(A) = \{\langle x|A|x\rangle : |x\rangle \in \mathbb{C}^d, \ \langle x|x\rangle = 1\}$. The celebrated Hausdorf-Töplitz theorem states that $W(A)$ is a convex set and therefore $W(A) =$

$\{\operatorname{tr} A\sigma : \sigma \in \Omega_d\}$. Let us now recall the well-known result for the distinguishability of unitary channels. Let $U \in \mathcal{U}_d$ and $\Phi_U : \rho \mapsto U\rho U^\dagger$ be a unitary channel. Then

$$\|\Phi_U - \Phi_{\mathbf{1}}\|_\diamond = 2\sqrt{1-\nu^2}, \tag{4}$$

where $\nu = \min_{x \in W(U^\dagger)} |x|$.

Proposition 1. *Let* $U = H_2 \operatorname{diag}(1, e^{i\phi}) H_2^\dagger$, $\phi \in [0, 2\pi)$ *and let* Φ_U *and* $\Phi_{\mathbf{1}}$ *be two unitary channels. The following equation holds*

$$\min_{E \in \mathcal{DU}_2} \|\Phi_{UE} - \Phi_{\mathbf{1}}\|_\diamond = \|\Phi_U - \Phi_{\mathbf{1}}\|_\diamond \tag{5}$$

Proof. We know that $\|\Phi_U - \Phi_{\mathbf{1}}\|_\diamond = 2\sqrt{1-\nu^2}$, where $\nu = \min_{x \in W(U^\dagger)} |x|$. For $U = H_2 \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix} H_2^\dagger$ we obtain that $\nu = \frac{|1+e^{i\phi}|}{2}$. The condition $\min_{E \in \mathcal{DU}_2} \|\Phi_{UE} - \Phi_{\mathbf{1}}\|_\diamond = \|\Phi_U - \Phi_{\mathbf{1}}\|_\diamond$ is equivalent to prove that

$$\max_{E \in \mathcal{DU}_2} \nu_E = \nu = \frac{|1+e^{i\phi}|}{2}, \tag{6}$$

where $\nu_E = \min_{x \in W(U^\dagger E)} |x|$. We can prove that

$$\min_{|x\rangle \in \mathcal{C}^2 \langle x|x\rangle = 1} |\langle x|U^\dagger|x\rangle| = \min_{\rho \in \Omega_2} |\operatorname{tr}(U^\dagger \rho)|. \tag{7}$$

For that, to calculate ν_E we obtain

$$\max_{E \in \mathcal{DU}_2} \nu_E = \max_{E \in \mathcal{DU}_2} \min_{\rho \in \Omega_2} |\operatorname{tr} \rho U E| \tag{8}$$

For that, our task is reduce to show that

$$\forall E \in \mathcal{DU}_2 \ |\operatorname{tr} \rho U E| \leq \nu. \tag{9}$$

Let us define $E = \begin{pmatrix} E_0 & 0 \\ 0 & E_1 \end{pmatrix}$ and let us take $\rho = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$. From spectral theorem, let us note U as

$$U = \lambda_1 |x_1\rangle\langle x_1| + \lambda_2 |x_1\rangle\langle x_2|, \tag{10}$$

where for eigenvector $\lambda_1 = e^{\mathbf{i}\phi}$ the eigenvector is of the form $|x_1\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$, whereas for $\lambda_2 = 1$ we have $|x_2\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$.

Then we have

$$\begin{aligned} &\forall E \in \mathcal{DU}_2 \ |\operatorname{tr} \rho U E| = \frac{1}{2} \left| \operatorname{tr} H_2 \operatorname{diag}(1, e^{i\phi}) H_2^\dagger E \right| = \\ &\frac{1}{2} \left| \operatorname{tr} \left((e^{i\phi} |x_1\rangle\langle x_1| + 1 |x_2\rangle\langle x_2|) E \right) \right| = \frac{1}{2} \left| e^{i\phi} \langle x_1|E|x_1\rangle + \langle x_2|E|x_2\rangle \right| = \\ &\frac{1}{2} \left| \frac{E_0 + E_1}{2} + e^{i\phi} \frac{E_0 + E_1}{2} \right| = \frac{\left|1 + e^{i\phi}\right|}{2} \left| \frac{E_0 + E_1}{2} \right| \leq \nu, \end{aligned} \tag{11}$$

which completes the proof. □

Remark 1. *The probability of correct discrimination of von Neumann measurements $\mathcal{P}_U$ and $\mathcal{P}_{\mathbf{1}}$ for $U = H_2 \operatorname{diag}(1, e^{i\phi})H_2^\dagger$, where $\phi \in [0, 2\pi)$ is given by*

$$p \leq \frac{1}{2} + \frac{|1 - e^{i\phi}|}{4}. \tag{12}$$

2.3. **Form of discriminator.** On the other hand, for Hermiticity-preserving Φ, we have the following alternative formula for the diamond norm

$$\|\Phi\|_\diamond = \max_\rho \|(\mathbb{1} \otimes \sqrt{\rho})J(\Phi)(\mathbb{1} \otimes \sqrt{\rho})\|_1 . \tag{13}$$

The state ρ, for which $\|\Phi\|_\diamond = \left\|(\mathbb{1} \otimes \sqrt{\rho})J(\Phi)(\mathbb{1} \otimes \sqrt{\rho})\right\|_1$, will be called a *discriminator*.

Theorem 2. *Let us $\mathcal{P}_U, \mathcal{P}_{\mathbf{1}}$ be two von Neumann measurements, $U = H_2 \operatorname{diag}(1, e^{i\phi})H_2^\dagger$, $\phi \in [0, 2\pi)$ such that $0 \notin W(U)$. If there exists the discriminator $\rho \in \Omega_2$ of the form $\rho = \frac{1}{2}\rho_1 + \frac{1}{2}\rho_2$ such that*

(1) $\rho_1, \rho_2 \in \Omega_2$
(2) $\Pi_1\rho_1\Pi_1 = \rho_1$,
(3) $\Pi_2\rho_2\Pi_2 = \rho_2$,
(4) $\operatorname{diag}(\rho_1) = \operatorname{diag}(\rho_2)$,

where Π_1, Π_2 are the projectors on the subspaces spanned by the eigenvectors corresponding to λ_1 and λ_2 of U. Then we have

$$||\mathcal{P}_U - \mathcal{P}_{\mathbf{1}}||_\diamond = 2\sqrt{1 - \left|\frac{1 + e^{i\phi}}{2}\right|^2}. \tag{14}$$

Proof. From 1 we have

$$\begin{aligned} ||\mathcal{P}_U - \mathcal{P}_{\mathbf{1}}||_\diamond &= \min_{E \in \mathcal{DU}_2} ||\mathcal{P}_{UE} - \mathcal{P}_{\mathbf{1}}||_\diamond = \min_{E \in \mathcal{DU}_2} ||\Delta\left(\Phi_{E^\dagger U^\dagger} - \Phi_{\mathbf{1}}\right)||_\diamond \leq \\ &\leq \min_{E \in \mathcal{DU}_2} ||\Delta||_\diamond ||\Phi_{E^\dagger U^\dagger} - \Phi_{\mathbf{1}}||_\diamond \leq \min_{\mathcal{DU}_2} ||\Phi_{E^\dagger U^\dagger} - \Phi_{\mathbf{1}}||_\diamond. \end{aligned} \tag{15}$$

The diamond norm between two unitary channels can be calculated by

$$\min_{E \in \mathcal{DU}_2} ||\Phi_{E^\dagger U^\dagger} - \Phi_{\mathbf{1}}||_\diamond = \min_{E \in \mathcal{DU}_2} 2\sqrt{1 - \min_{\rho \in \Omega_2} |\operatorname{tr}(UE\rho)|^2}. \tag{16}$$

Hence, it is enough to calculate the formula

$$\min_{E \in \mathcal{DU}_2} 2\sqrt{1 - \min_{\rho \in \Omega_2} |\operatorname{tr}(UE\rho)|^2}. \tag{17}$$

Recall, let us note U as

$$U = \lambda_1|x_1\rangle\langle x_1| + \lambda_2|x_1\rangle\langle x_2|, \tag{18}$$

where for eigenvector $\lambda_1 = e^{\mathbf{i}\phi}$ the eigenvector is of the form $|x_1\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$, whereas for $\lambda_2 = 1$ we have $|x_2\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$. Consider two cases:

1° Consider $\phi = \pi$. Then U is of the form

$$U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{19}$$

Then $0 \in W(U)$. From Proposition 3 in [1] the measurements $\mathcal{P}_U, \mathcal{P}_{\mathbf{1}}$ are perfectly distinguishable if and only if there exists $\rho \in \Omega_2$ such that

$$\operatorname{diag}\left(U^\dagger \rho\right) = 0. \tag{20}$$

Hence, for U we have

$$0 = \operatorname{diag}\left(U^\dagger \rho\right) = \operatorname{diag}\left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \rho_{1,1} & \rho_{1,2} \\ \rho_{2,1} & \rho_{2,2} \end{pmatrix}\right) = \operatorname{diag}\begin{pmatrix} \rho_{2,1} & \rho_{2,2} \\ \rho_{1,1} & \rho_{1,2} \end{pmatrix}. \tag{21}$$

It implies that $\rho_{2,1} = \rho_{1,2} = 0$. Therefore $\rho \in \Omega_2$ for which $\mathcal{P}_U, \mathcal{P}_{\mathbf{1}}$ are perfectly distinguishable is of the form

$$\rho = \begin{pmatrix} \rho_{1,1} & 0 \\ 0 & \rho_{2,2} \end{pmatrix}, \tag{22}$$

where $\rho_{1,1}, \rho_{2,2} \geq 0$ and $\rho_{1,1} + \rho_{2,2} = 1$.

2° Consider $\phi \neq \pi$. The spectrum of U is of the form $\sigma(U) = \{e^{\mathbf{i}\phi}, 1\}$. Hence $0 \notin W(U)$. Let us note U as

$$U = \lambda_1 |x_1\rangle\langle x_1| + \lambda_2 |x_1\rangle\langle x_2|. \tag{23}$$

Based on Lemma 5 in [1] let us take $\rho_1 = |x_1\rangle\langle x_1|$ and $\rho_2 = |x_2\rangle\langle x_2|$. Obviously, $\rho_1, \rho_2 \in \Omega_2$ and satisfies the requirements of theorem. Hence, the discriminator $\rho \in \Omega_2$ has the form

$$\rho = \frac{1}{2}\rho_1 + \frac{1}{2}\rho_2 = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \tag{24}$$

For $\rho = \frac{1}{2}\rho_1 + \frac{1}{2}\rho_2$ we obtain

$$\begin{aligned}
||\mathcal{P}_U - \mathcal{P}_{\mathbf{1}}||_\diamond &= \sum_{i=1}^{2} \sqrt{\left(\langle u_i|\rho|u_i\rangle + \langle i|\rho|i\rangle\right)^2 - 4|\langle i|\rho|u_i\rangle|^2} = \\
&\sum_{i=1}^{2} \sqrt{\left(\langle i|U^\dagger \rho U|i\rangle + \langle i|\rho|i\rangle\right)^2 - 4|\langle i|\rho U|i\rangle|^2} = \\
&\sum_{i=1}^{2} \sqrt{4\langle i|\rho|i\rangle^2 - 4\left|\frac{\lambda_1\langle i|\rho_1|i\rangle + \lambda_2\langle i|\rho_2|i\rangle}{2}\right|^2} = \\
&\sum_{i=1}^{2} \sqrt{4\langle i|\rho|i\rangle^2 - 4\langle i|\rho|i\rangle\left|\frac{\lambda_1 + \lambda_2}{2}\right|^2} = \\
&\sum_{i=1}^{2} 2\langle i|\rho|i\rangle\sqrt{1 - \left|\frac{\lambda_1 + \lambda_2}{2}\right|^2} = \\
&2\sqrt{1 - \left|\frac{\lambda_1 + \lambda_2}{2}\right|^2} = 2\sqrt{1 - \left|\frac{1 + e^{i\phi}}{2}\right|^2},
\end{aligned} \tag{25}$$

which completes the proof. □

Remark 2. *Let* $\rho_0 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. *Then*

$$\max_{\rho} \|(\mathbb{1} \otimes \sqrt{\rho})J(\mathcal{P}_U - \mathcal{P}_{\mathbf{1}})(\mathbb{1} \otimes \sqrt{\rho})\|_1 = \|(\mathbb{1} \otimes \sqrt{\rho_0})J(\mathcal{P}_U - \mathcal{P}_{\mathbf{1}})(\mathbb{1} \otimes \sqrt{\rho_0})\|_1 \tag{26}$$

for $U = H_2 \operatorname{diag}(1, e^{i\phi})H_2^\dagger$, *where* $\phi \in [0, 2\pi)$.

Scheme 1. *Let us consider the problem of discrimination of von Neumann measurements* $\mathcal{P}_U$ *and* $\mathcal{P}_{\mathbf{1}}$ *for* $U = H_2 \operatorname{diag}(1, e^{i\phi})H_2^\dagger$, *where* $\phi \in [0, 2\pi)$. *The schematic representation of theoretical setup is shown in Fig.* **??**.

From [1](Proposition 4) and Lemma 2 we have

$$|\psi\rangle = |\sqrt{\rho}^\top\rangle\rangle = \frac{1}{\sqrt{2}}|\mathbb{1}_2\rangle\rangle. \tag{27}$$

From Holevo-Helstrom theorem we constrain a measurement μ. *Let us define*

$$X = (\mathcal{P}_U \otimes \mathbb{1}_2)(|\psi\rangle\langle\psi|) - (\mathcal{P}_{\mathbf{1}} \otimes \mathbb{1}_2)(|\psi\rangle\langle\psi|) \tag{28}$$

where $|\psi\rangle$ *is defined in Remark 1. From Hahn-Jordan decomposition let*

$$X = P - Q \tag{29}$$

where $P, Q \geq 0$. *Observe, that* P *and* Q *are block-diagonal. Let us define projectors* Π_P *and* Π_Q *onto* $im(P)$ *and* $im(Q)$, *respectively. Then* Π_P *and* Π_Q *have the following forms*

$$\Pi_P = \begin{pmatrix} |x_p\rangle\langle x_p| & 0 \\ 0 & |y_p\rangle\langle y_p| \end{pmatrix} \tag{30}$$

and

$$\Pi_Q = \begin{pmatrix} |x_q\rangle\langle x_q| & 0 \\ 0 & |y_q\rangle\langle y_q| \end{pmatrix} \tag{31}$$

Hence, we define V_0 *such that*

$$\begin{aligned} V_0|x_p\rangle &= |0\rangle \\ V_0|x_q\rangle &= |1\rangle \end{aligned} \tag{32}$$

and V_1 *such that*

$$\begin{aligned} V_1|y_p\rangle &= |0\rangle \\ V_1|y_q\rangle &= |1\rangle \end{aligned} \tag{33}$$

The explicit form of V_0 *and* V_1 *we can see in* `one-qubit.nb`. *Finally, we obtain the measurement* μ *of the form*

$$\begin{aligned} \mu(0) &= |0\rangle\langle 0| \otimes V_0|0\rangle\langle 0|V_0^\dagger + |1\rangle\langle 1| \otimes V_1|0\rangle\langle 0|V_1^\dagger \\ \mu(1) &= |0\rangle\langle 0| \otimes V_0|1\rangle\langle 1|V_0^\dagger + |1\rangle\langle 1| \otimes V_1|1\rangle\langle 1|V_1^\dagger \end{aligned} \tag{34}$$

This scheme is only theoretical. It is not possible to implement it on Rigetti architecture. For this, we will consider two approaches. The first one is described by using postselection whereas the second one is considered by implementation of controlled unitary gate.

2.4. **Construction of V_0 and V_1.** For each $\phi \in \mathbb{R}$, the controlled unitary V_0 and V_1 have the following form

$$V_0 = \begin{pmatrix} i\sin\left(\frac{\pi-\phi}{4}\right) & -i\cos\left(\frac{\pi-\phi}{4}\right) \\ \cos\left(\frac{\pi-\phi}{4}\right) & \sin\left(\frac{\pi-\phi}{4}\right) \end{pmatrix}, \tag{35}$$

$$V_1 = \begin{pmatrix} -i\cos\left(\frac{\pi-\phi}{4}\right) & i\sin\left(\frac{\pi-\phi}{4}\right) \\ \sin\left(\frac{\pi-\phi}{4}\right) & \cos\left(\frac{\pi-\phi}{4}\right) \end{pmatrix}. \tag{36}$$

Scheme 2. *(By using postsellection)*

(1) We prepare input state $|\psi\rangle = \frac{1}{\sqrt{2}}|\mathbb{1}_2\rangle\rangle$.
(2) We prepare one of two unitary channel Φ_U *or* $\Phi_{\mathbf{1}}$.
(3) We implement unitary V_0 *or* V_1.
(4) We prepare the measurement Δ *in standard basis (already exists on Rigetti architecture) on each qubits.*
(5) We calculate the probability of correct discrimination in the following way

$$p = ... \tag{37}$$

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Scheme 3. *(By using controlled unitary)*

1. *We prepare input state* $|\psi\rangle = \frac{1}{\sqrt{2}}|\mathbb{1}_2\rangle\rangle$.
2. *We prepare one of two unitary channel* Φ_U *or* $\Phi_{\mathbf{1}}$.
3. *We implement controlled unitary* $V_0 \oplus V_1$.
4. *We prepare the measurement* Δ *in standard basis (already exist on Rigetti architecture) on each qubits.*
5. *We calculate the probability of correct discrimination in the following way*

$$p = \frac{|j = 0 \wedge \Phi_U| + |j = 1 \wedge \Phi_{\mathbb{1}}|}{\textit{all cases}} \tag{38}$$

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